Quadratic Quandary: Find the Equation of a Parabola

For GSP5 ACTIVITY NOTES

INTRODUCE

Show the presentation sketch **Quadratic Quandary Present.gsp** and ask students whether they see any symmetry in the shape of the graph. Ask a volunteer to adjust either the horizontal or vertical line (by dragging the point on the respective axis) in order to show the symmetry. Note the value of x that corresponds to the position of the line.

Then have another student volunteer move the line aside, show the reflected point, and move the line so that the reflected point lies on the opposite side of the graph as the original. Ask students what it means to have the reflected image of point P also lie on the parabola. Ask students to discuss whether the reflected image will remain on the parabola if point P is dragged, and ask them to justify their answers. Have the student volunteer drag P to test the conjectures. Ask students to note the value of x.

Finally, have another volunteer move the line aside, show the reflected graph, and move the line so that the reflected graph and the original coincide. Again, have students note the value of x, and compare all three values.

Ask which method (dragging without any reflections visible, dragging with a point reflection visible, or dragging with the reflection of the entire graph visible) allows for the most precise determination of the x-value of the axis of symmetry.

DEVELOP

Pass out the worksheets, and explain that in this activity students will not only approximate the axis of symmetry, but that they will also construct the exact axis, and then use it to find the exact vertex as well.

Tell students to begin the activity and to write out their answers to the questions as they work.

While students work, circulate around the room. Pay particular attention to students' answers to the questions, encouraging them to make their answers as precise and well reasoned as they can.

SUMMARIZE

Bring the class together, and review any difficulties students had in answering the questions. You may also want to review any difficulties that came up during the construction.

Point out to students that they had a pretty good estimate of the position of the axis of symmetry before beginning the construction. Ask, ***Since you already had an answer, why should you bother doing the construction?*** This discussion can bring out the importance of finding precise answers rather than estimates. Try to make sure that students realize that sometimes an estimate is all that's needed, but that sometimes a precise answer is important.

Pay special attention to reviewing Q6 and Q7. Have several students explain why they need to construct point *P* and measure its coordinates in order to determine the value of *a*. Also have students discuss why they can choose different locations for point *P*, but all get the same answer (for the original parabola) for *a*.

EXPLORE MORE

These challenges are appropriate for students who already have some familiarity with creating custom tools. The required constructions, and the tools that result from them, are included on the later pages of **Quadratic Quandary Present.gsp.**

ANSWERS

Q1 If students have not changed the original parabola (by pressing the *New Parabola* button), their result should be between 3.0 and 3.1.

Q2 The placement of the line is not accurate; explanations will vary.

Q3 The horizontal line allows you to locate two intersections, spaced equally on either side of the axis of symmetry. These intersections allow you to locate the axis precisely.

Q4 Answers will vary. An algebraic explanation might be based on the fact that the y-values of the two intersections are the same, so that the values of $(x - h)^2$ must also be the same, and that the only way this can happen for two different values of x is if those two values of x are greater than h and less than h by the same amount, respectively. A geometric explanation might be based on the fact that because the axis is vertical, it must go through the midpoint of any horizontal segment between two intersections. A dynamic explanation might focus on moving the horizontal line toward the vertex and observing that the segment gets shorter and shorter until its length is zero at the vertex.

Q5 Answers will vary.

Q6 To calculate a, substitute the coordinates of point P into the equation. Also substitute the values of h and k. Solve the resulting equation for a.

$$a = \frac{(y_P - k)}{(x_P - h)^2}$$

Q7 In the first part of this question, students check to make sure that their calculations are correct even when the shape of the parabola changes. In the last part of the question, all students should have the same result for a even though they've used different positions for point P. Explaining this is an important discussion topic in the whole-class summary.