

◆ Addition ◆ Subtraction ◆ Multiplication ◆ Division

## Overview of Skill Development

Each module has four groups of activities that occur from one lesson to the next. These are called tracks. Each module has four different tracks: Facts, Place Value, Operations, and Story Problems.

### Facts Track

The fact activities are designed to promote instructional economy and a high level of student proficiency. Instructional economy is ensured because the modules do not treat each fact as a separate entity to be memorized, but rather as a member of both a number family and a fact series.

**Number families** consist of three numbers that go together to form a basic fact. In the *Addition* module, students work with number families that look like this:

$$\boxed{5} \left\{ \begin{array}{l} 4 \\ 1 \end{array} \right.$$

The number in the box is referred to as the “big number.” The numbers inside the bracket are referred to as the “small numbers.” The students learn that “the big number is the number you end up with when you add the two small numbers.” Learning number families is instructionally economical because each number family translates into two addition facts. The number family above, for instance, translates into  $4 + 1 = 5$  and  $1 + 4 = 5$ .

$$\boxed{5} \left\{ \begin{array}{l} 4 \quad \dots \quad 4 + 1 = 5 \\ 1 \quad \dots \quad 1 + 4 = 5 \end{array} \right.$$

The *Subtraction* module uses the same number families as the *Addition* module. Learning the subtraction facts, therefore, involves a variation of something familiar. This is doubly efficient learning. In the *Subtraction* module, students learn that “there are two subtraction facts for each number family. Subtraction facts always start with the big number.” At this point, the students use the number family 5, 4, 1 to generate four facts:

$$4 + 1 = 5, 1 + 4 = 5, 5 - 4 = 1, \text{ and } 5 - 1 = 4.$$

A different type of number family is used in the *Multiplication* module. The multiplication number family is shown in the shape of a division problem.

$$\begin{array}{r} 5 \\ 3 \overline{)15} \end{array}$$

This distinguishes multiplication facts from addition and subtraction facts, and it will promote the relationship between multiplication and division. The students learn that the number in the box is the “big number” and the numbers outside the bracket are the “small numbers.” They also learn that “when you multiply the two small numbers, you end up with the big number.” The number family 3, 5, 15 becomes  $5 \times 3 = 15$  and  $3 \times 5 = 15$ .

The *Division* module makes use of the number families taught in *Multiplication*, so once again, the fact learning is doubly efficient. The students learn that “when you write the division sign, the big number goes under the sign. One small number goes before the sign. The other small number goes above the last digit of the big number.”

$$\begin{array}{r} 5 \\ 3 \overline{)15} \end{array}$$

The number family 3, 5, 15 becomes  $5 \times 3 = 15$ ,

$$3 \times 5 = 15, 3\overline{)15}, \text{ and } 5\overline{)15}.$$

Another way in which **Corrective Mathematics** shows the relationship among various facts is through **fact series** exercises. Facts are presented in order, to show their relationship to counting. For example, in the series  $6 + 1 = 7$ ,  $6 + 2 = 8$ , and  $6 + 3 = 9$ , every time a number is counted in the second addend (1, 2, 3), the number is counted in the sum (7, 8, 9). By teaching fact relationships, individual facts are easier to master and recall. Here are sample fact series from each of the four modules:

**Addition**  $10 + 1 = 11$

$$10 + 2 = 12$$

$$10 + 3 = 13$$

$$10 + 4 = 14$$

$$10 + 5 = 15$$

**Subtraction**  $15 - 10 = 5$

$$14 - 10 = 4$$

$$13 - 10 = 3$$

$$12 - 10 = 2$$

$$11 - 10 = 1$$

**Multiplication**  $5 \times 1 = 5$

$$5 \times 2 = 10$$

$$5 \times 3 = 15$$

$$5 \times 4 = 20$$

$$5 \times 5 = 25$$

**Division**

$$\begin{array}{r} 6 \\ 6\overline{)36} \end{array}$$

$$\begin{array}{r} 7 \\ 6\overline{)42} \end{array}$$

$$\begin{array}{r} 8 \\ 6\overline{)48} \end{array}$$

$$\begin{array}{r} 9 \\ 6\overline{)54} \end{array}$$

$$\begin{array}{r} 10 \\ 6\overline{)60} \end{array}$$

## Facts Practice

Once the students have a number of firm reference points and know how to use these reference points to figure out closely related facts, the modules provide a number of exercises designed to increase student proficiency with individual facts. Students work with facts in a variety of teacher-directed and independent activities, both oral and written.

Two of the most enjoyable of these activities are the **Fact Game**, in which groups of students compete, and the **Timing Format**, in which individual students earn points for speed and accuracy on fact worksheets. The Fact Game is an extremely important way of providing students with the practice they need to master facts. Because drill and practice can be boring to students, you need to reinforce student improvement and use games or challenges to motivate students.

The speed requirements specified in the modules are quite low. Whenever possible, require students to increase the rate at which they work fact exercises. If students cannot answer 40 facts per minute at the end of a module, continue drilling those facts even though students have completed the module.

Also, Facts-Practice Blackline Masters are provided in each of the four operations modules for optional facts practice. These may be used anytime during the day.

Cumulative review blackline masters are provided for *Subtraction*, *Multiplication*, and *Division*.

## Fact Mastery Tests

In addition to providing the amount of practice students need to become proficient on facts, the four modules also provide a vehicle for monitoring student proficiency. Fact Mastery Tests appear frequently in each of the modules. Students are tested for speed and accuracy. Instructions that accompany the tests provide criteria for evaluating test scores and remediation for students who fail.

## Special Facts

**Corrective Mathematics** also teaches fact relationships usually omitted in other instructional programs. Division facts are taught in most programs, but division remainder facts are not. For example, students are taught  $5 \overline{)30}$  but are not taught that 5 goes into 31, 32, 33, and 34, 6 times. **Corrective Mathematics** devotes a great deal of time to division remainder facts.

Another example from **Corrective Mathematics** is the way in which problems such as  $18 + 8 = 26$  are taught. Students learn to silently compute the answer to this type of problem. Teaching these problems as “facts” aids in the solution of many addition and multiplication problems such as the following:

$$\begin{array}{r} 18 \\ + 8 \\ \hline 26 \end{array} \quad \begin{array}{r} 29 \\ \times 9 \\ \hline 261 \end{array} \quad \begin{array}{r} 59 \\ 19 \\ + 28 \\ \hline 106 \end{array} \quad \begin{array}{l} 18 \\ 18 \end{array}$$

## Operations Track

Each module teaches a coherent problem-solving routine that allows students to handle a wide variety of computational problems. The routines provide for all subtypes of problems that students might encounter, such as borrowing from zero in subtraction and multiplying by a number with a zero in the ones column.

### Addition

The addition operation is taught in stages. First, the students add columns of single-digit numbers. Then, they are taught to add columns of numbers that have more than one digit but that do not require renaming (carrying). When carrying is introduced, the sum for the ones column is given, and the students write only the number being carried in a box at the top of the tens column. Later, the sum is no longer written, and then, the carrying box is dropped. The

operation is expanded so that carrying occurs not only from the ones column but also from the tens and hundreds columns. By the end of *Addition*, students can solve problems of this type:

$$\begin{array}{r} 1818 \\ 1943 \\ 2775 \\ + 559 \\ \hline \end{array}$$

### Subtraction

The *Subtraction* module begins with problems that do not require regrouping (borrowing). Students learn basic conventions, such as starting with the ones column and subtracting the bottom number from the top number. Next, students are introduced to four preskills for borrowing. They learn which column to borrow from, how to rewrite a number after borrowing, when to borrow, and how to subtract after borrowing. Finally, students learn how to work four types of borrowing problems: borrowing from one column, borrowing from zero, borrowing from consecutive columns, and borrowing from as many as three consecutive columns. By the end of the *Subtraction* module, student can solve problems of these types:

$$\begin{array}{r} 6824 \\ - 1904 \\ \hline \end{array} \quad \begin{array}{r} 4926 \\ - 3749 \\ \hline \end{array} \quad \begin{array}{r} 4000 \\ - 245 \\ \hline \end{array}$$

### Multiplication

The *Multiplication* module begins with noncarrying column problems and horizontal problems with counters. Then, students are introduced to carrying. They write the number being carried in a box at the top of the tens column. Next, students are introduced to multiplying by 2-digit numbers. Initially, students multiply only by the tens number (the ones number has already been done for them). Finally, students work the entire problem. By the end of *Multiplication*, students can solve problems like these:

$$\begin{array}{r} 265 \\ \times 93 \\ \hline \end{array} \quad \begin{array}{r} 241 \\ \times 50 \\ \hline \end{array} \quad \begin{array}{r} 503 \\ \times 48 \\ \hline \end{array}$$

## Division

The division operation is first introduced for single-digit divisor problems. Students underline the part of the dividend that is at least as big as the divisor. Next, they work the underlined problem and find the remainder for that part. Then, they bring down the next digit and work the new problem in the same way. They continue in this manner until they have written a number above the last digit of the dividend. This signals that the problem is finished.

The single-digit divisor strategy is first shown with problems that have 1- and 2-digit answers. Early problems do not have answers with zeros. Later problems present 3-digit answers, answers with zero in the middle, and answers with zero as the final digit. Special exercises focus on these troublesome types of division problems.

For 2-digit divisors, the procedure is the same as that for single-digit divisors, except the students round off the divisor and the underlined part of the problem to the nearest tens number. For example, if the problem is  $63\overline{)483}$ , students write the rounded-off problem as  $6\overline{)48}$ . The rounding-off sometimes leads to trial answers that are either too large or too small. Students are taught to determine whether the remainder is too large, and if it is, to make the answer larger. If the remainder is too small (a negative number), students make the answer smaller. By the end of the module, students can solve problems of these types:

$$\begin{array}{r} 44\overline{)5900} \\ 24\overline{)2165} \\ 34\overline{)3618} \\ 75\overline{)3052} \end{array}$$

## Problem-Solving Routines

The most important feature of the operations portions of these modules is that the problem-solving routines are introduced only after you have taught all preskills necessary for errorless student performance. For the students, the routines are simply procedural chains of familiar discriminations and responses, and success is ensured.

The care with which each necessary preskill is introduced is well demonstrated by the operations activities in the *Subtraction* module. Before the students learn the routine for subtracting with borrowing in one column, they master these preskills:

- **Rewriting numbers by borrowing.** Given a number with one digit slashed, the students learn to borrow from the slashed digit. The students write the borrowed amount in front of the digit immediately to the right of the slashed digit.

$$\begin{array}{r} \text{Given} \qquad \qquad 3572 \\ \text{Students write} \quad 2 \\ \qquad \qquad \qquad 3\text{^}572 \end{array}$$

- **Subtracting multidigit numbers without borrowing.** Given a multidigit subtraction problem, the students learn to subtract the bottom digit from the top digit in each column, starting with the ones column.

$$\begin{array}{r} \text{Given} \qquad \qquad 841 \\ \qquad \qquad \qquad - 410 \\ \hline \end{array}$$

Students begin with the ones column and write

$$\begin{array}{r} 841 \\ - 410 \\ \hline 431 \end{array}$$

- **Minusing zero when zero is not written.** Given a subtraction problem in which the minuend has more digits than the subtrahend, the students learn that the absence of a digit in the subtrahend means they must subtract zero.

$$\begin{array}{r} \text{Given} \qquad \qquad 348 \\ \qquad \qquad \qquad - \quad 7 \\ \hline \end{array}$$

Students write

$$\begin{array}{r} 348 \\ - \quad 7 \\ \hline 341 \end{array}$$

- **Determining when and where to borrow.** Given a partial subtraction problem, the students learn that “if you’re minusing more than you start with, you have to borrow.” The students use a slash to indicate the position of the digit they would borrow from.

$$\begin{array}{r} \square 2 \square \\ - \square 7 \square \\ \hline \end{array} \quad \begin{array}{r} \square \square 5 \square \\ - \square 3 \square \\ \hline \end{array}$$

$$\begin{array}{r} \square 2 \square \\ - \square 7 \square \\ \hline \end{array} \quad \begin{array}{r} \square \square 5 \square \\ - \square 3 \square \\ \hline \end{array}$$

- **Subtracting when borrowing has been done.** Given a problem in which borrowing has been done for the students, the students learn to subtract accurately.

$$\begin{array}{r} \phantom{2} \\ \text{Given} \quad \begin{array}{r} \boxed{3} 14 \\ - 18 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \phantom{2} \\ \text{Students write} \quad \begin{array}{r} \boxed{3} 14 \\ - 18 \\ \hline 16 \end{array} \end{array}$$

- **Practice borrowing.** Given a problem requiring borrowing, students learn to rewrite the digits in the minuend.

$$\begin{array}{r} \text{Given} \quad \begin{array}{r} 52 \\ - 36 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{Students write} \quad \begin{array}{r} 4 \\ \phantom{4} 512 \\ - 36 \\ \hline \end{array} \end{array}$$

Only after the students have mastered all six preskills are they introduced to the borrowing routine.

1. The students read the problem in the ones column.

2. The students determine whether they have to borrow.
3. If borrowing is necessary, they
  - A. Determine which number to borrow from.
  - B. Slash that number, and write the number that is left.
  - C. Write a 1 in front of the number to the right of the number borrowed from.
  - D. Reread the problem in the ones column, and subtract.

The same care is taken later in the *Subtraction* module when teaching the preskills required for borrowing from consecutive columns and borrowing from zero. The same care is also taken in *Addition*, *Multiplication*, and *Division* modules. In every case, all necessary preskills are taught before problem-solving routines are introduced.

## Story Problems Track

One of the major strengths of the *Corrective Mathematics* program is that *Corrective Mathematics* teaches a precise strategy for determining which mathematics operation is required by a given story problem—a feature not typically shared by other mathematics programs.

Story problems are introduced in the *Addition* module through three major exercises: working with pictures, working with sentences in columns, and working with sentences in paragraphs.

*Addition* story problems include distracters, numbers that appear but are not used in computing the answer. Two types of distracters are presented: distracters that involve the wrong class of object and those that involve the wrong verb. Here is an example of a story problem with a distracter that is from the “wrong class of objects” (rosebushes).

There are 528 apple trees. There are 4108 cherry trees. There are 180 rosebushes. There are 600 oak trees. How many trees are there?

Here is an example of a story problem with two verbs that distract, that are irrelevant to the operation (*built, rode*).

261 teachers went swimming. 493 students built forts. 97 parents went swimming. 135 students rode bikes. 2580 children went swimming. How many people went swimming?

Although students learn in the *Subtraction* module that certain verbs generally indicate whether to add (*find, get, buy*) or subtract (*lose, give away, break*), they quickly learn that they cannot rely solely on the verb to determine the appropriate operation. For example, the following problem calls for addition, even though *give away* would seemingly call for subtraction.

Bill gives away 4 toys. John gives away 2 toys. How many toys did the boys give away?

Because using the verb to determine whether addition or subtraction is called for is not a viable strategy for many story problems, the *Subtraction* module quickly teaches this discrimination strategy: If the problem gives the big number, it's a subtraction problem; if the problem does not give the big number, it's an addition problem. (The "big number" is the minuend in a subtraction problem and the sum in an addition problem.) The strategy is illustrated by the following problems.

Mr. Yamada had 36 books. Last week he bought more books at the used bookstore. Now he has 58 books. How many books did he buy last week?

In this problem, the big number, 58, is given. Therefore, the problem is a subtraction problem and translates into

$$\begin{array}{r} 58 \\ - 36 \\ \hline \end{array}$$

In the second problem, the big number (how many windows in all) is not given.

An office building has 2365 clean windows. The window washers have to wash 90 dirty windows. How many windows in all does the building have?

Therefore, the problem is an addition problem and translates into

$$\begin{array}{r} 2365 \\ + 90 \\ \hline \end{array}$$

In the *Multiplication* module, the students are taught that "if you use the same number again and again, you multiply."

There are 9 alarm clocks, 9 wall clocks, and 9 grandfather clocks in the shop. How many clocks are there in all?

In this problem, the same number is used again and again. The problem is a multiplication problem and translates into

$$9 \times 3 =$$

If the same number is not used again and again, the problem is not a multiplication problem. It must be an addition or a subtraction problem.

There are 5 green flowers, 4 red flowers, and 2 blue flowers in bloom. How many flowers are there in all?

Because the big number is not given, the problem is an addition problem and translates into

$$\begin{array}{r} 5 \\ 4 \\ + 2 \\ \hline \end{array}$$

Students also learn that the words *each* and *every* signal that the same number is being used again and again. This problem, therefore, is a multiplication problem.

There are 9 books on each shelf. There are 3 shelves. How many books are there in all?

In the *Division* module, the discrimination strategy is expanded. The students learn to apply two tests to story problems.

1. If the same number is used again and again, the problem is either a multiplication problem or a division problem. If the same number is not used again and again, it's an addition or a subtraction problem.
2. In problems involving multiplication or division, the problem requires division if the big number is given and multiplication if the big number is not given. In problems involving addition or subtraction, the problem requires subtraction if the big number is given and addition if the big number is not given.

In the problem below, the same number is used again and again, so the problem is either multiplication or division.

Every day Mattie read 3 books. Mattie read 18 books in all. How many days did Mattie read books?

The big number is given, so the problem is a division problem that translates to

$$3 \overline{)18}$$

The strategy ensures that the students will attend closely to all the words in a story problem, even in the *Addition* module where no discrimination between operations is possible.

An additional strength of the Story Problems track of the modules is that the students are taught to apply their discrimination strategies to a wide variety of problem types, such as in the *Subtraction* module.

- **Simple Action.** A “subtraction verb,” *broken*, calls for subtraction.

Ann found 206 pencils. 78 of the pencils were broken. How many of the pencils were not broken?

- **Complex Action.** An “addition verb,” *built*, calls for subtraction.

There were 143 cabins at the lake. This year more cabins were built. Now there are 160 cabins. How many more cabins were built at the lake?

- **Classification.** Numbers for the smaller classes are added.

The shop had 86 apples and 90 oranges. How many pieces of fruit did the shop have?

- **Comparison.** Younger age is subtracted from older age.

Ms. Savas is 42 years old. Ms. Hark is 70 years old. How many years older is Ms. Hark?

Furthermore, the specific preskills for each problem type are carefully taught. For instance, before being presented with addition and subtraction classification problems, the students are taught the class name for the big number. In a problem involving hammers, tools, and saws, students are taught that *tool* is the name for the big number because hammers are tools and saws are tools.

## Place-Value Track

In the *Addition*, *Subtraction*, and *Multiplication* modules, students learn about place value. They are taught to read numbers as long as five digits and to write these numbers, putting commas in appropriate places. They learn to deal with numbers that include one or more zeros. They receive practice in identifying the digits in a number as belonging to the ones, tens, hundreds, or thousands columns. The students then learn to write dictated numbers in columns with the digits properly aligned.

426  
32  
12  
634







# Scope and Sequence Chart

## Multiplication

	1	5	10	15	20	25	30	35	40	45	50	55	60	65	
<b>Facts</b>															
Determine the product of two 1-digit numbers.	[Teach]														
Write two multiplication facts for any two 1-digit numbers.	[Teach]														
Say a series of consecutively ordered facts. For example, $5 \times 5 = 25$ ; $5 \times 6 = 30$ .		[Teach]													
Determine the sum of a 2-digit number and a 1-digit number.		[Teach]													
Determine the product of two 1-digit numbers, one of which is zero.															
<b>Place Value</b>															
Say the number for a 3-digit numeral.															
Say the number in each column of a 2-digit number.															
Say the number for a 4-digit numeral.															
Say the number for a 5-digit numeral.															
<b>Operations</b>															
Determine the product of a 2- or 3-digit number and a 1-digit number. No regrouping required.															
Determine the product of a 2-digit number and a 1-digit number. Regrouping required.															
Determine the product of a 3-digit number and a 1-digit number. Regrouping required for two columns.															
Determine the product of two 2-digit numbers.															
Determine the product of a 2-digit number and a 3-digit number.															
Determine the product of a 2-digit number and a 1-digit number. The multiplier has a zero in the ones column.															
<b>Story Problems</b>															
Determine the product or sum in a story problem with two 1-digit numbers.															
Determine the product or sum in a story problem with a 2- or 3-digit number and a 1-digit number.															
Determine the product, sum, or difference in a story problem with a 2- or 3-digit number and a 1-digit number.															
Determine the product, sum, or difference in a story problem with a 2-digit number and a 2- or 3-digit number.															

Key: Teach [Blue bar]

Review [Black bar]

