Making a Kaleidoscope: Exploring Rotations

For GSP5 ACTIVITY NOTES

INTRODUCE

Be prepared to project the sketch for viewing by the class. Expect to spend about 15 minutes.

1. To start the activity, you may wish to pass out kaleidoscopes to the class and ask students to describe what they see. ***How many copies of each particular shape do you see? How are these copies the same? How are they different?*** Listen as students state their observations; don't clarify any thinking at this time.
2. Explain, ***Today you are going to make a kaleidoscope with Sketchpad by rotating and animating a quadrilateral. I'll do a brief demonstration, and then you'll make one on your own.***
3. Open **Making a Kaleidoscope.gsp.** Go to page "Kaleidoscope." Model worksheet steps 1–8. Here are some points to emphasize as you demonstrate.

Concentric is Latin in origin and comes from the words *con,* meaning "together," and *centrum,* meaning "center."

- In worksheet step 2, explain that circles that share a common center are called *concentric circles.* ***All three circles have point C as their center.*** Then model constructing a point on each circle, noting that each circle should be highlighted before placing the point.
- In worksheet step 4, emphasize the importance of selecting the points in the proper order to form a quadrilateral. ***What is a quadrilateral?*** [A polygon with four sides] ***How is a quadrilateral named?*** [In order of its vertices]

You can also double-click point *C* with the **Arrow** tool to mark it as the center for rotations.

- In worksheet step 5, model how to select point *C* and choose **Transform | Mark Center.** Explain that this means point *C* will be the center of rotation. ***Quadrilateral CDEF will turn, or rotate, about this point.*** A brief animation indicates that the point has been marked.
- ***We will tell Sketchpad to rotate this quadrilateral by 90 degrees. Where do you predict the new rotated quadrilateral will appear?*** Take students' predictions, and then select the quadrilateral interior and choose **Transform | Rotate.** In the Rotate dialog box, show students where to enter 90 for degrees. Point out how Sketchpad shows a preview of the rotated image. Click **Rotate.**
- Rotate the quadrilateral interior two more times, each time having students predict where the new image will appear. When you're done, change the color of the quadrilaterals so that each of the four is a different color.

- Emphasize that if students make any mistakes as they construct the kaleidoscope, they can back up one or more steps by choosing **Edit | Undo.**

4. If you want students to save their Sketchpad work, demonstrate choosing **File | Save As,** and let them know how to name and where to save their files.

DEVELOP

Expect students at computers to spend about 30 minutes.

5. Assign students to computers and tell them where to locate **Making a Kaleidoscope.gsp.** Tell students to work through steps 1–11 and do the Explore More if they have time. Encourage students to ask their neighbors for help if they are having difficulty with the construction.

6. Let pairs work at their own pace. As you circulate, here are some things to notice.

   - In worksheet step 2, remind students to wait until each circle is highlighted before clicking to construct a point on it.
   - If students make an error during the construction, remind them to use **Edit | Undo** to back up one or more steps. They can also use **Edit | Redo** to retrace their steps.
   - As students are rotating the quadrilaterals in worksheet steps 5–7, ask them to think about the rotations. ***In which direction does the quadrilateral rotate?*** [Counterclockwise] ***How far has the original quadrilateral rotated if it rotated 90 degrees and then 90 degrees again?*** [180°] ***And if it rotates another 90 degrees?*** [270°] ***What point is common to all four quadrilaterals?*** [Point *C,* the center of rotation]
   - Listen as students compare the four quadrilaterals in worksheet step 9. If students comment only that the quadrilaterals are different colors, ask questions to trigger further observations. ***What changed when the quadrilateral was rotated? What stayed the same?***
   - In worksheet step 10, students may be surprised that the shapes change as they animate. ***Why do you think the quadrilaterals change shape?*** Help students see that the three points animate at the same speed, so a point on a smaller circle makes a full rotation faster than one on a larger circle.

- If students have time for the Explore More, they will construct another kaleidoscope by rotating a quadrilateral 60° each time. Have students predict how many quadrilaterals they will get. Some students may have time to design their own kaleidoscopes on page "Make Your Own." You may want them to print these products to share with the class.

7. If students will save their work, remind them where to save it now.

SUMMARIZE

Project the sketch. Expect to spend about 15 minutes.

8. Gather the class. Students should have their worksheets with them. Ask, ***When you rotate a figure about a center point, what stays the same and what changes?*** Students may observe that the figure is the same size and shape but is oriented in a new way. Ask students to look at what happened to each vertex of *CDEF* when it was rotated three times. Each point and its rotated images all sit on the same circle. This means that all four points are the same distance from point *C*.

9. Go to page "Kaleidoscope" and construct quadrilateral *CDEF*. ***You started your kaleidoscope by rotating CDEF by 90 degrees.*** Do the rotation. ***You then rotated two more times by 90 degrees.*** Do the rotations. ***How many degrees of rotation is that in all?*** [270°] ***What do you think would happen if you rotated the fourth quadrilateral by 90 degrees?*** Students may make the following responses.

   It looks like all four quadrilaterals are evenly spaced. So I think that if you rotated again, the quadrilateral would land right on CDEF.

   Every quadrilateral on the kaleidoscope looks like it's been rotated 90 degrees from the one next to it. So, if you rotate the fourth quadrilateral by 90 degrees, you'd get to the quadrilateral you started with, CDEF.

   We rotated 270 degrees so far. There are 360 degrees in a circle. So, if we rotate another 90 degrees, that's $270 + 90 = 360$ *degrees. That means we'll be right back at CDEF.*

10. If time permits, discuss the Explore More. Have students share the kaleidoscopes they made following worksheet steps 12–14 and explain why they got six quadrilaterals when rotating by 60°. Discuss students' responses to step 15. Have students share any kaleidoscopes they made in step 16.

EXTEND

1. ***What questions occurred to you about kaleidoscopes or rotations?*** Encourage curiosity. Here are some sample student queries.

   How many degrees should we rotate if we want eight shapes in our kaleidoscope?

   What does a kaleidoscope look like with a rotation of 10 degrees or even 1 degree?

   Can you rotate different shapes, like a triangle?

   What happens if you rotate the quadrilateral by 11 degrees, which is not a factor of 360?

   Can you rotate a shape by more than 360 degrees?

   Is it possible to rotate the quadrilateral clockwise?

   Can you fix the speed so all the points move at the same speed?

2. Students can construct a kaleidoscope by starting with a blank sketch and constructing their own circles by following these steps.

   - Use the **Compass** tool to construct three circles with the same center. For the second and third circles, be sure the center point is highlighted before clicking to construct a new circle.
   - Label the center point *A*, using the **Text** tool.
   - Hide all the points except point *A:* Select the points and choose **Display | Hide Points.**

   Students may wonder why they should hide the three points in the last step. These are *radius points.* If a radius point is dragged, it changes the size of the circle. By contrast, a point constructed on an existing circle simply moves around the circumference of the circle without changing its size. You may wish to show students what happens to the kaleidoscope by constructing one with radius points. The animation is very different.

3. ***Suppose our kaleidoscope started with four concentric circles. What type of shape would we construct and rotate?*** [The shape will have five sides; it will be a pentagon.] Students may want to try making a kaleidoscope using four or more concentric circles.

ANSWERS

9. The quadrilaterals are all the same size and shape but are oriented differently.

13. Students should get six shapes. There are 360° in a circle. Dividing 360° by the number of degrees in each rotation gives the number of shapes: $360° \div 60° = 6$.

15. Answers will vary. As one example, if students repeatedly rotate a shape by 30°, a total of 12 copies of the shape will fit in the kaleidoscope.

16. Answers will vary. Check students' work.