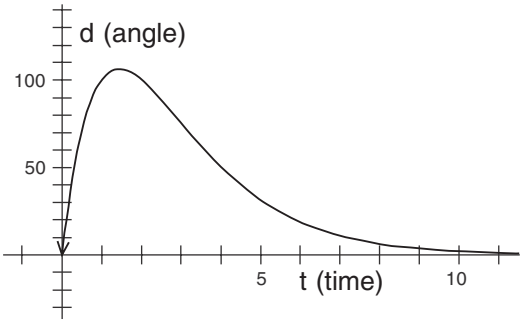
Instantaneous Rate

For GSP5

Many schools have doors equipped with automatic closers. When you push such a door, it opens quickly, and then the closer closes it again, more and more slowly until it finally closes completely.

DOOR ANGLE AS A FUNCTION OF TIME



1. Open **Instantaneous Rate.gsp.** Press the *Open Door* button to operate the door. The door opens and closes, and the graph shows the angle of the door (d) in degrees as a function of time (t) in seconds.
2. Drag point t_1 back and forth along the time axis, and watch how the angle of the door changes and how the point on the graph corresponds to the door's angle. Observe the values of the t_1 and d_1 measurements as you drag.

Q1 For what values of t_1 is the angle increasing? How can you tell?

Q2 What is the maximum angle the door reaches? At what time does this occur?

THE DOOR AT TWO DIFFERENT TIMES

The value Δt is the separation between the two values of time (t_1 and t_2).

3. To find the rate of change of the door's angle, you need to look at the door's position at two different times. Press the *Show t2* button to see a second point on the time axis slightly separated from point t_1. Drag point t_1 back and forth, and observe the behavior of the new points on the graph. To change the separation of the two times, press the button labeled *1.0* and then the button labeled *0.1*.
4. Make the separation of the two points smaller than 0.1. Can you still see two distinct points on the graph? Can you see the values of t_2 and d_2 change as you make Δt smaller? Experiment with dragging the Δt slider, to change the separation of the two values of time directly.

Q3 What is the largest separation you can get by moving the slider? What is the smallest separation you can actually observe on the graph?

Q4 As you make Δt smaller, can you observe changes in the numeric values of t_2 and d_2 even when you can no longer observe any changes on the graph?

THE RATE OF CHANGE OF THE DOOR'S ANGLE

Hint: Divide the change in the angle by the change in the time.

5. Set Δt to 0.1, and then use the numeric values of t_1, d_1, t_2, and d_2 to calculate the rate of change of the door's angle at any particular time. (Use Sketchpad's Calculator to perform this calculation.)

Q5 What are the units of the rate of change? What does the rate of change tell you about the door's motion?

When you press the *Show Rate* button, a dotted line appears connecting the two points on the graph.

6. Press the *Show Rate* button to check your result.

Q6 What is the relationship between the dotted line and the rate of change you calculated?

Q7 Move t_1 back and forth. How can you tell from the rate of change whether the door is opening or closing? How can you tell whether its rate is fast or slow?

After double-clicking the table, it shows two rows of numbers, with the first row permanent and the second row changing as the measurements change.

Q8 Use the buttons to set t_1 to 1.0 and Δt to 0.1. What is the rate of change?

7. Select the numeric values of t_1, d_1, t_2, d_2, Δt and the rate of change. With these six measurements selected, choose **Number | Tabulate.** Double-click the table to make the current entries permanent.

THE LIMIT OF THE RATE OF CHANGE

You may want to press the *0.1* button and then the *0.01* button again to check the motion of the dotted line.

8. Set the time interval (Δt) to exactly 0.01. Note the new value of the rate of change. Could you see the dotted line move as you reduced the time interval? Double-click the table to permanently record these new values.

Q9 How does this rate of change compare to the value when Δt was 0.1?

9. Similarly, record in the table values for time intervals of 0.001, 0.0001, 0.00001, and 0.000001.

Q10 What happens to the value of the rate of change as the time interval becomes smaller and smaller? What value does the rate of change appear to approach?

Q11 Can you see the dotted line move as Δt changes from 0.001 to 0.0001?

10. Set the value of t_1 to 3 seconds (by pressing the $t \rightarrow 3$ button), and collect more data on the rate of change of the door's angle. Collect one row of data for each time interval from 0.1 second to 0.000001 second.

The *average rate of change* is the rate of change between *two* different values of *t.* The *instantaneous rate of change* is the exact rate of change at *one* specific value of *t.* Because you must have two different values to calculate the rate of change, one way to measure the instantaneous rate of change is to make the second value closer and closer to the first and then find the *limit* of the average rate of change as the interval gets very small.

The instantaneous rate of change of a function—that is, the limit of the average rate of change as the interval gets close to zero—is called the *derivative* of the function.

Q12 What is the derivative of the door's angle when t_1 is 3 seconds?