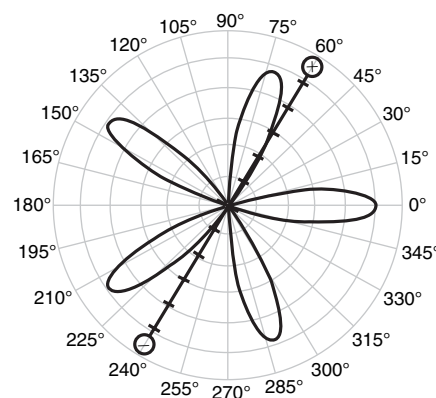


Cartesian Graphs and Polar Graphs

Cartesian coordinates are also called *rectangular* coordinates.

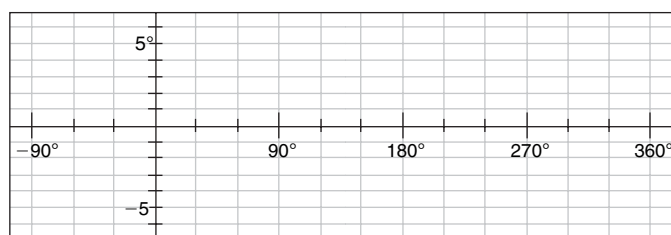
You have probably done lots of graphing in Cartesian (x, y) coordinates, but relatively little in polar (r, θ) coordinates. In this activity you will explore various functions plotted in both coordinate systems. You will find connections between the two types of graphing and use your understanding of Cartesian graphing to better understand polar graphing.



SKETCH AND INVESTIGATE

Start by investigating functions of the form $f(t) = a \cos(bt)$. You can think of these as $y = a \cos(bx)$ in the Cartesian plane and $r = a \cos(b\theta)$ in the polar plane.

- Q1** Start with $a = 5$ and $b = 3$. Use what you know about graphing in the Cartesian plane to make an approximate sketch of $y = 5 \cos(3x)$ on the following grid.



What do the parameters a and b control?

1. Open **Cartesian Polar.gsp**. This sketch contains the plot of the function $y = a \cdot \cos(b \cdot x)$. When you open the sketch, parameters a and b are both set to 1, so the function being plotted is simply $y = \cos x$.
 2. Adjust parameters a and b to check your plot from Q1. Was your sketch accurate?
 3. Drag the purple pointer back and forth along the number line. Note how the red bar shows the input (x) value and the green “bowtie” shows the output (y) value of the function. The + and - signs on the red bar show where y is positive and negative, respectively. Move the pointer back to 0° .
- Q2** In a moment, you’ll reveal the corresponding polar curve, $r = 5 \cos(3\theta)$. First, make a wild guess about what it will look like, and write down your guess.
4. After you have made a guess, press the *Show Polar* button. Compare the result with your guess.

To adjust a parameter, double-click it, or select it and press the + or - key on your keyboard.

5. Drag the purple pointer back and forth slowly, this time looking for connections between the Cartesian and polar graphs. Note that the “bowtie” is always in the same relative position on the two output axes, but the red bar corresponding to x in the Cartesian plane slides right and left, and the red bar corresponding to θ in the polar plane spins around.

- Q3** Which points on the polar graph correspond to x -intercepts on the Cartesian graph?
- Q4** Which points on the polar graph correspond to maximum points on the Cartesian graph? Which points on the polar graph correspond to minimum points on the Cartesian graph? Is there any connection between these points?
- Q5** Dragging the purple pointer from 0° to 360° , the polar graph repeats itself, with an interesting twist. What is different about the second copy?
- Q6** Make a prediction as to how changing parameter a will affect the polar graph. Now adjust parameter a in the sketch, and see if you’re right. What does the a -value appear to control in the polar sketch?
- Q7** Make a prediction as to how changing parameter b will affect the polar graph. Adjust parameter b in the sketch, and see if you’re right. What does the b -value appear to control in the polar sketch? Be sure to try both odd and even values.

Don’t be surprised if the answer to Q7 is a little more complicated than you may have expected!

EXPLORE MORE

- Q8** Explain the pattern you discovered in Q7. (*Hint:* Your answer to Q5 is relevant here. Think about what has to be true in order for the polar graph to repeat itself at 180° and for which values of b this can happen.)
- Q9** Go to page 2 of **Cartesian Polar.gsp**. There you’ll see $f(x) = \frac{2}{\cos(x)}$, which you could also write as $f(x) = 2 \sec(x)$. Predict what the polar graph will look like. After you’ve recorded your prediction, press the *Show Polar* button to reveal the answer. Can you explain analytically why the graph looks this way?
- Q10** Go to page 3 of **Cartesian Polar.gsp**. On this page you can enter your own functions to try them out. You can use the values of parameters a and b in these functions if you like. Try several different kinds of functions, and record your results.