⑦ Translations and Reflections: Define a Custom Transformation

In this tutorial, you'll use a combination of transformations to create a pattern of footprints. You'll start from just one footprint and apply translations and reflections to obtain the entire set of footprints.

Sketchpad Skills | Introductory Movie 🎬

Translate by a Vector 🎬

When you apply a translation to a picture, the picture moves a certain distance in a certain direction. A vector is a geometric object that specifies a distance and direction, so you'll use a vector to translate a footprint.

1. Open Sketchpad and choose **Help | Picture Gallery.**

2. Click the **Transformation** link and find the picture of a footprint. Drag and drop (or copy and paste) the footprint picture into your sketch.

3. Construct a vertical segment to the left of the footprint. (Hold the Shift key as you construct your segment to make it vertical.)

4. Label the endpoints *A* and *B.*

5. Select points *A* and *B* and choose **Transform | Mark Vector.** A brief animation appears.

6. Select the picture and choose **Transform | Translate.** In the dialog box, click **Translate.**

7. Leave the new footprint selected and translate it again. Repeat a few times to make a series of footprints.

8. Drag each endpoint of $\overline{AB}$ and observe how the footprints behave. How does the length and direction of the segment relate to the pattern of the footprints? In real life, how could you create this pattern with your own feet?

Reflect Across a Mirror Line 🎬

Now that you have a collection of right footprints, you'll reflect them over a mirror line to obtain the corresponding set of left footprints.

9. Choose the **Line** tool—press and hold the **Segment** tool until the straightedge menu appears, move the cursor to the line icon, and release.

10. Click points *A* and *B* to construct $\overleftrightarrow{AB}$. Leave the line selected and choose **Display | Line Style | Dashed.**

11. Move $\overleftrightarrow{AB}$ just to the left of your footprints. Double-click the line to mark it as a mirror (or select the line and choose **Transform | Mark Mirror**).

12. Select the pictures (click each one, or use a selection rectangle) and choose **Transform | Reflect.**

13. Drag the mirror line and observe the behavior of the reflected footprints. In real life, how could you create this pattern of footprints?

Define a Glide-Reflect Transformation 🎬

Now you'll combine the reflection and the translation into a single custom transformation called a *glide reflection.* By repeatedly applying a glide reflection, you can create a sequence of footprints that represents a typical walking pattern.

14. Select all pictures except for the original picture and choose **Edit | Clear Pictures** (or press Delete on your keyboard). If you are not sure which was the original, you can use the **Information** tool to find out.

15. Construct a point close to $\overleftrightarrow{AB}$. Label it *C.*

16. Select point *C,* and translate it as in step 6. Label the new image point *C'.*

17. Leave point *C'* selected and reflect it as in step 12. Label the new image point *C''.*

18. Deselect all objects. Select only points *C* and *C''* and choose **Transform | Define Custom Transform.** Name your new transformation *Glide Reflect* and click **OK.**

19. Select points *C,C',* and *C''* and choose **Display | Hide Points.**

20. Select the picture and choose **Transform | Glide Reflect.**

21. Leave the new picture selected and glide-reflect it again. Repeat many times (or use the keyboard shortcut shown next to **Glide Reflect** in the Transform menu) to make a series of footprints.

22. Drag points *A* and *B* and $\overleftrightarrow{AB}$ and observe how the footprints behave.

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23. Redo this tutorial, but this time in step 9, construct a mirror line that is separate from points *A* and *B.* What happens to the footprints pattern when the direction of the translation vector is not