Hikers: Solving Through Multiple Representations

For GSP5 ACTIVITY NOTES

INTRODUCE

Project the sketch for viewing by the class. Expect to spend about 10 minutes.

1. Open **Hikers Present.gsp** and go to page "Problem." Enlarge the document window so it fills most of the screen. Read the problem aloud. Pause when you come to the first occurrence of mi/h, and ask students how to read it and what it means.
2. You might have two volunteers act out the problem for the class. Then go to page "Simulation" and press *Start/Stop Simulation.* ***Between what times do the hikers meet?*** [Between 3 and 4 hours].
3. ***What might we do to find the time more precisely?*** Encourage many suggestions. Go to page "Table".
4. ***Do the distances corresponding to time 0 make sense?*** Help students see that the distances are from the trailhead, not the distances traveled, so Maria begins at 12 miles.
5. ***What should go on the row corresponding to time 1?*** Elicit the idea that Edna will be 1.5 miles from the trailhead and Maria will be $12 - 2 = 10$ miles from the trailhead. Demonstrate how to double-click the parameters to change their values.

DEVELOP

Expect students at computers to spend about 25 minutes.

6. Assign student pairs to computers and show them how to find **Hikers.gsp.** Distribute the worksheet. Ask students to work through step 12 and do the Explore More if they have time.
7. Let pairs work at their own pace. As you circulate, here are some things to notice.
   - Some students may say that the hikers meet 6 miles from the trailhead, because 6 appears in both columns (or because 6 is the halfway point). Ask, ***How many hours after they left were they 6 miles from the trailhead? Can you say that's the same time? What does it mean to meet?***
   - Students may have difficulty reading the table to see when the hikers meet. ***Who was closer to the trailhead after 2 hours? After 3 hours? After 4 hours?***
   - In worksheet step 3, if students don't see **Graph | Plot as (x, y),** they probably still have the previous point selected. Tell them to click in blank space before they plot each point.

- If the lines that students trace in worksheet steps 7 and 8 do not pass through the plotted points from step 3, tell them to change their expressions in step 6, erase their traces, and try again.
- The coloring in worksheet step 9 is intended to help students associate the graphs directly with the table.
- In worksheet step 10, students may not realize that the point they seek is the intersection. Help them think of one variable at a time. ***What points represent the two hikers at a distance 9 miles from the trailhead? What are their times when they are there? What points represent the hikers 6 miles from the trailhead? What are their times? What point represents when they are at the same place at the same time?***
- As needed, help students realize in worksheet step 11 that to find exact times and distances, they must convert to fractions.

SUMMARIZE

Expect to spend about 10 minutes.

8. Reconvene the class. Select some pairs to present their sketches (or use those in **Hikers Present.gsp**). Discuss worksheet steps 5–11.
9. ***Which approach do you prefer: a table, a graph, or an equation?*** Student preferences will vary. Students should realize that numerical information can be represented in multiple ways: arithmetically, algebraically, and graphically. Encourage comparisons of the methods, such as the fact that the table and graph give only estimates, whereas the equation could give an exact answer.
10. ***What have you learned?*** You may wish to have students respond individually in writing to this prompt, or have volunteers respond verbally. Bring out these objectives.
    - The same situation can be represented with tables, graphs, or equations.
    - Making a table can help in finding expressions to graph.
    - Knowing expressions to graph can help in finding an equation to solve.

EXTEND

What other questions might we ask? Encourage all student curiosity. Mathematical questions of interest include these.

Why are the graphs straight lines?

Why does the point of intersection represent where the hikers met?

Is there an easier way to find an exact solution?

What if the hikers paused to rest? Could we still tell when they met?

Are there ways other than tables, graphs, and equations to represent the situation?

What if there were no Maria? If Edna went over one day and came back the next, would there necessarily be a point at which she was at the same place at the same time of day?

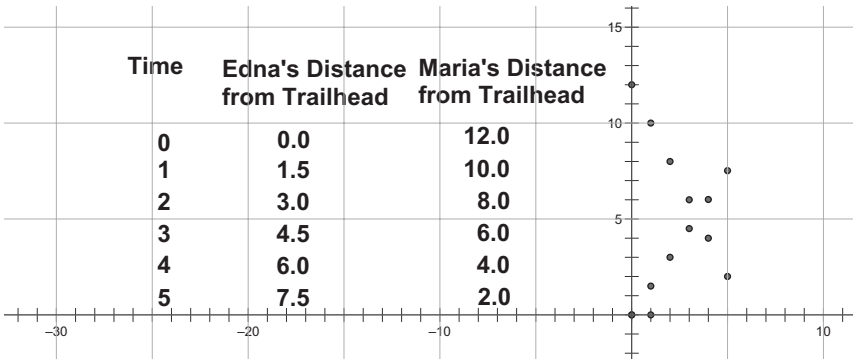
ANSWERS

1.

Time (hours)	Edna's Distance (miles from trailhead)	Maria's Distance (miles from trailhead)
0	0.0	12.0
1	1.5	10.0
2	3.0	8.0
3	4.5	6.0
4	6.0	4.0
5	7.5	2.0

2. The hikers will pass each other between hour 3 and hour 4. Any students who put in extra rows between hour 3 and hour 4 may predict a smaller interval.

3. The points representing each hiker's distances lie in a straight line, and those lines will cross, though not at a data point. The sketch should look like this.



6. Edna: $1.5x$; Maria: $12 - 2x$

7. The traced point moves in a straight line, passing through the plotted points representing Edna's distance.

8. The traced lines intersect when *Time* is approximately 3.4 hours.

10. Answers may vary. The point of intersection will be approximately (3.43, 5.14), indicating a time of 3.43 hours and a distance of 5.14 miles from the trailhead.

11. $1.5x = 12 - 2x$

    $3.5x = 12$

    $$x = \frac{12}{3.5} = \frac{12}{\frac{7}{2}} = \frac{24}{7} \text{ hours}$$

    $$1.5\left(\frac{24}{7}\right) = \frac{3}{2} \cdot \frac{24}{7} = \frac{36}{7} \text{ miles}$$

12. $\frac{24}{7}$ hours $= 3\frac{3}{7}$ hours

    $= 3$ hours $+ \frac{3}{7} \cdot 60$ minutes

    $= 3$ hours and about 25 minutes

    Some students may go further to add about 43 seconds.

13. Answers will vary. Students familiar with solving systems of equations simultaneously may write two equations and solve by substitution (equivalent to what is done in worksheet step 8) or by elimination.

    $$\begin{array}{r} 1.5x - y = 0 \\ 2x + y = 12 \\ \hline 3.5x \quad\quad = 12 \end{array}$$

    $$x = \frac{12}{3.5} \approx 3.43 \text{ hours}$$

    $$y \approx 1.5(3.43) \approx 5.14 \text{ miles}$$

    Alternatively, students might think about closing speed. The hikers together need to cover 12 miles at a combined speed of $1.5 + 2 = 3.5$ miles per hour. Doing so will take $\frac{12}{3.5}$ hours.

    Solutions could also be estimated by finding the intersection of the graphs of equations $y = 3.5x$ and $y = 12$.