This is a challenging and rewarding activity for students who have some experience with iteration. The idea comes from Michael Barnsley (*Fractals Everywhere*, Morgan Kaufmann, 2000).

Q1 Each of the four functions is associated with a fixed point.

r values	Fixed point
0.00–0.01	(0, 0)
0.01–0.08	(−0.61, 1.87)
0.08–0.15	(0.15, 0.63)
0.15–1.00	(2.66, 9.96)

Q2 The fourth function transforms a point on one leaf to the corresponding point on the next higher leaf.

Q3 $t_1 = 0.00$ $t_2 = 0.01$ $t_3 = 0.08$ $t_4 = 0.15$ $t_5 = 1.00$

Q4 The first function transforms a point anywhere on the fern into a point on the lowest portion of the stem. The second function transforms a point anywhere on the fern to the corresponding point on the lowest-left leaf. The third function transforms a point anywhere on the fern to the corresponding point on the lowest-right leaf.

Q5 The first fixed point is the base of the stem, and the fourth is the very tip of the fern. The second is the point on the bottom-left leaf that's in the same relation to both that leaf and the entire fern, and the third is the point on the bottom-right leaf that's in the same relation to that leaf and the entire fern.

Q6 If the first transformation is never used, the fern appears without a stem and each leaf appears without its stem. The first transformation takes the point to the lowest part of the stem, where the 85% transformation can move it up the stem. Thus the entire stem is produced.

Q7 The second transformation takes a point to the corresponding part of the lowest-left leaf. Subsequent 85% transformations generate the other left-hand leaves. Similarly, the third transformation is responsible for generating the right-hand leaves. Finally, the 85% transformation generates higher leaves from lower leaves, moving toward the tip of the fern.

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Q8 The 0.85 in function 4 makes each succeeding leaf 85% of the size of the previous leaf. The 0.44 in function 3 and the 1.6 in function 2 determine the height at which the leaves begin on the two sides. The numbers clustering around 0.20 in functions 2 and 3 make the side leaves about 1/5 the size of the main leaf.

Q9 To make the leaves opposite instead of alternating, use the same value for f in the second and third functions.

Q10 Answers, and resulting shapes, will vary. You may want to ask students to print their most interesting shapes to share with the class.

Q11 The three functions use these coefficients.

$a = 0.50$	$c = 0.00$	$e = 3.00$
$b = 0.00$	$d = 0.50$	$f = 0.00$
$a = 0.50$	$c = 0.00$	$e = -3.00$
$b = 0.00$	$d = 0.50$	$f = 0.00$
$a = 0.50$	$c = 0.00$	$e = 0.00$
$b = 0.00$	$d = 0.50$	$f = 5.00$

Q12 These functions produce the Sierpiński triangle.

Q13 The four functions use these coefficients.

$a = 0.44$	$c = 0.00$	$e = 0.00$
$b = 0.00$	$d = 0.44$	$f = 0.00$
$a = 0.44$	$c = 0.00$	$e = 0.56$
$b = 0.00$	$d = 0.44$	$f = 0.00$
$a = 0.44$	$c = 0.00$	$e = 0.00$
$b = 0.00$	$d = 0.44$	$f = 0.56$
$a = 0.44$	$c = 0.00$	$e = 0.56$
$b = 0.00$	$d = 0.44$	$f = 0.56$

The resulting figure is a *Sierpiński square*, sometimes called a *Sierpiński carpet*.