# Unit Circle and Right Triangle Functions

For GSP5

There are several different ways to define trigonometric functions like sine and cosine. One set of definitions is based on right triangles, and another set is based on a *unit circle* (a circle with a radius of exactly one unit). In this activity you'll explore the relationship between these two ways of defining trigonometric functions.

THE UNIT CIRCLE

Arc Angle = 46.68°

1. Open **Unit Circle Right Triangle.gsp.** Measure the arc angle of arc *BC* on the unit circle. Label the measurement *Arc Angle*.
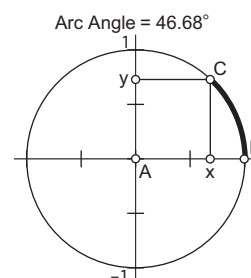
Q1 Drag point *C* around the circle and observe the angle measurement. What are the smallest and largest values that you observe? Leave *C* in Quadrant I when you finish.

2. Measure the *y*-coordinate of point *C* and label it *sin in circle*. Measure the *x*-coordinate and label it *cos in circle*.

3. Construct a line through *A* and *C,* and measure the slope of the line. Label this measurement *tan in circle*.

Q2 Drag *C* again. What are the smallest and largest values that you observe for the sine, cosine, and tangent of the arc angle? At what angles do these values occur?

THE REFERENCE TRIANGLE

Ratios of triangle sides provide another way to define trigonometric functions. You can use the mnemonic *SOH CAH TOA* to recall the ratios:

SOH: The ratio for *Sine* is *Opposite* over *Hypotenuse.*

CAH: The ratio for *Cosine* is *Adjacent* over *Hypotenuse.*

TOA: The ratio for *Tangent* is *Opposite* over *Adjacent.*

To measure ∠*E,* select points *D, E,* and *F.* Then choose **Measure | Angle.**

4. Measure ∠*E* for the right triangle.

5. Measure the *Adjacent* side by selecting points *D* and *E* and choosing **Measure | Coordinate Distance.** Do the same for the other two sides. Label your measurements *Opposite, Adjacent,* and *Hypotenuse.*

m∠DEF = 30.87°

6. Use Sketchpad's Calculator to calculate *Opposite/Hypotenuse.*
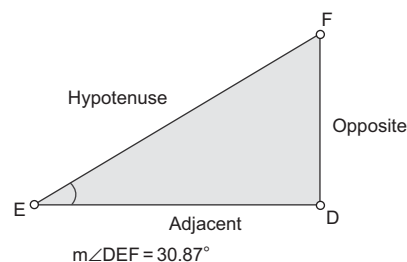
Q3 According to the *SOH CAH TOA* mnemonic, to which trigonometric function does this calculation correspond?

7. Label your calculation *sin in triangle.* Calculate each of the other two ratios and label them appropriately.

Q4 Drag point *F.* What are the smallest and largest values that you observe for the sine, cosine, and tangent in the right triangle? At what angle do the maximum and the minimum occur for each?

COMPARE THE DEFINITIONS

To compare these definitions, you'll combine the two models.
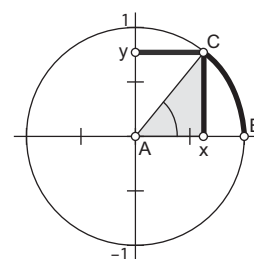
8. Select points *A* and *E,* and choose **Edit | Merge Points.** Also merge points *C* and *F.* The right triangle is now attached to the inside of the unit circle.

Q5 Drag point *C* and observe the two angle measurements (the arc angle and the angle in the triangle). When do these measurements agree? When do they disagree?

Q6 Drag point *C* and observe the two sine measurements. Explain why the values are equal in certain quadrants but not in others.

Q7 When do the two cosine measurements agree, and when do they disagree? Why?

Q8 When do the two tangent measurements agree? Explain.

Q9 Why is the sine of 150° the same value as the sine of 30°? Why is the sine of 210° the opposite of the sine of 30°? (*Hint:* Think about how each relates to either a coordinate or a ratio, and compare these.)

Q10 Describe possible advantages and disadvantages for each method of defining the trigonometric functions.

EXPLORE MORE

Q11 Based on the different definitions, which might be better to determine the flight path of an airplane? The position of a person on a Ferris wheel? The height of a building? Explain.

Q12 Could you always use a single definition? Explain.

Q13 Drag point *C.* What happens to the tangent at 90°? Explain. What does this mean in terms of the graph of the tangent function at 90°?