Analytic Conics

For GSP5

You can interpret conic sections both geometrically and analytically (using equations). Both representations are useful and interesting. Ideally you should learn both the geometric and analytic representations and never lose the connection between the two.

CIRCLE, ELLIPSE, AND HYPERBOLA IN STANDARD POSITION

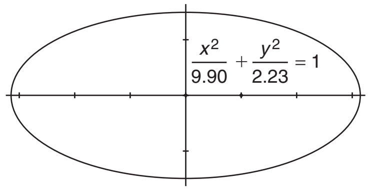
A circle is simply a special case of an ellipse.

The equations for circles, ellipses, and hyperbolas are simplest when they are in standard position. In standard position, the center is at (0, 0) and the axes are horizontal and vertical. You can express the equations of these three conics in this form:

$$\frac{x^2}{A} + \frac{y^2}{B} = 1$$

To edit a parameter, use the **Arrow** tool to double-click it, and then type a new value. You can also change the value using the **+** or **–** key on the keyboard, or you can select the parameter and choose **Display | Animate.**

1. Open the Standard Form page of **Conics.gsp.** At the top of the screen is a standard equation of an ellipse or a hyperbola. You cannot edit the equation directly, but you can edit parameters A and B (above their corresponding values in the equation). Experiment by changing the parameters and observing the effects on the shape of the curve.



Q1 From the values of A and B, how can you determine whether the curve is an ellipse, a circle, or a hyperbola?

Q2 From the equation of an ellipse, how can you determine which is the major axis? What are the major and minor radii?

Q3 From the equation of a hyperbola, how can you determine which is the transverse axis? What are the transverse and conjugate radii?

Q4 For what values of A and B is there no solution to the equation?

Q5 Why can't you generate a parabola from an equation in this form?

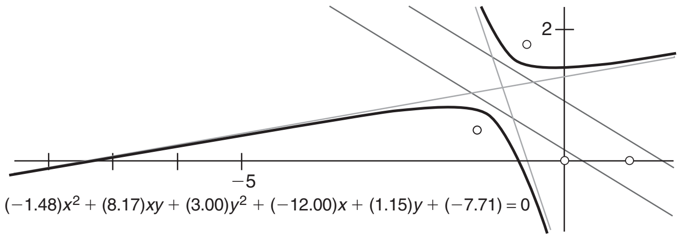
GENERAL SECOND-DEGREE EQUATIONS

You can represent any conic section by a second-degree Cartesian equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

2. Open the 2nd-Degree page of **Conics.gsp.** At the top of the screen is a general second-degree equation in x and y. You can change the equation by editing parameters A through F, which are above their corresponding values in

the equation. The graph of the equation is in blue. Other objects (such as foci, center points, directrices, and asymptotes) appear when appropriate.



3. To begin, remove the *xy* term from the equation by setting parameter *B* to zero. Experiment by changing the other parameters and observing the effect.

Q6 What general statements can you make about the graph when *B* is zero?

Q7 The following table contains descriptions of conic sections. Either systematically or by guessing, find an equation that will produce each locus.

Locus	Equation
Circle	
Ellipse	
Hyperbola	
Parabola	
Line	
Two intersecting lines	
Two parallel lines	
Point	
No solution	

Q8 is best answered by describing the ranges of possible values for the coefficients.

Q8 Still assuming that *B* is zero, describe the general forms of the equations for the circle, ellipse, hyperbola, and parabola.

Q9 The solution is a line if you reduce the equation to the first degree by setting the first three parameters to zero. Is it possible to have a second-degree equation with a solution that is a single line? How?

4. Change parameter *B* to a nonzero number. Experiment again with changing the other parameters.

Q10 Review the generalizations that you made in Q6. Are they still true when $B \neq 0$?

