INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 30 minutes.

1. Introduce the Sketchpad model. Open **Circles Squares Two Unknowns.gsp** and go to page "Game 1." ***We've got a problem to solve. There are rows of squares and circles. The squares all have a hidden value, a whole number 0 through 8. Every square has the same value. The circles also have a whole number value 0 through 8. We don't know what the value of the square is or what the value of the circle is—that's what we need to figure out!***

   We can drag any row of squares and circles across the screen. When I drag a row across the line, the computer will add together the value of the squares and circles. Drag the first row across the divider line. The numeral 0 will change to 9, indicating the sum of the two numbers. With the aid of the class, write this sum as

2. ***Do we now know the values of the circle and square?*** Take responses. Students will propose that there are many combinations of two numbers that add up to 9. Ask students to make a list of these values. Remind students that neither symbol is greater than 8.

   Ask volunteers to share square/circle values from their lists. Write these on the board in an organized list.

1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1

3. ***How do you think we can figure out the actual values of the square and circle?*** Let students suggest the idea of dragging another row across the divider line. Allow them to choose which row to drag. In the example that follows, we assume students pick the second row.

   - Drag the second row  to the right of the divider. A sum of 11 appears.

- ***How can this new information help us?*** Give students time to check their list of square/circle values to determine which pair of numbers satisfies the new statement.
- Make a new column on the board and label it □□●. In this column, ask the class to write the sum of each pair of numbers. Only one pair gives the desired sum: square = 2 and circle = 7.

□	●	□□●
1	8	10
2	7	11
3	6	12
4	5	13
5	4	14
6	3	15
7	2	16
8	1	17

- Press *Show Answers* to confirm the values of the two symbols. Drag the first two rows of symbols back to the left of the divider.
- ***We solved this problem by dragging over the first and second rows of symbols. Suppose instead we drag over the third and fourth rows of symbols. If we solve the problem with that information, will we get the same answer?*** Some students may realize that the answer will be the same regardless of which rows are picked. Other students may not be sure. Drag over the third and fourth rows, and give students time to work with the results before discussing as a class.

DEVELOP

Continue to project the sketch. Expect to spend about 30 minutes.

4. Go to page "Game 2." Drag over the first row of symbols. The sum is 26. ***In this game, the range of possible values for the square and circle has changed. Now, each symbol represents a whole number from 0 through 20. What should we do first to solve this problem?***

   Some students are likely to suggest making a list again. With the class, list all the square and circle combinations. There are 15 possible pairs, starting with square = 6, circle = 20 and ending with square = 20, circle = 6. (Note that it is possible that the values of square and circle are the same.) As before, this approach will allow students to solve the problem. The list, however, is quite long, prompting the question of whether there's a more efficient way to solve the problem.

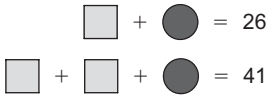
5. ***Our list is really long. Can you think of another way to solve the problem that might be faster?*** Ask for permission to erase the list, and then allow plenty of time for students to consider this question. Some may see a strategy more quickly than others.

6. The steps that follow describe a way to guide discovery of this strategy in a way that includes writing addition statements. Some students will find this way of working helpful in continuing to solve problems. Judge for yourself how much you want to guide students. Alternatively, invite a volunteer to the computer to demonstrate the ideas expressed previously and in the next steps.

   - Drag the second row of symbols (the two squares and circle) across the divider line. Their sum is 41. ***Now that we have two rows of information, I'd like you to write each row as a number sentence using circles, squares, and plus and equal signs.***

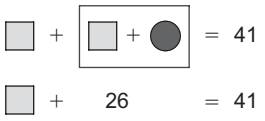
Solving for two unknown values in two statements is an important concept that students will encounter again in algebra. These statements are known as *simultaneous equations* in algebra.

   - Ask a volunteer to write the statements on the board.



   - ***What do these statements have in common and what's different about them?*** Elicit from students that each statement contains a square and a circle, and that the second statement contains an extra square.

   - Draw a box around the square and circle in the second statement and ask, ***What do you know about this sum?*** From the first statement, students know that the sum is 26. Let students suggest replacing the two symbols with 26.

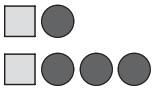


   - ***Now can you figure out the value of the square?*** One way to think about it is to ask what number plus 26 equals 41. [The square is equal to 15.]
   - ***Now that you know the value of the square, can you figure out the value of the circle?*** [Because square + circle = 26, and the square is equal to 15, the circle is equal to 11.]

   - Check the answer to the problem by pressing *Show Answers*.

Solving More Problems

7. Create a new problem by pressing *New Challenge*. Drag the following two rows of symbols over the line.

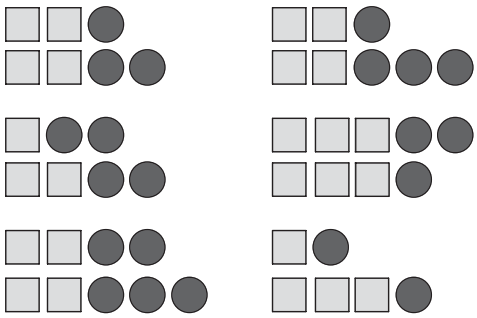


In this case, students can replace the square and circle in the second statement with a number. They'll be left with a statement that gives the sum of two circles. If students determine, for instance, that circle + circle = 10, then circle = 5.

Some students will quickly realize that the values of the circles and squares can always be found by dragging the first two rows—and that those rows are the easiest to use. Make sure they explore other pairs of rows.

8. Continue creating new problems. For each problem, press *New Challenge* and solve by selecting different rows of symbols. Explain that by choosing different pairs of rows each time, students can not only take on new challenges, but also look for a good general strategy for finding the unknowns.

   Some good pairs of rows to choose follow. Present several of these pairs so that students have the opportunity to make a conjecture about why these pairs are useful. Alternatively, you may let the class decide on the rows to drag and observe whether some students suggest pairs like these.



SUMMARIZE

Continue to project the sketch. Expect to spend about 30 minutes.

9. Lead a discussion in which students develop and describe the general strategy for solving for the unknowns. Remind the class of the question that was posed earlier: ***Is there a way to figure out the values of the square and circle that is faster than making a list?***

   If you chose the pairs of rows as the class solved more problems, ask, ***For each problem you've solved, I picked two rows of symbols. How do you think I chose those rows? Was there something nice about the rows we used? Was there something the same about all the pairs of rows?***

If students chose the pairs of rows, ask, ***I noticed that some of you figured out pairs of rows that were easy to work with. How did you choose those rows? Was there something nice about those rows? Was there something the same about all the pairs you chose?***

The discussion should elicit these ideas.

- Good pairs of rows differed by one or more squares, or by one or more circles (and not by both squares and circles).
- Good pairs of rows had some symbols in common.
- By replacing the shared symbols with their numerical value, it was easy to find the value of the "extra" symbol or symbols.

10. ***Can you think of a pair of rows that would make it really hard for us to solve the problem?*** Take responses and try one pair with the sketch. A harder-to-solve pair of rows might be the seventh and eighth rows. It's still possible to figure out the values of the two symbols, but not as easy.

Creating the Problems

11. Explain that students will each create a problem, on paper, for a classmate to solve. Have students brainstorm ideas for presenting their problems.
    - Will they draw many rows of symbols or just two rows? For simplicity, it is probably easier if students show just two rows. (Students may use rows from the sketch or create their own combinations of symbols.)
    - How will the answer to the problem be shown? Will it be written on a separate sheet of paper or on the same sheet, perhaps hidden behind a folded edge?

    Students should create their problems on their own, creating the rows of symbols and the values of the symbols. For variety's sake, students can draw symbols other than the square and circle.

    Students should exchange problems with a classmate and observe as their partners solve the problems.

EXTEND

1. For students who would benefit from more individualized work with solving for unknowns, provide opportunities to use pages "Game 1" or "Game 2" at a later time.

2. Pairs of students can create challenges for each other by changing the values of the symbols on page "Make Your Own" of **Circles Squares Two Unknowns.gsp.** As one student looks away, the other student double-clicks the value of the circle and then the square, changing each number to a new value. Pressing *Hide Answers* conceals the values of the two symbols. The sketch is now ready for use by the other student. When she is convinced that she knows the value of the two symbols, she should explain her reasoning to her partner and then press *Show Answers* to check her work.

   Let students know that to change the value of the square or circle on page "Make Your Own," they will double-click the number with the **Arrow** tool. In the dialog box that appears, they will enter a new number in the value field and click **OK.**