



Student Pages



Name _____

1 A Weighty Problem

Jared plays mini-league football. He weighs 65 pounds, but when he weighs in wearing his uniform, the scale shows 72 pounds. Jared's father, who was also a football player, weighs 260 pounds.

How much would Jared's father's uniform weigh if the ratio of uniform weight to body weight is the same for both Jared and his father?



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Name _____

2 Tourney Time

Last week, there was a basketball elimination tournament at the community center. Eight teams entered the tournament. The Lions beat the Tigers in the first game. The Hawks beat the Eagles in the second game. The Giants lost to the Rebels in the third game, and the Wildcats lost to the Bulldogs in the fourth game. In the fifth game, the winner of the first game beat the winner of the fourth game. In the sixth game, the winner of the third game beat the winner of the second game. In the final game, the winner of the sixth game lost.

Which team won the championship?



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Teacher Pages



1 A Weighty Problem

Mathematics Concepts and Skills

Focus: Ratios and Equal Ratios
Proportions

Related Topic: Decimal Operations

Problem-Solving Strategies

- ♣ Make a Table
- ♣ Write an Equation

About the Mathematics

This problem involves the concept of equal ratios. A ratio is a comparison of two quantities. Equal ratios make the same comparison. One method for finding equal ratios is to multiply or divide both terms of the ratio by the same nonzero number. Since a proportion is an equation stating that two ratios are equal, another method for solving the problem is to use a proportion and set the cross products equal.

The only prerequisite for working on this problem is familiarity with the basic definition and notations of ratio. Use the lesson to explore the meaning of equal ratios and ways of finding them. Alternatively, if students already have experience with these concepts, the problem provides an opportunity for informal assessment of skills.

After distributing the problem to the class, have students work individually or in pairs to find a solution. Once they have had a chance to work, ask students to share their strategies and solutions, or encourage students to find other ways to solve the problem. Use the problem-solving strategies presented in this lesson as a basis for class discussion. Try to relate the Key Questions to students' methods that may differ from the ones presented here.

Name _____

1 A Weighty Problem

Jared plays mini-league football. He weighs 65 pounds, but when he weighs in wearing his uniform, the scale shows 72 pounds. Jared's father, who was also a football player, weighs 260 pounds.

How much would Jared's father's uniform weigh if the ratio of uniform weight to body weight is the same for both Jared and his father?



Problem-Solving Strategies

♣ Make a Table

Have students begin by finding the weight of Jared's uniform.

$$72 \text{ total pounds} - 65 \text{ pounds for Jared} = 7 \text{ pounds for Jared's uniform}$$

The ratio of uniform weight to body weight, then, is 7 to 65.

To find an equal ratio with a second term (body weight) of 260, multiply both terms of the ratio by the same nonzero number. Make a table to show the numbers.

		$\times 2$	$\times 3$	$\times 4$
Uniform Weight	7	14	21	28
Player's Weight	65	130	195	260

A 260-pound player would need a uniform weighing 28 pounds.

Teacher Tip

This strategy, sometimes called *scaling up*, is convenient when one weight is a multiple of the other.

❖ Write an Equation

Let W represent the weight of the father's uniform.
Write a proportion to show that the ratios are equal.

$$\frac{7}{65} = \frac{W}{260}$$

In a proportion, the cross products are equal. The cross products for the proportion shown are 7×260 and $65 \times W$.

$$65W = 1,820$$

$$\frac{65W}{65} = \frac{1,820}{65} \quad \text{Divide both sides by 65.}$$

$$W = 28$$

The father's uniform weighs 28 pounds.

Teacher Tip

The rule for proportions is sometimes stated as "the product of the means equals the product of the extremes." The first and fourth terms, 7 and 260, are the extremes; the second and third terms, 65 and W , are the means.

Key Questions

1. How much does Jared weigh? (65 pounds)
2. How much does Jared's uniform weigh?
(72 pounds – 65 pounds = 7 pounds)
3. What is the ratio of Jared's uniform weight to his body weight? (7:65)
4. How much does Jared's dad weigh?
(260 pounds)
5. How many times greater is the father's weight than Jared's weight? (4 times;
 $4 \times 65 \text{ pounds} = 260 \text{ pounds}$)
6. How many times greater must the weight of the father's uniform be? (4 times) What is the weight? ($4 \times 7 \text{ pounds} = 28 \text{ pounds}$)
7. If W represents the weight of the father's uniform, what is the ratio of his uniform weight to his body weight? ($W:260$)
8. What is a proportion? (An equation stating that two ratios are equal)
9. What two ratios must be equal? (7:65 and $W:260$)

10. How do you solve a proportion? (Find the cross products, set them equal, and then solve for the value of the variable.)

Assessing Understanding

Use the following problem to assess students' understanding of the mathematical concepts and strategies in this lesson.

[Problem] Jared is buying balloons for a team party. A small bag contains 25 balloons, 4 of which are orange. In a super-size bag, the ratio of orange balloons to other colors is the same. There are 105 balloons of other colors. How many orange balloons are in the super-size bag? (20)

Key Questions

1. How many orange balloons are in a small bag? (4)
2. How many balloons of other colors are in a small bag? (21; $25 - 4 = 21$)
3. What is the ratio of orange to other colors of balloons? (4:21)
4. How many times greater is the number of other color balloons in the super-size bag than the number in the small bag? (5 times)
5. If the ratios are equal, how many orange balloons are in the super-size bag? ($5 \times 4 = 20$)
6. If B represents the number of orange balloons in the super-size bag, what two ratios can you use to write a proportion? (4:21 and $B:105$)
7. What are the cross products? (4×105 and $21 \times B$)

Extending the Mathematics

Extending the Mathematics provides opportunities for students to consider the problem with a new condition. This section may also provide opportunities to introduce other mathematical concepts.

[Problem 1] The quarterback on Jared's team weighs 80 pounds. If the ratio of uniform weight to body weight is the same as in the original problem, how much does the quarterback's uniform weigh, to the nearest tenth of a pound? (*8.6 pounds*)

Key Questions

1. Is the quarterback's weight a multiple of Jared's weight? (*No*)
2. What will W represent in this problem? (*The weight of the quarterback's uniform*)
3. What proportion can you write? ($7:65 = W:80$)
4. What are the cross products? (7×80 and $65 \times W$)
5. What place must you look at to round to the nearest tenth? (*Hundredths*)

[Problem 2] The ratio of girls to boys at a football game is 5:8. If there are 247 girls and boys at the game, how many girls are there? (*95*)

Key Questions

1. What is the ratio of girls to all the girls and boys at the game? ($5:13$)
2. What is the total number of girls and boys at the game? (*247*)
3. By what number can you multiply 13 to get 247? (*19*)
4. What multiplication gives the number of girls at the game? ($5 \times 19 = 95$)
5. If G represents the number of girls, what is the ratio of girls to all the girls and boys at the game? ($G:247$)
6. What proportion must you solve? ($5:13 = G:247$)
7. What are the cross products? (5×247 and $13 \times G$)

2 Tourney Time

Mathematics Concepts and Skills

Focus: Data Analysis
Tree Diagrams

Related Topics: Exponents
Exponential Form
Powers of 2

Problem-Solving Strategies

- ❖ Make a Drawing
- ❖ Use Logical Reasoning

About the Mathematics

This problem presents students with data that they must organize and analyze to reach a conclusion. The tournament consists of three rounds, with the winners and losers given in a series of statements. Students can use a special type of tree diagram to represent the information so that the relationships become clear. The problem shows how visual presentation can convey key ideas at a glance.

Use this problem during a study of statistical graphing or at any time during the year as an application of thinking skills. Students who have experience with exponents and powers of 2 will have greater insight about the numbers of starting teams needed for this type of tournament.

After distributing the problem to the class, have students work in pairs or small groups to find a solution. Once they have had a chance to work, ask students to share their strategies and solutions, or encourage students to find other ways to solve the problem. Use the problem-solving strategies presented in this lesson as a basis for class discussion. Try to relate the Key Questions to students' methods that may differ from the ones presented here.

Name _____

2 Tourney Time

Last week, there was a basketball elimination tournament at the community center. Eight teams entered the tournament. The Lions beat the Tigers in the first game. The Hawks beat the Eagles in the second game. The Giants lost to the Rebels in the third game, and the Wildcats lost to the Bulldogs in the fourth game. In the fifth game, the winner of the first game beat the winner of the fourth game. In the sixth game, the winner of the third game beat the winner of the second game. In the final game, the winner of the sixth game lost.

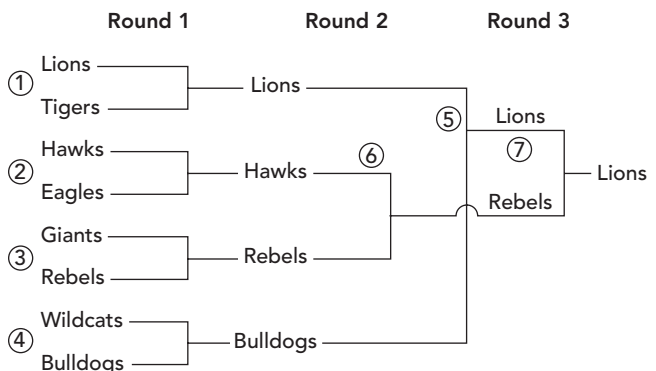
Which team won the championship?



Problem-Solving Strategies

❖ Make a Drawing/Use Logical Reasoning

Students can make a tree diagram showing the team pairs. Draw a branch to the next round for the winning team in each game.



The Lions won the tournament.

Teacher Tip

Diagrams like the one used for this problem often appear in the sports section of newspapers. Collect a few examples to display for this lesson.

Key Questions

1. How many games were played in the first round of the tournament? (4)
2. How many rounds were needed to complete the tournament? (3)
3. How many games were played in all? (7)
4. Explain how you determined the opponents for game 5. (*The problem states that in game 5, the winner of game 1 (the Lions) beat the winner of game 4 (the Bulldogs).*)
5. Explain how you determined the opponents for game 6. (*The problem states that in game 6, the winner of game 3 (the Rebels) beat the winner of game 2 (the Hawks).*)
6. Explain how you determined the champion team. (*The problem states that in the final game, the winner of game 6 (the Rebels) lost. So their opponent, the Lions, must have won.*)

Assessing Understanding

Use the following problem to assess students' understanding of the mathematical concepts and strategies in this lesson.

[Problem] Sixteen tennis players entered an elimination tournament. Marsha won the tournament in the final match. How many matches in all were played in the tournament? (15)

Key Questions

1. How many matches were played in the first round? (8)
2. How many rounds were needed to determine a champion? (4)
3. How many players were in the second round? (8) How many matches were there? (4)
4. How many players were in the third round? (4) How many matches were there? (2)
5. How many matches were played in all? (15)
6. What do you observe about the number of matches and the number of players entered in the tournament? (*The total number of matches is one less than the number of players.*)

Extending the Mathematics

Extending the Mathematics provides opportunities for students to consider the problem with a new condition. This section may also provide opportunities to introduce other mathematical concepts.

[Problem 1] In a single-elimination basketball tournament, teams played six rounds of games to determine a winner. How many teams entered the tournament? (64) How many games were played in all? (63 games)

Key Questions

1. How many teams were in round 6? (2)
How many games were played? (1)
2. How many teams were in round 5? (4)
How many games were played? (2)
3. How many teams were in round 4? (8)
How many games were played? (4)
4. Explain how you work backward from each round to determine the number of teams and games in the previous round. (*Double the number of teams and the number of games from round N to get the numbers for round $N - 1$.*)
5. How many teams and games were in each of rounds 3, 2, and 1? (*Round 3: 16 teams and 8 games; round 2: 32 teams and 16 games; round 1: 64 teams and 32 games*)
6. How many games were played in all?
($32 + 16 + 8 + 4 + 2 + 1 = 63$)
7. What do you observe about the numbers of teams and games? (*The number of games is one less than the number of teams.*)

[Problem 2] Recall that the exponential expression 2^3 means $2 \times 2 \times 2$. In this expression, 2 is called the *base* and 3 is the *exponent*. The exponent tells the number of times the base is used as a factor.

- a. Express 16, 32, and 64 in exponential form.
(2^4 , 2^5 , 2^6)
- b. Look back at the answers to the problems in this lesson. If the number of teams in an elimination tournament is expressed as 2^N , how many rounds will there be in the tournament? (*N rounds*) How many games will be played in all? ($2^N - 1$)

Key Questions

1. What is the value of 2^3 ? ($2^3 = 2 \times 2 \times 2 = 8$)
2. If $2^3 = 8$, what must be the exponential form for 16? Why? (*Since $16 = 8 \times 2$, $16 = 2^3 \times 2 = 2 \times 2 \times 2 \times 2 = 2^4$*)
3. How many factors of 2 do you need to get the product 32? (*Five: $2 \times 2 \times 2 \times 2 \times 2 = 32$*)
4. How many rounds and games were there for eight teams? (*3 rounds; 7 games*)
5. How many rounds and games were there for 16 teams? (*4 rounds; 15 games*)
6. Predict how many rounds and games there would be for 128 teams in a basketball elimination tournament. (*Since $128 = 2^7$, there would be 7 rounds and 127 games.*)

Teacher Tip

Discuss with students that for an elimination tournament to have an even number of teams for each round, the starting number of teams must be a power of 2.