CIRCLE, ELLIPSE, AND HYPERBOLA IN STANDARD POSITION

Not all texts use the phrase *standard position* in the same way. In this case, the x- and y-axes are the axes of the conic section. Usually, the denominators are in the form a^2 and b^2. This sketch uses a different (closely related) form in order to make negative terms possible while using only two parameters.

Q1 Ellipse: $A > 0$, $B > 0$, and $A \neq B$
Circle: $A = B > 0$
Hyperbola: A and B have opposite signs.

Q2 The major axis matches the variable above the larger parameter. For example, if $B > A$, then the y-axis is the major axis, because y is above B. The major radius is the square root of the larger parameter, and the minor radius is the square root of the smaller.

Q3 The transverse axis is determined by the variable above the positive parameter. For example, if $A > 0$, then the x-axis is the transverse axis. The transverse radius is the square root of the positive parameter, and the conjugate radius is the square root of the negative parameter.

Q4 There is no solution if $A = 0$, if $B = 0$, or if A and B are both negative.

Q5 If the axis of a parabola is horizontal or vertical, then either the x^2 term or the y^2 term must have a zero coefficient. That is not possible when the equation is in this form.

GENERAL SECOND-DEGREE EQUATIONS

The general equation used with this file can represent any second-order Cartesian equation in x and y. For simplicity, the xy-coefficient is set to zero for most of the activity, which is how the equation appears in most textbook introductions.

Q6 When $B = 0$, the locus is a conic section and the axes of the conic are either horizontal or vertical.

Q7 The responses in the table can vary endlessly. Below are some correct example responses. As a guide to evaluating the answers, refer to the answers to Q8.

Locus	Equation
Circle	$3x^2 + 3y^2 - 8x + y = 5$
Ellipse	$2x^2 + 3y^2 - 8x + y - 5 = 0$
Hyperbola	$-2x^2 + 3y^2 - 8x + y - 5 = 0$
Parabola	$3y^2 - 8x + y - 5 = 0$
Line	$-8x + y - 5 = 0$
Intersecting lines	$x^2 - 9y^2 + 4x + 6y + 3 = 0$
Parallel lines	$3y^2 + y - 5 = 0$
Point	$x^2 + 9y^2 + 4x - 6y + 5 = 0$
No solution	$x^2 + 9y^2 + 4x - 6y + 6 = 0$

Encourage students to consider these curves as the intersection of a plane and a cone. How can that intersection create the curve in question? One apparent conflict arises in the case of two parallel lines. Consider a cylinder, the limiting case of a cone when the apex goes to a point at infinity. In this case, a plane can intersect it on two parallel lines.

Q8 Certain other conditions must be satisfied in order to produce real solutions, but these relations hold true:

Circle: $A = C$

Ellipse: $A \neq C$, and A and C have the same sign.

Hyperbola: A and C have opposite signs.

Parabola: $A = 0$ or $C = 0$

Q9 The solution to a second-degree equation is a line when the equation is a quadratic equation of one variable with a double root. Example: $x^2 + 6x + 9 = 0$. Remind students that if a plane is tangent to a cone, the set of intersection points is a line.

Q10 The generalizations do not hold up when $B \neq 0$. In this case, the curve is rotated and interpreting the equation gets much more complicated. This sketch may become useful again later if you pursue this subject.