Zooming Decimals: Precision and Place Value
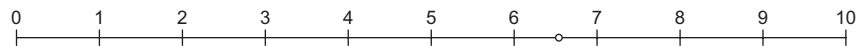
INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 20 minutes.

1. Open **Zooming Decimals.gsp.** Go to page "Model 1." Distribute the worksheet.

2. Ask students to describe what they see. The model displays a number line labeled from 0 to 10. A red point sits on the line. Drag the point to show that it can move anywhere on the number line. Then choose **Edit | Undo** one or more times to return the point to its original position.

3. ***What can you say about the location of the red point?*** Give students time to record their answers alongside "First" on the worksheet. Take responses and record them on the board. Sample responses follow.
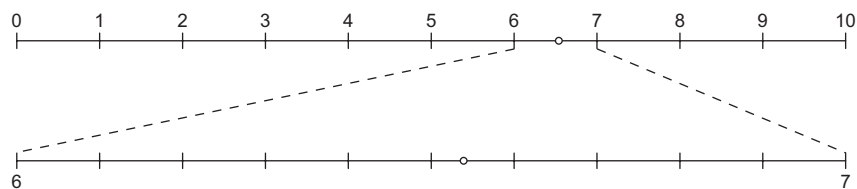
   The point is between 6 and 7.

   The point is around $6\frac{1}{2}$.

   The point is around 6.5.

   The point is between 6.5 and 6.75; it's more than halfway to 7, but it's not three-quarters of the way.

4. ***The point sits somewhere between 6 and 7. How do you think we can find its location more precisely?*** Take responses. Students may suggest dividing the interval between 6 and 7 into more parts.

5. ***Let's take a closer look at what's happening between 6 and 7.*** Press the first *Zoom* button. A number line will appear directly below the 6−7 interval and slowly expand, as if "zooming in" on this interval.

6. ***How does this number line relate to the one above it?*** Elicit the idea that the new number line represents a magnified, or "zoomed," view of the interval where the point lies, between 6 and 7. The dashed lines connecting the two number lines show which portion of the original number line is shown on the number line below it. Explain that the point sitting on the new number line is "the same" as the one above it:

Both points lie at the same location. This may not be immediately clear to students because the points do not sit one directly below the other.

7. ***What do the tick marks that sit between 6 and 7 represent? How far is it from one tick mark to the next?*** Now, an interval of one has been divided into ten equal parts, so there is an increase of one-tenth, or 0.1, from tick mark to tick mark. Point to each tick mark between 6 and 7, asking the class to count as you go along: *six and one-tenth, six and two-tenths, ...*

8. ***What can you say about the location of the point now?*** Give students time to record their responses alongside "Second" on the worksheet. Take responses and record them on the board. Sample responses follow.

    Now we can estimate the location more accurately.

    We were right that the point is a little closer to 7 than it is to 6.

    The point is between $6\frac{5}{10}$ *and* $6\frac{6}{10}$.

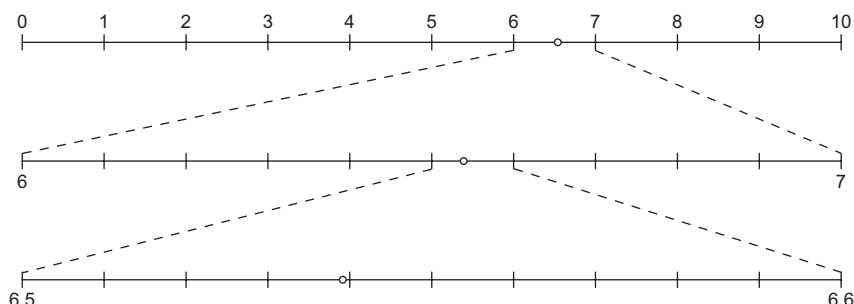   *The point is between 6.5 (six point five) and 6.6 (six point six).*

    The point is around 6.55.

    Give the class time to discuss estimates of the point's location.

    ....
    If you want students to estimate the location of the point to the tenths or hundredths place only, stop here. Press *Reset,* and try a new problem by dragging the point to a new location.

9. ***When we zoom in, we gain precision; we can describe the location of the point more accurately. What do you think we'll see if we zoom in again, this time on the interval between 6.5 and 6.6?*** Take responses. Students may or may not predict that the interval will be divided into ten smaller parts, with each part representing a tenth of a tenth—a hundredth.

10. Press the next *Zoom* button. The new interval 6.5 to 6.6 is shown. Elicit the idea that again an interval has been divided into ten equal parts, but this time a tenth has been divided, not one whole unit. ***What is a tenth of a tenth?*** Read the location of each tick mark with the class: *six and fifty-one hundredths, six and fifty-two hundredths,* and so on.

11. Ask students to use this magnified view to make a more precise estimate of the point's location. Students should write their answers on the worksheet alongside "Third." Take responses and record them. Here are samples of student thinking.
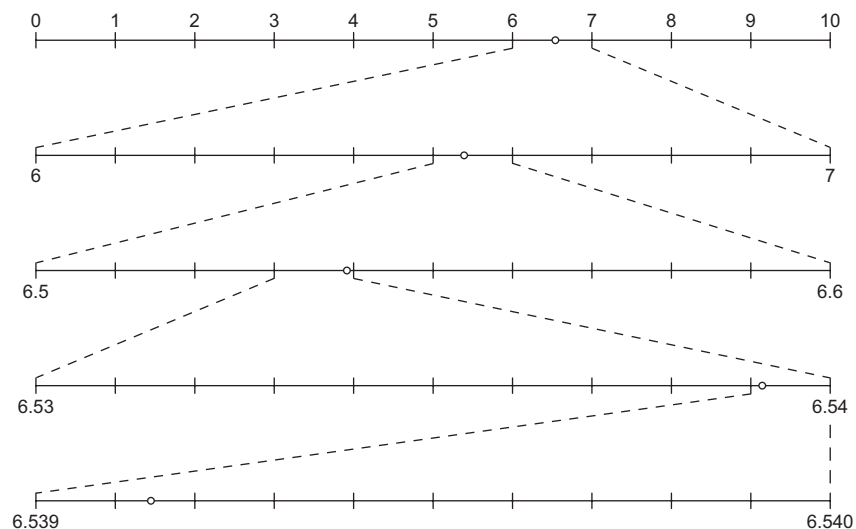
   The point is closer to 6.5 than 6.6.

   The point is between six and fifty-three hundredths and six and fifty-four hundredths.

   The point is between 6.53 (six point five three) and 6.54 (six point five four).

   The point is very close to 6.54. I'd say it's probably about 6.539.

12. If you want to continue to thousandths and ten-thousandths, repeat the sequence of steps two more times. Press the next *Zoom* button, watch the interval expand, discuss what the tick marks represent, and ask students to estimate the location of the point. Students' final estimates of the point's location will likely be that it lies between 6.5391 and 6.5392. To view the location of the point, reported to eight decimal places, press *Show Location.*

DEVELOP

Continue to project the sketch. Expect to spend about 30 minutes.

13. Have students look at all five estimates on their worksheets. **What is different about the estimates you made using each number line?** Students should explain that each time they viewed a new number line a more detailed scale was shown, allowing them to name the location of the point more precisely.

14. Press *Reset* to hide all but the top 0−10 number line. Drag the point to a new location. Repeat the same steps, having students record their estimates.

15. Press *Show Location.* Ask a volunteer to drag the point that sits on the top number line. Students will observe that all five points move simultaneously, because each point represents the same location.

To speed up the movement of all the points, choose **Display | Show Motion Controller** and click the up arrow repeatedly to increase the speed.

16. To make the movement steadier, press *Animate Point.* **Look at how all five points are moving. What do you notice?** Here are some sample responses.

The points all move at different speeds.

The point on the first number line moves the slowest. The point on the fifth number line moves the fastest.

Every time the point on the fourth number line moves all the way across, the point on the third number line moves one tick mark to the right. That's because the fourth number line is divided into thousandths and the third number line is divided into hundredths. Every time the point has gone 10 thousandths, it has gone a hundredth.

Every time the point on the fifth number line moves all the way across, the point on the fourth number line moves one tick mark. That's because the fifth number line is divided into ten-thousandths and the fourth number line is divided into thousandths. There are 10 ten-thousandths in a thousandth.

SUMMARIZE

Continue to project the sketch. Expect to spend about 10 minutes.

17. Present problems such as the following. Take responses and have students write each response using decimal notation.

A point is closer to 3.6 than it is to 3.7. What are some possible locations of the point? Here are two sample responses.

Three and sixty-four hundredths. [3.64]

Three and six hundred twenty-five thousandths. [3.625]

Name a point that is closer to 5.0 than it is to 5.1.

Name a point that is closer to 99.9 than it is to 100.1.

Name a point that is closer to 30.45 than it is to 30.47.

18. ***Suppose I name two decimals for you. Do you think it's always possible to name another decimal that lies somewhere between them?*** Students' experiences with magnifying the number line and viewing ever-finer scales may lead them to believe (correctly) that it is always possible to do so. Provide an opportunity for them to communicate their reasoning and craft explanations in their own words.

EXTEND

The colored diamonds serve as a way to indicate where the hidden point sits along the five number lines.

1. Page "Model 2" contains a related model to explore. Students are given the numerical location of an unseen point and must mark the point's location along the five progressively scaled number lines. To start, students should press *Units* to reveal a number line scaled from 0 to 10. Students should decide on which unit interval the point is located and drag the colored diamond so that it covers that interval. Continuing in this manner with the remaining four number lines, students should identify the intervals on which the hidden point is located and mark them with the colored diamonds. Pressing *Show Answer* reveals the locations of the point. Pressing *New Problem* generates a new number.

2. On page "Model 1," spend time identifying the location of points on intervals other than 0−10. To change the endpoints of the top number line, double-click *Left Endpoint* = 0, enter a whole number in the dialog box, and click OK.

3. For students who would benefit from more individualized work, provide opportunities to use the decimal model alone or in pairs.

4. Discuss the concept of calibrated scales in tools people use. Begin by discussing measuring tools like tape measures and thermometers that students are familiar with. Continue by discussing reasons people use estimates to the tenths, hundredths, thousands, and ten-thousandths places. Include the idea that the "best" estimate is the one with the level of precision needed in a particular situation: ***When you bake a cake, do the directions tell you to bake from 30.5 to 39.5 minutes?***