Quadratic Quandary: Find the Equation of a Parabola

For GSP5

If you are given the graph of a parabola but not its equation, how can you find its axis of symmetry and vertex? In this activity you'll approximate the location of a parabola's axis of symmetry and then use a geometric construction to pinpoint both the axis of symmetry and the parabola's vertex.
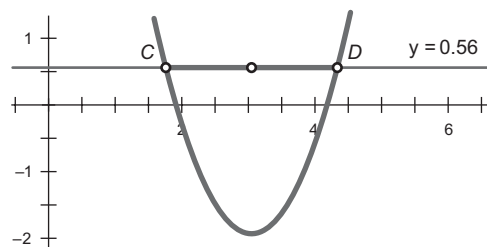
ESTIMATE THE AXIS OF SYMMETRY

1. Open **Quadratic Quandary.gsp.** You'll see a parabola and two lines, each of which is parallel to one of the axes.

2. Drag the vertical line. As you drag, the equation of the line changes.

Q1 Drag the vertical line as close as you can to the parabola's axis of symmetry. Record the equation of the line.

Q2 How accurately do you think you placed the line? Do you think it is exact? Why or why not?

CONSTRUCT THE AXIS OF SYMMETRY

Now construct the axis of symmetry and see how close your estimate was.

Q3 Drag the vertical line to the side. Before you continue, think about this question: How could you use the horizontal line to help you construct the exact location of the axis of symmetry?

3. Drag the horizontal line so it intersects the parabola in two places. Select the line and the parabola and choose **Construct│Intersections.** Label the intersection points C and D.

4. Construct $\overline{CD}$. With $\overline{CD}$ selected, choose **Construct│Midpoint.**

5. Select the midpoint and the x-axis. Choose **Construct│Perpendicular Line.**

Q4 Why is this line the axis of symmetry for the parabola?

6. Select the line and choose **Measure│Equation.**

Q5 Look back at your answer to Q1. How close was your approximation of the axis of symmetry?

CONSTRUCT THE VERTEX

7. Use the axis of symmetry to construct the vertex of the parabola.

Select the vertex and hold the Shift key while choosing **Measure | Abscissa & Ordinate.**

8. Select the vertex and measure its abscissa (*x*-value) and ordinate (*y*-value). Label these coordinates *h* and *k*.

9. Press *New Parabola.* Each time that you press this button, you'll get a new parabola. If you don't see your axis of symmetry and vertex, drag the horizontal line so that it intersects the parabola.

10. Construct point *P* on the graph, and measure its abscissa and ordinate.

Q6 Recall that the vertex form of a parabola is $y = a(x - h)^2 + k$. You already know the values of *h* and *k*. How can you use the coordinates of point *P* to find the value of *a*?

11. Use Sketchpad's Calculator to compute the value of *a* based on your answer to Q6. Check your result by graphing $y = a(x - h)^2 + k$ and seeing whether the plotted function matches the parabola already on the screen.

Q7 Press *Original Parabola* to return to the very first parabola. Do your results for *a, h,* and *k* appear correct? Check your results with those of other students. Did you get the same result for *a*? Did you use the same point *P*? How could you and your classmates get the same value for *a* even though you were using different points for *P*?

EXPLORE MORE

12. Create a custom tool to find the vertex of any parabola. In this version of the tool, you'll have three given objects: the quadratic graph, a point to determine the horizontal line, and the *x*-axis. (The main difference from the construction you've already done is that you'll have to construct the horizontal line yourself, as a line parallel to the *x*-axis through a given point.) In a new sketch, graph several parabolas and test your tool on them.

Hint: To construct the horizontal line without using the *x*-axis, translate the given point by 1 cm at 0°.

13. Create a custom tool that's easier to use by eliminating the *x*-axis as a given. With this tool you can click only the graph and one point.

14. Create a custom tool that's even easier to use by using a point on the parabola itself. (*Hint:* Merge the given point to the parabola.) With this tool you need to click only once, on the parabola.

15. Extend your tool so that it also displays the equation of the parabola.