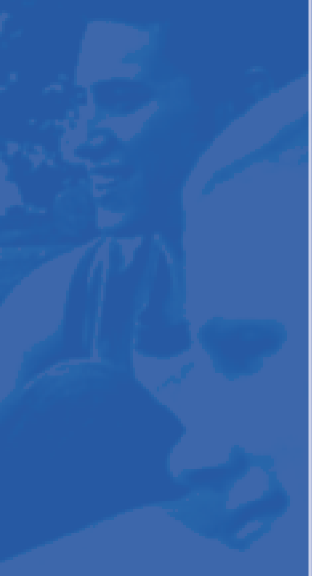

The Research Base for

IMPACT Mathematics

Macmillan/McGraw-Hill
Glencoe

The McGraw·Hill Companies

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The Research Base for *IMPACT Mathematics*

Introduction

In today's technology-driven society, mathematical skills and understanding are more important than ever. The need to use mathematics with fluency and comfort occurs daily—not just in the scientific and technical community, but in the workplace and in daily life. Those who understand and can use mathematics will have significantly enhanced opportunities and options for shaping their careers and futures. The past three decades have seen an increased recognition of the importance of mathematics for every student and the accompanying need for creating uniform national standards in mathematics education. The National Council of Teachers of Mathematics (NCTM) has led this reform movement from its beginning.

NCTM is the world's largest mathematics education organization, with more than 100,000 members and 250 affiliates throughout the United States and Canada. Between 1989 and 1995, NCTM released a trio of publications on curriculum and evaluation, assessment, and professional standards to articulate goals for mathematics teachers and policymakers. Since the release of these publications, they have given focus, organization, and fresh ideas to efforts to improve mathematics education.

History of the Mathematics Education Reform Movement

1989	National Council of Teachers of Mathematics (NCTM) publishes *Curriculum and Evaluation Standards for School Mathematics*
1991	NCTM publishes *Professional Standards for Teaching Mathematics*
1995	NCTM publishes *Assessment Standards for School Mathematics*
1995	NCTM appoints the Commission on the Future of Standards to oversee the *Standards 2000* project
1997 to 1999	The Standards 2000 Writing Group, with input from Association Review Groups, the NCTM Research Advisory Committee, the National Research Council, and more than 650 individuals and 70 groups, writes the *Principles and Standards for School Mathematics*
2000	NCTM publishes *Principles and Standards for School Mathematics*
2006	NCTM publishes *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*
2008	The National Math Advisory Panel presents its final report, *Foundations for Success,* with recommendations for mathematics education reform.

All *Standards* documents are available at *www.nctm.org*.

In 2000, NCTM released *Principles and Standards for School Mathematics.* This publication represents the culmination of five years of development by the NCTM's Commission on the Future of the Standards and their Standards 2000 Writing Group. The Standards 2000 Writing Group included teachers, teacher educators, administrators, researchers, and mathematicians with a wide range of expertise. The first draft was released in 1998. Over 650 individuals and more than 70 groups, including a committee of experts from the National Research Council, provided assistance and feedback. The final version of *Principles and Standards for School Mathematics* was released in 2000.

The *Principles* set forth important overall characteristics of mathematics programs and the *Standards* describe the mathematical content that students should learn. Together, *Principles and Standards for School Mathematics* constitute a vision to guide educators in the continual improvement of mathematics education in classrooms, schools, and educational systems.

The vision for mathematics education described in *Principles and Standards for School Mathematics* was highly ambitious. Achieving it required mathematics curricula based on the principles and standards; committed, competent, and knowledgeable teachers who could integrate instruction with assessment; and administrative policies that supported learning and access to technology.

In 2006, NCTM released a comprehensive project—*Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* (NCTM). Building on *Principals and Standards for School Mathematics* (NCTM, 2000), the National Council of Teachers of Mathematics offered a beginning point for a more focused and coherent mathematics curriculum.

The nine-member writing team compiled and examined state mathematics curricula as well as the curricula of other nations. The writing team also examined the writings of researchers to build a starting point for teachers to focus their efforts.

This approach moves away from goals, standards, expectations, and objectives and toward a smaller number of targeted grade-level concepts and skills. The concept of focal points is intended to be a launching point for the development of new assessment models and new curriculum for the teaching of mathematics.

The National Mathematics Advisory Panel presented in their final report, *Foundations for Success*, that there is a need for coherence. In general the curriculum in the United States reviews and extends mathematical concepts already presented at earlier grade levels. In contrast, curriculum in other countries is more likely to present fewer topics at a greater depth (The National Mathematics Advisory Panel, 2008).

Reaching the NCTM *Curriculum Focal Points* with *IMPACT Mathematics*

Prekindergarten through Grade 8 Mathematics: A Quest for Coherence promotes high expectations for students' learning, effective methods of supporting the learning of mathematics by all students, and sufficient and appropriate focused curricula for students and teachers. A school's or district's choice of math curriculum can determine whether students meet mathematics standards.

Glencoe/McGraw-Hill, one of the nation's largest textbook developers, has integrated the focus of *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* with the *IMPACT Mathematics* series. The grade-level curriculum focal points are listed in each Teacher Edition on pages vi–vii for reference. This series was designed as a coherent approach to mathematics using the following:

- Different topical strands such as algebra, geometry, numbers and operations, and data and probability that are highly interconnected;
- Central mathematical ideas that are organized and integrated so that students can see how the ideas build on, or connect with, other ideas;
- Foundational ideas such as fractions, decimals, percentages, exponents, ratios and proportions;
- Activities to develop mathematical thinking and reasoning skills, including making conjectures and developing sound deductive arguments;
- Opportunities for experiences that demonstrate mathematics' usefulness in modeling and predicting real-world phenomena;
- Guidance for teachers on the depth of study warranted at particular times and when closure is expected for particular skills or concepts; and
- Emphasis on the mathematics processes and skills that support the quantitative literacy of students, such as judging claims, finding fallacies, evaluating risks, and weighing evidence.

The *IMPACT Mathematics* program was also designed to meet all six of the principles set forth in *Principles and Standards for School Mathematics* and five of the mathematical processes that reinforce each set of three focal points for each grade level:

- **Equity** *IMPACT Mathematics* encourages high student achievement. Teacher support materials provide activities for differentiated instruction, pacing for individual levels of achievement, and daily intervention opportunities. For example, in the Teacher Guide, *Reaching All Learners* sections give teachers suggestions on how to tailor instruction to meet the diverse needs of students. Each *Differentiated Instruction* page in the Teacher Guide outlines different problem-solving techniques students may use depending on their individual learning styles, and *Additional Examples* provide extra practice problems for struggling students. *IMPACT Mathematics* also provides a *Classroom Manipulative Kit*, which is an excellent tool to use with kinesthetic learners.

- **Curriculum** In developing *IMPACT Mathematics* we have relied on our collective experiences as teachers, parents, and former students. Our main goal is to offer a curriculum that respects the background and knowledge of middle school teachers, recognizes the competence and energy of middle school students, and addresses the need for intellectually challenging and inclusive mathematics materials. With *IMPACT Mathematics*, we have combined the best of what is known as "reform" curricula with the best of "traditional" curricula, incorporating more active involvement on the part of students in making sense of important mathematical ideas.

  With middle grades teachers and students in mind, we have created a comprehensive curriculum for Grades 6 through 8 that completes a full year of algebra by the end of Grade 8. While the number and operations, geometry, and data and probability strands were created especially for this program, the algebra strand is based on the highly successful Australian program, *Access to Algebra*, developed by Curriculum Corporation.

  The rewarding and interesting introduction to algebra offered by this program can help develop and maintain students' ongoing interest in all areas of mathematics. The materials created for *IMPACT Mathematics* follow the *Access to Algebra* material in style: use of narrative and realistic contexts, personalization in the form of cartoons in which middle grades students explain how they approach problems, and opportunities for students to choose or create their own problems.

- **Teaching** Glencoe recognizes that the teacher plays a vital role in students' academic achievement. Each course of *IMPACT Mathematics* includes a two-volume Teacher's Guide with corresponding volumes of *Teaching Resources* and *Assessment Resources,* providing in-depth teacher support. At the beginning of each chapter, the Teacher's Guide contains a *Chapter Organizer* that provides all chapter-planning information in a convenient and easy-to-use format. Included in the *Chapter Organizer* are: *The Big Picture,* which summarizes the chapter goals and illustrates how the chapter connects to others across the curriculum; a *Planning Guide* that provides starting points for developing weekly lesson plans with lesson objectives, pacing guidelines, and materials needed; *Assessment Opportunities and Assessment Resources,* including standard assessment, ongoing assessment, and alternative assessment, portfolios, and journaling ideas; *Additional Resources*; and a *Mathematical Background* section that provides in-depth explanations of key mathematical concepts for the chapter. The Teacher's Guide also provides the following within each lesson: a *Lesson Planner* that contains a summary of each investigation along with materials needed and assessment opportunities; *Teacher Tips* that give classroom-tested "tricks of the trade" from experienced teachers; *Reaching All Learners* sections that provide ideas for adjusting instruction for both advanced and less prepared students; and *Lesson Notes*.
- **Learning** *IMPACT Mathematics* is designed to actively engage students in their own learning. To facilitate the learning and teaching process, *IMPACT Mathematics* is designed around a three-step instructional cycle—*Introduce, Develop & Understand*, and *Assign & Assess*. For the first step, *Introduce*, each multiday lesson begins with a class discussion, activity, or problem, such as the *Explore* activities or *Think & Discuss* questions provided in the student materials, to introduce the mathematics and help set a context for learning. For the second step, *Develop & Understand*, each *IMPACT Mathematics* lesson is composed of in-class *Investigations* that provide a mix of worked-out examples, direct modeling through cartoons, and interactive problem sets. *Investigations* engage students in mathematical tasks that will lead to their understanding of important ideas. Each *Investigation* includes carefully sequenced problem sets for developing mathematical ideas. *Share & Summarize* questions at the end of each *Investigation* offer students an opportunity to share what they did and what they learned. In the final step, *Assign & Assess, On Your Own Exercises* at the end of each lesson include ample opportunity for students to practice and apply the

ideas learned in the lesson. They also include exercises that allow students to use their knowledge in new situations and exercises that help student skills stay sharp with mixed review problems.

- **Assessment** The assessment tools in *IMPACT Mathematics* are broader than those in traditional mathematics programs. They include a variety of assessment vehicles to inform instruction, measure student success, and accommodate the various ability levels and learning styles of the students. *IMPACT Mathematics* assesses problem-solving, reasoning, communication, connections, concepts, applications, representational strategies, and procedures. Assessment opportunities with *IMPACT Mathematics* include standard assessment, ongoing assessment, and alternative assessment. Standard assessment options include a *Review and Self-Assessment* for each chapter of the student text, *Quick Quizzes* in the teacher's guide for each lesson, and two forms of a chapter test in the *Assessment Resources* book. Ongoing assessment options include *Share & Summarize* tasks in the student text, *On the Spot Assessment* suggestions in the teaching notes, and informal assessment forms in the *Assessment Resources* book. Alternative assessment options include *In Your Own Words* items in the student text and *Performance-Based Assessment* included in the *Assessment Resources*, which are perfect for inclusion in student journals or portfolios.

  An NSF grant (National Science Foundation grant #ESI-9726403) has supported the many years of research, development, and evaluation that form the basis of the high-quality, performance-based assessments that comprise the assessment section of *IMPACT Mathematics*. Each assessment task is carefully constructed to assess the broad domain of mathematical performance that national and local standards specify. Tasks go through a development and review process to ensure validity and usability for student evaluation and continued improvement in instruction.

- **Technology** *IMPACT Mathematics* encourages student involvement. *Lab Investigations* extend explorations through the use of computer software. The *Virtual Activities for Middle School Mathematics* CD-ROM provides interactive computer-based learning opportunities for students. The *What's Math Got To Do With It? Real Life Videos* share real-life examples of mathematical concepts in a video format. Research on instructional software has generally shown positive effects on students' achievement in mathematics as compared with instruction that does not incorporate such technology (National Mathematics Advisory Panel, 2008).

The *IMPACT Mathematics* program was also designed to meet all of the standards in ten content areas from *Principles and Standards for School Mathematics*.

Research-Based Instructional Strategies Used in *IMPACT Mathematics*

IMPACT Mathematics not only responds to the goals set by *Principles and Standards in School Mathematics* and *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence*, but it also uses the latest research on best practices in mathematics education. Educational research serves as a basis for many of the assertions throughout *IMPACT Mathematics* about what is possible for students to learn about certain content areas at certain levels and under certain pedagogical conditions.

IMPACT Mathematics utilizes several research-based instructional strategies that reinforce the five mathematical processes that support the grade-level focal points:

1) **Problem Solving** Research shows us that teachers cannot simply transfer knowledge to students by lecturing. Students must take an active role in their own learning. Mathematics programs must therefore include ample opportunity for students to explore, question, discuss and discover. Teachers are not removed from the educational process; the learning experience includes a balance of implicit and explicit instruction. *Implicit instruction* occurs when students grapple with problems and construct conceptual knowledge (Shulman & Keislar, 1996; Pressley, Harris, & Marks, 1992). *Explicit instruction* occurs when teachers and textbooks clearly explain problem-solving strategies to students in a direct, low-inference fashion (Duffy, 2002).

   Problem solving means engaging in a task for which the method of solution is not known in advance. Students draw on their knowledge and past experience to find a solution. Problem solving is a process; it is not simply carrying out an algorithm. Genuine problem solving is not a linear, step-by-step process; it is dynamic and cyclic (James et al., 1993).

Problem solving is an integral part of mathematics. Application of mathematics in other disciplines requires problem solving. It can stimulate student interest in mathematics. Problem solving helps students build mathematical understanding.

Good problem solvers

- have developed an organized base of mathematical knowledge (Lester & Kehle, 2003; Schoenfeld & Herrmann, 1982),
- move flexibly among steps in the process as they work through a problem (Kroll & Miller, 1993),0
- constantly monitor their understanding and adjust their strategies as they work (this is called *metacognition)* (Bransford et al., 1999; Garofalo & Lester, 1985; Schoenfeld, 1985), and
- focus on structural features, not superficial features, of the problem (Lester & Kehle, 2003).

Important differences between successful and unsuccessful problem solvers lie in their beliefs about problem solving and about their ability in problem solving (Kroll & Miller, 1993). Students may have mistaken or unhelpful beliefs, attitudes, feelings, or dispositions toward problem solving (Lesh & Zawojewski, 2007).

IMPACT Mathematics offers a balance of real-world applications, hands-on exercises, direct instruction, written exercises, and practice that enables students to develop both conceptual understanding and procedural knowledge. Lessons often begin with class discussions, activities and/or real-world problems for students to solve; then students use multiple representations to explore new concepts. Implicit instruction techniques in the *Explore* activities and the *Think & Discuss* questions in student materials provide ample opportunity for student exploration and learning. *On Your Own Exercises* for students reinforce their learning. There is an explicit instruction component in each chapter of *IMPACT Mathematics*. All lessons and investigations in the program include detailed and concise explanations of mathematical concepts and problem-solving strategies in conjunction with teacher resources to strengthen explicit instruction.

2) **Reasoning** Curriculum is a blueprint for guiding student acquisition of concepts, procedures (skills), dispositions, and ways of reasoning (Clements, 2007). A curriculum that enables students to reach high mathematics standards is

- balanced—focuses on conceptual understanding and procedural fluency;
- comprehensive—includes all the important content strands of mathematics as well as computation and other procedural skills;
- aligned with state- and national-level standards, external assessments, and instruction; and
- coordinated and coherent within and across grades—ideas are well developed and build on or connect with other ideas within and across grades (Apthorp et al., 2001).

Students compare things, determining and describing how two or more objects or processes are alike and/or different. They classify things, grouping things that are alike into categories based on characteristics. The mental operations of comparing and classifying are central to thinking (Marzano et al., 2001). These skills are components of analysis. They are essential in problem solving to determine how the current problem and previously solved problems are alike and different. Teachers must explicitly guide students to identify similarities and differences, beginning with concrete examples and moving toward abstract knowledge (Marzano et al., 2001; Reeves & Weisberg, 1993).

Teachers must listen to students' reasoning and be able to make sense of students' mathematical thinking (Mewborn, 2003; Thompson & Thompson, 1994, 1996), providing corrective feedback as needed to help students correct misconceptions. Teachers should explain to students what they are doing incorrectly and what they are doing correctly. Having students continue working on a task until they succeed also increases achievement (Marzano et al., 2001; Bangert-Drowns et al., 1991; Crooks, 1988).

IMPACT Mathematics was developed to be a balanced and comprehensive curriculum that is coordinated within and across grades. This program allows students to develop their reasoning skills by providing numerous opportunities to analyze, solve problems, and continue working on a task or answering a question until students have succeeded.

3) **Communication** Discourse, reading, and writing are three of the major activities through which students acquire mathematical knowledge. Becoming fluent in the language of mathematics requires unique knowledge and skills, since mathematical writing is dense, terse, and comprised of symbols as well as words (Barton & Heidema, 2002).

Discourse Discourse means students talking about mathematics—explaining their ideas, justifying their methods, evaluating others' methods, and posing questions—in order to strengthen their understanding. This talk can take place between students or between student and teacher. Discourse, like reading and writing, means using mathematical language, which is the foundation of mathematical reasoning. Discourse is part of communicating mathematics, and it is a critical classroom practice along with establishing norms and building relationships (Franke et al., 2007).

Discourse (like writing) helps students make sense of mathematics. Orally explaining, summarizing, and conjecturing helps students clarify their thinking as well as communicate it to students and teachers. Discourse is how students, teachers, and subject matter interact (Franke et al., 2007). Discourse contributes to a sense of community of learners where everyone is both teaching and learning (Whitin & Whitin, 2000). It also provides teachers with a window into student understanding (Franke et al., 2007).

The ways in which teachers talk about concrete situations and mathematical relationships make a significant difference in whether students can understand the concepts beyond the concrete examples (McNair, 1998). Questioning and discussion should focus on students' thinking and strategies, building on them to reach greater clarity and precision (Kilpatrick et al., 2001). Discourse should include discussion of connections to other problems, alternative representations and methods, and the nature of mathematical justification (Kilpatrick et al., 2001).

Whole-class discussion can be enhanced by using three strategies—*explain* your reasons, *build* upon others' ideas, and *go beyond* a specific example to a general case (Sherin, Mendez, & Louis, 2000). Talking helps students move from concrete objects to related mathematical concepts (this process is called *mathematizing*). This abstracting process is central to mathematical learning (Lampert & Cobb, 2003; Sfard, 2000).

Reading Reading helps students understand concepts, work problems, organize ideas, extend their thinking, and view mathematics as a valuable subject (Martinez & Martinez, 2001). Reading mathematics means making sense of everything that is on a page in a mathematics textbook, worksheet, or computer screen (Barton & Heidema, 2002). Instructional strategies can help students become more proficient in reading mathematics. Reading is one of the major activities through which students acquire mathematical knowledge. Reliance on reading increases as grade level increases. Mastering mathematical vocabulary is key and has been shown to increase achievement in arithmetic and problem solving (Helwig, Rozek-Tedesco, Tindal, Heath, & Almond, 1999; Earp & Tanner, 1980).

Mathematics text should be read with pencil and paper—working problems, drawing graphs or diagrams, and noting questions. Students should pause at designated points in the reading for discussion, sharing, and clarification of concepts (Reehm & Long, 1996). Teachers should model for students how to monitor and reflect on their reading. Modeling by thinking aloud is effective (Barton & Heidema, 2002; Reehm & Long, 1996).

Teachers can model reading strategies by

- reading a problem aloud, paraphrasing it, and talking through how to figure out word meanings,
- by asking if everyone is clear on a word's meaning, and
- by asking questions about the meaning of the problem (Kenney et al., 2005).

Reading word problems requires special skills: understanding the problem situation, identifying the main idea, and extracting relevant information (Musthafa, 1996).

Vocabulary research has shown that

- vocabulary should be taught both directly and indirectly (National Reading Panel, 2000; White, Graves, & Slater, 1990),
- repetition and multiple exposures to vocabulary words are effective (National Reading Panel, 2000; Leung, 1992),

- learning vocabulary in context is valuable (National Reading Panel, 2000), and
- computer technology can help students learn vocabulary (National Reading Panel, 2000; Davidson, Elcock, & Noyes, 1996).

Writing Writing is an important means of communication. It is also effective in helping students learn mathematical concepts (Abel & Abel, 1988). Frequent, regular writing in mathematics class has been linked to improved quality of writing, improved student attitudes toward mathematics, and teachers' increased insight into their teaching (Miller & England, 1989).

Writing, along with reading and discourse, means using mathematical language, which is the foundation of mathematical reasoning. Writing makes a record of mathematical thinking. It includes writing study notes, vocabulary definitions, explanations, descriptions, predictions, and justifications, along with attitudes, confusions, and questions. Writing can take the form of notes, outlines, answers to exercises, journal entries, short writing assignments, or longer project reports.

Writing helps students make sense of mathematics (Countryman, 1992). The thinking skills involved in writing are major aspects of doing mathematics. Through writing, students clarify and organize their thinking. Writing is an effective medium for learning mathematics and should be an integral part of every mathematics class (Abel & Abel, 1988). Writing reinforces what students have learned, helping to retain it in long-term memory.

Writing is effective in helping learn mathematical concepts. Writing by students gives teachers valuable information on student learning (Powell, 1997). When students write explanations about how to solve numerical problems, they are doing much more than acquiring content and demonstrating mastery of a benchmark—they are communicating and problem solving (Urquhart & McIver, 2005).

Opportunities for students to practice and enhance their communication skills are present throughout the *IMPACT Mathematics* program. Students are encouraged to write about mathematical ideas in the *In Your Own Words* features. Through these activities, students describe the concepts covered within the book using their own real-life experiences. These activities also encourage creativity and expression while reinforcing the lesson concepts. Reading development is focused on by special attention to mathematics vocabulary; sections highlight relevant terms and concepts in context in a recognizable format throughout the text. The worked out Examples in *IMPACT Mathematics*, which introduce problem-solving techniques in a step-by-step manner, also strengthen reading skills and help further conceptual knowledge.

4) **Making Connections** Activation of prior knowledge through cues or questions is critical to learning. Asking students questions before a learning activity helps them process information (Pressley et al., 1992). Questioning and cueing account for a large percentage of classroom activity. Teachers often underestimate the number of questions they ask (Marzano et al., 2001). Questions and cues are teaching strategies to help students recall and use what they already know about a topic. Cues are hints about what students will do or learn, such as a brief description of a hands-on lab activity before students begin the lab. Questions help students focus on what they already know and are part of classroom discourse. Teachers should use higher level questions rather than lower level questions. Questions that require students to analyze or apply information produce more learning than questions that ask students to simply recall or recognize information (Marzano et al., 2001; Redfield & Rousseau, 1981). Teachers need to fully understand the concepts that make up mathematics and underlie mathematical procedures. They need to understand the interconnections among topics (Mewborn, 2003; Swafford, Jones, & Thornton, 1997).

   IMPACT Mathematics intertwines concepts and continuously refers to material in previous chapters and in students' personal experiences to make mathematics more relevant. Each multi-day lesson in *IMPACT Mathematics* begins with a class discussion, activity, or problem designed to introduce the mathematics and provide a context for learning. To help guide the introduction, *Explore* activities and *Think & Discuss* questions provided in the student materials remind students of what they already know about the upcoming chapter and give cues to what they will be learning about. In the Teacher Guide, a *Big Picture* chart at the beginning of each chapter shows how the main ideas in a particular chapter connect to past and future learning. *Connect & Extend* problems show where the lesson concepts are related to other mathematical topics and strands presented previously or preview what is to come. *Remember* markers in the student edition remind students about information they learned in previous lessons. *Math Link* and *Real-World Link* segments engage students with interesting real-life facts and examples of the mathematical concepts presented in the text, helping make the concepts more relevant to them.

5) **Designing and Analyzing Representations** Representation in mathematics means an arrangement of symbols, alphanumeric characters, icons, or objects that stand for something else (Goldin, 2003). Manipulatives are physical objects that represent mathematical quantities. Drawings include student-made drawings and textbook drawings of problem situations. Diagrams can be more abstract than drawings and show relationships, such as Venn diagrams. Representations help students move from concrete to more generalized (abstract) thinking. Students' creation of or interaction with external representations can help teachers assess students' internal understanding. Graphic representations are a means of communication.

Graphs are visual representations of data, functions, or relationships. Graphs communicate mathematical ideas. Even more importantly, they are tools for building mathematical understanding.

Symbols include numeric, alphabetic, and graphic elements, for example, the decimal number system, fraction notation, variables, function notation, and the square root symbol. Symbols play a crucial role in mathematics. They facilitate communication of mathematical ideas and are central to the process of generalization.

Visuals—such as complex diagrams and elaborate drawings—used in conjunction with verbal description increase students' chances of learning, understanding, and remembering relationships and properties of mathematics concepts. Student-created representations can help teachers understand individual student thinking, be instructional tools to illustrate various solution methods, and provide information on patterns of understanding within a group of students (Kalathil & Sherin, 2000). Visuals are often the only way to effectively communicate ideas that explain central concepts needed to understand algebra and geometry. Research shows that students are better able to organize and group ideas when visuals illustrate different and common characteristics (Hegarty, Carpenter, & Just, 1991). Also, the mental images that high-quality visuals encourage are an indispensable tool for recalling information, especially compared to information presented with only text or lower-quality visuals (Willows & Houghton, 1987). Long-term use of manipulatives by teachers knowledgeable in their use increases student achievement and improves student attitudes toward mathematics (Sowell, 1989). Graphs used to communicate need to be precise and clear; graphs created to build meaning may be creative and may include ambiguity (Monk, 2003). Graph-plotting technology helps students shift attention from the process of making graphs to analyzing and manipulating the graph itself (Yerushalmy & Schwarz, 1993).

IMPACT Mathematics includes high-quality charts, tables, graphs, art, and photographs throughout the text. Visuals are often accompanied by caption questions and ideas for effective use of models and manipulatives. Cartoons are also frequently used to model mathematical reasoning and problem-solving strategies in a format that is understandable and interesting for middle school students.

Manipulatives and calculators can be powerful tools for teaching and learning mathematics. There is, however, much discussion and controversy about the appropriateness of their use. In *IMPACT Mathematics*, manipulatives and calculators are used to support content learning. More specifically, *IMPACT Mathematics* considers the important mathematical ideas first and then determines whether manipulatives or calculators can be used in learning those ideas more completely.

Summary

Glencoe/McGraw-Hill is committed to the idea that curricula should strive to reach all of *Principles and Standards for School Mathematics*, thereby providing road maps that help teachers guide students to increasing levels of sophistication and depths of knowledge. The NCTM *Principles and Standards for School Mathematics* were developed to accomplish several goals, including guiding the development of curriculum frameworks, assessments, and other instructional materials. In addition, *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* provides a focused, grade-level specific list of targeted concepts and skills for educators to use as a launching point in the teaching of mathematics. Attaining the vision of *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* will require the talents, energy, and attention of many individuals, including students, teachers, school administrators, policymakers, teacher educators, parents, mathematicians, local communities, and curriculum developers. Glencoe is proud to provide the *IMPACT Mathematics* program as an informed road map to excellence in mathematics education in the 21st century.

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