Jump Along: Factor Families on the Number Line

For GSP5

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Open **Jump Along Factor Families.gsp.** Go to page "Jump to 12." Ask students to describe what they see. A rabbit is sitting at 0 on a number line.
2. Tell students, ***Watch what happens when I press* Jump Along.** Press *Jump Along.* ***What did the rabbit do?*** Take responses. If necessary, press *Jump Along* again so that students can watch the movement again. The rabbit makes 4 jumps of size 1 and lands at 4.

Although it is possible to enter a non-integer value for the *Number of Jumps* or *Jump By* parameter, students will use only integer values in this activity.

3. Press *Erase Traces* to clear the path of the rabbit's jumps.
4. Change the *Jump By* number to 2 by double-clicking *Jump By* with the **Arrow** tool, entering 2, and pressing OK. (Alternatively, you can select *Jump By* with the **Arrow** tool and press either + or − on the keyboard to increase or decrease the value.)

   Press *Jump Along*. Ask students to describe what the rabbit did. Make sure everyone sees that *Number of Jumps* tells them how many times the rabbit jumped and *Jump By* tells the size of each jump.
5. Distribute the worksheet. Point to 12 on the number line. ***How can the rabbit get to 12 if it starts at 0?*** Take a suggestion and model it with the rabbit. Have students describe the rabbit's movement using language such as, "The rabbit made 6 jumps of size 2 and landed at 12."
6. ***Let's change the color of the jumps each time.*** Select the point below the rabbit and choose **Display | Color** to pick a different color. ***Now, when we press* Jump Along, *the rabbit's jumps will be a new color.*** Ask students for a different pair of numbers that will land the rabbit at 12, and try the numbers using the new color. ***When you work with the sketch, change the color each time you try a different way to jump to the target.***
7. Together, look at the table in worksheet step 2. Model how to fill in the first row using one of the jumps to 12 you made as a class. ***Let's look at the last column of the table. Can you write a multiplication sentence that explains how the rabbit made its way to 12?*** As a class, develop the multiplication sentence (for example, $6 \times 2 = 12$). ***Why are we multiplying 6 by 2?*** [There were 6 jumps of size 2.]

   Note that students may also suggest $2 \times 6 = 12$ as a valid multiplication statement. Explain, ***That is certainly another correct way to represent 6 jumps of size 2. Let's agree as a class to write the number of jumps first.*** Establishing this convention with students will facilitate conversations at computers and in class discussions.

This is an example of the *commutative property* of multiplication.

8. If you will have students print their sketches, model how to choose **File | Print Preview,** and in the dialog box that appears, set the image to fit on one page and then click Print.

DEVELOP

Expect students at computers to spend about 30 minutes.

9. Assign students to computers and tell them where to locate **Jump Along Factor Families.gsp.** Tell students to work through steps 1–7. Encourage students to ask a neighbor for help if they have questions about using Sketchpad.

   Explain that in worksheet step 2, everyone will try to reach the target number of 12. In worksheet steps 4 and 6, however, students will choose a target from the list of numbers provided.

10. Let pairs work at their own pace. As you circulate, listen to students' conversations. Here are some things to notice.

    - In worksheet steps 2, 4, and 6, students should change the color of the rabbit's traces each time. Tell students to check the *Jump By* and *Number of Jumps* numbers carefully before pressing *Jump Along.* If students make an error and pick numbers that are not factors of the target number, they will not be able to erase the trace of just the last jump. *Erase Traces* deletes all traces.
    - If students happen to enter numbers that cause the rabbit to jump beyond the visible portion of the screen, the extra jumps will not be visible if students scroll across the screen after the rabbit has finished jumping.
    - Traces will not be saved when students go to another page of the sketch or if they save their sketch. If you are having students print their sketches, be sure they print the current page before moving on to the next target number.
    - Ask, ***How do you know when you've found all the ways to reach the target number?*** Some students will be systematic; others will have a more informal approach.

      We started with jumps of size 1; they always work. Then we thought about jumps of size 2, then jumps of size 3, and so on.

      We just tried different numbers until we couldn't find anymore.

We tried numbers starting from 1. We knew 1×12 *worked, so we also knew* 12×1 *worked. We kept going until the number sentences started to repeat. For example, we found* $3 \times 4 = 12$, *so we knew* $4 \times 3 = 12$ *worked. Our next number to try was 4, but we already had it in* $4 \times 3 = 12$. *We knew there weren't any other ways. Every number in our Number of Jumps column of the table also appeared in the Jump By column.*

SUMMARIZE

Project the sketch. Expect to spend about 20 minutes.

11. Gather the class. Students should have their worksheets with them. Go to page "Jump to 12." Begin by presenting an example in order to elicit students' understanding of the *commutative property of multiplication.* (You may or may not wish to mention this term.) Use the model to display 2 jumps of size 6. ***How does knowing 2 jumps of size 6, or*** $\boldsymbol{2 \times 6 = 12}$, ***help you find another way to reach 12?*** Take responses. Have a volunteer come up and model $6 \times 2 = 12$, using a different trace color. Sample responses follow.

    You can put in the same numbers but reverse them—6 jumps of size 2.

    The actions that the rabbit takes are related. Two jumps of size 6 is related to 6 jumps of size 2.

    It doesn't matter what order you multiply 2 and 6 in. The answer is the same; so 2 jumps of size 6 and 6 jumps of size 2 will both land at 12.

    Whenever you multiply two numbers, it doesn't matter what order you multiply them in. The product is always the same.

12. Have volunteers come up to the computer and finish the remaining ways for the rabbit to land at 12, using a different color for each way. ***What patterns did you notice?*** Students may make the following observations.

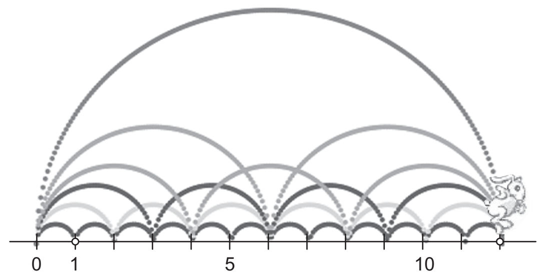
    When the size of the jump increased, it took fewer jumps to get to 12.

    The opposite was true, too. When the size of the jump got smaller, it took more jumps to get to 12.

    All of the jumps the rabbit took fit under the biggest jump of all—1 jump of size 12.

    The pattern made by the jumps is symmetric. If I drew a vertical line through the number line at 6, I could reflect one half of the pattern across the line to show the other half.

The rabbit traced 1 half circle in orange, 2 half circles in pink, 3 half circles in light purple, 4 half circles in dark purple, 6 half circles in light blue, and 12 half circles in red. These numbers are all factors of 12.



13. ***How can you list the factors of 12 just by looking at your table?*** Students can list all of the factors of 12 (1, 2, 3, 4, 6, and 12) just by reading the numbers in either of the first two columns of their table.

14. Ask students to share their results from worksheet steps 4 and 6. In particular, ask them how the target numbers in step 6 differ from the target numbers in step 4. Sample responses follow.

    There are only two ways to get to each of the targets in step 6.

    In step 6, the rabbit can either jump by 1, or it can jump the whole way in one jump. So, for 13, it's either 13×1 *or* 1×13.

    The targets in step 4 have lots of factors. The numbers in step 6 only have two factors. One of those factors is 1, and the other is the number itself.

    The targets in step 6 are prime numbers.

EXTEND

15. For students who are ready to work with larger numbers, rescale the number line by dragging the point at 1 so that it is closer to 0. With more of the number line visible, students can reach targets larger than 25.

16. ***What other questions can you ask about jumping along the number line?*** Encourage student curiosity. Here are some sample student queries.

    How many different ways can the rabbit jump to 100, starting at 0?

    Do you think there are more ways to reach large target numbers than smaller target numbers?

    What target number under 100 has the most number of ways for the rabbit to jump?

What if the Jump By *number is less than 0? What happens then?*

Is it possible that there's only one way for a rabbit to jump to a particular target number?

I noticed that for most targets, the rabbit could reach them in an even number of ways. There are eight ways to reach 24, six ways to reach 20, four ways to reach 10, and so on. Why is that? Are there any target numbers that can be reached in an odd number of ways?

ANSWERS

2.

Number of Jumps	Jump By	Multiplication Number Sentence
1	12	$1 \times 12 = 12$
2	6	$2 \times 6 = 12$
3	4	$3 \times 4 = 12$
4	3	$4 \times 3 = 12$
6	2	$6 \times 2 = 12$
12	1	$12 \times 1 = 12$

3. Answers will vary. Students may make the following observations.
   - The larger the *Jump By* number, the smaller the number of jumps, and vice versa: the smaller the *Jump By* number, the larger the number of jumps.
   - Students may notice the commutative property of multiplication at work. For example, if 3 jumps of 4 works, then so will 4 jumps of 3.
4. Answers will vary. Check students' tables.
5. Answers will vary. See observations listed in answer for 4.
6. Answers will vary. Check students' tables.
7. Answers will vary. Students should notice that there are only two ways for the rabbit to reach the target number.