Taylor Series

Description: Students approximate the sine function using a Taylor series. They construct one point corresponding to the first partial sum and iterate to find points corresponding to the subsequent terms. Their final result is a graph to any desired depth.

Technology Strength: By creating parameters and using iteration, students can create a Taylor series for the sine function and explore how adding terms to a Taylor series approximation increases the accuracy of the approximation.

Objectives: Create a Taylor series; explore how adding terms to a Taylor series approximation increases the accuracy of the approximation

Prerequisites: Familiarity with the sine and cosine functions

Suggested Grade Level: 11 to 12

Sketchpad Level: Challenging

Suggested Duration: 30 minutes

Suggested Classroom Setting: Whole Class, Student Pairs. This activity, designed for use by student pairs, can be easily modified for whole-class use.

Preparation: Review the Activity Notes. Preview the student sketch. Work through the steps on the worksheet and make a copy of the worksheet for each student. The presentation sketch can be used as a demonstration.

Materials: None

Student Worksheet(s): Taylor Series

Student Sketch: None

Presentation Sketch: Taylor Series Present.gsp

Vocabulary: Series, Taylor series, partial sums, locus

Sketchpad Version: GSP5

Using the Sketch:

In a new sketch, students create a coordinate system and construct a point on the *x*-axis. They create four parameters to represent the initial values of the parameters in the Taylor series formula, and use these parameters and the *x*-value of the point to calculate the next values from the initial values.

Students plot a point representing the next value of the sum of the series and iterate each of the parameters to its next value. By finding the terminal point of the point image, they can construct a locus that shows the graph of the Taylor series approximation at the current depth of iteration.

Students describe the shape of the resulting graph, describe how the result changes as they change the depth of iteration, and determine the number of terms needed to give a reasonable approximation of the first period of the sine graph.

In the Explore More section, students evaluate another Taylor series expansion, this time for the cosine function.
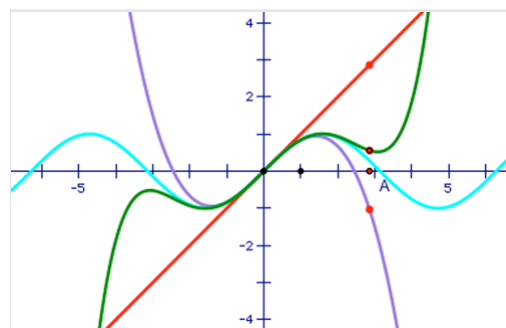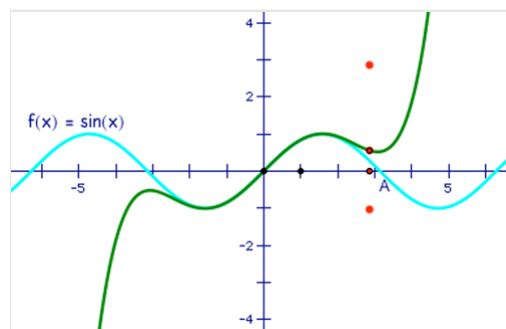
Sketch Tips:

Sketch Tips show skills needed in this activity, and the step at which the skill is first used.

Sketch Tip	Tip Sheet or Tip Video
Step 2: Create a new number (parameter) using **Number	New Parameter**	Creating a Parameter
Step 3: Change the value of a number (parameter)	Changing Parameters
Step 4: Calculate an expression using **Number	Calculate**	Using the Calculator
Step 4: Click a value in the sketch to enter it into the Calculator	Using the Calculator
Step 5: Plot a point by selecting two values and using **Graph	Plot as (x,y)**	Plotting Points
Step 6: Iterate a construction using **Transform	Iterate**	Using Iteration
Step 10: Construct a locus using **Construct	Locus**	Constructing a Locus