Place-Value Counter: Get to the Target

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Open **Place Value Counter Target.gsp.** Go to page "Counter." Tell students, ***Today you're going to be solving puzzles that use this special counter.*** Press the + button in the ones place repeatedly, pausing between presses, while the class observes. Press the + button in the tens place a few times; then the + button in the hundreds place. (Leave the rest of the buttons, including the − buttons, for students to investigate on their own.) Invite students' comments about the counter. Does it remind them of an odometer? Are there other places where they have seen a display that counts up (or down) using large numbers?
2. Press *Reset to 0* to clear the counter. ***I want to reach 99. How can I do that?*** Some students may suggest pressing buttons to "count by 10's to 90, and then count by 1's." ***Is there another way we can reach 99 that requires fewer button presses?*** [A more efficient approach, starting at 0, is to press the + button in the hundreds place once, and then press the − button in the ones place once.]
3. Distribute the worksheet. ***Your goal is to reach each target number in as few button presses as possible.*** Solve the first problem together. For each button pressed, record the updated number on the counter. Make sure students understand that this way of recording will let them come back and review the presses they used. In the case of 42, the recording should be written as 10, 20, 30, 40, 41, 42. There are lines for recording two trials for reaching each target. If students think that they can get to the target using fewer presses than they used the first time, they should try the problem again and list the numbers in the new solution. ***At the end of each line, write the number of presses used.***

DEVELOP

Expect students at computers to spend about 30 minutes.

4. Assign students to computers and tell them where to locate **Place Value Counter Target.gsp.** Tell students to work on all the targets in steps 1–12 and do the Explore More if they have time.
5. Let pairs work at their own pace. As you circulate, observe and listen to students' conversations. Here are some things to notice.
   - In worksheet step 2, students may reach 48 using the sequence 10, 20, 30, 40, 41, 42, 43, 44, 45, 46, 47, 48. If so, challenge them to find a shorter sequence. Students may recall that reaching 99 in only two

presses required them to jump past the target. They can use the same strategy here to obtain the shorter sequence 10, 20, 30, 40, 50, 49, 48.

- In worksheet step 2, when students have reached the target, ask, ***Why did you jump to 40 for target 1, but jump to 50 for target 2?*** Students may explain that 42 is closer to 40 than to 50, whereas 48 is closer to 50 than to 40.
- In worksheet step 3, students will likely realize that jumping to 60 (as opposed to 70) is the most efficient way to reach 61. The question is, what's the best way to jump to 60? Some students may only consider the sequence 10, 20, 30, 40, 50, 60. If so, ask, ***Sometimes we've seen that it helps to jump past a number. Would it help to jump past 60?*** Going to 100 and then back by 10's (100, 90, 80, 70, 60) produces a shorter sequence.

The rule works for all numbers between 1 and 99, except 51 through 55. There is no need, however, to raise these exceptions now unless a student does.

- When most of the class has completed worksheet step 4, consider checking in with the class. ***You've reached targets that are between 1 and 99. What advice would you give someone for reaching a target in that range?*** Take responses. One good idea is this student's: *First, check whether the number is closer to 0 or 100. If it's closer to 100, jump to 100. Otherwise, stay at 0. Then check which multiple of 10 is closest to the number. Jump to that number, moving either forward or backward by 10's. Then go either forward or backward by 1's to reach the target.*
- In worksheet steps 5 and 6, reaching the targets reveals an interesting property of target numbers ending in a 5. The standard rounding rule says that a number ending in a 5 should be rounded up. But if students round 15 up to 20, their sequence—10, 20, 19, 18, 17, 16, 15—takes seven presses. Rounding 15 down to 10 yields the sequence 10, 11, 12, 13, 14, 15—six presses. By comparison, in worksheet step 6, the fewest number of presses to get to 85 requires rounding 85 up to 90 first, yielding the sequence 100, 90, 89, 88, 87, 86, 85. Students can try reaching other two-digit targets ending in 5 to develop the idea that numbers closer to 0 (15, 25, 35, 45) should be rounded down to the nearest 10, whereas numbers closer to 100 (65, 75, 85, 95) should be rounded up to the nearest 10. In the case of 55, the number can be rounded either up or down; the number of presses is the same.

Students' strategies will likely be more tentative and incomplete. Give students plenty of time to experiment before expecting them to develop a general approach.

- In worksheet steps 7–10, students develop strategies for reaching three-digit numbers. One general approach is to round the target to the nearest thousand, then to the nearest hundred, and then to the nearest ten. For example, the target in step 10 is 782, which is 1000 when rounded to the nearest thousand, 800 when rounded to the nearest hundred, and 780 when rounded to the nearest ten. These three rounded numbers are landmarks to reach, as in this sequence: 1000, 900, 800, 780, 781, 782.

SUMMARIZE

Project the sketch. Expect to spend about 20 minutes.

6. Gather the class. Students should have their worksheets with them. Choose a problem from the worksheet and ask students to share the number of presses they used to get to the target. Facilitate discussion and have students come to the computer to share their solutions. Make sure students communicate the reasoning behind their presses, and highlight any common strategies presented if students don't point them out to their classmates.

7. To focus the discussion, you might ask questions such as these.

   How can we get to 99? How can we get to 999? How can we get to 9999?

   If the target is a one-digit number, how do you know whether your first jump should be to 10?

   If the target is a two-digit number, how do you know whether your first jump should be to 100?

   If the target is a three-digit number, how do you know whether your first jump should be to 1000?

   Can you give me several two-digit numbers, other than those on the worksheet, that you would reach most quickly by using only the + buttons? How can you tell by looking at the numbers that you'll use only the + buttons?

8. Solving the counter problems required no prior knowledge of rounding; the activity, in fact, helps to develop an intuitive sense of the logic behind the rounding rules. One rounding rule however does not fit so neatly into this activity. When a number ends in a 5, the standard rounding rule says to round up. For the targets in worksheet steps 5 and 6, however, students saw that when a number ended in 5, it did not always make sense to "round up" and jump to a higher number first.

EXTEND

What other questions about getting to targets using the counter occur to you? Here are sample student queries.

What two-digit number takes the most presses to reach?

What three-digit number takes the most presses to reach?

Suppose we weren't allowed to press certain buttons. How would that change things?

Suppose you weren't allowed to press a button two times in a row. How would that change things?

Can we predict correctly how many presses it will take?

Is there a way to predict whether a number will take the same number of presses whether you jump higher or lower?

ANSWERS

1. 10, 20, 30, 40, 41, 42
2. 10, 20, 30, 40, 50, 49, 48
3. 100, 90, 80, 70, 60, 61
4. 100, 90, 89, 88, 87
5. 10, 11, 12, 13, 14, 15
6. 100, 90, 89, 88, 87, 86, 85
7. 100, 110, 120, 130, 131, 132, 133, 134
8. 100, 200, 190, 180, 170, 171, 172
9. 1000, 900, 800, 810, 820, 830, 840, 839, 838, 837, 836
10. 1000, 900, 800, 790, 780, 781, 782
11. 1000, 2000, 3000, 4000, 3900, 3800, 3700, 3600, 3610, 3620, 3619
12. 10000, 9000, 8000, 7990, 7980, 7970, 7969, 7968
13. Answers will vary.