It's important that students understand that a parabola remains a parabola regardless of the form its equation is written in. Any parabola's equation can be written in many different forms. The forms we study are the ones that are especially simple or that convey special information.

The factored form of a parabola is different from the standard form and the vertex form, because parabolas that don't cross or touch the *x*-axis cannot be written in factored form. When you summarize this activity, you might discuss the existence of roots and the possibility of categorizing parabolas based on whether they have 0, 1, or 2 distinct real roots. If students have studied the discriminant, this would be a good connection to make.

The activity is shorter if you use page 2 of the sketch document and start with the Exploring Families of Parabolas section of the worksheet.

SKETCH AND INVESTIGATE

Q1 The plotted point follows a curved, parabolic path. The exact shape (narrow or wide, opening up or down) depends on the values of the sliders.

EXPLORING FAMILIES OF PARABOLAS

Q2 If *a* is positive, the parabola opens upward; if *a* is negative, the parabola opens downward. The larger the absolute value of *a,* the narrower the parabola. The closer *a* is to zero, the wider the parabola.

Q3 When you drag *a,* the two *x*-intercepts remain fixed. One *x*-intercept exactly matches the value of slider r_1, while the other exactly matches the value of slider r_2.

Q4 Dragging either *r* slider changes the *x*-intercept corresponding to that slider.

Q5 A parabola with two equal roots has its vertex on the *x*-axis. Such roots are called *double roots.*

Q6 a. $y = 0.04(x + 4)(x - 6)$
b. $y = 2(x + 5)(x - 1)$
c. $y = 0.3(x - 0)(x + 3)$ or $y = 0.3x(x + 3)$
d. $y = 1(x - 3)(x + 1)$ or $y = (x - 3)(x + 1)$
e. $y = 2(x + 4)(x - 1)$

It is very important that students find the equations of these parabolas using paper-and-pencil calculations and use Sketchpad to check their answers.

Q7 The ball will land 120 feet away. The equation is

$y = \left(\frac{-1}{90}\right)(x - 0)\,(x - 120)$ or $y = \left(\frac{x}{90}\right)(x - 120)$.

EXPLORE MORE

Q8 Because of the symmetry parabolas exhibit, the x-coordinate of the vertex is the average of the two roots, or $(r_1 + r_2)/2$. To find the y-coordinate of the vertex, substitute $(r_1 + r_2)/2$ for x in the original equation:

$$y = a\left(\frac{r_1 + r_2}{2} - r_1\right)\left(\frac{r_1 + r_2}{2} - r_2\right)$$

Simplifying, you get:

$$y = \frac{-a}{4}(r_1 - r_2)^2$$

Q9 The vertex will trace out a parabola whose vertex is at $(r_2, 0)$, opens in the opposite direction of the given parabola, and is the same shape. The equation is $y = -a(x - r_2)\,(x - r_2)$ or $y = -a(x - r_2)^2$.

WHOLE-CLASS PRESENTATION

Use **Factored Form Present.gsp** to present this activity to the whole class, following the directions and using the buttons in the sketch.

RELATED ACTIVITIES

Parabolas in Standard Form, Parabolas in Vertex Form, Changing Quadratic Function Forms