A Geometric Approach to *e^iπ*

Description: Students explore *e^iπ*. They first explore the limit of (1+*x*/*n*)^*n* for large values of *n* to find it approaches *e^x*. They then replace *x* with *iπ* and use a geometric approach to multiplication on the complex plane to find the limit and evaluate *e^iπ*.

Technology Strength: By using iteration to construct the sequence for *e^x* in the complex plane and by adjusting a slider to simulate a limit, students can use a geometric approach to find the value of *e^iπ*.

Objectives: Use a limit definition of *e* to approximate integral powers of *e*; use a limit definition of *e*, along with multiplication on the complex plane, to find the value of *e^iπ*.

Prerequisites: Completion of the activity *Multiplication of Complex Numbers*; some exposure to the concept of limits

Suggested Grade Level: 11 to 12

Sketchpad Level: Beginning

Suggested Duration: 45 minutes

Suggested Classroom Setting: Whole Class, Student Pairs. This activity, designed for use by student pairs, can be easily modified for whole-class use.

Preparation: Review the Activity Notes. Preview the student sketch. Work through the steps on the worksheet and make a copy of the worksheet for each student.

Materials: None

Student Worksheet(s): A Geometric Approach to *e^iπ*

Student Sketch: eipi.gsp

Presentation Sketch: None

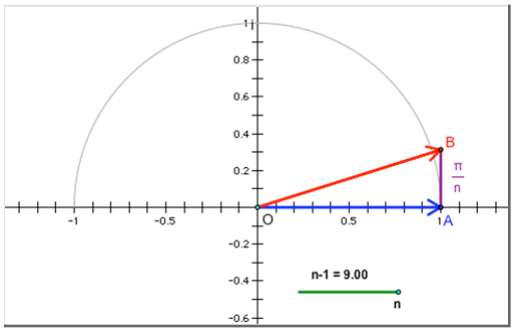
Vocabulary: Complex numbers, complex plane, argument

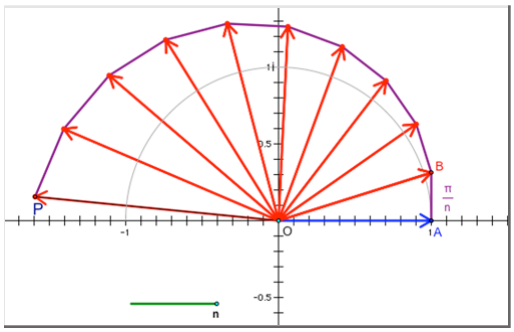
Sketchpad Version: GSP5

Using the Sketch:

Students first use a new sketch to calculate the terms in the sequence (1+1/*n*)^*n* using increasing powers of 10 for the value of *n* and discover that their calculations approach *e* (approximately 2.71). They edit their calculations and use the limit definition of *e* to approximate e^3.

Students then open a prepared sketch. On page 1, students are given two points plotted on the complex plane. Point *A* is at (1, 0) and represents the value 1 and Point *B* is at (1, *π*/10) and represents the value 1 + *iπ*/10. They use these two points and a slider to construct an iterated image and then adjust the slider so they can approximate the value of *e*^*iπ*. On page 2 of the sketch, they use the same process to investigate *e*^*Iθ*, where *θ* is any number.





Sketch Tips:

Sketch Tips show skills needed in this activity, and the step at which the skill is first used.

Sketch Tip	**Tip Sheet or Tip Video**
Step 2: Calculate an expression using **Number | Calculate**	Using the Calculator
Question 3: Edit a function by double-clicking with the **Arrow** tool	Creating and Editing Functions
Step 6: Iterate a construction using **Transform | Iterate**	Using Iteration
Step 11: Change the value of a number (parameter)	Changing Parameters