Balancing with Balloons: Solving Equations with Negatives

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Students should have already completed the activity Balancing, which introduces the Sketchpad balance model using only positive weights. If not, ask, ***Have you ever seen a balance like this one? How does it work?*** Make sure students understand that the heavier side will go down and that the two sides will balance only if they have the same weight. You should also show examples involving only positive weights before introducing the balloons.

2. Open **Balancing with Balloons.gsp** and go to page "Balance." Enlarge the document window so it fills most of the screen. Explain, ***Today you're going to use a Sketchpad balance to model solving equations that include negative numbers and negative variables. Now there are balloons that pull up on the balance in addition to the weights that push down on the balance.*** Drag a 1-weight to one side of the balance, and then drag a -1-balloon to the same side. ***A negative one balloon pulls up just as much as a positive one-weight pushes down, so the scale remains balanced.*** Demonstrate that the same is true for an x-weight and a $-x$-balloon.

3. Drag one x-weight to the left side and four 1-weights to the right side. They should balance (x is set equal to 4 on this page). ***What does this tell you about the weight of x?*** [It equals 4.] ***How would you write this as an equation?*** [$x = 4$] Use the **Text** tool to write the equation so that the equal sign is aligned with the center of the balance. Use a large font for visibility.

For a new text caption, use the **Text** tool. To move a caption, use the **Arrow** tool. To edit a caption, either click once with the **Text** tool or double-click with the **Arrow** tool.

4. Add a -1-balloon on the left side and ask, ***What can I do to the right side to keep the balance?*** Students may suggest adding a -1-balloon to the right or removing a 1-weight from the right. Acknowledge both approaches, and then say, ***What happens if I do add a negative one balloon to the right?*** Besides balancing the scale, elicit the idea that a 1-weight and a -1-balloon cancel each other out. Drag the balloon so it is directly above the weight, select both of their points, and drag them simultaneously out to the storage bin.

5. Write the equation $x - 1 = 3$. Use a new text caption so that you can align the equal signs. Hide the original equation and ask, ***What if we were trying to solve this equation? What could we do to isolate x, that is, to get x by itself?*** [Add 1 to both sides.] Drag a 1-weight to both sides

of the equation. Then select both the weight and balloon on the left side, remove them, and unhide the original equation.

6. ***Today you'll be solving equations like these. What is the original equation? The problems get more complex as you go, and most will require more than one step. Be sure to record the equations and the solutions steps on your worksheets.*** If you wish to give more guidance, the page "Example" has an animation of how to solve the equation $-x + 4 = 2x - 2$ (press *Show Steps*). You might do the example together with the class, ask student pairs to review it on their own, or mention it only to student pairs if they need support.

DEVELOP

Expect to spend about 25 minutes.

7. Assign pairs to computers and tell them where to locate the sketch **Balancing with Balloons.gsp.** Distribute the worksheet. Tell students to work through step 6 and do the Explore More if they have time.
8. Let pairs work at their own pace. As you circulate, here are some things to notice.
   - Watch that students are systematically doing the same thing to both sides of the balance. Some students might go directly to moving all but one x from one side and then manipulating the other side until they find the solution. This will work, but misses the point. Encourage students to articulate what they're doing in terms of mathematical operations (subtracting and dividing both sides of the equation by the same thing).
   - In worksheet step 4, equation C requires students to replace a -5-balloon with five -1-balloons in order to divide it into two groups. Students may also just do the division mentally, which is fine.
   - In worksheet step 5, check that students understand that the solution to equation C is negative. The problem asks what this means in terms of the model. One way of thinking about it is that the x's are balloons, which would mean that $-x$-balloons are weights. The main point of the question is to make students think, but don't focus too much on this limitation of the model.
   - In worksheet step 6, students will need to remove $-x$-balloons. Generally, students will find it easier to use positive x-weights to cancel out $-x$-balloons. If students don't, they will need to divide

by a negative number on the last step, which is difficult to explain with this model. One way to think about it is that you isolate one $-x$-balloon, and so the solution for x must be the opposite of that value.

- Some students are likely to work through the equations more quickly than others. Encourage them to experiment with the Explore More to build equations of their own.

SUMMARIZE

Expect to spend about 10 minutes.

9. Bring the class back together to discuss the strategies they discovered. ***What have you learned about solving equations?*** Bring out these objectives.
   - Equations might be solved by balancing, similar to working with a pan balance.
   - Adding and subtracting the same number from both sides of an equation does not change the solution.
   - Dividing both sides of an equation by the same number does not change the solution. (You might remind students that they cannot divide by zero.)
10. ***What other questions can you ask that you may or may not be able to answer?*** Encourage all student curiosity. Mathematical questions of interest include these.
    - What does it mean to have a negative value for the solution?
    - Can you also multiply both sides of an equation by the same thing?
    - Can you use balancing if the numbers aren't integers?

EXTEND

For practice with solving simple linear equations without the balance, randomly generated linear equations appear on page "Practice." Students who already have strong equation-solving skills should enjoy creating complex equations of their own on page "Balance."

ANSWERS

1. $x - 5 = 2$

2. Add a 5-weight to both sides, and then remove the 5-weight and -5-balloon from the left side.
   $x = 7$

3. $3 = 2x - 1$
   $4 = 2x$
   $x = 2$

4. $2x + 1 = -5$
   $2x = -6$
   $x = -3$

5. The solution to equation C is negative. In terms of the model, x's are balloons (and $-x$-balloons are weights). Mostly, this illustrates a limitation of the model.

6. Equation D: $7 - x = 2x + 1$
   $7 = 3x + 1$
   $6 = 3x$
   $x = 2$

   Equation E: $x - 1 = -3x - 5$
   $4x - 1 = -5$
   $4x = -4$
   $x = -1$

   Equation F: $-3x + 2 = -x + 10$
   $2 = 2x + 10$
   $-8 = 2x$
   $x = -4$

   Equation G: $-x - 11 = -4x - 2$
   $3x - 11 = -2$
   $3x = 9$
   $x = 3$

7. Answers will vary.

8. Answers will vary.