Barnsley's Fractal Fern

Description: Students create several functions that transform a point and then iterate the transformation, choosing randomly among the functions, to create a fractal fern and other fractal shapes.

Technology Strength: By dragging a seed point, students can see the effect of a pair of mapping functions. By iterating the mapping, students observe the fixed point of several mappings. By choosing randomly among mappings, students generate a fractal fern.

Objectives: Use a pair of functions to transform a point, use several such function pairs to create an iterated function system, and vary the functions and iteration to produce a variety of interesting shapes

Prerequisites: Preferably, experience using iteration to create a fractal such as the Koch curve or the Sierpinski triangle

Suggested Grade Level: 11 to 12

Sketchpad Level: Challenging

Suggested Duration: 60 minutes. Students can complete the construction itself in a class period, but will benefit both from exploring and answering questions about the finished construction and from creating several related constructions described in the Explore More section.

Suggested Classroom Setting: Whole Class, Student Pairs. This activity, designed for use by student pairs, can be easily modified for whole-class use.

Preparation: Review the Activity Notes. Preview the student sketch. Work through the steps on the worksheet and make a copy of the worksheet for each student.

Materials: None

Student Worksheet(s): Barnsley's Fractal Fern

Student Sketch: Fractal Fern.gsp

Presentation Sketch: Fractal Fern Present.gsp

Vocabulary: Affine transformation, seed, fixed point, iterated function system
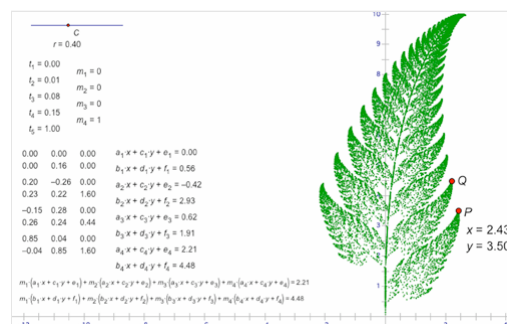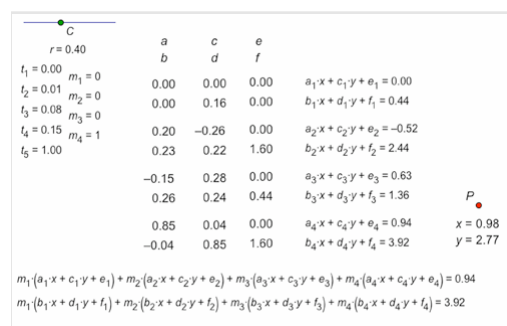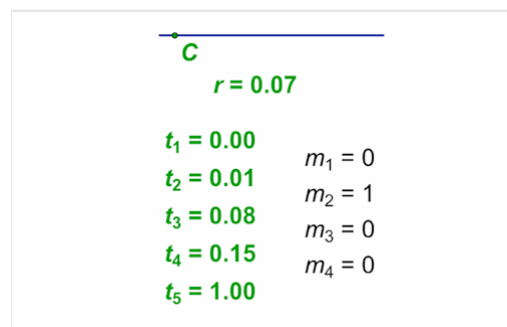
Sketchpad Version: GSP5

Using the Sketch:

Students construct a point on a segment and measure its value. To use this value (between 0 and 1) to choose among four transformations, students create parameters to divide the interval [0, 1] into four parts and then use the Between custom tool to determine which of the four parts contains the point. The result is four calculations corresponding to the four parts of the interval; the interval calculation corresponding to the position of the point has a value of 1 and the other three interval calculations have values of 0.

Students use the Affine Transformation custom tool on parameters already in the sketch to create four transformations that take a point in the coordinate plane and produce a transformed image. They then use the interval calculations to create a combined calculation that chooses one of the four transformations based on the position of the point.

Students apply this combined transformation to a seed point and iterate the transformation, choosing randomly among the four transformations based on the position of the point on the segment. The result is the Barnsley fractal fern.

In the Explore More section, students modify this iterated function system to produce a variety of shapes including the Sierpinski triangle and the Sierpinski carpet.

Sketch Tips:
Sketch Tips show skills needed in this activity, and the step at which the skill is first used.

Sketch Tip	Tip Sheet or Tip Video
Step 3: Create a new number (parameter) using **Number | New Parameter**	Creating a Parameter
Step 4: Choose a custom tool from the Custom Tools menu	Using Custom Tools
Step 6: Measure the coordinates of a point using **Measure | Abscissa, Measure | Ordinate**, or **Measure | Coordinates**	Measuring Coordinates
Step 8: Calculate an expression using **Number | Calculate**	Using the Calculator
Step 9: Plot a point by selecting two values and using **Graph | Plot as (x,y)**	Plotting Points
Step 11: Iterate a construction using **Transform | Iterate**	Using Iteration