Bugs in Groups: Dividing into Groups of Equal Size

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Open **Bugs in Groups.gsp,** and go to page "24 Bugs." Start with 5 as the value of the *Group Size* parameter.
   - Press *1* … *2* … *3* … and let students observe the 24 bugs moving around.
   - Press *Red Light!* to make the bugs move into groups. Provide time for students to observe that the bugs arrange themselves in equal-sized groups except for four leftover bugs.
   - Press *1* … *2* … *3* … to scatter the bugs.
   - Change the group size to 6 by selecting the *Group Size* parameter with the **Arrow** tool and pressing **+** on the keyboard one time.
   - Press *Red Light!* again. ***What did you see?*** Among other observations, students should note that the number of bugs in each full group is equal to the value of the *Group Size* parameter.

Students often like to try large values for the group size. The sketch accepts any value, including negative numbers, zero, and non-integer numbers. Because it doesn't make sense to have negative group sizes, or non-integer group sizes, encourage students to use natural numbers less than or equal to the number of bugs in all.

2. Ask, ***Are there other group sizes that leave no bugs out of full groups?*** Invite a volunteer to the computer to experiment with one or two other group sizes in response to suggestions from the class. Have the volunteer model pressing **+** or **–** on the keyboard one or more times to change the value of the *Group Size* parameter each time.

   Highlight the language that students use, and clarify vocabulary. The term *groups* can here refer to *equal-sized groups*, or *full groups*. The terms *remainder* and *left over* can be used to refer to any bugs that appear in a group that is not full. Experiment with enough values to elicit the observations that sometimes there are no bugs left over when equal groups are made and that the bugs always make as many equal-sized groups as they can.

3. Distribute the worksheet and read it through with the class. Let students know they can use more than one copy of the table if they need to. Make sure students understand that they should record every group size they try and the results. Go to page "Explore More" and point out that in worksheet step 5 students will use this page. Explain the differences in this model: Students can change the total number of bugs, and the buttons are different. Model pressing *Scatter* and then changing the total number of bugs by double-clicking *Bugs in All* and entering a new value in the dialog box that appears. Press *Group.*

DEVELOP

Expect students at computers to spend about 30 minutes.

4. Assign students to computers and tell them where to locate **Bugs in Groups.gsp.** Encourage students to ask a neighbor for help if they have questions about using Sketchpad.

5. Let students work at their own pace. Here are some things to be aware of as you observe and ask students to tell you about their strategies and thinking.

Notice that students generally do not employ repeated subtraction when solving division problems of this type.

- The problems students are solving are set in a division context and provide an opportunity for students to develop intuitive understandings about the relationship between multiplication and division. Students' own thinking, at this stage, is likely to involve multiplication strategies in which students "build up" to the number of bugs in all. In predicting or making sense of the number of groups, students may reason as in this example involving 24 bugs grouped by 6: *Six and six is 12, plus two more sixes is 12 more. That's 24.*
- Some students may try all possible group sizes, in order, starting from 1. Others may choose group sizes randomly. And others may be guided by their thinking about number composition and multiples.
- Students may intuitively construct the distributive property for multiplication over addition. *There are six groups of four. The three groups on my side are three times four—that's 12. The three groups on your side are also three times four. So, that's 12 plus 12—24 bugs.*
- Students may construct and apply the commutative property on their own. *We made 12 groups of 2 bugs. If we split the bugs in each group, we could make 2 groups of 12 bugs.*
- Students may notice and explain that they are using multiplication to figure out what will happen when they divide.
- Some students may overlook the possibility of a group size of one. Others may consider the possibility but debate whether it "counts" because "a group is more than one." Simply ask students to consider that a group size of one leaves no *remainder* bugs; each "group" is full—it has one bug.
- Students may also debate whether one group with all the bugs "counts" as grouping the bugs. This reasoning fits with students' experiences forming teams or dividing up objects into groups.

Again, simply ask students to entertain the criteria that no bugs are left out when the bugs form one group.

6. Students who have time should do the Explore More, which offers them the opportunity to investigate questions they pose. (Note that students often like to try large values for the *Bugs in All* parameter. The sketch accepts any value for the total number of bugs, but large numbers may make it hard to distinguish the bugs. Encourage students to use natural numbers of 100 or less.)

SUMMARIZE

Project the sketch. Expect to spend about 20 minutes.

7. Gather the class. Students should have their worksheets with them. Facilitate a discussion and have **Bugs in Groups.gsp** available for modeling. Here are ideas the discussion may bring out. You may wish to show a transparency of the table and record students' data for the class to refer to.

   - For 24 bugs, students may say that there are no bugs left over when the group size is 2, 3, 4, 6, 8, or 12. There are also no left over, or *remainder,* bugs when the 24 bugs are grouped by 1's or by 24. (As mentioned in the Develop section, students may question whether a group of one is really a group, and whether the bugs are grouped when they are all in one group. Mathematically, 1 and 24 are factors of 24.)
   - Students may invent and apply the commutative property of multiplication. *We saw that the numbers work in pairs. A size of one makes 24 groups, and a size of 24 makes one group; a size of two makes 12 groups, and a size of 12 makes two groups; there are eight groups of three, and three groups of eight; there are six groups of four, and four groups of six.*
   - Students may be intrigued by the results of trying to group 11 bugs. *There were a lot of group sizes that had no bugs left out for 24, but only two sizes that worked for 11—1 and 11. One group with all the bugs and 11 groups that each had one bug. We wondered whether we could find other numbers like 11, so that's what we did when we used the "Explore More" page.*
   - Students may have noticed that they were using multiplication to figure out and explain the results of dividing the bugs into groups.

Highlight this idea, inviting several students to express their thinking in their own words.

8. If students had time to pose their own questions and explore them, have them share their questions and any findings, as time permits.

EXTEND

1. ***What other questions can you ask about dividing objects into groups of equal size?*** Encourage student curiosity and provide time for students to explore their questions using the sketch. Here are some sample student queries.

   Is there a way to know whether there will be bugs left over before you group them?

   Why can the number of groups and the size of the groups be switched?

   For certain numbers of bugs like 11 or 13, we can only make equal groups with no bugs left out if we put the bugs all in one group, or if we make groups of one. Why is that?

   Do odd numbers of bugs work differently than even numbers of bugs?

   Can you know whether there will be an odd number of bugs or an even number left over?

   If I list all the group sizes that leave no bugs out for 16 bugs, is there an easy way for me to make a list for 32 bugs?

   What would happen if we made the group size number in the sketch larger than the total number of bugs?

2. Have students go to page "26 Bugs." Pose this question: ***Now there are 26 bugs. Can you find a value for the group size that leaves exactly two remainder bugs? Can you find other values that leave exactly two remainder bugs?*** Encourage students to see whether there is a pattern to the possible values. [Group sizes of 4, 6, 8, 12, and 24 all leave two remainders. These are numbers that are factors of $26 - 2 = 24$ (but aren't also factors of 26).] ***Pose another problem: Can you find values that leave exactly three remainder bugs?*** Group sizes that have three remainders will be numbers that are factors of $26 - 3 = 23$; since 23 is a prime number, only a group size of 23 will have three remainders.)

3. Have students who would benefit from additional work dividing bugs into groups of equal size use the sketch, including the additional pages that display 23, 26, and 30 bugs.

ANSWERS

2. 1, 2, 3, 4, 6, 8, 12, and 24
4. 1 and 11
5. Answers will vary.