The Research Base for

California Math Triumphs

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This document is an extension of a paper developed in February of 2007 "Research Base of Effective Mathematics Instruction" by Dr. Rosemary Papa. That paper was an update of an earlier publication created by Interactive Educational Systems Design, Inc. They are responsible for all writing and references prior to 2004. The update incorporated new research findings and references through January 2007. This paper extends those studies to a Math Intervention series.

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The Research Base for *California Math Triumphs*

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Overview

The guide for the development of the *California Math Triumphs* mathematics intervention program states that the purpose of the program is to assist students who are two or more years below grade level in grades 4 to 7. The goal is to provide them with the skills to learn successfully and efficiently so that they can achieve with basic grade level materials. The program is intended as a course for students to accelerate their learning and be able to return successfully to the regular program. Assessment is diagnostic and imbedded so that the teacher can better monitor entrance and exit from the program. The writing guide lists the following key differences from the typical textbook:

- consumable volumes that allow for flexibility and personalized instruction;
- connections between concepts that reveal big ideas;
- truly differentiated instruction, not just differentiated examples;
- vocabulary instruction and English language support that goes beyond a mere list;
- presentation of small 'chunks' of content;
- numerous examples with different strategies;
- step-by-step exercises to walk through processes;
- communication practice—peer reviews, explanations, presentations, etc.; and,
- experiences that are engaging and motivating, including hands-on activities and assessment.

The sections that follow provide the research base for the two programs with specific examples as well as how the programs match that base and meet California Mathematics Intervention Standards.

Mathematical Proficiency for All Learners

Intervention

Before examining the results of studies themselves, it is useful to note that in the research regarding math intervention, Seethaler and Fuchs (2005) analyze the literature in terms of the efficacy of studies completed. They found that randomized, controlled designs were clearly underrepresented in the literature. They conclude that to truly assess efficacy, study

methodology may need to improve. In a related article, Augustyniak, Murphy and Phillips (2005) argue that the research on the definition of a math disability is lacking with respect to identification of core deficits. They identify the core areas needing further explanation as numerical skills, visual/spatial deficits, cognitive skill development (memory retrieval, working memory, speed of processing, attention regulation, problem-solving) and social cognition. Mazzocco (2005) reviewed research regarding practices of early identification and intervention for students with math difficulties. The commentary discusses the criteria and nature of math difficulties and notes the need for additional research.

The above being said, Butler, Beckingham, and Lauscher (2005) report on three case studies regarding the support of students with math learning challenges. Three eighth grade students were given assistance in self-regulating their learning. General strategies found to be successful include:

- engaging the students in constructive conversation;
- supporting students' reflection on their learning; and,
- the need for teachers to engage in dynamic, curriculum-based forms of assessment.

Fuchs, Fuchs, and Hamlett (2006) report on the validation of an intervention to improve math problem solving at the third grade level. The intervention (Hot Math) involved explicit instructions, self-regulation strategies, and tutoring. Results indicated positive short term results for problem-solving skills.

Stinson (2006) suggested that a focus on the discourse of achievement in mathematics rather than the discourses of deficiency and rejection could prove beneficial in reducing the well known achievment gap between white and black students. He suggests that the limited amount of research shows that enrichment activities, mentoring competent teachers, and black students identifying with the'good kids group' (p. 496), may enhance math achievement in African-American males.

Research also suggests a variety of instructional strategies that are effective to meet the needs of students with special needs—including those with physical disabilities, mental impairments, and/or learning disabilities; English Language Learners (ELL); and low-performing students

who require some special attention to bring out the best of their abilities. The research has found that effective instruction for special-needs students includes:

- setting clear goals for students (Bray & Turner, 1986, Cherkes-Julkowski & Gertner, 1989; Ferritti, 1989; Ferritti & Cavalier, 1991, as cited by Baroody, 1996; Schunk, 1985, as cited by Mastropieri, Scraggs, & Shinh, 1991);
- using a "big ideas" structure for concepts (Kameenui & Carnine, 1998, as cited by Fuson, 2003, p. 88);
- teaching content that is not too difficult (Bray & Turner, 1986, Cherkes-Julkowski & Gertner, 1989, Ferritti, 1989, Ferritti & Cavalier, 1991, as cited by Baroody, 1996; Baroody, 1996) and presented within meaningful contexts (Miller & Mercer, 1997, as cited by Allsopp, Lovin, Green, & Savage-Davis, 2003);
- laying ample groundwork by providing background knowledge (Bray & Turner, 1986, Cherkes-Julkowski & Gertner, 1989; Ferritti, 1989; Ferritti & Cavalier, 1991, as cited by Baroody, 1996; Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- modeling by teachers (Allsopp et al., 2003; Baroody, 1996; Blankenship, 1978, as cited by Mastropieri et al., 1991);
- sequencing instruction to go from the concrete to the abstract (Miller & Mercer, 1997, as cited by Allsopp et al., 2003);
- using mediated scaffolding (e.g., visual supports with cues, teachers' feedback on thinking, peer tutoring) (Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- discussing mathematics using language (Miller & Mercer, 1997, as cited by Allsopp et al., 2003);
- building in multiple practice opportunities (Miller & Mercer, 1997, as cited by Allsopp et al., 2003) and time for review by students (Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- using reinforcement (e.g., earning verbal praise) (Mastropieri et al., 1991); and,
- providing continual feedback (Miller & Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Blankenship, 1978; Schunk & Cox, 1986, as cited by Mastropieri et al., 1991).

Three of these elements of effective special needs instruction—modeling, mediated scaffolding, and feedback—are discussed in further detail below.

Modeling

Directly modeling both general problem-solving strategies and specific learning strategies using multisensory techniques has been shown to be useful with students having attention problems, cognitive-processing problems, memory problems, and metacognitive deficits, notes a summary of relevant research (Allsopp et al., 2003). A comparative study of 30 students suggests that direct modeling may be advantageous for students with slight mental retardation as well. One group of students who received direct modeling help (e.g., used blocks as physical manipulatives) and extra opportunities for purposeful practice employed "substantially fewer" inappropriate learning strategies than another group who did not receive such support (Baroody, 1996, pp. 81–82). Furthermore, it was found that one or two direct-modeling demonstrations enabled such students to correct basic arithmetic procedural strategies and improve their proficiency (Baroody, 1996).

In addition, a review and synthesis of 30 studies on mathematics instruction for learning-disabled students found that modeling and demonstration with corrective feedback improved problem-solving accuracy and generalization skills by the students (Blankenship, 1978, as cited by Mastropieri et al., 1991). For example, an instructional model in which teachers solved a problem, verbalized how they did it, and left the problem as a reference model improved learning-disabled students' computational skills in seven different experiments (Smith & Lovitt, 1975, Rivera & Smith, 1987, 1988, as cited by Mastropieri et al., 1991).

Mediated Scaffolding

A review of 30 studies found several types of scaffolding strategies to be effective in improving learning-disabled students' mathematical achievement in grades K–6 (Mastropieri et al., 1991):

- use of manipulatives teamed with pictorial representations (Peterson, Mercer, & O'Shea, 1988, as cited by Mastropieri et al., 1991);
- multisensory approaches—mixing visual, auditory, and/or kinesthetic methods. Verbalization (a specific practice in which teachers and students repeat aloud problems, instructions, and solution steps) also improved students' mathematical performance, found a comparative study of 90 such students in grades 6 through 8 (Schunk & Cox, 1986, as cited by Mastropieri et al., 1991); and,

- use of pre-organizers (e.g., read problem, underline numbers, decide on the operation sign and problem type) and/or post-organizers (e.g., read problem, check operation, check math statement, check calculations, write labels) to support students when solving word problems (Mastropieri et al., 1991).

Feedback

Ongoing feedback is crucial with special-needs students. Such students require continual monitoring and feedback on their efforts to be successful, several studies and meta-analyses have found (Miller & Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Schunk & Cox, 1986, as cited by Mastropieri et al., 1991).

Addressing Specific Mathematics Disabilities

A synopsis of relevant research noted that four different kinds of mathematics disabilities have been identified (Geary, 1994, as cited by Fuson, 2003). They, and what the research suggested as useful strategies to address them, are as follows:

- semantic memory disabilities: Students experience trouble with verbal and phonetic memory but may have normal visuospatial skills. Instruction that employs visual clues is most effective for these learners (Fuson, 2003);
- procedural deficits: Students use less advanced methods overall. Conceptually based instruction is especially helpful for these students (Fuson, 2003);
- visuospatial disabilities: Students struggle with concepts that use spatial relations, (e.g., place value). Instruction most helpful for these students includes extra cues to support visual processing, and focuses on methods that can be carried out in either direction (Fuson, 2003); and,
- problem-solving deficits: Such students benefit from problem-drawing supports, including visual representations and manipulatives (Fuson, 2003).

English Language Learners (ELL) and Special Needs Students

In his review of the research on how race, ethnicity, social class and language might affect student achievement in mathematics, Secada (1992) found a relationship between the amount of proficiency in a given language and mathematics achievement (Fernandez & Nielson, 1986,

Duran, 1988, Secada, 1991b, as cited by Secada, 1992). To support academic achievement for non-native speakers of English and other diverse learners, Secada recommended:

- intervening early;
- providing ongoing extra support materials and strategies;
- using a student's native language for instruction;
- using a structured curriculum or focus teaching on basic skills;
- using small-group instruction, preferably in cooperative learning settings; and
- carefully grouping students by specific ability, if necessary (Secada, 1992).

Ample research has concluded that students find more success and satisfaction in school if they are taught in ways that are responsive to their readiness levels (e.g., Vygotsky, 1986), interests (e.g., Csikszentmihalyi, 1997), and learning profiles (e.g., Sternberg, Torff, & Grigorenko, 1998) (as cited by Tomlinson, 2000). Differentiated instruction is how this translates into classroom practice. Every elementary classroom holds a wide range of learners. In most elementary classrooms, some students struggle with learning, others perform well beyond grade-level expectations, and the rest fit somewhere in between. Within each of these categories of students, individuals also learn in a variety of ways and have different interests. To meet the needs of a diverse student population, many teachers differentiate instruction (Tomlinson, 2000).

Differentiated instruction involves varying ones' teaching according to each learner in either (1) content, (2) instructional process, (3) students' products (e.g., papers, projects, computer models), and/or (4) learning environment (e.g., cooperative learning in small groups, grouped by ability) (Tomlinson, 2000). By definition, differentiated instruction always involves ongoing assessment linked to instructional decisions and planning (Tomlinson, 2000). Because differentiated instruction focuses on each learner's varying needs, it is especially well suited for special-needs students.

The quality of the curriculum and instruction used during differentiation is crucial. High quality curriculum focuses on what experts deem the most essential mathematical concepts and skills. High quality instruction incorporates lessons, tasks, and materials designed to ensure that students (1) grapple with essential concepts and skills, (2) find the learning experiences relevant and interesting, and (3) are engaged in active learning experiences (Tomlinson, 2000).

Research has also shown that flexible groupings can improve the mathematical achievement of special-needs students (Slavin, Madden, & Leavey, 1984, as cited by Mastropieri et al., 1991; Mastropieri et al., 1991; Secada, 1992; Slavin, Madden, Karweit, Livermon, & Dolan, 1990, as cited by Secada, 1992). Teachers can use flexible grouping to deliver a variety of differentiated learning environments in their classrooms, including small workgroups, cooperative learning groups, cross-grade groups, between-grade groups, grouping by ability for guided or independent practice, as well as whole class, and individual practice settings (Tomlinson, 2000).

Furthermore, Burris, Heubert & Levin (2006) found that students who have completed advanced math courses increase in all heterogeneous grouped students including minority and low SES students. The same conclusion was reached for all students at whatever initial achievement level. Initial high achievers performed the same as counterparts in homogeneous groups. Rates of participation and test scores improved in all groups.

According to an article by McElroy (2005), teachers need to expand their teaching tools to assist ELL students in content areas such as math. The article describes a website "Colorin Colorado" (sic) that teachers can use to work with ELL students. Material specific to ELL are presented along with teaching tips. While focused more on the language learning of ELL, several recommendations are made by Goldenberg (2006). The instructional practices seen as having a positive impact specific to math include:

- clear instructions and expectations;
- additional opportunities for practice; and,
- extended explanations.

In a well presented case, Abedi (2004) reports on the difficulty of assessment of math and reading for ELL, especially as related to the No Child Left Behind (NCLB) act. Factors are presented as issues including the sparse ELL populations in some states, subgroup lack of stability, linguistic complexity of assessment tools and lower ELL baselines requiring greater gains. The implications are that unless these issues are considered, schools with large ELL populations face unfair and undue pressure under NCLB. In a similar vein, the American Federation of Teachers (AFT, 2004) point out that NCLB challenges faced by ELL students

in math and reading include defining ELL subgroups and the paucity of natural language assessment. The AFT identified four changes needed:

- appropriate tests for ELL students;
- relevant and valid testing of ELL students in English;
- clarifying assessment of proficiency versus math and reading skills; and,
- clarifying existing policies regarding ELL immigrant and non-immigrant groups.

In yet another piece (Zehr, 2006), the changes brought by NCLB are debated as to their positive versus negative effects. The major point made is that many schools may be responding to NCLB in math and reading by narrowing the curriculum to focus only on test scores. With respect to the No Child Left Behind legislation, Kim & Sunderman (2005) suggets that use of the mean proficiency score may have a disparate effect on schools with low income children. Accountability rules for the sub groups can over identify racially diverse schools as failing to meet proficiency levels. The use of multiple indicators of performance may produce a more accurate assessment of student proficiency. Multiple measures could include a measure of improvement.

Much debate has surrounded the achievement issues in public versus private education. Charter schools, educational vouchers, etc., have been the responses of individuals and groups concerned about public school performance issues. Utilizing mathematics data from the 2003 NAEP data, Lubienski & Lubienski (2006) analyzed differences among students from public, charter and other types of private schools. They found that once demographic information was statistically controlled (ethnicity, gender and SES), no significant differences favoring private over public entities were found. In fact, public school fourth graders scored significantly higher than their private school counterparts. Their findings validate the importance of other in-school factors such as curriculum, teacher competence, etc.

How California Math Triumphs Reflects the Research on Mathematical Proficiency for All Learners

The guide for the development of the *California Math Triumphs* intervention program are quite explicit and accurately reflect the research base in terms of use by those developing the materials. A summary of the strategies identified in the research include:

- clear goals;
- vocabulary support;
- ELL methods;
- word problems;
- sequencing;
- graphics and visuals;
- student reflection;
- cooperative learning;
- math conversation and discourse;
- enrichment;
- scaffolded questions;
- tiered questions;
- writing about math;
- feedback; and,
- dynamic diagnostic and prescriptive assessment.

Each chapter in the series begins with clearly stated goals. 'Key Concepts' are presented at the beginning of each chapter with critical vocabulary highlighted. For example, in Chapter 2 (Volume 1A, Place Value), the key concept is "Place value is the value assigned to each digit based on its position in the number". The words, place value and digit, are highlighted and their meaning is explained in a 'Vocabulary' box to the side of the page. To accomplish the goals for English language learners (ELL), an 'English Learner Strategy' box is included with teaching tips for such students. This is consistent for all chapters.

Word problems are given in a sequenced manner with graphic and visual support for all materials. For example in Lesson 5.1 (Chapter 5, 1B), the explanation of division of 8 by 2 (a word problem dividing eight pretzels between a student and friend) is sequenced in a horizontal, vertical and fraction method. Number boxes, sentences and pictures are utilized.

Student reflection, cooperative learning, conversation, and discourse are encouraged throughout the *California Math Triumphs* chapters. In the Teacher Edition (Chapter 2, 1A), a strategy is given to divide students into small groups to create posters for discussion utilizing various numbers demonstrating place value.

Enrichment activities are given in all materials. In the *California Math Triumphs* series, a 'Math Challenge' box in each chapter provides puzzles and brain teasers for those seeking extra work.

Tiering and scaffolding of questions appear in all materials. A strategy in *California Math Triumphs* asks students to work through and write answers to questions to 'Understand, Plan, Solve and Check'. In a place value example (Chapter 2. 1A), students work with charts (hundreds, tens and ones) to make the greatest number value with the digits 3, 4, and 5.

Finally, diagnostic and prescriptive assessment and feedback are used extensively. A readiness quiz begins each chapter. As lessons are taught, practice questions are given to assess understanding and data-driven decision making. These questions cover both the math concepts and vocabulary. A 'Common Error Alert' is included in the Teacher Edition to assist in instruction. In addition, a 'Spiral Review' section assesses learning along with a concluding progress check. For all lessons, additional examples provide alternative ideas for concept presentation.

In summary, the development of the materials is based, to a large extent, on the relevant and current literature in the area of math instruction.

California Mathematics Intervention Standards

The California State Board of Education (California, 2006) developed the mathematics content standards to establish what they believe all students in California need to know with respect to mathematics. They were established to achieve six goals:

- develop fluency in basic computational skills;
- develop an understanding of mathematical concepts;
- become mathematical problem solvers who can recognize and solve routine problems readily and develop ways to reach a solution or goal where no routine path is apparent;
- communicate precisely about quantities, logical relationships and unknown values through the use of symbols, models, graphs and mathematical terms;

- reason mathematically by gathering data, analyzing evidence and building arguments to support hypotheses; and,
- make connections among mathematical ideas and between mathematics and other disciplines.

Volume I Place Value and Basic Number Skills

I-1. Counting
Number Sense (Grade One)
1.1 Count, read, and write whole numbers to 100.
1.2 Compare and order whole numbers to 100 by using the symbols for less than, equal to, or greater than (<, =, >).
1.3 Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (to 20) (e.g., 8 may be represented as 4 + 4, 5 + 3, 2 + 2 + 2 + 2, 10 − 2, 11 − 3).
1.4 Count and group objects in ones and tens (e.g., three groups of 10 and 4 equals 34, or 30 + 4).
2.1 Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory.
2.5 Show the meaning of addition (putting together, increasing) and subtraction (taking away, comparing, finding the difference).
Number Sense (Grade Two)
1.3 Order and compare whole numbers to 1,000 by using the symbols <,= , >.
3.1 Use repeated addition, arrays, and counting by multiples to do multiplication.
3.3 Know the multiplication tables of 2s, 5s, and 10s (to "times 10") and commit them to memory.
I-2. Place Value
Number Sense (Grade Two)
1.1 Count, read, and write whole numbers to 1,000 and identify the place value for each digit.
1.2 Use words, models, and expanded forms (e.g., 45 = 4 tens + 5) to represent numbers (to 1,000).
Number Sense (Grade Three)
1.3 Identify the place value for each digit in numbers to 10,000.
1.5 Use expanded notation to represent numbers (e.g., 3,206 = 3,000 + 200 + 6).
Number Sense (Grade Four)
1.1 Read and write whole numbers in the millions.
1.2 Order and compare whole numbers and decimals to two decimal places.

1.3 Round whole numbers through the millions to the nearest ten, hundred, thousand, ten thousand, or hundred thousand.
1.6 Write tenths and hundredths in decimal and fraction notations and know the fraction and decimal equivalents for halves and fourths (e.g., ½ = 0.5 or 0.50; 7/4 = 1 ¾ = 1.75).
I-3. Addition and Subtraction
Number Sense (Grade One)
2.1 Know the addition facts (sums to 20) and the corresponding subtraction facts and commit them to memory.
2.6 Solve addition and subtraction problems with one- and two-digit numbers (e.g., 5 + 58 = __).
2.7 Find the sum of three one-digit numbers.
Number Sense (Grade Two)
2.2 Find the sum or difference of two whole numbers up to three digits long.
2.3 Use mental arithmetic to find the sum or difference of two two-digit numbers.
Number Sense (Grade Three)
2.1 Find the sum or difference of two whole numbers between 0 and 10,000.
Number Sense (Grade Four)
3.1 Demonstrate an understanding of, and the ability to use, standard algorithms for the addition and subtraction of multidigit numbers.
I-4. Multiplication
Number Sense (Grade Three)
2.2 Memorize to automaticity the multiplication table for numbers between 1 and 10.
2.4 Solve simple problems involving multiplication of multidigit numbers by one-digit numbers (3,671 × 3 = __).
2.6 Understand the special properties of 0 and 1 in multiplication and division.
Number Sense (Grade Four)
4.1 Understand that many whole numbers break down in different ways (e.g., 12 = 4 × 3 = 2 × 6 = 2 × 2 × 3).
I-5. Division
Number Sense (Grade Four)
3.2 Demonstrate an understanding of, and the ability to use, standard algorithms for multiplying a multidigit number by a two-digit number and for dividing a multidigit number by a one-digit number; use relationships between them to simplify computations and to check results.

Volume II Fractions and Decimals

II-1. Parts of a Whole
Number Sense (Grade Two)
4.0 Students understand that fractions and decimals may refer to parts of a set and parts of a whole.
4.1 Recognize, name, and compare unit fractions from 1/12 to ½ .
4.3 Know that when all fractional parts are included, such as four-fourths, the result is equal to the whole and to one.
5.1 Solve problems using combinations of coins and bills.
5.2 Know and use the decimal notation and the dollar and cent symbols for money.
II-2. Equivalence of Fractions
Number Sense (Grade Three)
3.1 Compare fractions represented by drawings or concrete materials to show equivalency and to add and subtract simple fractions in context (e.g., ½ of a pizza is the same amount as 2/4 of another pizza that is the same size; show that 3/8 is larger than ¼).
3.2 Add and subtract simple fractions (e.g., determine that 1/8 + 3/8 is the same as ½).
Number Sense (Grade Four)
1.5 Explain different interpretations of fractions, for example, parts of a whole, parts of a set, and division of whole numbers by whole numbers; explain equivalence of fractions (see Standard 4.0).
Number Sense (Grade Five)
1.5 Identify and represent on a number line decimals, fractions, mixed numbers, and positive and negative integers.
Number Sense (Grade Six)
1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.
II-3. Operations on Fractions
Number Sense (Grade Five)
2.5 Compute and perform simple multiplication and division of fractions and apply these procedures to solving problems.
Number Sense (Grade Six)
2.1 Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
II-4. Decimal Operations
Number Sense (Grade Four)
2.0 Students extend their use and understanding of whole numbers to the addition and subtraction of simple decimals.

Number Sense (Grade Five)
2.0 Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.
Number Sense (Grade Six)
1.1 Compare and order positive and negative fractions, decimals, and mixed numbers and place them on the number line.
II-5. Positive and Negative Fractions and Decimals
Number Sense (Grade Two)
5.2 Know and use the decimal notation and the dollar and cent symbols for money.
Number Sense (Grade Three)
3.4 Know and understand that fractions and decimals are two different representations of the same concept (e.g., 50 cents is 1/2 of a dollar, 75 cents is ¾ of a dollar).
Number Sense (Grade Four)
1.6 Write tenths and hundredths in decimal and fraction notations and know the fraction and decimal equivalents for halves and fourths (e.g., ½ = 0.5 or 0.50; 7/4 = 1 ¾ = 1.75).
1.7 Write the fraction represented by a drawing of parts of a figure; represent a given fraction by using drawings; and relate a fraction to a simple decimal on a number line.
1.8 Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in "owing").
Number Sense (Grade Five)
2.1 Add, subtract, multiply, and divide with decimals; add with negative integers; subtract positive integers from negative integers; and verify the reasonableness of the results.
Number Sense (Grade Six)
2.3 Solve addition, subtraction, multiplication, and division problems, including those arising in concrete situations, that use positive and negative integers and combinations of these operations.
Number Sense (Grade Seven)
1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.

Volume III Ratios, Rates, and Percents

III-1. Ratio and Unit Conversion
Number Sense (Grade Three)
2.7 Determine the unit cost when given the total cost and number of units.
Algebra and Functions (Grade Three)
1.4 Express simple unit conversions in symbolic form (e.g., __ inches = __ feet × 12).

Measurement and Geometry (Grade Three)
1.4 Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).
III-2. Proportion and Percent
Algebra and Functions (Grade Three)
2.1 Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit).
2.2 Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).
Number Sense (Grade Five)
1.2 Interpret percents as a part of a hundred; find decimal and percent equivalents for common fractions and explain why they represent the same value; compute a given percent of a whole number.
Statistics, Data Analysis, and Probability (Grade Five)
1.3 Use fractions and percentages to compare data sets of different sizes.
Number Sense (Grade Six)
1.2 Interpret and use ratios in different contexts (e.g., batting averages, miles per hour) to show the relative sizes of two quantities, using appropriate notations (a/b, *a* to *b*, *a*:*b*).
1.3 Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$ find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.
1.4 Calculate given percentages of quantities and solve problems involving discounts at sales, interest earned, and tips.
III-3. Rates
Algebra and Functions (Grade Six)
2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).
2.2 Demonstrate an understanding that rate is a measure of one quantity per unit value of another quantity.
2.3 Solve problems involving rates, average speed, distance, and time.
Statistics, Data Analysis, and Probability (Grade Six)
3.3 Represent probabilities as ratios, proportions, decimals between 0 and 1, and percentages between 0 and 100 and verify that the probabilities computed are reasonable; know that if P is the probability of an event, $1\text{-}P$ is the probability of an event not occurring.

Algebra and Functions (Grade Seven)
4.2 Solve multistep problems involving rate, average speed, distance, and time or a direct variation.
Number Sense (Grade Seven)
1.6 Calculate the percentage of increases and decreases of a quantity.
1.7 Solve problems that involve discounts, markups, commissions, and profit and compute simple and compound interest.

Volume IV The Core Processes of Mathematics

IV-1. The Use of Symbols
Algebra and Functions (Grade Three)
1.0 Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple number relationships.
Algebra and Functions (Grade Four)
1.1 Use letters, boxes, or other symbols to stand for any number in simple expressions or equations (e.g., demonstrate an understanding and the use of the concept of a variable).
Algebra and Functions (Grade Five)
1.2 Use a letter to represent an unknown number; write and evaluate simple algebraic expressions in one variable by substitution.
Algebra and Functions (Grade Six)
1.2 Write and evaluate an algebraic expression for a given situation, using up to three variables.
Algebra and Functions (Grade Seven)
1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).
IV-2. Mathematical Fundamentals
Algebra and Functions (Grade Two)
1.1 Use the commutative and associative rules to simplify mental calculations and to check results.
Algebra and Functions (Grade Three)
1.5 Recognize and use the commutative and associative properties of multiplication (e.g., if $5 \times 7 = 35$, then what is 7×5? and if $5 \times 7 \times 3 = 105$, then what is $7 \times 3 \times 5$?).
Algebra and Functions (Grade Five)
1.3 Know and use the distributive property in equations and expressions with variables.

IV-3. Evaluating Expressions
Algebra and Functions (Grade Seven)
1.2 Use the correct order of operations to evaluate algebraic expressions such as $3(2x + 5)^2$.
1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.
IV-4. Equations and Inequalities
Algebra and Functions (Grade Four)
2.1 Know and understand that equals added to equals are equal.
2.2 Know and understand that equals multiplied by equals are equal.
Algebra and Functions (Grade Seven)
4.0 Students solve simple linear equations and inequalities over the rational numbers.
IV-5. Symbolic Computation
Number Sense (Grade Seven)
1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.

Volume V Functions and Equations

V-1. Functions
Algebra and Functions (Kindergarten)
1.1 Identify, sort, and classify objects by attribute and identify objects that do not belong to a particular group (e.g., all these balls are green, those are red).
Statistics, Data Analysis, and Probability (Grade One)
1.1 Sort objects and data by common attributes and describe the categories.
2.1 Describe, extend, and explain ways to get to a next element in simple repeating patterns (e.g., rhythmic, numeric, color, and shape).
Statistics, Data Analysis, and Probability (Grade Two)
2.1 Recognize, describe, and extend patterns and determine a next term in linear patterns (e.g., 4, 8, 12 . . .; the number of ears on one horse, two horses, three horses, four horses).
Algebra and Functions (Grade Three)
2.1 Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit).
2.2 Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).
Algebra and Functions (Grade Four)
1.5 Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.

Algebra and Functions (Grade Six)
2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).
V-2. Graphing
Statistics, Data Analysis, and Probability (Grade One)
1.2 Represent and compare data (e.g., largest, smallest, most often, least often) by using pictures, bar graphs, tally charts, and picture graphs.
Statistics, Data Analysis, and Probability (Grade Two)
1.1 Record numerical data in systematic ways, keeping track of what has been counted.
1.2 Represent the same data set in more than one way (e.g., bar graphs and charts with tallies).
Statistics, Data Analysis, and Probability (Grade Three)
1.3 Summarize and display the results of probability experiments in a clear and organized way (e.g., use a bar graph or a line plot).
Measurement and Geometry (Grade Four)
2.0 Students use two-dimensional coordinate grids to represent points and graph lines and simple figures.
2.1 Draw the points corresponding to linear relationships on graph paper (e.g., draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line).
Statistics, Data Analysis, and Probability (Grade Five)
1.4 Identify ordered pairs of data from a graph and interpret the meaning of the data in terms of the situation depicted by the graph.
1.5 Know how to write ordered pairs correctly; for example, (x, y).
V-3. Proportional Relationships
Algebra and Functions (Grade Three)
2.1 Solve simple problems involving a functional relationship between two quantities (e.g., find the total cost of multiple items given the cost per unit).
2.2 Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by 4s or by multiplying the number of horses by 4).
Number Sense (Grade Six)
1.3 Use proportions to solve problems (e.g., determine the value of N if $4/7 = N/21$ find the length of a side of a polygon similar to a known polygon). Use cross-multiplication as a method for solving such problems, understanding it as the multiplication of both sides of an equation by a multiplicative inverse.

V-4. The Relationship Between Graphs and Functions
Algebra and Functions (Grade Five)
1.5 Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.
Algebra and Functions (Grade Seven)
3.0 Students graph and interpret linear and some nonlinear functions.
3.1 Graph functions of the form $y = nx^2$ and $y = nx^3$ and use in solving problems.
3.3 Graph linear functions, noting that the vertical change (change in *y*-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of the graph.
3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.

Volume VI Measurement

VI-1. How Measurements Are Made
Algebra and Functions (Grade Three)
1.4 Express simple unit conversions in symbolic form (e.g., __ inches = __ feet × 12).
Measurement and Geometry (Grade Three)
1.4 Carry out simple unit conversions within a system of measurement (e.g., centimeters and meters, hours and minutes).
Algebra and Functions (Grade Six)
2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).
Measurement and Geometry (Grade Seven)
1.1 Compare weights, capacities, geometric measures, times, and temperatures within and between measurement systems (e.g., miles per hour and feet per second, cubic inches to cubic centimeters).
1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.
VI-2. Length and Area in the Real World
Measurement and Geometry (Grade Two)
1.3 Measure the length of an object to the nearest inch and/or centimeter.
Measurement and Geometry (Grade Three)
1.2 Estimate or determine the area and volume of solid figures by covering them with squares or by counting the number of cubes that would fill them.
1.3 Find the perimeter of a polygon with integer sides.

VI-3. Exact Measure in Geometry
Measurement and Geometry (Grade Four)
1.1 Measure the area of rectangular shapes by using appropriate units, such as square centimeter (cm^2), square meter (m^2), square kilometer (km^2), square inch ($in.^2$), square yard ($yd.^2$), or square mile ($mi.^2$).
2.2 Understand that the length of a horizontal line segment equals the difference of the *x*-coordinates.
2.3 Understand that the length of a vertical line segment equals the difference of the *y*-coordinates.
Measurement and Geometry (Grade Five)
1.1 Derive and use the formula for the area of a triangle and of a parallelogram by comparing each with the formula for the area of a rectangle (i.e., two of the same triangles make a parallelogram with twice the area; a parallelogram is compared with a rectangle of the same area by pasting and cutting a right triangle on the parallelogram).
1.2 Construct a cube and rectangular box from two-dimensional patterns and use these patterns to compute the surface area for these objects.
1.3 Understand the concept of volume and use the appropriate units in common measuring systems (i.e., cubic centimeter [cm^3], cubic meter [m^3], cubic inch [$in.^3$], cubic yard [$yd.^3$]) to compute the volume of rectangular solids.
VI-4. Angles and Circles
Measurement and Geometry (Grade Five)
2.1 Measure, identify, and draw angles, perpendicular and parallel lines, rectangles, and triangles by using appropriate tools (e.g., straightedge, ruler, compass, protractor, drawing software).
2.2 Know that the sum of the angles of any triangle is 180° and the sum of the angles of any quadrilateral is 360° and use this information to solve problems.
Measurement and Geometry (Grade Six)
1.2 Know common estimates of π (3.14, 22/7) and use these values to estimate and calculate the circumference and the area of circles; compare with actual measurements.
1.3 Know and use the formulas for the volume of triangular prisms and cylinders (area of base × height); compare these formulas and explain the similarity between them and the formula for the volume of a rectangular solid.
2.2 Use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.
Measurement and Geometry (Grade Seven)
3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.
3.4 Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

How California Math Triumphs Reflects the California Mathematics Intervention Standards

The linkage of the program to the California Standards is both clear and complete. Both the Student and Teacher Editions are explicit as to what standards are being met.

For example, each lesson begins with key concepts and the California State shaped icon in blue and gold identifying which standards are being addressed. In the Teacher Edition, a 'Chapter at a Glance' section outlines the entire chapter in terms of California Standards in each lesson along with a plainly written objective. For example, in Chapter 1 of Volume1, A, Standard 2NS1.1 is identified as key. That Standard, Grade 2, Number Sense refers to counting, reading and writing of whole numbers to 1000. The objective in plain terms is stated as count, read and model numbers to 1000. In Volume 1A, Chapter 5 (Division), two California Standards are noted, one from grade 3 (Number Sense 2.2) and one from grade 4 (Number sense 3.2). Since concepts drive the Math Intervention program, both standards are covered even if from different grade levels.

Volume 6, Chapter 1 presents the unique conceptual features of the program while covering triangles and quadrilaterals. The standards are presented contiguous with the California icon, but since the material is topic based, standards in both grades 5 and 7 are covered (5MG2.2, 5MG 2.2 and 7MR 1.1). This approach assures that students in the program get all material related to the standard as well as the necessary prerequisite and sequential material.

An outstanding additional feature in the program, which is strongly supported in the research literature, is the opportunity for practice questions directly related to the standard covered. This assures that students become familiar with the format utilized for subsequent testing.

In summary, the Student and Teacher Edition of the series provide a section on California Intervention Standards to be covered. There is no doubt as to what is being addressed are ubiquitously tied to the California Mathematics Standards.

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