Common Problems Experienced by Struggling Students

Students in need of remediation often exhibit similar problems. ***Corrective Mathematics*** provides the careful teaching and systematic practice that creates steady, measurable progress.

Learn more about five problems struggling students face and how ***Corrective Mathematics*** helps them overcome the difficulties that they have had with mathematics.

- Memorization of the basic number facts
- Standard algorithms
- Over reliance on estimation
- Mathematical reasoning and problem-solving competency
- Understanding of fractions

Common Problem—Memorization of the Basic Number Facts

Common Problem	How Corrective Mathematics Addresses It
Memorization of the Basic Number Facts Memorizing the "basic number facts," i.e., the sums and products of single-digit numbers and the equivalent subtraction and division facts, frees up working memory to master the arithmetic algorisms and tackle math applications. Research in cognitive psychology points to the value of automatic recall of the basic facts. Students who do not memorize the basic number facts will founder as more complex operations are required, and their progress will likely grind to a halt by the end of elementary school. There is no *real* mathematical fluency without memorization of the most basic facts. *The State of State Math Standards*	*Corrective Mathematics* promotes a high level of student proficiency by teaching each fact as a member of both a number family and a fact series, rather than separate entities to be memorized. • *Number families* consist of three numbers that go together to form a basic fact. In *Addition* and *Subtraction*, students work with number families that look like this: $4 + 1 = 5$ $1 + 4 = 5$ $5 – 1 = 4$ $5 – 4 = 1$ Learning number families is instructionally economical because each number family translates into four facts. • *Fact Series* exercises present facts in order to show their relationship to counting. For example: $6 + 1 = 7$ $6 + 2 = 8$ $6 + 3 = 9$ Every time a number is counted in the second addend (1, 2, 3), the number is counted in the sum (7, 8, 9). By teaching fact relationships, individual facts are easier to recall. • Once students have a number of firm reference points and know how to use these reference points to figure out closely related facts, the modules provide a number of exercises designed to increase proficiency and automaticity with individual facts, including *Fact Games* in which groups of students compete, the *Timing Format* in which individual students earn points, and *Blackline Masters* for optional facts practice. • *Special fact* relationships usually omitted in instructional programs are taught. For example, division facts are taught in most programs, but not division remainder facts. In *Corrective Mathematics* students are taught 5 goes into 30 six times and also that 5 goes into 31, 32, 33, and 34 six times.

Common Problem—The Standard Algorithms

Common Problem	How Corrective Mathematics Addresses It
The Standard Algorithms The standard algorithms are powerful theorems and they are *standard* for a good reason: They are guaranteed to work for all problems of the type for which they were designed. Knowing the standard algorithms, in the sense of being able to use them, is a foundational skill for an elementary schools students. Students who master these algorithms gain confidence in their ability to compute. They know they can solve any addition, subtraction, multiplication, or division problem without relying on a mysterious black box, such as a calculator. Moreover, the ability to execute the arithmetic operations in a routine manner helps student to think more conceptually and are well positioned to understand the meaning and uses of other algorithms in later years. For example, one benefit of the long division algorithm is that it requires estimation of quotients at each stage. If the next digit placed in the (trial) answer is too large or too small, that stage has to be done over again, and the error is made visible by the procedure. Number sense and estimation skills are reinforced in this way. The long division algorithm has applications that go far beyond elementary school arithmetic. At the middle school level, it can be used to explain why rational numbers have repeating decimals. Division is also central to the Euclidean Algorithm for the calculation of the common divisor or two integers. In high school algebra, the long division algorithm, in a modified form, is used for division of polynomials. At the university level, the algorithm is important in advanced abstract algebra. Experience with the long division algorithm thus lays the groundwork for advanced topics in mathematics. *The State of State Math Standards*	*Corrective Mathematics* teaches coherent routines that allow students to handle a wide variety of computation problems. Routines provide for all the various types of problems that students might encounter, such as borrowing from zero in subtraction and multiplying by a number with a zero in the ones column. Instruction is carefully sequenced and strongly scaffolded so that students learn all necessary components skills (preskills) prior to the introduction of the routine and can apply steps they are to follow in using a routine. For example: The division operation is first introduced for single-digit divisor problems. Students underline the part of the dividend that is at least as big as the divisor. Next, they work the underlined problem and find the remainder for that part. Then they bring down the next digit and work the new problem in the same way. They continue in this manner until they have written a number above the last digit of the dividend. This signals that the problem is finished. The single-digit divisor strategy is first shown with problems that have one- and 2-digit answers. Early problems do not have answers with zeros. Later problems present 3-digit answers, answers with zero in the middle, and answers with zero as the final digit. Special exercises focus on these troublesome types of division problems. For 2-digit divisors, the procedure is the same as that for single-digit divisors, except the students round off the divisor and the underlined part of the problem to the nearest tens number. For example, if the problem is $63\overline{)483}$, students write the rounded-off problem as $6\overline{)48}$. The rounding-off sometimes leads to trial answers that either too large or too small. Students are taught to determine whether the remainder is too large, and if it is, to make the answer larger. If the remainder is too small (a negative number), students make the answer smaller. By the end of the module, students can solve problem of these types: $44\overline{)5900}$ $24\overline{)2165}$ $34\overline{)3618}$ $75\overline{)3052}$

Common Problem—Over Reliance on Estimation at the Expense of Exact Arithmetic Calculations

Common Problem	How Corrective Mathematics Addresses It

Common Problem

Over reliance on estimation at the expense of exact arithmetic calculations

Fostering estimation skills is a commendable goal shared by all math programs. However, there is a tendency to overemphasize estimation at the expense of exact arithmetic calculations. For simple subtraction, the *correct answer* is the *only* reasonable answer. The notion of "reasonableness" might be appropriate in connection with measurement, but not in connection with arithmetic of small whole numbers. The main goal of elementary school math is to get students to think about numbers and to learn arithmetic. Hand calculations force students to develop an intuitive understanding of place value and of fractions.

The State of State Math Standards

How Corrective Mathematics Addresses It

Corrective Mathematics teaches arithmetic calculations through carefully sequenced and strongly scaffolded lessons.
The care with which each necessary preskill is introduced is well demonstrated by the operations activities in the *Subtraction* module. Before the students learn the routine for subtracting with borrowing in one column, they master these preskills:

- **Rewriting numbers by borrowing.** Given a number with one digit slashed, the students learn to borrow from the slashed digit. The students write the borrowed amount in front of the digit immediately to the right of the slashed digit.

  Given 3572

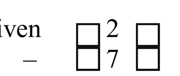
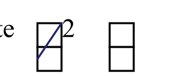
  Students write $\overset{2}{3}\,{}^{1}572$

- **Subtracting multidigit numbers without borrowing.** Given a multidigit subtraction problem, the students learn to subtract the bottom digit from the top digit in each column, starting with the ones column.

  Given $\begin{array}{r} 841 \\ -\ 410 \\ \hline \end{array}$

  Students begin with the ones column and write $\begin{array}{r} 841 \\ -\ 410 \\ \hline 431 \end{array}$

- **Determining when and where to borrow.** Given a partial subtraction problem, the students learn that "if you're minusing more than you start with, you have to borrow." The students use a slash mark to indicate the position of the digit they would borrow from.

  Given  Students write 

- **Subtracting when borrowing has been done.** Given a problem in which borrowing has been done for the students, the students learn to subtract accurately.

  Given $\begin{array}{r} \overset{2}{3}\,{}^{1}4 \\ -\ 18 \\ \hline \end{array}$

  Students write $\begin{array}{r} \overset{2}{3}\,{}^{1}4 \\ -\ 18 \\ \hline 1\ 6 \end{array}$

Common Problem—Mathematical Reasoning and Problem Solving Competency

Common Problem	How Corrective Mathematics Addresses It
Mathematical Reasoning and Problem-solving Competency Problem-solving is an indispensable part of learning mathematics. Children should be able to solve single-step word problems in the earliest grades and deal with increasingly more challenging, multi-step problems as they progress. Too often, programs fail to develop important prerequisites before introducing advanced topics. *The State of State Math Standards*	One of the major strengths of the ***Corrective Mathematics*** program is that ***Corrective Mathematics*** teaches a precise strategy for determining which mathematics operation is required by a given story problem—a feature not typically shared by other mathematics programs. • Although students learn in the *Subtraction* module that certain verbs generally indicate whether to add (find, get, buy) or subtract (lose, give away, break), they quickly learn that they cannot rely solely on the verb to determine the appropriate operation. For example, the following problem calls for addition, even though *give away* would seemingly call for subtraction. *Bill gives away 4 toys. John gives away 2 toys. How many toys did the boys give away?* • Because using the verb to determine whether addition or subtraction is called for is not a viable strategy for many story problems, the *Subtraction* module quickly teaches this discrimination strategy: If the problem gives the big number, it's a subtraction problem; if the problem does not give the big number, it's an addition problem. (The "big number" is the minuend in a subtraction problem and the sum is an addition problem.) The strategy is illustrated by the following problems. *Mr. Yamada had 36 books. Last week he bought more books at the used bookstore. Now he has 58 books. How many books did he buy last week?* • In this problem, the big number, 58, is given. Therefore, the problem is a subtraction problem and translates into $\begin{array}{r} 58 \\ \underline{-\ 36} \end{array}$ • In the second problem, the big number (how many windows in all) is not given. *An office building has 2365 clean windows. The window washers have to wash 90 dirty windows. How many windows in all does the building have?* • Therefore, the problem is an addition problem and translated into $\begin{array}{r} 2365 \\ \underline{+\ \ \ 90} \end{array}$ An additional strength of the Story Problems track of the modules is that the students are taught to apply their discrimination strategies to a wide variety of problem types. *Subtraction* module. Furthermore, the specific preskills for each problem type are carefully taught. For instance, before being presented with addition and subtraction classification problems, the students are taught the class name for the big number. For example, in a problem involving hammers, tools, and saws, students are taught that *tool* is the name for the big number because hammers are tools and saws are tools.

Common Problem—Fraction Development

Common Problem	How Corrective Mathematics Addresses It
Fraction Development In general, too little attention is paid to the coherent development of fractions and there is not enough emphasis on paper- and pencil- calculations. When fraction arithmetic is poorly developed in the elementary grades, students have little hope of understanding algebra as anything other than a maze of complicated recipes to be memorized. *The State of State Math Standards*	The development of fractions and decimals receives special attention in *Corrective Mathematics* as students are guided through a logical, coherent progression of steps. • *Basic Fractions* teaches what the numbers in a fraction tell. The bottom number tells how many parts in each whole, and the top number tells how many parts are used. In the fraction ¾, there are 4 parts in each whole and 3 parts are used. • Students learn the difference between parts of a whole and an entire whole. Later they learn to tell how many wholes a fraction equals by determining how many times bigger the top number is than the bottom number. • The module presents visual examples of what happens when fractions are added and worksheets provide a great deal of practice adding and subtracting fractions with like denominators. Before students add and subtract fractions with unlike denominators, they learn to make the bottom numbers the same by figuring out the fraction versions of 1 by which they must multiply each original fraction. • Students learn that equivalent fractions are created by multiplying a fraction by another fraction that equals 1. Two components skills exercises prepare students for equivalent fraction exercises. • The first component skill teaches students to identify fractions that equal 1 whole: A fraction equals 1 whole when you use the same number of parts that are in each whole. $\frac{4}{4} = \frac{7}{7} = \frac{9}{9} = 1$ • The second component skill teaches the concept that when you multiply by 1, you start and end with equal amounts. $\frac{3}{5} \times 1 = \frac{3}{5}$ • The initial exercises in which students are asked to find a missing number in an equivalent fraction are written in this form: $\frac{4}{5} \times \left(\quad\right) = \frac{\quad}{15}$ • The students will write a fraction equal to 1 in the parentheses. The equal sign indicates that we must end with an amount that equals the amount we start with. We must multiply $\frac{4}{5}$ by a fraction that equals 1. Students first figure out what number the denominator of the first fraction must be multiplied by to end up with the denominator of the second fraction. Five time what number equals 15? The answer is 3. The denominator of the fraction we're multiplying $\frac{4}{5}$ by is 3. Because we must multiply by a fraction that equals 1, the top number must also be 3. A fraction equals 1 when the top and the bottom numbers are the same. The students write $\frac{3}{3}$ in parentheses and then multiply the numerator of the initial fraction and the numerator of the fraction that equals one whole. The answer is 12.