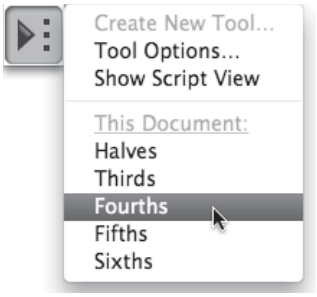
Dividing and Subdividing: Fractions on the Number Line

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

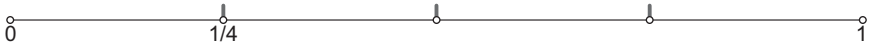
1. Open **Dividing and Subdividing.gsp.** Go to page "a." Explain, ***Today you're going to use some special Sketchpad fraction tools to find the locations of fractions between 0 and 1 on a number line.*** Point out the four identical number lines. Explain that the class will be working with only the part of the lines from 0 to 1. Press *Hide Lines* so that only that part remains.



Let's see how we can use our fraction tools to find $\frac{1}{4}$. We'll find these tools in the Custom Tools menu. Model pressing the **Custom** tool icon and choosing the **Fourths** tool. Move your pointer over the point labeled 0. Make sure the point is highlighted and then click. Move the pointer. As you do, you'll see that you're dragging a "stretchy fraction tool." Its length changes but is always divided into four equal parts by tick marks. Anchor the other endpoint of your stretchy fraction tool by dragging to the point at 1 and clicking when you see that the point is highlighted. Students should note that using the **Fourths** tool has divided the interval from 0 to 1 into four equal parts.

Explain that Sketchpad uses the form 1/4 to write $\frac{1}{4}$.

2. When students have identified the location of $\frac{1}{4}$, model labeling the point. Using the **Text** tool, double-click the point. Type 1/4 and click **OK.** If necessary, drag the label using the **Text** tool, so that the label is below the point.



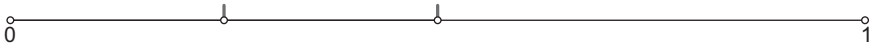
Ask students to name the locations shown by the other two tick marks. They should note that the location of $\frac{2}{4}$ is also the location of $\frac{1}{2}$.

3. ***Suppose we didn't have the* Fourths *tool.*** Press and hold the **Custom** tool icon to show the list of tools. (You might write the list of stretchy fraction tools on the board.) ***Is there another way we could divide the interval from 0 to 1 to find $\frac{1}{4}$? We can use the same tool more than once, or we can use a combination of tools.***

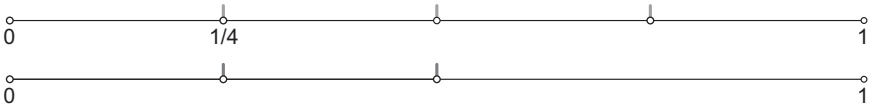
You can also label the point at $\frac{1}{2}$ to help students keep track of the divisions.

Students may suggest using the **Halves** tool to divide the whole (the interval from 0 to 1) in half, and then using the **Halves** tool again to divide the interval from 0 to $\frac{1}{2}$ in half. If this is not suggested, hint,

Suppose we use the* Halves *tool. What can we do with it? When the class is ready, choose the **Halves** tool. On the second number line, divide the interval from 0 to 1 in half. Then, using the **Halves** tool again, click the points at 0 and $\frac{1}{2}$.



4. Now that you've found the location of $\frac{1}{4}$ in two different ways, use the **Arrow** tool to drag one number line onto or near the other to help compare the results.



5. Demonstrate that students can choose **Edit | Undo** to back up one or more steps if they change their minds about what they want to do.
6. If you want students to save their Sketchpad work, demonstrate choosing **File | Save As,** and let students know where to save.

DEVELOP

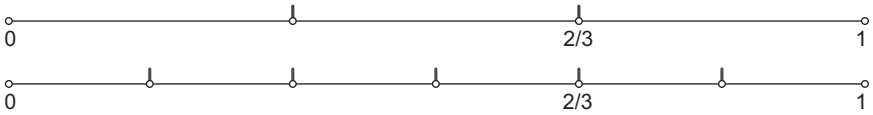
Expect students at computers to spend about 35 minutes.

7. Assign students to computers and tell them where to locate **Dividing and Subdividing.gsp.** Distribute the worksheet. Tell students to work through step 2 and do the Explore More tasks if they have time.

   In the worksheet table, it is important for students to keep track of the custom tools they use and the order in which they use them. Explain to students that they should complete the table carefully so that they can refer back to it later. Encourage students to find and record several ways to locate each fraction.

8. Let students work at their own pace. As you circulate, here are some things to notice.
   - As students create divisions and subdivisions of the 0–1 interval, they should label as many points as they find useful.
   - In worksheet step 2b, students will likely use the **Thirds** tool. To find $\frac{2}{3}$ in a different way, students might experiment and notice that the **Sixths** tool divides the interval into twice as many parts as does the

Thirds tool, with some points in common. In particular, $\frac{2}{3}$ and $\frac{4}{6}$ have the same location.



As students solve more problems, they will begin to see that it is helpful to consider the factors of the denominator when looking for different ways to locate the fractions.

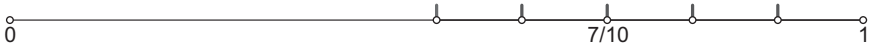
- In worksheet step 2c, some students may struggle to locate $\frac{1}{6}$ in a different way after using the **Sixths** tool to do it. Hint, ***The whole is divided equally into six parts. It's also divided evenly into other fractional parts. Can you see those? What are those parts?*** [Halves and thirds] ***How can you use that information to locate $\frac{1}{6}$ with different tools?*** Let students investigate this. [The **Halves** and **Thirds** tools can be used together, in either order.]
- Students who are looking for additional ways to find a fraction can change the order in which they use the tools. In worksheet step 2d, for example, students can locate $\frac{3}{8}$ by dividing the interval into fourths and dividing each fourth in half, or by dividing the interval into halves and then dividing each half into four equal parts.

If students have been introduced to reducing fractions, note whether they make the connection between this use of the tools and that procedure.

- In worksheet steps 2f and 2i, students may use their knowledge of equivalent fractions to find the fractions using only one tool one time. For students who need a hint, suggest, ***There's a way to locate $\frac{8}{10}$ using one tool one time.*** Using the **Fifths** tool, students might construct $\frac{4}{5}$ to locate $\frac{8}{10}$; likewise, they might construct $\frac{2}{5}$ to locate $\frac{6}{15}$.

Be alert to students' thinking about benchmark fractions.

- Students may realize another way to be economical in their use of the fraction tools. In worksheet step 2e, for example, students may begin by dividing the whole interval in half. Because the fraction $\frac{7}{10}$ is larger than $\frac{1}{2}$, students may then divide only the interval from $\frac{1}{2}$ to 1 into tenths, using the **Fifths** tool.



9. If you want students to save their Sketchpad work, demonstrate choosing **File | Save As,** and let them know how to name and where to save their files.

SUMMARIZE

Project the sketch. Expect to spend about 45 minutes.

10. Gather the class. Students should have their worksheets with them. Focus the discussion on the ideas here that are most appropriate for your students.

    - If students have previously made paper fraction strips by folding, ask them how subdividing the interval from 0 to 1 using the stretchy fraction tools compares to folding.

It's not necessary that students learn the term *lowest terms* or a procedure for reducing fractions at this point. Keep the focus on equivalency.

    - Explore fractions that are not in lowest terms; these are opportunities for students to develop their understanding of equivalent fractions. Invite students who used the **Fifths** tool to locate $\frac{8}{10}$ and $\frac{6}{15}$ to explain their solutions. Propose that the class find more fractions this way. ***I'm wondering about $\frac{6}{8}$. Can we find it using one tool one time?*** Students will likely see that the **Fourths** tool can be used because the location of $\frac{3}{4}$ is also the location of $\frac{6}{8}$. Challenge the class to locate $\frac{10}{20}$, $\frac{4}{12}$, $\frac{12}{16}$, and $\frac{50}{60}$ using one tool one time.
    - ***Sometimes you were able to locate a fraction by using one tool two times. I saw you locate $\frac{1}{9}$ that way. What other fractions can you locate that way?*** [$\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, and $\frac{1}{36}$, as well as any other fraction with one of these denominators] ***What do you notice about the denominators in all of these fractions?*** [They are square numbers.] ***Why does using one tool twice create fractional parts whose denominators are square numbers?*** Students may give examples such as this one: *Thirds divided into thirds makes 3 times 3 parts. That's how you get a square number. You multiply a number times itself.*
    - ***Can you tell by looking at a fraction whether you can make it using one tool? By using two tools? How?***
    - ***Can you tell whether you have located a fraction in all the ways possible using the fraction tools you have? How?***
    - If students worked on the Explore More tasks, discuss them. Alternatively, do one or more of the tasks as a class now.

EXTEND

1. ***What questions occurred to you about locating fractions?*** Encourage all curiosity. Here are sample student queries.

   Why is $\frac{1}{2}$ of $\frac{1}{3}$ the same as $\frac{1}{3}$ of $\frac{1}{2}$?

   What would happen if we had different stretchy fraction tools available? For instance, what if we had tools for primes only? That would be Halves, Thirds, Fifths, Sevenths, Elevenths, and so on. Would that work, too?

   When I found $\frac{1}{6}$, I used the **Halves** *and* **Thirds** *tools. The numbers 2 and 3 are both factors of 6. Why does this always work, that you can use the factors? Why do the factors of the denominator show how you can divide parts into parts to locate a fraction?*

2. Use page "Explore More 1" to provide more fractions between intervals other than 0–1 for students to locate. To change the label for an endpoint of an interval, use the **Arrow** tool to double-click the point, type a new label in the dialog box that appears, and click **OK.** To change the fraction students are to locate, edit the caption at the top of the page.

ANSWERS

2. a. Use the **Fourths** tool once, or use the **Halves** tool twice.
   b. Use the **Thirds** tool once, or use the **Sixths** tool once.
   c. Use the **Sixths** tool once; or use both the **Halves** and **Thirds** tools, in either order.
   d. Use the **Halves** tool repeatedly to find eighths; or use both the **Halves** and **Fourths** tools, in either order.
   e. Use both the **Halves** and **Fifths** tools, in either order.
   f. Use both the **Halves** and **Fifths** tools, in either order; or, because $\frac{8}{10}$ is equal to $\frac{4}{5}$, use the **Fifths** tool only.
   g. Use either the **Halves** and **Thirds** tools, the **Thirds** and **Fourths** tools, or the **Halves** and **Sixths** tools.
   h. Many combinations of the **Halves**, **Thirds**, **Fourths**, and **Sixths** tools work.
   i. Use both the **Thirds** and **Fifths** tools, in either order; or, because $\frac{6}{15}$ is equal to $\frac{2}{5}$, use the **Fifths** tool only.

j. Use the **Thirds** tool twice.

k. Use the **Halves** tool repeatedly or use both the **Halves** and **Fourths** tools.

3. Answers will vary.

4. One way to construct $\frac{4}{3}$ is to use the **Halves** tool to locate 1 and then use the **Thirds** tool to subdivide the interval from 1 to 2.

5. One-seventh is between $\frac{1}{6}$ and $\frac{1}{8}$, both of which can be found using the given custom tools; find both fractions and say that $\frac{1}{7}$ is between them. Alternatively, $\frac{1}{7}$ is equal to $\frac{2}{14}$. A fraction close to $\frac{2}{14}$ is $\frac{2}{15}$, which can be constructed by using the **Thirds** and **Fifths** tools. For an even better approximation, consider that $\frac{1}{7}$ is equal to $\frac{3}{21}$. A fraction close to $\frac{3}{21}$ is $\frac{3}{20}$, which can be constructed by using the **Fourths** and **Fifths** tools.