SKETCH AND INVESTIGATE

Q1 As students drag point C, they should notice that the area of the rectangle changes but its perimeter remains constant. Because CB and CD are radii of the same circle, the sum of two sides of the rectangle, $AC + CD$, is equal to AB. Thus, AB is half the perimeter of the rectangle. As long as this length is kept constant, the perimeter of the rectangle will be constant.

Q2 A square is the rectangle with the greatest area for a given perimeter.

Q3 The coordinates of the high point of the graph show the side length and area of the maximum-area rectangle. The side length at this point verifies that the rectangle with the maximum area is a square.

Q4 The low points on the graph show where the area of the rectangle is zero. This happens when AC is zero and when $AC = AB$.

You might want to discuss with students why the locus graph of (side length, area) of a rectangle is a parabola.

EXPLORE MORE

1. Regular polygons have maximum area for a given perimeter. Polygons with more sides are more efficient. The circle is the closed planar figure that gives maximum area for a given perimeter.
2. The area of the rectangle can be represented by the equation $A = x[(1/2)P - x]$. The graph is a parabola with roots 0 and $(1/2)P$. So the x-value of the maximum point is $(1/4)P$. Since the side length of the maximum area rectangle is 1/4 the rectangle's perimeter, the rectangle must be a square.