◆ Basic Fractions

## Overview of Skill Development

Concept of Fractions

Basic Fractions teaches what the numbers in a fraction tell. The bottom number tells how many parts in each whole, and the top number tells how many parts are used.

In the fraction $\frac{3}{4}$, there are 4 parts in each whole, and 3 parts are used.

Students are shown how to apply these rules in exercises that require them to fill in the numbers to represent a picture and in exercises that require them to write a diagram that represents a numerical fraction. Exercises include diagrams showing fractions that
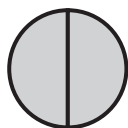
◆ equal less than 1 whole
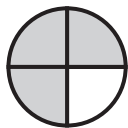
$$\frac{3}{4} \qquad \frac{2}{3}$$
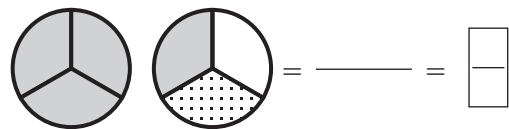
◆ equal 1 whole

$$\frac{2}{2} \qquad \frac{5}{5}$$

◆ equal more than 1 whole

$$\frac{5}{3}$$

Operations on Fractions

The modules present visual demonstrations of what happens when fractions are added. The initial exercises presenting adding fractions with like denominators show diagrams like this:

$$= \frac{}{} = \frac{}{}$$

The students write 3 on the bottom because there are 3 parts in each whole. Then they write the 4 + 1 on the top. The 4 represents the 4 shaded parts, and the 1 represents the dotted part. The students then add the 4 + 1 and write 5. The solved problem looks like this:

$$= \frac{4 + 1}{3} = \boxed{\frac{5}{3}}$$
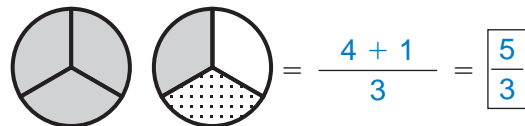
There are no diagrams in the next exercise. The students are shown problems such as this:

$$\frac{3}{4} + \frac{2}{4} = \frac{}{} = \frac{}{}$$

The student writes

$$\frac{3}{4} + \frac{2}{4} = \frac{3 + 2}{4} = \boxed{\frac{5}{4}}$$

Worksheet exercises include several addition and subtraction problems with like denominators and some problems with unlike denominators. In *Basic Fractions,* students simply cross out the problems with unlike denominators because they cannot add or subtract when the fractions do not have the same number of parts in each whole. Addition and subtraction of fractions with unlike denominators are taught in *Fractions, Decimals, and Percents.*

After students have had a great deal of practice adding and subtracting fractions, problems requiring multiplication are introduced. Extensive practice is provided to enable students to master the difference between the procedure to solve multiplication problems (top times top and bottom times bottom) and the procedures used to solve addition and subtraction problems. Delaying the introduction of multiplying fractions until students have practiced adding and subtracting fractions decreases the chances of students becoming confused.

$$\frac{4}{5} \times \frac{3}{4} = \frac{}{}$$

$$\frac{4}{5} \times \frac{3}{4} = \frac{12}{}$$

$$\frac{4}{5} \times \frac{3}{4} = \frac{12}{20}$$

Then *Basic Fractions* teaches students how to convert a whole number to a fraction by writing a denominator of 1. You make 9 into a fraction by writing 9 over 1:

$$\frac{9}{1}$$

This critical component skill is practiced before problems that require multiplying a whole number and a fraction are presented. When this type of problem is presented, students are taught to convert the whole number to a fraction and then to multiply top times top and bottom times bottom. This example is from Lesson 27.

EXERCISE 2

Multiplication

a. (Write on the board:)

$$3 \times \frac{5}{7} = \underline{} \qquad \frac{2}{3} \times 8 = \underline{}$$

b. (Point to the 3 in $3 \times \frac{5}{7}$)

- Let's change the number into a fraction so that we can multiply. Tell me how to write this number as a fraction. (Signal.) *3 over 1.*
- (Change 3 to $\frac{3}{1}$.)
- Now we can multiply. What's the answer for the top? (Signal.) *15.*
- (Write to show:)

$$\frac{3}{1} \times \frac{5}{7} = \frac{\boxed{15}}{}$$

- What's the answer for the bottom? (Signal.) *7.*
- (Write to show:)

$$\frac{3}{1} \times \frac{5}{7} = \frac{15}{\boxed{7}}$$

- What's the answer to the problem? (Signal.) *15 sevenths.*

Parts and Wholes

Basic Fractions teaches students the difference between parts of a whole and an entire whole. In early lessons, diagrams show fractions representing several wholes.

$$\bigoplus \bigoplus = \frac{8}{4}$$

Later, students learn to tell how many wholes a fraction equals by determining how many times bigger the top number is than the bottom number of a fraction. If the top number is 2 times bigger, the fraction equals 2 wholes. If the top number is 6 times bigger, the fraction equals 6 wholes. The following is part of an exercise from Lesson 42.

EXERCISE 3

Fractions to Wholes

a. (Write on the board:)

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$$\frac{12}{4} \qquad \frac{15}{3} \qquad \frac{20}{2}$$

$$\frac{30}{5} \qquad \frac{20}{4}$$

b. These fractions are the same as numbers. We're going to figure out the amount that each fraction equals.
- Here's a rule about fractions: A fraction equals 2 wholes when the top is 2 times bigger than the bottom.
- A fraction equals 5 wholes when the top is 5 times bigger than the bottom.
- A fraction equals 6 wholes when the top is 6 times bigger than the bottom.
c. When does a fraction equal 3 wholes? (Signal.) *When the top is 3 times bigger than the bottom.*
- When does a fraction equal 9 wholes? (Signal.) *When the top is 9 times bigger than the bottom.*
- When does a fraction equal 7 wholes? (Signal.) *When the top is 7 times bigger than the bottom.*

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d. (Point to $\frac{12}{4}$.)
- Let's figure out the number that this fraction equals. What's the bottom number? (Signal.) *4.*
- Tell me how many times bigger than 4 is the top. Get ready. (Signal.) *3.*

▶ **To Correct**

- (If the student has been taught to multiply by "count-bys," say:) Figure out how many times you would count by 4 to reach 12. (Pause.) What's the answer? (Signal.) *3.* ◀

▶ **To Correct**

- (If the student knows any other forms of multiplication, say:) Tell me 4 times how many equals 12. (Pause.) What's the answer? (Signal.) *3.* ◀
e. The top is 3 times bigger than the bottom. So how many wholes does the fraction equal? (Signal.) *3.*
- Yes, it equals 3 wholes.
- (Write to show:)

$$\frac{12}{4} = 3$$

Mixed Numbers

Converting mixed numbers to fractions is introduced after students have learned and practiced finding the number of wholes in a fraction. To convert a mixed number to a fraction, the students learn first to write the fractional equivalent of the whole number and then to add the number of parts in the remaining whole. This example is from Lesson 45.

EXERCISE 4

Expanding Fractions

a. (Write on the board:)

$$2\frac{1}{3} =$$

b. We're going to change this mixed number into a fraction.

- (Point to the $\frac{}{3}$.)
- The bottom number of the fraction tells us that we will have 3 parts in each whole. So I write a 3.
- (Write to show:)

$$2\frac{1}{3} = \frac{}{3}$$

- (Touch the 2.)
- How many wholes do we have? (Signal.) *2.*
- A fraction equals 2 wholes when the top is 2 times bigger than the bottom. What's the bottom of the fraction we're writing? (Signal.) *3.*
- The top must be 2 times bigger than 3. Tell me that number for the top. Get ready. (Signal.) *6.*
- (Write to show:)

$$2\frac{1}{3} = \frac{6}{3}$$

- We used the 2 wholes.
- (Touch the $\frac{1}{}$.)
- How many parts are left? (Signal.) *1.*

- (Write to show:)

$$2\frac{1}{3} = \frac{6 + 1}{3}$$

- We have 6 plus one parts. How many parts do we have altogether? (Signal.) *7.*
- (Write to show:)

$$2\frac{1}{3} = \frac{6 + 1}{3} = \frac{7}{}$$

- How many parts are in each whole? (Signal.) *3.*
- (Write to show:)

$$2\frac{1}{3} = \frac{6 + 1}{3} = \frac{7}{3}$$

- What fraction does 2 and one third equal? (Signal.) *7 thirds.*

Equivalent Fractions

Students learn that equivalent fractions are created by multiplying a fraction by another fraction that equals 1. Two component skills exercises prepare students for equivalent fraction exercises.

The first component skill teaches students to identify fractions that equal 1 whole: A fraction equals 1 whole when you use the same number of parts that are in each whole.

$$\frac{4}{4} = \frac{7}{7} = \frac{9}{9} = 1$$

The second component skill teaches the concept that when you multiply by 1, you start and end with equal amounts.

$$\frac{3}{5} \times 1 = \frac{3}{5}$$

The initial exercises in which students are asked to find a missing number in an equivalent fraction are written in this form:

$$\frac{4}{5} \times \left(\right) = \frac{}{15}$$

The students will write a fraction equal to 1 in the parentheses. The equal sign indicates that we must end with an amount that equals the amount we start with. We must multiply $\frac{4}{5}$ by a fraction that equals 1. The students first figure out what number the denominator of the first fraction must be multiplied by to end up with the denominator of the second fraction. Five times what number equals 15? The answer is 3. The denominator of the fraction we're multiplying $\frac{4}{5}$ by is 3. Because we must multiply by a fraction that equals 1, the top number must also be 3. A fraction equals 1 when the top and the bottom numbers are the same. The students write $\frac{3}{3}$ in parentheses and then multiply the numerator of the initial fraction and the numerator of the fraction that equals 1 whole. The answer is 12.

$$\frac{4}{5} \times \left(\right) = \frac{12}{15}$$

Scope and Sequence Chart

Basic Fractions

Skill	1	5	10	15	20	25	30	35	40	45	50	55
Determine the number of parts in a whole.	Teach	Review										
Represent denominators with pictures.	Teach	Review										
Determine the number of parts used.	Teach	Review										
Identify the fractions represented by pictures.		Teach	Review									
Name fractions.		Teach	Review									
Represent fractions with pictures.		Teach	Review									
Identify fractions equal to 1 from pictures.			Teach / Review									
Identify fractions as more, less, equal to 1.				Teach / Review								
Complete fractions equal to 1.				Teach / Review								
Identify the missing number in an addition or subtraction problem involving fractions with common denominators.				Teach / Review								
Write addition and subtraction problems from representations of fractions.				Teach / Review								
Rewrite fractions with common denominators that are added or subtracted. Do not reduce sum or difference.					Teach / Review							
Write whole numbers as fractions.						Teach / Review						
Multiply two fractions. Do not reduce product.						Teach / Review						
Multiply a whole number by a fraction. Do not reduce product.							Teach / Review					
Add or subtract column problems with common denominators.								Teach / Review				
Multiply by 1.									Teach			
Reduce fractions to whole numbers.										Teach / Review		
Expand mixed numbers to fractions.											Teach / Review	
Generate a series of fractions equal to 1.											Teach	
Complete equivalent fraction equations.												Teach

Key: Teach ▬▬▬▬▬   Review ▬▬▬▬▬