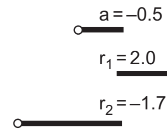
Parabolas in Factored Form

For GSP5

If you use a parabola to model a thrown ball, you might want to know exactly where it hits the ground. Or if you model a profit function with a parabola, you might be interested in the *break-even point*—the cutoff between profitability and loss. In both cases you're interested in the *roots* of the equation—where the function's value is zero. In this activity you'll explore a quadratic form that's based on the roots.

SKETCH AND INVESTIGATE

1. Open **Factored Form.gsp.** This sketch contains three sliders (a, r_1, and r_2). Adjust each slider by dragging the point at its tip. Observe how the values change as you drag.

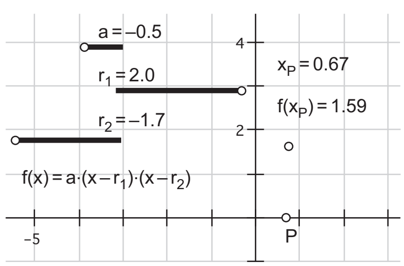


First you'll create a function, evaluate it for an input variable, and plot the resulting point.

2. Use the slider values to define the function $f(x) = a(x - r_1)(x - r_2)$. To do so, choose **Number | New Function.** The New Function dialog box appears. If necessary, move it so that you can see the measurements of a, r_1, and r_2.

To enter a, r_1, and r_2, click their measurements in the sketch. To enter x, click the x key in the dialog box.

3. Enter $a*(x-r_1)*(x-r_2)$ and click OK. Sketchpad creates the function.
4. Measure the x-coordinate of point P by choosing **Measure | Abscissa.**
5. Calculate $f(x_P)$, the value of function f evaluated at x_P. To do so, choose **Number | Calculate.** Click on the function object $f(x)$ and then on measurement x_P. Finally click OK. The value of the function appears.
6. To plot the point, select x_P and $f(x_P)$ and choose **Graph | Plot as (x, y).**



To turn tracing on or off, select the plotted point and choose **Display | Trace Plotted Point.**

Q1 Drag P back and forth along the x-axis to change the input variable for the function. How does the plotted point behave? Turn on tracing for the plotted point to better observe its behavior. What shape does this function trace out?

Traces are temporary in Sketchpad. Instead of using traces, you'll now make the graph permanent so you can compare the graphs of different functions.

7. Turn off tracing for the plotted point. Then choose **Display | Erase Traces** and drag *P* to make sure the traces no longer appear.
8. Plot a permanent graph of the function by selecting the function and choosing **Graph | Plot Function.** Drag *P* back and forth to make sure that the graph really corresponds to the path of the plotted point.

EXPLORING FAMILIES OF PARABOLAS

By dragging point *P*, you explored how x and y vary for *one particular function* with specific values of a, r_1, and r_2. Now you'll change the values of a, r_1, and r_2, which *changes the function itself,* allowing you to explore whole families of parabolas.

Q2 Adjust slider a and describe its effect on the parabola. Discuss the effect of a's sign (whether it's positive or negative), its magnitude (how big or small it is), and anything else that seems important.

Q3 Dragging a appears to change all the points on the parabola but two: the x-intercepts of the parabola (the roots). Adjust all three sliders and observe the effect that each has on the x-intercepts. How are the locations of the x-intercepts related to the values of the sliders?

Q4 Adjust slider r_1. What happens to the parabola as r_1 changes? What happens as r_2 changes?

Q5 Adjust the sliders so that $r_1 = r_2$. Describe the resulting parabola.

Figure out the equations using pencil and paper only. Once you think you have the equation, you can use the sketch to check your result.

Q6 For each description below, write an equation for a parabola in factored form $f(x) = a(x - r_1)(x - r_2)$. Check your answers by adjusting the sliders.

a. x-intercepts at $(-4, 0)$ and $(6, 0)$; vertex at $(1, -1)$

b. x-intercepts at $(-5, 0)$ and $(1, 0)$; contains the point $(3, 32)$

c. x-intercepts at $(0, 0)$ and $(-3, 0)$; contains the point $(2, 3)$

d. same x-intercepts as $y = 2(x - 3)(x + 1)$; contains the point $(0, -3)$

e. same shape as $y = 2(x - 3)(x + 1)$; x-intercepts at $(-4, 0)$ and $(1, 0)$

Q7 You throw a baseball and it flies in a parabolic path across a field. If the ball reaches its apex (highest point) 60 feet away from you, and the apex is 40 feet above the ground, how far away from you will the ball land? What is the equation of the ball's flight in factored form? (Assume that the ball starts at the point $(0, 0)$.)

EXPLORE MORE

Q8 When you have a parabola in the form $f(x) = a(x - h)^2 + k$, it's easy to find its vertex, but harder to find its roots. The opposite is true with the form $f(x) = a(x - r_1)(x - r_2)$. Explain what you do know about the vertex of parabolas in this form. Can you write an expression for the x-coordinate of the vertex in terms of r_1 and r_2? The y-coordinate?

9. Use the expressions you just wrote for the coordinates of the vertex to plot the vertex. If you do this properly, the plotted point will remain at the vertex regardless of how you drag the sliders. You can use Sketchpad's Calculator to calculate the x- and y-coordinates of the vertex using the expressions you found in the previous question. You can then select the two calculations and choose **Graph | Plot as (x, y).**

Q9 If you successfully plotted the vertex, try this: Turn on tracing for the vertex by choosing **Display | Trace Plotted Point.** Then adjust slider r_1. What shape does the vertex trace? Can you write an equation for this curve in terms of a and r_2?