SKETCH AND INVESTIGATE

In **Q1,** the secant line that you construct is an approximation of the derivative at $x = x_P$. (Note that this is an arbitrary choice. The line could also be used to approximate the derivative at $x = x_Q$.) Finding the value of the derivative at $x = x_P$ requires finding the limit of the slope of the line *PQ* as point *Q* approaches point *P*. By moving point *Q* as close as possible to point *P*, you can minimize the error in the approximation of the derivative.

By placing the points close together and moving them along the function plot, the locations of point *S* trace the approximate value of the derivative at $x = x_P$. Note that the difference between the *x*-coordinates of points *P* and *Q* may change as you move them, so the precision of the approximation may change slightly as you move the points.

In **Q2,** the trace created is a parabola.

The trace will differ according to whether $x_Q > x_P$ or $x_Q < x_P$. This difference, as suggested in **Q3** through **Q5,** is a result of the slope of the function not being symmetric around point *P*. The discrepancy is greater if the points are further apart, and less if they are closer. If $x_Q > x_P$, you are calculating the limit in the derivative from the right side, sometimes called the *right-hand derivative.* If $x_Q < x_P$, you are calculating the limit from the left.

In **Q6,** students can adjust *h* to make the locus match the trace they made with the *Animate* button. The traces will match when the value of *h* is equal to the difference between the *x*-coordinates of points *P* and *Q* when the *Animate* button was used.

Symbolically, $h = x_Q - x_P$. The value of *h* can be positive or negative, so you can match either the trace created when $x_Q > x_P$, or when $x_Q < x_P$.

EXPLORE MORE

Students are asked to predict what the plot of the derivative will look like by examining the slope of the tangent line. For **Q7,** point *S* will have a negative *y*-coordinate if the slope of the line is negative, a positive *y*-coordinate if the slope of the line is positive, and a *y*-coordinate of 0 if the line is horizontal.

In **Q8,** focus first on the actual slope of the line and then the changes in the slope of the line. If students have difficulty with this question (they may make the error of thinking that the slope of the line is decreasing when it is actually the value of the function that is decreasing), you can ask them to think about how to determine when point *S* will be moving up and when it will be moving down. For instance, when the slope of the line decreases, the point will move down. When the slope of the line increases, the point will move up. In **Q9,** the locus is a cubic.

EXTENSION

This document provides an opportunity for students to move point *P* themselves and draw conclusions about the motion of point *S* (and the increasing/decreasing behavior of the derivative) before trying to sketch a derivative graph on their own. Also consider having students record their observations and conclusions in writing, to help them make precise verbal descriptions of the relationships between the plot of a function and the plot of its derivative.

As another activity, have students create a independent point in the plane, and turn on tracing for this point. With point *S* hidden, run an animation of point *P*, have students move this independent point to try to trace the plot of the derivative. Then, without erasing the traces, run the animation again with point *S* showing and its trace on, and have students compare their trace with the trace created.

To explore a symmetric difference quotient follow these steps:

1. Calculate $x_P - h$ and $f(x_P - h)$.
2. Plot the point $\left(x_P - h, f\left(x_P - h\right)\right)$. Label this *R*.
3. Construct the line *QR* and measure its slope (instead of *PQ*).
4. Select x_P and the slope measurement, *slopeRQ,* in that order.
   Choose **Plot as (x, y)** from the Graph menu.