SKETCH AND INVESTIGATE

Q1 To calculate the next value of i, calculate $i + 2$.

Q2 For the next value of *num*, calculate $-num \cdot x^2$.

Q3 To calculate the next value of *den*, calculate $den \cdot (i + 1)(i + 2)$.

Q4 The initial value of the sum, before adding the first term, should be zero. To calculate the next value of *sum*, calculate $sum + num/den$.

Q5 The initial values should be $i = 1$, $den = 1$, and $sum = 0$. Because *num* depends on x, set it to 2.

Q6 The locus is a straight line—the graph of $f(x) = x$. Only the first term of the series contributes to this graph.

Q7 Now the shape is that of a cubic function, generated by the first two terms: $f(x) = x - x^3/6$.

Q8 The terms are polynomials of order 1, 3, 5, 7, and so forth. The high-order terms have large denominators, so the low-order terms predominate for small values of x; the high-order terms introduce corrections for larger values of x.

Q9 About 8 terms gives a reasonable approximation for a single period; about 16 terms are required for two periods.

The Taylor series expansion of the sine function doesn't converge rapidly enough to be of use in calculators and in computer algorithms. Other, faster-converging series are used for these purposes.

If you did want to use the Taylor series for these calculations, you'd get the best results by first transforming the argument (the x-value) so that it is between 0 and $\pi/2$ and then performing the calculation. Ask students how they would do this—they should figure out how to do the transformation, and they should describe how to keep track of whether the resulting value should be positive or negative.

A related follow-up exercise would be to ask students to use Sketchpad's calculator to determine the number of digits of accuracy they obtain for $\sin(\pi/2)$ for various depths of iteration. As the following Sketchpad-generated table shows, eight iterations gives a result accurate to ten decimal places.

terms	value	error · 10^6
1	1.57080	570796.32679
2	0.92483	−75167.77071
3	1.00452	4524.85553
4	0.99984	−156.89860
5	1.00000	3.54258
6	1.00000	−0.05626
7	1.00000	0.00066
8	1.00000	−0.00001
9	1.00000	0.00000

PRESENT

A nice touch, when presenting this activity, would be to create a fifth parameter, *depth,* and to do the iteration to a depth determined by this parameter. Then you can animate the resulting iteration with a button that changes the *depth* parameter.

EXPLORE MORE

To generate the Taylor series approximating the cosine function, you need only change the initial value of *i* to 0 and the initial value of *num* to 1.

DEMONSTRATE

You can use the sketch **Taylor Series Present.gsp** in a classroom demonstration, using the buttons to move through the demonstration. This sketch also shows the finished constructions.