Patty Paper Parabolas

The paper-folding technique in this activity is a simple and impressive way to generate parabolas. In fact, since all it requires is a single sheet of paper, this method can be downright addictive. Got a blank sheet of paper? Fold a parabola!

(This activity originally appeared in *Exploring Conic Sections with The Geometer's Sketchpad* by Daniel Scher, available from Key Curriculum Press.)

CONSTRUCT A PHYSICAL MODEL

To model the folding process before students begin, follow the first few steps of the Presenter Notes.

Q1 Point *A* is the focus and the bottom edge of the paper is the directrix.

Q2 The curve would appear narrower.

Q3 The crease is the perpendicular bisector of segment *AB*.

CONSTRUCT A SKETCHPAD MODEL

10. In this step, students must decide how to construct the crease line in Sketchpad. If possible, allow them to get the satisfaction of figuring the construction out by themselves. It helps if they've already realized (perhaps from Q3) that the crease is the perpendicular bisector of segment *AB*.

    As another hint, you might suggest to students that they take a fresh sheet of patty paper or notebook paper, mark two random points, fold one onto the other, and then unfold the paper. Ask them, "What is the geometric relationship of the crease line to the two points?"

Q4 Construct the crease line between *A* and *B* in three steps: (a) construct segment *AB;* (b) construct the midpoint of the segment; and (c) construct the perpendicular bisector—the perpendicular to the segment through the midpoint.

Q5 The curve passes between point *A* and the line, and opens upward. It appears to be a parabola.

Q6 As point *A* approaches the horizontal line, the curve appears narrower.

Q7 As point *A* moves away from the horizontal line, the curve appears wider.

Q8 When point *A* moves below the horizontal line, the curve opens downward instead of upward.

PLAY DETECTIVE

Q9 The point of tangency is directly above point *B*.

20. In this step students must construct the perpendicular to the horizontal line through point *B*. They may need some encouragement to observe carefully the relationship between point *B* and the point of tangency. If they continue to struggle even after they've observed that the point of tangency is always directly above point *B*, you could ask them to think about what construction includes all points directly above point *B*.

Q10 To create the new line, construct a perpendicular to the horizontal line through point *B*. The point of tangency is the intersection of this line with the crease line.

It's interesting to consider whether a parabola has *asymptotes;* that is, if there are lines that the curve approaches but never crosses. By observing the tangent line as point *B* moves farther and farther away from the focus, we see that the curve becomes more and more perpendicular to the directrix. So if there are asymptotes, they must be perpendicular to the directrix.

But given any perpendicular line, there is a point on it that is equidistant from the focus and the directrix. Thus the curve crosses the line, and the line cannot be an asymptote.

PROVE IT

Q11 To prove that point *D* traces a parabola, you must prove that $AD = BD$.

Q12 You can use SAS. Segments *AC* and *BC* are equal because point *C* is the midpoint of segment *AB*. Angles *ACD* and *BCD* are equal because both are right angles. And segment *CD* is equal to itself.

Q13 The two triangles are congruent, so their corresponding sides are equal: $AD = BD$. Because segment *BD* is perpendicular to the horizontal line, its length is the distance from point *D* to the line. Therefore point *D* is equidistant from the focus (point *A*) and the directrix (the horizontal line).

EXPLORE MORE

22. Since the circle is tangent to the line at point *B*, the radius *CB* is perpendicular to the line. Thus segment *CB* represents the distance of point *C* from the line. Segments *CB* and *CA* are both radii of the circle and equal in length. Therefore *C* traces a parabola with a focus at point *A*.

23. As point *A* travels farther and farther off screen, the portion of the circle that we see gets straighter and straighter. When point *A* finally stops moving, we're left with a construction that looks like the Patty Paper Parabola—a single focus (point *B*) and a nearly straight directrix.

24. $\angle GDF \cong \angle BDC$, as they are vertical angles. Since $\triangle ADC \cong \triangle BDC$, we have $\angle ADC \cong \angle BDC$. Putting these two statements together gives $\angle GDF \cong \angle ADC$.

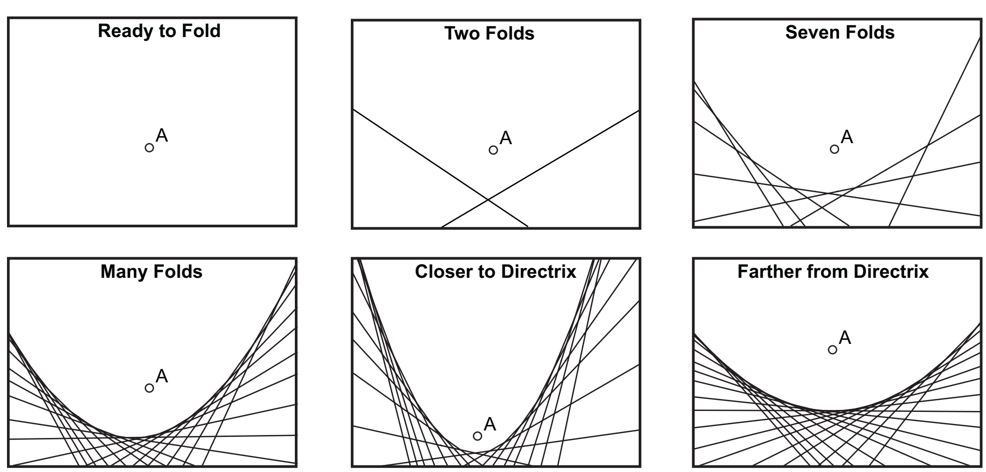
WHOLE-CLASS PRESENTATION

This activity is most effective when students begin by actually folding the patty paper. To do it this way, distribute patty paper and hand out the first page of the student worksheet. After students have finished folding their parabolas, continue with the Construct a Parabola section of the Presenter Notes.

Alternatively, you can use an overhead projector and a transparency to do the folding. In this case, use the Fold a Parabola section of the Presenter Notes.

If there's no overhead available, use a large sheet of paper, mark point *A*, and use the bottom edge of the paper as the directrix.

The various stages in the folding process are illustrated here:



In this presentation you'll use geometric methods to construct a parabola—first by paper-folding and then by using the equivalent construction with Sketchpad.

FOLD A PARABOLA

It's best if students fold their own patty paper at their desks, following the first page of the student worksheet. In this case, skip this section and start with Construct a Parabola.

If you don't have an overhead projector, you can do the folding on a large sheet of paper. Use the bottom edge of the paper as the directrix, and trace each fold with a marker.

Before you begin, prepare a transparency by marking a vertex *A* and a directrix.

1. On the overhead projector, show the transparency. Describe the objective: to find the pattern created when you fold various points of the line onto point *A*. Explain that *A* is called the *vertex* and the line is called the *directrix*.
2. Have a student fold the transparency so that a point on the directrix falls on point *A*. Straighten out the transparency; the fold will be visible. Have another student fold a different directrix point onto point *A*. Draw attention to this fold also. Have students continue folding until there are six or seven folds.

Q1 Ask students, "Do you see a pattern yet?" Ask a student to come to the projector and use a finger to trace the observed pattern.

Q2 Ask, "How do you think the pattern would change if the vertex were closer to the directrix?" Solicit conjectures and discuss them.

3. Observe that folding is a slow way to test student conjectures, and that it's time to move to Sketchpad.

CONSTRUCT A PARABOLA

You can do the Sketchpad construction in a blank sketch, or you can use the presentation sketch. To do the construction in a blank sketch, follow the steps in the Construct a Sketchpad Model section of the student worksheet, and adapt the steps and questions below.

4. Open **Patty Paper Parabolas Present.gsp** and show the vertex and directrix.

Q3 Show point *B* and ask students to describe where the crease will appear if *B* is folded to the vertex. After discussing, show the crease.

Q4 Ask, "What geometric construction would we need to make the crease?" After discussing, show the construction objects.

5. Say, "We can use this construction to generate many folds very quickly." Animate *B*, ask students to predict the shape of the traces, and then trace.

Extend the presentation by using other sections of the student worksheet, or by exploring other pages of the sketch.

Q5 Ask, "How will the shape change if the vertex is closer to the directrix?" After discussion, move the vertex, erase the old traces, and then trace again.

Q6 Ask, "How will the shape change if the vertex is farther from the directrix?" After discussion, move the vertex, erase the old traces, and then trace again.