A Rectangle with Maximum Area

For GSP5

Suppose you had a certain amount of fence and you wanted to use it to enclose the biggest possible rectangular field. What rectangle shape would you choose? In other words, what type of rectangle has the most area for a given perimeter? You'll discover the answer in this investigation. Or, if you have a hunch already, this investigation will help confirm your hunch and give you more insight into it.

SKETCH AND INVESTIGATE

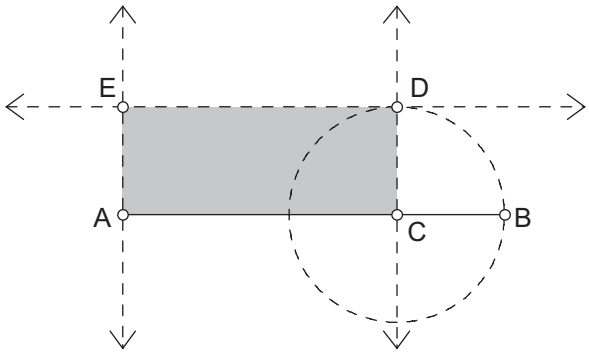
1. Construct $\overline{AB}$.

Select $\overline{AB}$, point *A*, and point *C*. Then, in the Construct menu, choose **Perpendicular Line.**

2. Construct $\overline{AC}$ on $\overline{AB}$.
3. Construct lines perpendicular to $\overline{AB}$ through points *A* and *C*.

Be sure to release the pointer—or click a second time—with the pointer over point *B*.

4. Construct circle *CB*.
5. Construct point *D* where this circle intersects the perpendicular line.



6. Construct a line through point *D*, parallel to $\overline{AB}$.
7. Construct point *E*, the fourth vertex of rectangle *ACDE*.

Select the vertices of the rectangle in consecutive order. Then, in the Construct menu, choose **Quadrilateral Interior.**

8. Construct interior *ACDE*.
9. Measure the area and perimeter of this polygon.
10. Drag point *C* back and forth and observe how this affects the area and perimeter of the rectangle.

Select point *A* and point *C*. Then, in the Measure menu, choose **Distance**. Repeat to measure *AE*.

11. Measure *AC* and *AE*.

Q1 Without measuring, state how *AB* is related to the perimeter of the rectangle. Explain why this rectangle has a fixed perimeter.

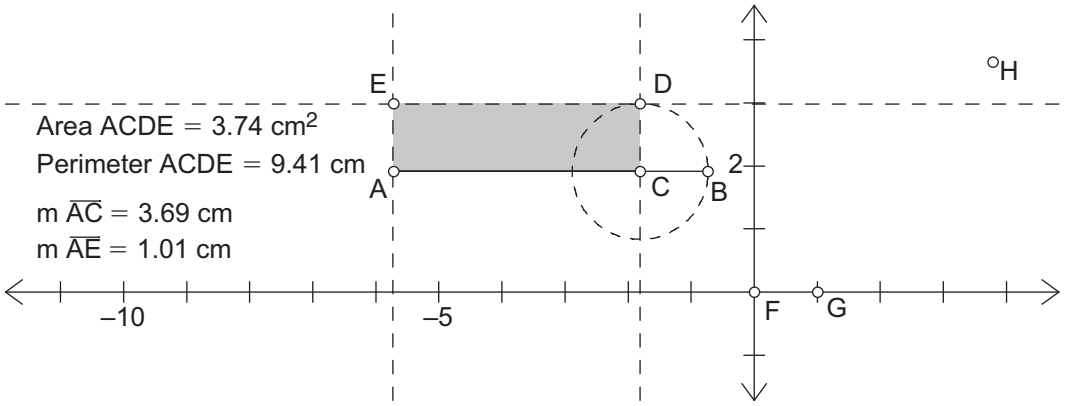
Q2 As you drag point *C*, observe what rectangular shape gives the greatest area. What shape do you think that is?

In steps 12–14, you'll explore this relationship graphically.

Select, in order, measurements *AC* and Area *ACDE*. Then choose **Plot as (x, y)** from the Graph menu. If you can't see the plotted point, drag the unit point at (1, 0) to scale the axes.

12. Plot the measurements for the length of $\overline{AC}$ and the area of *ACDE* as (*x*, *y*). You should get axes and a plotted point *H*, as shown in the following step.

13. Drag point *C* to see the plotted point move to correspond to different side lengths and areas.



Select point *H* and point *C;* then, in the Construct menu, choose **Locus.**

14. To see a graph of all possible areas for this rectangle, construct the locus of plotted point *H* as defined by point *C*. It should now be easy to position point *C* so that point *H* is at a maximum value for the area of the rectangle.

You may wish to select point *H* and measure its coordinates.

Q3 Explain what the coordinates of the high point on the graph are and how they are related to the side lengths and area of the rectangle.

15. Drag point *C* so that point *H* moves back and forth between the two low points on the graph.

Q4 Explain what the coordinates of the two low points on the graph are and how they are related to the side lengths and area of the rectangle.

EXPLORE MORE

16. Investigate area/perimeter relationships in other polygons. Make a conjecture about what kinds of polygons yield the greatest area for a given perimeter.

17. What's the equation for the graph you made? Let *AC* be *x* and let *AB* be (1/2)*P*, where *P* stands for the perimeter (a constant). Write an equation for area, *A*, in terms of *x* and *P*. What value for *x* (in terms of *P*) gives a maximum value for *A*?