GETTING STARTED

Q1 The four calculations seem to approach a value roughly equal to 2.71.

Q2 A good approximation is 2.71828.

Q3 A good approximation of e^3 is 20.0855.

SKETCH AND INVESTIGATE

Q4 To multiply $1 + i\pi/10$ by itself, square its length (OB) and double its argument (AOB). To compute the tenth power of $1 + i\pi/10$, raise its length to the tenth power and multiply its argument by 10.

Q5 The successive powers of $1 + i\pi/10$ are represented in counterclockwise order by the vectors that originate at point O.

Q6 The vector OP represents $(1 + i\pi/10)^{10}$. The coordinates of point P are $(-1.59, 0.16)$. Thus, $(1 + i\pi/10)^{10}$ equals $-1.59 + 0.16i$.

Q7 The value of $(1 + i\pi/n)^n$ approaches -1 as n grows larger.

EXPLORE MORE

Q8 The value of $e^{i\pi/2}$ is i.

Q9 The value of $e^{i\pi/3}$ is approximately $0.5 + 0.87i$ $\left(\text{or } \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and the value of $e^{i\pi/4}$ is approximately $0.71 + 0.71i$ $\left(\text{or } \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$.

Q10 As the approximations get better and better, the length of the vector approaches 1.

Q11 When $k = 2$, the angle is $\frac{\pi}{2}$. When $k = 3$, the angle is $\frac{\pi}{3}$. When $k = 4$, the angle is $\frac{\pi}{4}$.

Q12 The vector representing $e^{i\theta}$ has a length of 1 and makes an angle of θ with the x-axis. Thus, its real component is $\cos\theta$ and its complex component is $\sin\theta$.

Q13 When $\theta = \pi$, $e^{i\pi} = \cos\pi + i\sin\pi = -1$.