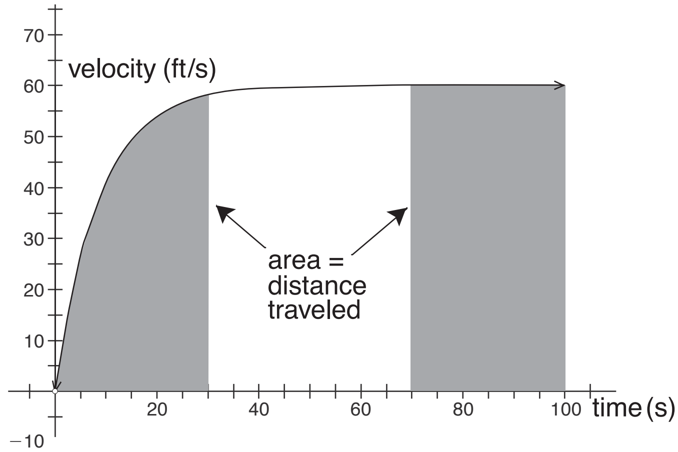
One Type of Integral

For GSP5

Consider a car starting up from a red light. Its velocity increases for a while and then levels off at 60 ft/s. From a graph of the velocity, how could you determine the distance the car has traveled in a given period of time?

The distance traveled from $t = 70$ s to $t = 100$ s is equal to the area of the shaded rectangle on the right, because *rate* $\times$ *time* is equal to *height* $\times$ *width.*

Recall that *distance* = *rate* $\times$ *time*. The velocity is constant in the interval on the far right, from $t = 70$ s to $t = 100$ s, so you can multiply rate by time (60 ft/s $\times$ 30 s) to find that the car travels 1800 ft during this period of time.



The definite integral of a function corresponds to the area under the graph of the function.

Finding how far the car travels during the time from $t = 0$ s to $t = 30$ s is harder, because the velocity is changing. The process of finding this distance, when the velocity is changing, is called finding the *definite integral* of the velocity.

THE DISTANCE A CAR TRAVELS

1. Open **Definite Integral.gsp.** Press the *Drive Car* button to start the car. Observe how the car's velocity behaves, starting from zero and ending at 60 ft/s.
2. To find the total distance the car travels during any period of time, you'll have to estimate the area under the curve. Press the *Show Grid* button to display a grid you can use to estimate the area.

Q1 How wide is each square of the grid? What are the units?

Q2 How high is each square of the grid? What are the units?

Q3 What is the area of each square of the grid? What are the units?

Q4 How many squares are in the shaded region on the right? How can you use this result to confirm the distance traveled from $t = 70$ s to $t = 100$ s?

Now you'll count squares to find the area for the shaded region on the left.

To keep track, you can use the **Point** tool to put a point inside each square you count.

3. Count the number of complete squares totally contained within this region.
4. Now estimate the area of the squares that are partially within the shaded region. For each such square, estimate whether it is more than half shaded or less than half shaded. Count only the squares that are more than half shaded. Add this number to the number of complete squares you counted in step 3.

Remember to multiply the number of squares by the value represented by each square.

Count the complete squares first, and then count all the partial squares that are more than half shaded.

Q5 What is your estimate of the number of squares in the shaded region on the left?

Q6 What is the distance the car traveled from $t = 0$ s to $t = 30$ s?

5. Select the *Square Size* parameter, and then press the **–** key on the keyboard to change the size of the squares to 2.0.

6. Count the squares in the shaded region on the left.

Q7 How many squares do you get this time? What is the area of each square?

Q8 Based on this count, what is the distance the car traveled from $t = 0$ s to $t = 30$ s?

Q9 Do you think this estimate is more or less accurate than the previous one? Why? How could you make it still more accurate?

Q10 Estimate the area from $t = 30$ s to $t = 70$ s. Add your results for the three areas to find the total distance the car traveled in 100 s.

DEFINITE INTEGRALS FOR OTHER FUNCTIONS

7. On page 2 of the sketch, you'll find another function, $y = 8 \cdot 0.7^x$. Estimate the definite integral for this function, using the domain from $x = 1.00$ to $x = 7.00$.

Q11 What is your estimate of the definite integral for this function?

Q12 Double-click the **Arrow** tool on the *Square Size* parameter, and change the value of the parameter to 0.5. What is your new estimate of the definite integral for this function?

Q13 Change the value of the parameter to 0.1. If you had to count such small squares, what kind of function would you prefer to have? Why?

EXPLORE MORE

Be careful to determine the area of each rectangle correctly.

On page 3 of the sketch, you can set both the height and width of the rectangles. Use several different-size rectangles to estimate the definite integral for this function.

Using the grid on page 4 of this sketch, plot some other functions of your choice and estimate the definite integrals on the specified intervals. Here are some possible choices:

What should you do about cells below the *x*-axis?

$f(x) = \sin(x)$ from $x = 0$ to $x = \pi$

$f(x) = \sin(x)$ from $x = \pi$ to $x = 3\pi/2$

$g(x) = x^2 - 2x - 1$ from $x = 1$ to $x = 2$

$g(x) = x^2 - 2x - 1$ from $x = 2$ to $x = 3$