THE DISTANCE A CAR TRAVELS

This activity uses the term *velocity* rather than *speed,* because direction is important when accumulating the values. If the driver puts the car in reverse, the velocity is negative, and the distance traveled is reduced—but the speed (which is the absolute value of the velocity) is positive, even when the car is traveling backward.

1. Some students may expect that the velocity of the point on the screen corresponds to the velocity of the car. It's important for students to realize that the car's velocity corresponds to the vertical position on the graph, and *not* to the velocity of the point on the graph. It may be useful to have the class observe the animation and interpret the graph as a group.

Q1 Each grid square is 5 s wide.

Q2 Each grid square is 5 ft/s high.

Q3 Each grid square represents 25 ft of travel.

Q4 The region on the right has $6 \cdot 12 = 72$ squares. Because each square represents 25 ft of travel, the car travels $72 \cdot 25 = 1800$ ft during this period of time.

4. This step asks students to count only the squares that are more than half shaded. Discuss with students why this method might work, and why it's easier than estimating a value for every partially filled square. Can students imagine a function for which this results in a bad approximation?

Q5 Answers will vary. There are approximately 44 whole squares and 7 squares that are more than half shaded, for a total of 51 squares in the left region.

Q6 Answers will vary. The car traveled approximately 1275 ft.

6. Because the squares are quite small and there are many of them, counting them one at a time is tedious. Encourage students to find ways to make the process more efficient (perhaps counting by 10's or 20's).

Q7 Answers will vary. There are approximately 320 or 330 squares in the left region. Each square now represents 4 ft of travel.

Q8 Answers will vary. A count of 325 squares corresponds to 1300 ft of travel.

Q9 This estimate is more accurate. The smaller squares allow a more accurate measurement, because they fit the curve better. You could obtain a more accurate result by making the squares still smaller.

Q10 There are about 597 squares between $t = 30$ s and $t = 70$ s, representing a distance of 2388 ft. The total distance traveled is about 1300 ft + 2388 ft + 1800 ft = 5488 ft.

DEFINITE INTEGRALS FOR OTHER FUNCTIONS

Q11 Based on 14 squares, the definite integral is about 14.

Q12 Based on 55 squares, the definite integral is about 13.75.

Q13 If the function were constant, we could just find the area of the rectangle instead of counting. If the function were a linear function, we could find the area of a trapezoid. Either method would be much easier than trying to count every single square.

EXPLORE MORE

When students count the squares on page 3 of the sketch, answers will vary but should not be too far from the actual value by symbolic integration, which is 22.7 (to three significant digits).

Student answers for the suggested functions and domains will vary. Here are precise values:

$$\int_0^n \sin x \, dx = 2.0 \qquad \int_0^{\frac{3\pi}{2}} \sin x \, dx = 2.0$$

$$\int_1^2 x^2 - 2x - 1 \, dx = -1\frac{2}{3} \qquad \int_2^3 x^2 - 2x - 1 \, dx = \frac{1}{3}$$

Make sure that students have figured out that when a square is below the x-axis, they must subtract it instead of adding it.