



The Research Base for

Math Triumphs



Macmillan/McGraw-Hill
Glencoe

The McGraw-Hill Companies

Table of Contents

Introduction	5
Mathematical Proficiency for All Learners	5
Intervention	5
Modeling	8
Mediated Scaffolding	8
Feedback	9
Addressing Specific Mathematics Disabilities	9
Differentiated Instruction	9
English Language Learners (ELL)	11
How <i>Math Triumphs</i> Reflects the Research on Mathematical Proficiency for All Learners	12
References	14

The Research Base for *Math Triumphs*

Introduction

The guide for the development of the *Math Triumphs* program states that the purpose of the program is to assist students who are two or more years below grade level in grades K–8 mathematics. The goal is to provide them with the skills to learn successfully and efficiently so that they can achieve with basic grade level materials. The program is intended as a course for students to accelerate their learning and be able to return successfully to the regular program. Assessment is diagnostic and imbedded so that the teacher can better monitor entrance and exit from the program. The writing guide lists the following key differences from the typical textbook:

- consumable volumes that allow for flexibility and personalized instruction;
- connections between concepts that reveal big ideas;
- truly differentiated instruction, not just differentiated examples;
- vocabulary instruction and English language support that goes beyond a mere list;
- presentation of small chunks of content;
- numerous examples for different strategies;
- step-by-step exercises to walk through processes;
- communication practice – peer reviews, explanations, presentations, etc.; and,
- experiences that are engaging and motivating including hands-on activities and assessment.

These differences were chosen based on best practices found throughout education literature, as described in the section that follows.

Mathematical Proficiency for All Learners

Intervention

It is useful to note that in the research regarding math intervention, Seethaler and Fuchs (2005) analyze the literature in terms of the efficacy of studies completed. They found that randomized, controlled designs were clearly underrepresented in the literature. They conclude that to truly assess efficacy, study methodology might need to improve. In a related article, Augustyniak, Murphy and Phillips (2005) argue that the research on the definition of a

math disability is lacking with respect to identification of core deficits. They identify the core areas needing further explanation as numerical skills, visual/spatial deficits, cognitive skill development (memory retrieval, working memory, speed of processing, attention regulation, problem solving) and social cognition. Mazzocco (2005) reviewed research regarding practices of early identification and intervention for students with math difficulties. The commentary discusses the criteria and nature of math difficulties and notes the need for additional research.

The above being said, Butler, Beckingham, and Lauscher (2005) report on three case studies regarding the support of students with math learning challenges. Three eighth grade students were given assistance in self-regulating their learning. General strategies found to be successful included:

- engaging the students in constructive conversation;
- supporting students in reflection on their learning; and,
- the need for teachers to engage in dynamic, curriculum-based forms of assessment.

Fuchs, Fuchs, and Hamlett (2006) report on the validation of an intervention to improve math problem solving in third grade. The intervention (Hotmath) involved explicit instructions, self-regulation strategies, and tutoring. Results indicated positive, short-term results for problem-solving skills.

Stinson (2006) suggested that a focus on the discourse of achievement in mathematics rather than the discourses of deficiency and rejection could prove beneficial in reducing the well-known achievement gap between white and African-American students. He suggests that the limited amount of research shows that enrichment activities, mentoring competent teachers, and helping students identify with the 'good kids group' (p. 496) might enhance math achievement in African-American males.

Research also suggests a variety of instructional strategies that are effective to meet the needs of students with special needs—including those with physical disabilities, mental impairments, and/or learning disabilities; English Language Learners (ELL); and low-performing students

who require some special attention to bring out the best of their capabilities. Effective instruction for special-needs students that the research has found includes:

- setting clear goals for students (Bray & Turner, 1986, Cherkas-Julkowski & Gertner, 1989, Ferritti, 1989, Ferritti & Cavalier, 1991, as cited by Baroody, 1996; Schunk, 1985, as cited by Mastropieri, Scraggs, & Shinh, 1991);
- using a “big ideas” structure for concepts (Kameenui & Carnine, 1998, as cited by Fuson, 2003, p. 88);
- teaching content that is not too difficult (Bray & Turner, 1986, Cherkas-Julkowski & Gertner, 1989, Ferritti, 1989, Ferritti & Cavalier, 1991, as cited by Baroody, 1996; Baroody, 1996) and presented within meaningful contexts (Miller & Mercer, 1997, as cited by Allsopp, Lovin, Green, & Savage-Davis, 2003);
- laying ample groundwork by providing background knowledge (Bray & Turner, 1986, Cherkas-Julkowski & Gertner, 1989, Ferritti, 1989, Ferritti & Cavalier, 1991, as cited by Baroody, 1996; Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- modeling by teachers (Allsopp et al., 2003; Baroody, 1996; Blankenship, 1978, as cited by Mastropieri et al., 1991);
- sequencing instruction to go from the concrete to the abstract (Miller & Mercer, 1997, as cited by Allsopp et al., 2003);
- using mediated scaffolding (e.g., visual supports with cues, teachers’ feedback on thinking, peer tutoring) (Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- discussing mathematics using language (Miller & Mercer, 1997, as cited by Allsopp et al., 2003);
- building in multiple practice opportunities (Miller & Mercer, 1997, as cited by Allsopp et al., 2003) and time for review by students (Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- using reinforcement (e.g., earning verbal praise) (Mastropieri et al., 1991); and,
- providing continual feedback (Miller & Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Blankenship, 1978; Schunk & Cox, 1986, as cited by Mastropieri et al., 1991).

Three of these elements of effective special-needs instruction—modeling, mediated scaffolding, and feedback—are discussed in further detail below.

Modeling

Directly modeling both general problem-solving strategies and specific learning strategies using multisensory techniques has been shown to be useful with students having attention problems, cognitive processing problems, memory problems, and metacognitive deficits, notes a summary of relevant research (Allsopp et al., 2003). A comparative study of 30 students suggests that direct modeling may be advantageous for students with slight mental retardation as well. One group of students who received direct modeling help (e.g., used blocks as physical manipulatives) and extra opportunities for purposeful practice employed “substantially fewer” inappropriate learning strategies than another group who didn’t receive such support (Baroody, 1996, pp. 81-82). Furthermore, it was found that one or two direct-modeling demonstrations enabled such students to correct basic arithmetic procedural strategies and improve their proficiency (Baroody, 1996).

In addition, a review and synthesis of 30 studies on mathematics instruction for learning-disabled students found that modeling and demonstration with corrective feedback improved problem-solving accuracy and generalization skills by the students (Blankenship, 1978, as cited by Mastropieri et al., 1991). For example, an instructional model in which teachers solved a problem, verbalized how they did it, and left the problem as a reference model improved learning-disabled students’ computational skills in seven different experiments (Smith & Lovitt, 1975, Rivera & Smith, 1987, 1988, as cited by Mastropieri et al., 1991).

Mediated Scaffolding

A review of 30 studies found several types of scaffolding strategies to be effective in improving learning-disabled students’ mathematical achievement in grades K–6 (Mastropieri et al., 1991):

- use of manipulatives teamed with pictorial representations (Peterson, Mercer, & O’Shea, 1988, as cited by Mastropieri et al., 1991);
- multisensory approaches—mixing visual, auditory, and/or kinesthetic methods; verbalization (a specific practice in which teachers and students repeat aloud problems, instructions, and solution steps) also improved students’ mathematical performance, found a comparative study of 90 such students in grades 6–8 (Schunk & Cox, 1986, as cited by Mastropieri et al., 1991); and,

- use of preorganizers (e.g., read problem, underline numbers, decide on the operation sign and problem type) and/or postorganizers (e.g., read problem, check operation, check math statement, check calculations, write labels) to support students when solving word problems (Mastropieri et al., 1991).

Feedback

Ongoing feedback is crucial with special-needs students. Such students require continual monitoring and feedback on their efforts to be successful, several studies and meta-analyses have found (Miller & Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Schunk & Cox, 1986, as cited by Mastropieri et al., 1991).

Addressing Specific Mathematics Disabilities

A synopsis of relevant research noted that four different kinds of mathematics disability have been identified (Geary, 1994, as cited by Fuson, 2003). They, and what the research suggested as useful strategies to address them, are as follows:

- semantic memory disabilities: students experience trouble with verbal and phonetic memory but may have normal visuospatial skills; instruction that employs visual clues is most effective for these learners (Fuson, 2003);
- procedural deficits: students use less advanced methods overall; conceptually based instruction is especially helpful for these students (Fuson, 2003);
- visuospatial disabilities: students struggle with concepts that use spatial relations (e.g., place value); instruction most helpful for these students includes extra cues to support visual processing, and focuses on methods that can be carried out in either direction (Fuson, 2003); and,
- problem-solving deficits: such students benefit from problem-drawing supports, including visual representations and manipulatives (Fuson, 2003).

Differentiated Instruction

Ample research has concluded that students find more success and satisfaction in school if they are taught in ways that are responsive to their readiness levels (e.g., Vygotsky, 1986), interests (e.g., Csikszentmihalyi, 1997), and learning profiles (e.g., Sternberg, Torff, & Grigorenko, 1998, as cited by Tomlinson, 2000). Differentiated instruction is how this translates into classroom

practice. Every classroom holds a wide range of learners. In most classrooms, some students struggle with learning, others perform well beyond grade-level expectations, and the rest fit somewhere in between. Within each of these categories of students, individuals also learn in a variety of ways and have different interests. To meet the needs of a diverse student population, many teachers differentiate instruction (Tomlinson, 2000).

Differentiated instruction involves varying one's teaching according to each learner in either (1) content, (2) instructional process, (3) students' products (e.g., papers, projects, computer models), and/or (4) learning environment (e.g., cooperative learning in small groups, grouped by ability) (Tomlinson, 2000). By definition, differentiated instruction always involves ongoing assessment linked to instructional decisions and planning (Tomlinson, 2000). Because differentiated instruction focuses on each learner's varying needs, it is especially well suited for special-needs students.

The quality of the curriculum and instruction used during differentiation is crucial. High quality curriculum focuses on what experts deem the most essential mathematical concepts and skills. High quality instruction incorporates lessons, tasks, and materials designed to ensure that students (1) grapple with essential concepts and skills, (2) find the learning experiences relevant and interesting, and (3) are engaged in active learning experiences (Tomlinson, 2000).

Research has also shown that flexible groupings can improve the mathematical achievement of special-needs students (Slavin, Madden, & Leavey, 1984, as cited by Mastropieri et al., 1991; Mastropieri et al., 1991; Secada, 1992; Slavin, Madden, Karweit, Livermon, & Dolan, 1990, as cited by Secada, 1992). Teachers can use flexible grouping to deliver a variety of differentiated learning environments in their classrooms, including small workgroups, cooperative learning groups, cross-grade groups, between-grade groups, grouping by ability for guided or independent practice, as well as whole class and individual practice settings (Tomlinson, 2000).

Furthermore, Burris, Heubert & Levin (2006) found that students who have completed advanced math courses increase in all heterogeneous grouped students including minority and low SES students. The same conclusion was reached for all students at whatever initial achievement level. Initial high achievers performed the same as counterparts in homogeneous groups. Rates of participation and test scores improved in all groups.

English Language Learners (ELL)

In his review of the research on how race, ethnicity, social class and language might affect student achievement in mathematics, Secada (1992) found a relationship between the amount of proficiency in a given language and mathematics achievement (Fernandez & Nielson, 1986, Duran, 1988, Secada, 1991b, as cited by Secada, 1992). To support academic achievement for non-native speakers of English and other diverse learners, Secada recommended:

- intervening early;
- providing ongoing extra support materials and strategies;
- using a student's native language for instruction;
- using a structured curriculum or focus teaching on basic skills;
- using small-group instruction, preferably in cooperative learning settings; and
- carefully grouping students by specific ability, if necessary (Secada, 1992).

According to an article by McElroy (2005), teachers need to expand their teaching tools to assist ELL students in content areas such as math. The article describes the website 'Colorin Colorado' (sic) that teachers can use to work with ELL students. Materials specific to ELL are presented along with teaching tips. While focused more on the language learning of ELL, several recommendations are made by Goldenberg (2006). The instructional practices seen as having a positive impact specific to math include:

- clear instructions and expectations;
- additional opportunities for practice; and,
- extended explanations.

Abedi (2004) reports on the difficulty of assessment of math and reading for ELL, especially as related to the No Child Left Behind (NCLB) act. Factors are presented as issues including the sparse ELL populations in some states, subgroup lack of stability, linguistic complexity of assessment tools and lower ELL baselines requiring greater gains. The implications are that unless these issues are considered, schools with large ELL populations face unfair and undue pressure under NCLB. In a similar vein, the American Federation of Teachers (AFT, 2004) point out that NCLB challenges faced by ELL students in math and reading include defining

ELL subgroups and the paucity of natural language assessment. The AFT identified four changes needed:

- appropriate tests for ELL students;
- relevant and valid testing of ELL students in English;
- clarifying assessment of proficiency versus math and reading skills; and,
- clarifying existing policies regarding ELL immigrant and non-immigrant groups.

How *Math Triumphs* Reflects the Research on Mathematical Proficiency for All Learners

The guides for the development of the *Math Triumphs* program are quite explicit and accurately reflect the research base. A summary of the strategies identified in the research for struggling learners in mathematics include:

- clear goals;
- vocabulary support;
- specific ELL support;
- word problems;
- sequencing;
- explicit instruction in various problem-solving strategies;
- extended explanations;
- multiple opportunities for practice;
- graphics and visuals;
- student reflection;
- cooperative learning;
- math conversation and discourse;
- enrichment;
- scaffolded questions;
- tiered questions;
- writing about math;
- feedback; and,
- dynamic diagnostic and prescriptive assessment.

Each chapter in the series begins with clearly stated goals. Key Concepts are presented at the beginning of each chapter with critical vocabulary highlighted. For example, in Lesson 1.1 (Grade 5), the key concept states that “Place value is the value assigned to each digit based on its position in the number.” The words *place value* and *expanded form* are highlighted and their meaning is explained in a vocabulary box to the side of the page. To accomplish the goals for English language learners (ELL), an English Learner Strategy box (Lesson 7.1, Grade 2 Teacher Guide) is included with teaching tips for these students. This is consistent for all chapters.

Word problems are given in a sequenced manner with graphic and visual support for all materials. For example, in Lesson 3.1 (Grade 5) the explanation of division of 8 by 2 (a word problem dividing eight pretzels between a student and friend) is sequenced in horizontal, vertical, and fraction method. Number boxes, sentences, and pictures are utilized. To help students better understand perimeter (Lesson 8.5, Grade 4), number sentences, sentences, and diagrams are shown.

Student reflection, cooperative learning, conversation and discourse are encouraged throughout the *Math Triumphs* chapters. In the Teacher Edition (Lesson 6.1, Grade 4), a strategy is given to divide students into pairs to create sets of cards to utilize various numbers demonstrating place value. To engage auditory and logical learners, a strategy is given to have students explain verbally how to find the quotient of division problems (Lesson 3.3, Grade 5 Teacher Guide).

Enrichment activities are given in all materials. In *Math Triumphs*, a Math Challenge box (Lesson 3.2, Grade 2 or Lesson 8.1, Grade 3) in each chapter provides puzzles and brain teasers for those seeking extra work.

Tiering and scaffolding of questions appear in all materials. A strategy in *Math Triumphs* asks students to work through and write answers to questions in Understand, Plan, Solve and Check. In a multiplication example (Lesson 2.3, Grade 4), students draw a diagram to help them understand and solve the word problem. In a geometry example (Lesson 8.3, Grade 3), students use pattern blocks to build a given figure.

Finally, diagnostic and prescriptive assessment and feedback are used extensively. A readiness quiz begins each chapter. As lessons are taught, practice questions are given to assess understanding and data-driven decision making. These questions cover both the math concepts and vocabulary. A Common Error Alert is included in the Teacher Edition (Lesson 5.6, Grade 1) to assist in instruction. In addition, a Spiral Review section (Grades 3 – 8) assesses learning along with a concluding progress check. For all lessons, additional examples provide alternative ideas for concept presentation.

In summary, the development of the materials is based, to a large extent, on the relevant and current literature in the area of mathematics instruction.

References

Abedi, J. (2004, January/February). The no child left behind act and English language learners: Assessment and accountability issues. *Educational Researcher*, 33 (1), 4-14.

Allsopp, D., Lovin, L., Green, G., & Savage-Davis, E. (2003). Why students with special needs have difficulty learning mathematics and what teachers can do to help. *Mathematics Teaching in the Middle School*, 8, 308-314.

American Federation of Teachers. (2004, March). English language learners and 'adequate yearly progress' calculations: A catch-22. Washington, D.C.: American Federation of Teachers.

Augustyniak, K., Murphy, J., & Phillips, D.K. (2005). Psychological perspectives in assessing mathematics learning needs. *Journal of Instructional Psychology*, 32 (4), 277-286.

Baroody, A.J. (1996). Self-invented addition strategies by children with mental retardation. *American Journal on Mental Retardation*, 10, 72-89.

Bray, N.W. & Turner, L.A. (1986). The rehearsal deficit hypothesis. In R. Ellis & N.W. Bray (Eds.), *International review of research in mental retardation* (Vol. 14, pp. 55-111). New York: Academic Press.

Burris, C.C., Heubert, J.P., & Levin, H.M. (Spring, 2006). Accelerating mathematics achievement using heterogeneous grouping. *American Educational Research Journal*. 43 (1), 105-136.

Butler, D.L., Beckingham, B., & Lauscher, H.J. (2005). Promoting strategic learning by eighth grade students struggling in mathematics: A report of three cases studies. *Learning Disabilities Research and Practice*, 20, 3, 156-174.

Cherkes-Julkowski, M. & Gertner, N. (1989). *Spontaneous cognitive processes in handicapped children*. New York: Springer-Verlag.

Csikszentmihalyi, M. (1997). *Finding flow: The psychology of engagement with everyday life*. New York: Basic Books.

Ferritti, R.P. (1989). Problem solving and strategy production in mentally retarded persons. *Research in Developmental Disabilities*, 10, 19-31.

Fuchs, L.S., Fuchs, D., & Hamlett, C.L. (2006). Extending responsiveness to intervention to math problem solving at third grade. *Teaching Exceptional Children*, 39 (4), 59-63.

Fuson, K.C. (2003). Developing mathematical power in whole number operations. In J. Kilpatrick, W.G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 68-94). Reston, VA: National Council of Teachers of Mathematics.

Goldenberg, C. (2006, April). *Improving achievement for English learners: Conclusions from 2 research reviews*. Retrieved on September 23, 2006 from: www.colorincolorado.org/articles/cgoldenberg_april06.php.

Kim, J.S. & Sunderman, G.L. (November, 2005). Measuring academic proficiency under the No Child Left Behind Act: Implications for educational equity. *Educational Researcher*, 34 (8), 3-13.

Lubienski, S.T. & Lubienski, C. (2006). School sector and academic achievement: A multilevel analysis of NAEP mathematics data. *American Educational Research Journal*, 43 (4), 651-698.

Mastropieri, M.A., Scraggs, T.E., & Shinh, S. (1991). Mathematics instruction for learning disabled students: A review of research. *Learning Disabilities Research and Practice*, 6 (2), 89-98.

Mazzocco, M.M. (2005). Challenges in identifying target skills for math disability screening and intervention. *Journal of Learning Disabilities*, 38 (4), 318-323.

McElroy, E.J. (2005, November/December). Teacher to teacher: Supporting English language learners. *Teaching pre K-8*, 36 (3), 8-9.

Secada, W.G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623-660). New York: Macmillan.

Seethaler, P.M. & Fuchs, L.S. (2005, May). A drop in the bucket: Randomized control trials testing reading and math interventions. *Learning Disabilities Research and Practice*, 20 (2), 98-102.

Smith, D.D. & Lovitt, T.C. (1975). The use of modeling techniques to influence the acquisition of computational arithmetic skills in learning disabled children. In E. Ramp, & G. Senf (Eds.), *Behavior analysis: Areas of research and application* (pp. 283-308). Hillsdale, NJ: Erlbaum.

Stinson, D.W. (2006). African-American male adolescents, schooling (and mathematics): Deficiency, rejection and achievement. *Review of Educational Research*, 76, 4, 477-506.

Tomlinson, C.A. (2000). *Differentiation of instruction in the elementary grades*. ERIC Digest. Champaign, IL: ERIC Clearinghouse on Elementary and Early Childhood Education (ERIC Document Reproduction Service No. ED443572).

Vygotsky, L. (1986). *Thought and language*. Cambridge, MA: MIT Press.

Zehr, M.A. (2006, March). "No child" effect on English-learners mulled: Teachers welcome attention, fault focus on test scores. *Education Week*, 25 (25), 14-15.