Right or Left: Adding and Subtracting Integers

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 5 minutes.

1. Open **Right or Left.gsp** and go to page "Addition." Enlarge the document window so it fills most of the screen.
2. Explain, ***Today you're going to use Sketchpad to add and subtract integers on a number line.*** You might remind students that integers are the set of whole numbers and their opposites, and ask them to give real-world examples of integers.
3. ***First I'll show you how the Sketchpad model works.*** Model worksheet steps 1–5. Here are some tips.
   - Introduce the addition problem in the sketch. ***What addition problem is shown?*** [8 + 5] ***Are the numbers in the problem positive or negative?*** [Both positive] ***Will the answer be to the right or to the left of the first number?*** [Right] ***Will it be to the right or left of zero on the number line?*** [Right] Ask a few volunteers to share their predictions. Then press *Add*. Press *Reset,* and then press *Add* a second time.

In this model each integer is represented by a vector. The term *arrow* is used because students may not be familiar with the term *vector* yet. A vector has both magnitude and direction, so practice with this model will help students understand how the signs of the addends come into play.

   - ***What does the first arrow represent?*** [8] ***What does the second arrow represent?*** [5] ***Why do both arrows point to the right?*** [Adding two positive numbers] ***How can you find the answer to* 8 + 5 *using the model?*** Students may make a response such as the following: *The second arrow starts from the end of the first arrow, so where the second arrow ends is the answer. The second arrow ends at 13.*
   - Now press *Reset,* and drag the circles to model 2 + (−6). ***By dragging the circles, I can change the problem. What is the problem now?*** [2 + (−6)] ***Why does the second arrow point left?*** [It represents a negative number.] ***Why does an arrow that represents a negative number point left?*** [Negative numbers are to the left of zero on a number line] ***How many units long is each arrow?*** [The arrow representing 2 is 2 units long; the arrow representing −6 is 6 units long.] ***Predict whether the answer will be to the right or to the left of zero.***
   - ***If I press* Show Steps*, we can see the solution modeled in steps.*** Press *Show Steps,* and then press each numbered button to see the model step by step. ***What is the answer to* 2 + (−6)*? How do you know using the model?*** Here is a sample student response: *The answer is −4*

because that is where the second arrow ended up after placing it at the end of the first arrow.

- You might consider doing another sample problem without pressing *Reset*. Students may gain more insight from leaving the arrows visible on the number line as they drag the circles to change the problem.

4. Ask students to consider the relationship between addition and subtraction of integers as they work. ***As you are adding and subtracting integers using the Sketchpad model, think about how adding and subtracting integers are related. How are they similar? How are they different?***

DEVELOP

Expect students at computers to spend about 35 minutes.

5. Assign students to computers and tell them where to locate **Right or Left.gsp.** Distribute the worksheet. Tell students to work through step 25 and do the Explore More if they have time. Encourage students to ask their neighbors for help if they are having difficulty with Sketchpad.

6. Let pairs work at their own pace. As you circulate, here are some things to notice.

   - In worksheet step 3 and for all activity questions, encourage students to write clear and detailed explanations using complete sentences. By clearly describing what they observe, students acquire a strong mental image of operations with integers. If time is limited, you might have students write their explanations for homework.

   - In worksheet step 8, have students predict what will happen in the Sketchpad model before pressing any buttons. ***What will the model of $-6 + (-3)$ look like? Why?*** Try to get students to concentrate on the behavior of the model rather than on the numeric answer.

   - In worksheet steps 10 and 11, encourage students to explore these questions by dragging to change the values of the integers without pressing *Reset*. Students can quickly view several problems before making a conjecture.

   - In worksheet step 12, students must interpret different parts of the Sketchpad model. As you walk around, observe students to be sure they understand each part and can model any problem they are given. When students successfully model all the problems, ask them to look for patterns. Students may notice that the sign of the answer is the

same as the sign of the longer arrow. They may not recognize this as the integer with the greater absolute value; that's okay. Students are focusing on the visual model at this time. Discuss absolute value later.

- In worksheet step 14, students start subtracting integers. The concept of additive inverse is not explicitly named, but it plays a prominent role in the model. Ask students to think about why the second integer is flipped in a subtraction problem.
- As students are creating new subtraction problems, ask them to predict what the model will do each time before pressing the action buttons. ***Can you predict what the model of this subtraction problem will look like? Why do you think it will act that way?*** Students can test their conjectures using the step-by-step buttons.
- In worksheet step 24, ask students what patterns they see and how they could predict the answer from the two numbers being subtracted.
- In worksheet step 25, students are asked to write an addition problem using the same first number and the same answer. Students can test their addition problems by going to page "Addition." Switching back and forth between the two pages will reinforce the idea of using addition to rewrite a subtraction problem: To find the answer to a subtraction problem, you add the additive inverse (opposite) of the second number.
- If students have time for the Explore More, they will investigate the behavior of addition and subtraction independent of specific values, and they will use special cases to identify the position of zero on the number line.

SUMMARIZE

Project the sketch. Expect to spend about 20 minutes.

7. Gather the class. Students should have their worksheets with them. Begin the discussion by opening **Right or Left.gsp** and going to page "Addition." Work through the different types of addition problems with the class.
   - Have volunteers model the problems they recorded for worksheet step 9. ***What happens in the model when you add two negative integers?*** Students may make this sample response: *When adding two negative integers, the arrows both point left, so the answer is always*

negative. ***How does this compare to adding two positive integers?*** Students may reply that in both cases the arrows point in the same direction. With positive integers, the arrows point right. With negative integers, the arrows point left.

- Next have volunteers model the problems in worksheet step 12. ***What happens in the model when you add a positive and a negative integer?*** Students may make the following response: *If the negative integer is greater, the arrow pointing left will be longer, so the answer will be negative. If the positive integer is greater, the arrow pointing right will be longer, so the answer will be positive.*
- At this point, you may wish to introduce the term *absolute value* and the absolute value symbol. **Absolute value** ***is the distance a number is from zero. What represents the absolute value of a number in this model?*** Help students see that the length of an arrow is the distance from zero. ***What is the absolute value of −2?*** [2] ***What is the absolute value of 2?*** [2] Work through several problems with the class, each time focusing on the length of the arrow. Students should understand that opposites, or additive inverses, have the same absolute value. ***Can the absolute value of a number ever be negative?*** Students should realize that because distance is a positive value, the absolute value can never be negative.
- ***When adding a positive and a negative integer, how can you look at the numbers and tell whether the answer will be positive or negative?*** Students may make the following responses.

  The sign of the number with the longer arrow will be the sign of the answer.

  The sign of the number with the greater absolute value will be the sign of the sum.

8. Go to page "Subtraction." Have volunteers model subtracting two positive integers, a negative and a positive integer, a positive and a negative integer, and two negative integers. ***How are adding and subtracting integers related? How are they similar? How are they different?*** Students may respond with the following answers.

   When you subtract two integers, you flip the second number, so its arrow points the other way. You don't do that with addition.

In subtraction, after you flip the second number, the model is similar to addition. The answer is where the second arrow ends.

Subtraction is just adding the second number flipped.

In subtraction you are adding the opposite of the second number.

9. If time permits, discuss the Explore More. Have students explain how they determined the position of zero.

10. ***Explain the different ways you can get a negative answer when you subtract two integers.*** You may wish to have students respond individually in writing to this prompt. Here are the possible ways: If both integers are positive, the second integer must be greater than the first one. If the first integer is negative and the second integer is positive, the difference will be negative. If both integers are negative, the second integer's absolute value must be smaller than that of the first integer.

EXTEND

1. ***When you add two integers, does the order matter? In other words, is*** $-3 + 5$ ***the same as*** $5 + (-3)$***?*** Have students use the sketch to explain their answers. The order does not matter when you add two integers. The arrows determine how far you go and in which direction. It doesn't matter whether you follow the first arrow and then the second or whether you follow the second arrow and then the first.

2. ***When you subtract two integers, does the order matter? In other words, is*** $-3 - (-5)$ ***the same as*** $-5 - (-3)$***? Explain in terms of the model why your answer makes sense.*** The order does matter when you subtract integers, because only the second arrow is flipped. More sophisticated students will observe that the order matters only if the second integer is nonzero, because flipping zero has no effect.

3. ***What questions occurred to you while you were adding and subtracting integers?*** Encourage curiosity. Here are some sample student queries.

   Why do you flip the second arrow when subtracting?

   If you subtract a positive number from any number, is the result always to the left of the first number?

   Can this model work for multiplying and dividing integers?

ANSWERS

3. In their final positions, the second arrow starts from where the first arrow ends, and the answer (13) is at the end of the second arrow.

6. Answers will vary. Problems should include only positive integers.

7. Each bottom arrow is exactly the same size and direction as the corresponding top arrow.

8. The sum of $-6 + (-3)$ is -9.

9. Answers will vary. Problems should include only negative integers.

10. Whether adding two negative or two positive integers, both arrows go the same way, taking the sum farther away from the center of the number line (farther from zero). The difference is that the arrows go to the right when the numbers are positive and to the left when they are negative.

11. When you add two negative integers, you cannot get a positive sum. Both numbers take the sum in the negative direction from zero, so the sum must be negative.

12. $7 + (-4) = 3$ $\quad -4 + 7 = 3$

    $-6 + 2 = -4$ $\quad 2 + (-6) = -4$

    $-3 + 7 = 4$ $\quad 3 + (-7) = -4$

    $2 + (-5) = -3$ $\quad -2 + 5 = 3$

13. When you add a positive and a negative integer, the integer that has the greater absolute value tells you whether the answer will be positive or negative. In other words, the sign of the answer is the same as the sign of the longer arrow.

16. During the animation, the arrow for 5 flips from the right to the left. This shows which way the second arrow must go in order to subtract it from the first.

18. Answers will vary. This step creates the additive inverse by flipping the arrow.

19. Problems will vary. Problems should include those in which the second integer (*subtrahend*) is greater than the first integer (*minuend*).

20. If both integers are positive, the answer will be positive if the first integer is greater, and it will be negative if the second integer is greater.

21. Some students will write direct observations, and others will interpret those observations. Typical answers will be similar to these.

    Observation: In this problem, $4 - (-3)$, the second arrow starts out pointing to the left, so when it flips, it turns around and points to the right.

    Interpretation: The second number starts out negative, so when it flips, it becomes positive.

22. Problems will vary. The first integer is positive and the second integer is negative, so after flipping, both arrows point to the right. The answer must be positive.

23. Problems will vary. The first integer is negative and the second integer is positive, so after flipping, both arrows point to the left. The answer must be negative.

24. $7 - (-4) = 11$ $\quad -4 - 7 = -11$

    $-3 - 8 = -11$ $\quad -3 - (-8) = 5$

    $2 - (-7) = 9$ $\quad -2 - 7 = -9$

    $-6 - (-5) = -1$ $\quad -5 - (-6) = 1$

25. $7 + 4 = 11$ $\quad -4 + (-7) = -11$

    $-3 + (-8) = -11$ $\quad -3 + 8 = 5$

    $2 + 7 = 9$ $\quad -2 + (-7) = -9$

    $-6 + 5 = -1$ $\quad -5 + 6 = 1$

    Subtracting is the same as adding the opposite.

26. For addition, the sum moves in the same direction and at the same speed as the movement of either integer. For subtraction, the difference moves in the same direction and at the same speed as the movement of the minuend (a), but it moves in the opposite direction as the movement of the subtrahend (b).

27. For addition, move one integer until the sum is equal to the other integer. For subtraction, move the two integers to the same location so the difference is at zero, or move the subtrahend (b) until the difference equals the minuend (a).