THE UNIT CIRCLE

Q1 The value of *Arc Angle* ranges from 0° to 360°.

Q2 The sine ranges from −1 to 1, at angles of 270° and 90°, respectively. The cosine ranges from −1 to 1, at angles of 180° and 0°, respectively. The tangent has no limit in either direction, but the measurement is limited by the resolution of the objects on the screen.

THE REFERENCE TRIANGLE

Q3 The calculation corresponds to the sine function.

Q4 The smallest value for sine is 0 and occurs at 0°. The largest value is 1 and occurs at 90°. The smallest value for cosine is 0 and occurs at 90°. The largest value is 1 and occurs at 0°. The smallest value for the tangent occurs at 0°. The tangent has no upper limit, and it gets very large as the angle approaches 90°. At both 0° and 90°, the triangle is degenerate, with various points and sides coinciding.

COMPARE THE DEFINITIONS

Q5 The measurements agree only in the first quadrant. In the other quadrants the arc angle is more than 90°, but the angle in the triangle remains between 0° and 90°.

Q6 The definitions agree in Quadrants I and II because the y-value is positive there. The definitions disagree in the other two quadrants because the measured length of a line segment is always positive.

Q7 The cosine values agree in Quadrants I and IV, but disagree in Quadrants II and III. In these two quadrants the x-value is negative, but the distance measured in the triangle remains positive.

Q8 The tangent values agree in Quadrants I and III. In Quadrant I the coordinates (for the unit circle definition) and the distance measurements (for the right triangle definition) are all positive, so the two functions agree. In Quadrant III both coordinates are negative, so their ratio is positive, matching the right triangle definition. In the other two quadrants one coordinate or the other is negative, resulting in values that the right triangle cannot produce.

Q9 Explanations will vary. This is a good place to introduce the idea of the *reference triangle* within the unit circle and to observe that the opposite side for both 30° and 150° corresponds to the same y-value. For 210°, the opposite side corresponds to a negative y-value, so the value of $\sin 210°$ is the opposite of that of $\sin 30°$.

Q10 Answers will vary. A big advantage of the unit circle method is the ability to work with angles that are beyond 90°. An advantage of the right triangle method is that it's easier to apply when the angle is not in standard position. (Though students don't know this yet, the unit circle method will allow them to explore topics, such as uniform circular motion, which would not be possible with only a right triangle definition.)

EXPLORE MORE

Q11 Answers will vary. Analyzing the flight path of a plane or the position of a person on a Ferris wheel both benefit from using angles beyond 90°. For the height of a building, a right triangle definition is sufficient.

Q12 You could condense the two methods into one by considering the right triangle method to be a special case of the unit circle in Quadrant I.

Q13 At 90° the line AC is vertical, so its slope (and the tangent of 90°) is undefined. The result is that the tangent graph has an asymptote at 90°.

RELATED ACTIVITIES

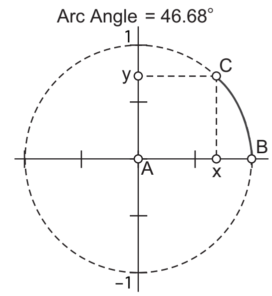
The definitions are introduced in Right Triangle Functions and in Unit Circle Functions.

Use this presentation to relate the two ways of defining the trig functions.

THE UNIT CIRCLE

Angles in this sketch are in degrees rather than radians.

Leave point *C* in motion while students answer these three questions.



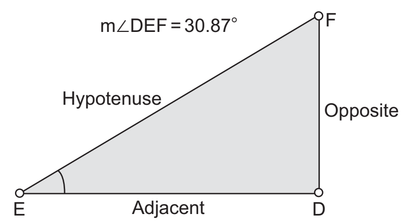
1. Open **Unit Circle Right Triangle Present.gsp.** Show the unit circle and animate point *C*. Show the arc angle.

Q1 Ask students what are the largest and smallest values they observe for the arc angle. (0° to 360°)

Q2 Ask what measurements are needed in the unit circle to define the sin, cos, and tan functions. (in order: *y*, *x*, and equivalently either *y*/*x* or the slope of *AC*)

Q3 Show these measurements and ask students to observe the largest and smallest values for each of the measurements. Review again which is sin, cos, and tan.

THE REFERENCE TRIANGLE



2. Show the right triangle and measure ∠*DEF*.

Q4 Drag point *F* and ask students to observe the largest and smallest values for the angle.

Q5 Show the length measurements and ask students what ratios must be calculated to find the sine, cosine, and tangent.

Q6 Show the ratios and have students confirm which is sine, cosine, and tangent. Drag point *F* and have students observe the largest and smallest values for each ratio.

COMPARE THE DEFINITIONS

3. To compare the definitions, combine the models. Press *Merge Triangle to Circle.*

Q7 Drag point *C* (keeping it in Quadrant I), and ask students to compare the four measurements from each triangle.

Q8 Ask students to make conjectures about what will happen if *C* leaves Quadrant I.

Q9 Drag point *C* slowly through the other three quadrants, and ask students to describe what they observe about each of the four measurements. Encourage them to explain their observations.

Q10 Why does the sine of 150° in the circle have the same value as the sine of 30° in the triangle? Why is sin 210° the opposite of sin 30°?

Q11 Ask for advantages and disadvantages of each way of defining the functions.