CORE-PLUS MATHEMATICS PROJECT



CCSS GUIDE TO CORE-PLUS MATHEMATICS

Contemporary Mathematics in Context

CORE-PLUS MATHEMATICS PROJECT

Authors

Christian R. Hirsch (Director) Western Michigan University

James T. Fey (Emeritus) University of Maryland

Eric W. Hart Maharishi University of Management

Harold L. Schoen (Emeritus) University of Iowa

Ann E. Watkins California State University, Northridge

with

Beth E. Ritsema Western Michigan University

Rebecca K. Walker Grand Valley State University

Brin A. Keller Michigan State University

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Send all inquiries to: McGraw-Hill Education 8787 Orion Place Columbus, OH 43240

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Robin Marcus University of Maryland

Arthur F. Coxford (deceased) University of Michigan

Principal Evaluator

Steven W. Ziebarth Western Michigan University

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Robert E. Megginson University of Michigan

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Technical and Production Coordinator

James Laser Western Michigan University



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INTRODUCTION



Background

Core-Plus Mathematics is an international-like, problem-based, inquiry-oriented, technology-rich four-year curriculum for high schools. It shares many of the features that characterize curricula of mathematically high-achieving foreign countries against whose standards the Common Core State Standards for Mathematics (CCSS) were benchmarked.

The CCSS Edition of *Core-Plus Mathematics* builds on the strengths of the 1st and 2nd editions whose development and evaluation were funded by a series of grants from the National Science Foundation. These editions were shaped by continuing advances in research on student learning, by evolving school mathematics standards for what students should know and be able to do, and by multi-year field trials of the complete courses in 49 school districts. Included were schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Missouri, Ohio, South Carolina, Texas, and Wisconsin with diverse student populations.

Each text is the product of a three-year cycle of research and development, pilot testing and revision, and field-testing and refinement prior to publication. Research and evaluation reports on the published programs are available at www.MHEonline.com/coreplusmath.

Curriculum Overview

The first three courses of *Core-Plus Mathematics* provide a significant common core of broadly useful mathematics aligned with the CCSS and designed for all students. They were developed to prepare students for success in college, in careers, and in daily life in contemporary society. Course 4, *Preparation for Calculus*, continues the rigorous preparation of STEM-oriented (science, technology, engineering, and mathematics) students for success in college mathematics, especially calculus.

Table 1 provides a list of the eight units that comprise each course.

Table 1: Core-Plus Mathematics CCSS Edition Units

Course 1	 Unit 1 Patterns of Change Unit 2 Patterns in Data Unit 3 Linear Functions Unit 4 Discrete Mathematical Modeling Unit 5 Exponential Functions Unit 6 Patterns in Shape Unit 7 Quadratic Functions Unit 8 Patterns in Chance
Course 2	 Unit 1 Functions, Equations, and Systems Unit 2 Matrix Methods Unit 3 Coordinate Methods Unit 4 Regression and Correlation Unit 5 Nonlinear Functions and Equations Unit 6 Modeling and Optimization Unit 7 Trigonometric Methods Unit 8 Probability Distributions
Course 3	 Unit 1 Reasoning and Proof Unit 2 Inequalities and Linear Programming Unit 3 Similarity and Congruence Unit 4 Samples and Variation Unit 5 Polynomial and Rational Functions Unit 6 Circles and Circular Functions Unit 7 Recursion and Iteration Unit 8 Inverse Functions
Course 4 <i>Preparation</i> <i>for Calculus</i>	 Unit 1 Families of Functions Unit 2 Vectors and Motion Unit 3 Algebraic Functions and Equations Unit 4 Trigonometric Functions and Equations Unit 5 Exponential Functions, Logarithms, and Data Modeling Unit 6 Surfaces and Cross Sections Unit 7 Concepts of Calculus Unit 8 Counting Methods and Induction

Alignment and Coherence

Core-Plus Mathematics, CCSS Edition fully embraces the essence of the Common Core State Standards. Given its instructional design—problem-based and inquiryoriented—and its emphasis on mathematical modeling, *Core-Plus Mathematics* fully addresses the Common Core Standards for Mathematical Practice and the Content Standards in a coherent and connected manner. The correlation charts for Courses 1–4 on pages 7–31 indicate how the content and the mathematical practices in which students engage align with the Common Core State Standards for Mathematics.

The CCSS Pathway through *Core-Plus Mathematics*, CCSS Edition (pp. 32–40) outlines a carefully articulated scope and sequence of units and lessons that provide a coherent and connected development of CCSS content and practices. In each course, students advance their understanding and proficiency in algebra and functions, in geometry, in statistics and probability, and in discrete mathematical modeling, along with increasing facility with the mathematical practices.

Adaptive and Responsive

Adoption of the CCSS is a state-by-state decision, and implementation will vary by state. If a state chooses to adopt the CCSS, it may include an additional 15% of its own custom standards to complete its framework. To assist states in this case, a richer and more challenging *Core-Plus Mathematics* Pathway has been prepared. For details, see the Unit Planning Guides in the *Teacher's Edition* for each course.

Mathematical Modeling: The Core of the CCSS

The centrality of mathematical modeling to the CCSS is signaled by the fact that modeling with mathematics is both a Mathematical Practice (see Table 2) and a Conceptual Category. Additionally, specific standards related to mathematical modeling appear throughout the high school standards.

MP1:	Make sense of problems and persevere in solving them.	MP5:	Use appropriate tools strategically. ¹
MP2:	Reason abstractly and quantitatively.	MP6: MP7 [.]	Attend to precision.
MP3: MP4:	Construct viable arguments and critique the reasoning of others. Model with mathematics.	MP8:	structure. Look for and express regularity in repeated reasoning.

Table 2: Standards for Mathematical Practices

¹ To support development of MP5, the Core-Plus Mathematics program includes CPMP-Tools, a suite of freely available software including a computer algebra system (CAS), a spreadsheet, and dynamic geometry, statistics, and probability tools, together with custom apps specific to the curriculum. Mathematical modeling has been a central and unifying theme of each unit of *Core-Plus Mathematics* since its initial design and development over 20 years ago. Figure 1 illustrates the modeling process as developed in *Core-Plus Mathematics*. Depending on the unit, the model may be deterministic or stochastic and involve continuous or discrete quantities. Our approach to mathematical modeling and its connections to reasoning, sense making, and mathematical habits of mind have been refined with each new edition.





In creating the CCSS edition of *Core-Plus Mathematics*, we have used mathematical modeling as an effective way of connecting the Mathematical Practices and the Content-related Standards across Conceptual Categories. As suggested in Figure 1, the process of mathematical modeling focuses throughout on making sense of problems in context (MP1) and using mathematics to model authentic problems in everyday life, society, and careers (MP4).



Simulating bungee jumping

To model a real-world problem situation entails first representing the situation mathematically, drawing on mathematics in the content standards and flexibly using mathematical practices MP2, MP5, and MP7 (see Figure 2).



Figure 2: Connecting Mathematical Practices and the Content Standards

Once a mathematical model has been constructed, it is analyzed in terms of its faithfulness to the real-world situation and modifications may result from quantitative, spatial, or abstract reasoning about the "goodness of fit" (MP2), taking into account arguments provided by others (MP3). Once a satisfactory model has been arrived at, a solution can be attempted. This solution process may involve further mathematical reasoning (MP2), strategic use of tools (MP5), making use of structure (MP7), and communicating precisely the results with attention to needed precision in calculations (MP6). Once a mathematical solution is determined, it must be interpreted back in the real-world setting, again using practices MP2 and MP6.

After the solution is interpreted, answering the question "Does it make sense?" is essential and draws once again on MP2 and MP6. Looking back at and evaluating the solution involves mathematical reasoning (MP2) and often leads to identification of mathematical structure in repeated calculations, algebraic manipulations, or reasoning patterns (MP2, MP7, MP8).

In summary, the benefits of modeling with mathematics across a wide range of contexts lie not only in the solutions to the problems, but also in the new ideas and relationships that are discovered—the essence of the content standards.



Testing coordinate models of transformations



COMMON CORE STATE STANDARDS

The course-level correlation charts for Courses 1 through 4 on the following pages showpage numbers in *Core-Plus Mathematics* whose content aligns to the Common Core State Standards for Mathematics.



Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Course 1	Course 2	Course 3	Course 4
1. Make sense of problem	ns and persevere in solvir	ng them.	
Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8
Examples: 8–10, 56–58, 61 #11, 93 #8, 111, 131, 159, 239–242, 297 STM, 413 #3, 420–421	Examples: 61–64, 121 #7, 124 #11, 141 Check Your Understanding (CYU) 280–285, 314 #13, 364–367, 372 #17, 474 #2, #3	Examples: 32–33, 149 #14, 233 #7, 360 #15, 432 CYU, 435 #7, 440 #8, 450 #33, 459–461, 470 #4, 554 #22	Examples: 71 #22, 178–180, 231 CYU, 288 #7, 300–301, 318–321, 370–374, 446–449, 474–479, 581–585, 601–604
2. Reason abstractly and	d quantitatively.		
Throughout Units 1–3, 5, 8	Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8
Examples: 27–31, 116–123, 194–197, 250–254, 307–311, 369 CYU, 469–472	Examples: 70–71, 104–117, 145 #2, 223 #15, 331, 349–350 #17, 472 #4, 489–497, 552–553	Examples: 118 #1, #2, 145–150, 220 #18, 338–339, 353 #1, 562 CYU, 468–471	Examples: 68 #10, 146 #3, 157–161, 239–242, 305–306, 340–343, 375–380, 415–418, 468–473
3. Construct viable argu	ments and critique the re	asoning of others.	
Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8
Examples: 180 #30, 194 #4, 335–337, 348 #29, 374–377, 387, 493 #2, 504 #13, 517 CYUb	Examples: 22 #14, 54–55, 111 #7, 123 #10, 168 #9, 170–180, 179 #8, 207 #3, #4, 490–491	Examples: 2–15, 166 #5, 171 #5, 204–208, 233 #7, 352 #9, 405–406, 487 #11, 566–567	Examples: 124 #19, 287–294, 315 #5, 316 #10, 326 #22, 410 #6, 448 #8, 584–585, 589 #8, 595–604
4. Model with mathema	tics.		
Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8
Examples: 161–167, 280 #10, 323–332, 363–369, 383, 463–468, 551–561	Examples: 158–160, 232–242, 260–268, 439–442, 518–519, 522–531, 569 CYU, 573–576	Examples: 81–88, 132–143, 218 #12, 231 #5, 242–247, 360 #14, 435–437, 495–506	Examples: 126 #23, 157–161, 182–187, 401–405, 412 #9, 440–446, 459 #29, 479, 490–494

Course 1	Course 2	Course 3	Course 4
5. Use appropriate tools	strategically.		
Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8
Examples: 52–56, 137 #16, 161–167, 281 #13, 297, 480–482, 568–570	Examples: 49–57, 97 #18, 280–298, 385, 405 #12, 498–501, 507 #11, 581	Examples: 93–95, 220 #19, 254 #13, 260–265, 462–467, 475 #11, 519–522	Examples: 138–144, 427 #32, 444–446, 460 #31, 496 #20, 514–515, 581–585
6. Attend to precision.			
Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8
Examples: 20–21 #15, 175 #17, 374–377, 387 #10, 390 #17, 391 #19, 466 #5, 482 #9	Examples: 32–33, 92 #7, 172 #6, 261 #3, 382–383, 467–477, 531	Examples: 123 #16, 165–168, 245–247, 283–296, 321–323, 581–583	Examples: 66 #7, 110 #5, 239 #10, 342, 471–473, 495 #15, 513 #3c, 566 #5
7. Look for and make use	of structure.		
Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8	Throughout Units 1–8
Examples: 178 #24, 195, 239–247, 291–297, 303 STM, 328 STM, 468, 478	Examples: 26–29, 42 #21, 60 STMd, 96 CYU, 178 #6, 332–340, 351 #22, 481 #20	Examples: 60 #5, 62–71, 112–117, 334, 348–352, 387 #28, 489–495, 506 #19, 597 #29	Examples: 59–64, 141–144, 188–193, 202 #14, 252–258, 260 #14, 262 #24, 345–348, 367–370, 440–446, 458 #28, 460 #31
8. Look for and express	regularity in repeated rea	soning.	
Throughout Units 1–7	Throughout Units 1–7	Throughout Units 1–8	Throughout Units 1–8
Examples: 27–35, 152 #1e, 332–334, 404–406, 437–438 #6, 473–478, 497 #5	Examples: 23 #20, 40 #15, 210–216, 371 #13, 570–572, 578 #14	Examples: 62 #4, #5, 103 #3, 329–331, 341 #12, 482–489, 507, 539–542	Examples: 107 #7, 165–168, 205 #30, 245 #26, 318–321, 337 #7, 477 #10, 479–483, 513 #3, 581–585

Number and Quantity				
🔞 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
The Real Number System, N-RN				
Extend the properties of exponents to rational exp	ponents.			
N-RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{\frac{1}{3}}$ to be the cube root of 5 because we want $(5^{\frac{1}{3}})^3 = (5^{\frac{1}{3}})^3$ to hold, so $(5^{\frac{1}{3}})^3$ must equal 5.	332–337, 344, 351 #35, 358, 359			
N-RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.	335–337, 344 #35, 358, 585 #33	102 #30, 230 #40	156	
Use properties of rational and irrational numbers	-			
N-RN.3 Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.			70 #27	
Quantities*, N-Q				
Reason quantitatively and use units to solve probl	ems.			
N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	83 #9 & CYU, 110–112, 153 #3, 154 #4, 155 #5, 165–167, 170 #5, 171 #10, 190 STM, 191, 233, 292–303	5–9, 14, 24 #27, 25–29, 44 #26	435–436, 441–442	
N-Q.2 Define appropriate quantities for the purpose of descriptive modeling.	4–5, 324	5–9		
N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	49–51	460, 464 #8, 467, 471–477, 481 #18	236–242	

* Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*).

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4	
The Complex Number System, N-CN					
Perform arithmetic operations with complex numbers.					
N-CN.1 Know there is a complex number <i>i</i> such that $i^2 = -1$, and every complex number has the form $a + bi$ with <i>a</i> and <i>b</i> real.			354–356	210–218, 222 #20, 223 #25	
N-CN.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.			360 #14, 362 #25, 452	212–218, 222, 247, 312 #30– 31, 318–320, 328, 502 #43	
N-CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.				212, 220, 313– 317, 324 #8	
Represent complex numbers and their operations	on the comp	lex plane.			
N-CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.			360 #14	214–218, 220, 222–225, 313–327	
N-CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(1 - \sqrt{3}i)^3 = 8$ because $(1 - \sqrt{3}i)$ has modulus 2 and argument 120°.				214–218, 224, 313–327	
N-CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.				215	
Use complex numbers in polynomial identities and	d equations.				
N-CN.7 Solve quadratic equations with real coefficients that have complex solutions.			353–356, 358, 361 #19	209–214, 219, 221 #12, 224	
N-CN.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$.				222, 247 #31, 312 #31, 328 #30–31, 502 #43	
N-CN.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.			353–356, 358 #8, 362 #24	325 #12	

⁺ Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+).

🚥 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Vector and Matrix Quantities, N-VM				
Represent and model with vector quantities.				
N-VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v , $ v $, $ v $, v).				102–125, 129–137, 178–180, 423 #17
N-VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.				109–112, 130–132, 179
N-VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.				102–125, 133–137
Perform operations on vectors.				
N-VM.4 (+) Add and subtract vectors.				Unit 2
N-VM.4a Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.				109–116, 121 #9, 122 #12, 123 #16, 133– 137, 178–180
N-VM.4b Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.				109–126, 133– 137, 178–180
N-VM.4c Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of \mathbf{w} , with the same magnitude as \mathbf{w} and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.				121 #10, 122 #12, 133–137, 145 #2, 178–180
N-VM.5 (+) Multiply a vector by a scalar.				Unit 2
N-VM.5a Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.				106–110, 129–137, 145–146, 178–180
N-VM.5b Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $ c\mathbf{v} = c \mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $ c \mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along \mathbf{v} (for $c > 0$) or against \mathbf{v} (for $c < 0$).				106–110, 111 #6, 129–137, 145–146, 178–180
Perform operations on matrices and use matrices	in applicatio	ons.		
N-VM.6 (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	248–249, 256, 258, 262	74–100, 103–129, 157– 160, 231–251, 400–425	541 #6	
N-VM.7 (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.		82–86, 91, 92, 231–250		149 #10

🚥 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
N-VM.8 (+) Add, subtract, and multiply matrices of appropriate dimensions.		82–100, 103–129, 231–250		86 #10, 90 #22
N-VM.9 (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.		132–138, 148 #8, 150 #12	474 #10, 497 #3, 556 #28	
N-VM.10 (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.		132–138, 147 #7	556–557	
N-VM.11 (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.				149 #10, 151 #16
N-VM.12 (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.		231–250, 252–256	444 #15, 556 #28	71 #18, 86–87, 90 #22, 424 #20, 427 #34

Algebra

🚥 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Seeing Structure in Expressions, A-SSE				
Interpret the structure of expression.				
A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*	Units 1,3, 5, and 7	Units 1, 2, 3, 4, 5, and 7	Units 1, 2, 4, 5, 7, and 8	Units 1–8
A-SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.	157–158, 161–174, 176 #19, 178 #24, 474–489	336–344, 348, 353 #30–#33, 354 #37, 355 #38, 393–398, 488–497	112–117, 137– 155, 327–335, 341, 347–362, 364–388, 390–394, 458–464, 481–510	76–78, 138–147, 156–176, 188–207, 270–332, 440–460
A-SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.	26–45, 298–301, 491–508, 529 STM c	336, 342–343, 348, 355 #39, #41, 359–367, 396 #6	56–68, 319–345, 353, 364–388, 390–394, 458–494, 560–563, 597 #29, #30	138–147, 156–171, 248–262, 270–294, 334–392, 479–485

🙃 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
A-SSE.2 Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	494–498, 501–508	336–344, 348, 355 #41, 382–390, 395–398	192 #32, 258 #27, 281 #22, 332–362 #23, 392 #5, 559– 574, 577–583, 585 #3, 587 #8, 594 #20	188–207, 210–225, 271–331, 343–348, 555–561
Write expressions in equivalent forms to solve pro	oblems.			
A-SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*	Units 1, 3, 5, and 7	Units 1, 2, 3, and 5	Units 1, 2, 5, 7, and 8	Units 1, 2, 3, 4, 5, and 6
A-SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.	475–479, 491–508, 510–514, 518–523	278 #20, 321 #33, 336–344, 348, 353 #31, #32, 364–370	51 #31, 112– 117, 125 #26, 258 #27, 314 #28, 345 #24, 353–354, 356, 358, 361 #21, #22	200–201, 303–304, 306, 325 #12
A-SSE.3b Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.		355 #41	348–352, 357–359, 362, 511 #26	49, 50, 407–414, 421
A-SSE.3c Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1.15^{\frac{1}{12}})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	304–306, 311–312, 332– 337, 343–344, 348, 351, 358–359	377–390, 397–398	383 #14, 557 #33, 559–562	82–86, 334–359
A-SSE.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.★			63, 492–495, 501 #6, 534 #2, #3, 536	
Arithmetic with Polynomials and Rational Express	sions, A-APR			
Perform arithmetic operations on polynomials.				
A-APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.			327–335, 337 #3, 338 #5, 393 #9	75–78, 84–85, 99–100, 188–207
Understand the relationship between zeros and fa	actors of poly	nomials.		
A-APR.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	521 #19		345 #21 & #22, 385 #22	195 #7–197, 204
A-APR.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	521 #19	332–335, 352 #28, 355 #38, 359–370	101 #24, 319–345, 532 #22	188–193, 200–207, 254–256, 258

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4		
Use polynomial identities to solve problems.						
A-APR.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.		355 #39		188–207		
A-APR.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer <i>n</i> , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.				581–585, 589, 615		
Rewrite rational expressions.						
A-APR.6 Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x) + \frac{r(x)}{b(x)}$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.			376–379, 382–383, 385 #22, #23, 387–388	75–78, 193–197, 201, 227–246, 254, 256, 259 #9, 267–268		
A-APR.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.			369–379, 393 #9	227–241, 242 #20		
Creating Equations*, A-CED						
Create equations that describe numbers or relati	onships.					
A-CED.1 Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising</i> <i>from linear and quadratic functions, and simple rational</i> <i>and exponential functions.</i>	8, 47–51, 90, 200, 203 #9, 204 #10, #12, 209 #25, 210–211, 290–303, STM b, 301 CYU b, 307–311, 314 #21, 323–331, 462–482	359–370, 372 #16, 374 #23, 382–384, 387 #14, 390, 393–398	108–111, 127–157, 258 #26, 338 #6, 341 #13, 381 #4, 384 #19, 385 #20, 391 #2, 521, 561–564	4, 14–15, 94–95, 227–238, 335–339, 340–343, 349–353		
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	197–200, 204–207	30–46, 49–67, 69–72, 359– 370, 382–386, 393–398	132–155, 159 #6, 320–328, 364–376, 391–393, 458–461, 481–498, 573 #25	2–9, 14–17, 96, 138–154, 156–176, 182–187, 198– 200, 201 #12, 204 #27, 240 #12, 453 #10, 430–461		

🕮 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i>		30–45, 53– 54, 58–59, 61–67, 69–71, 139–141, 145–146, 148–152, 159	132–155, 159 #6	182–187, 198–200
A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.		25–46, 143– 144, 229 #38, 382–383, 387 #14, 390, 396 #11, 397 #11, 494 #4	50 #29, 58, 61 STM b, 63 #8, CYU, 171, 192 #33, 452 #37	221 #15, 231, 238 #3, 250 #3, 262 #23
Reasoning with Equations and Inequalities, A-REI				
Understand solving equations as a process of reas	soning and ex	plain the rea	soning.	
A-REI.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.		40 #16, 55–57, 139–141, 340–344, 352 #29–#33, 355 #41, 382–389, 462, 512 #25	58–59, 61 STM a, 65 #10 c, 12 a, 66 #14, 353, 478–479, 559–567, 569 #13, 583, 585, 587 #8, 588 STM c, f, 589, 590 #4, 591 #9, 592 #13	196–197, 235–236, 249–251, 302–304, 345–348, 407–410
A-REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.		193 #33, 352 #24, 359–375, 485 #32	115–117, 119 #6, 120 #9, 124 #21, 258 #26	247 #33, 249–253, 258, 261, 262, 428 #38
Solve equations and inequalities in one variable.				
A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	186–208, 210 #28, 232–235, 236 STM, 521 #14	30–35, 39 #13, #14, 45 #27, 47 #32, 156 #28, 357 #47, 388 #19	101 #22, 125 #27, 258 #26, 388 #31	
A-REI.4 Solve quadratic equations in one variable.	Unit 7	Unit 5	Unit 5	Units 3 and 6
A-REI.4a Use the method of completing the square to transform any quadratic equation in <i>x</i> into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	510-522	355 #41	347–353, 358 #4, 359 #10, 362 #23	406–414

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
A-REI.4b Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .	510-522	68 #31, 340– 344, 348, 355 #41, 359–375, 364–370, 388 #19, 391 #30, 433 #31, 451 #22, 486 #36, 585 #28	108–124, 281 #22, 345 #22, 346 #28, 347–362, 419 #37	190–193, 200–201, 209–222, 303–304, 306, 324 #8, 325 #12
Solve systems of equations.				
A-REI.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	197–200, 204–211, 236	67 #24		
A-REI.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	209 #26, 523 #26	49–67, 70–72, 130 #27, 132– 154, 159–160, 514 #29	125 #29, 132–155, 159 #6	185–187, 253–256, 258–259, 457, 459–460
A-REI.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.		356 #42, 365–375	115–117, 120 #9, 145 #5, 347	
A-REI.8 (+) Represent a system of linear equations as a single matrix equation in a vector variable.		139–154, 159–160	497, 498, 501 #7, 502 #8	186–187, 200, 203
A-REI.9 (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).		136–138, 147 #7, 151 #13	497, 502 #8	186–187, 200
Represent and solve equations and inequalities gr	aphically.			
A-REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	186–190, 197–200, 210 #28	1–23, 30–33, 37 #8, 42 #22, 326–335	46 #17, 108–124, 341 #13, 353–356, 361–362, 462–467	
A-REI.11 Explain why the <i>x</i> -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*		49–53, 57–60, 64 #12, 321 #35, 356 #42, 359–375	114 STM, 115–116, 122 #14, 137–143	
A-REI.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.		375 #24	127–132, 137–151, 152 #20, 227 #43, 314 #27, 389 #37	

Functions				
🕬 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Interpreting Functions F-IF				
Understand the concept of a function and use fun	ction notatio	n.		
F-IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.		326–335, 345–346, 349–350, 352, 354 #35, 387 #13	109–111, 320–345, 386 #25, 474 #9, 514–522, 556 #27, 558 #37, 601 #40	10–13, 18–19, 227–234, 252–254, 468–500
F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.		326–335, 345–346, 349–350, 352, 354 #35, 382, 387, 389, 390, 394, 398	66 #6, 109– 111, 323–345, 357 #2, 360 #17, 364–388, 462–467, 514–522	182–193, 227–231, 237, 240, 243, 468–500, 511–523
F-IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ for $n \ge 1$.	Informal 26–44, 150–161, 168, 175, 290–319, 322–329, 338–343		55 #6, 462–467, 481–510, 514–522, 533–536, 575 #37	13 #2, 20 #17, 353 #17, 495 #14, 496 #19, 599–605
Interpret functions that arise in applications in te	rms of the co	ontext.		
F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> *	2–17, 52–58, 152–170, 176 #18, 197–204, 290–303, 322–329, 346 #24, 462–481	2–23, 331– 333, 338, 346 #7, 353 #34	109–111, 127– 143, 320–345, 363–366, 368–376, 380–385, 425–427, 432–437, 577–589	10–51, 182–193, 227–231, 237, 240, 243, 284–286, 334, 386, 468–527
F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*	2–17, 26–39, 43 #22, 150– 159, 162–183, 186–191, 206 #18, 322–329, 462–472	326–335, 345–346, 349–350, 352, 354 #35, 382, 387, 389, 390, 394, 398	109–111, 138 #2, 338 #6, 364–386, 543–551, 553 #15, 565 #2, 577–582, 601 #40	10–13, 182–193, 227–231, 237, 240, 243
F-IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	155–156, 158, 161–168, 169 #3, 170 #6, 175 #16, 177 #22, 181–182	2–9, 387 #13		468–500

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Analyze functions using different representations	S.			
F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*	Units 1, 3, 5, and 7	Units 1, 4, and 5	Units 2, 5, 6, 7, and 8	Units 3, 4, 5, 6, and 7
F-IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.	150–182, 462–477	10–15, 22 #15, 30–33, 38 #10, 40 #16, 42 #22, 332– 335, 348, 352 #28	108–155, 332, 336–337, 347–352, 357–361, 391 #2	11–12, 14, 44, 46, 48, 484, 529
F-IF.7b Graph square root, cube root, and piecewise- defined functions, including step functions and absolute value functions.	346 #24	17–19	116, 124 #21, 383 #19, 158	18–19, 23, 28, 33, 37, 40, 46–47, 50, 58, 65, 69, 92, 97, 501 #38
F-IF.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.		326–356, 355 #38	320-344	188–193, 200, 202–203
F-IF.7d (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.		2–23	363 #30, 364–369, 381–387, 601 #40	227–237, 241–245, 253–256
F-IF.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	292–303, 307–311	382–383, 387 #13	124, 432–437, 441–442, 446 #24, 449 #30, 472, 559– 562, 565 #2, 573, 576 #43, 601 #43	5–6, 11–13, 15, 56, 61–62, 69, 71 #21, 72, 97, 284–286 344–345
F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	Units 5 and 7	Units 1 and 5	Units 1, 2, 5, 7, and 8	Units 3, 4, 5, 6, and 7
F-IF.8a Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	475–479, 510–514	332–340, 346–356	347–352, 357, 362 #23	303–304, 306–310, 406–408, 414, 421
F-IF.8b Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{\frac{t}{10}}$, and classify them as representing exponential growth or decay.	27–32, 36–40, 42, 44, 46, 71, 290–297, 352 #40	382–386	383 #19, 559–562	82–83, 86 #9, 340–348, 351 #10, 367–370, 375–386

🙃 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	314 #21, 345 #2, 352 #40	19 #7, 373 #19	157 #42	10–12, 468–500
Building Functions, F-BF				
Build a function that models a relationship betwe	en two quant	ities.		
F-BF.1 Write a function that describes a relationship between two quantities.*	Units 1, 3, 5, and 7	Units 1, 4, and 5	Units 1, 2, 5, 7, and 8	Units 1, 3, 5, and 7
F-BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.	26–44, 150–154, 158–159, 168, 289–319, 322–329, 338–343, 359	8 #9, 16 #2, 17 #3, 32–33, 36–37, 61 #1, 70–71, 100 #23, 139–141, 145–146, 159, 360–363, 368–369, 391, #32	481–510, 533–536	13 #2, 20 #17, 595–610
F-BF.1b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	470-471		327–335, 373–388, 433, 441 #11	32–78, 84–86, 99, 228–229, 237, 245 #25, 300–302, 305–310, 498 #27
F-BF.1c (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.				79–83, 85–90, 99, 100
F-BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*	26–44, 150–161, 168, 290–319, 322–329, 338–343		63 #4–#5, 458–479, 481–510, 535–536	13 #2, 20 #17, 353 #16, 595–607
Build new functions from existing functions.				
F-BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. <i>Include recognizing even and odd functions from their graphs and algebraic expressions for them.</i>	153, 155, 177 #22, 303, 316 #27, 359, 419 #11, 473–479, 529	12–14, 64, 278 #20, 346 #7, 352 #28	418 #33, 432–437, 441–443, 445 #19	26–50, 52–73, 97–98, 100, 202 #14, 290 #15, 454 #13

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
F-BF.4 Find inverse functions.			Unit 8	Unit 4
F-BF.4a Solve an equation of the form $f(x) = c$ for a simple function <i>f</i> that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ for $x > 0$ or $f(x) = \frac{(x+1)}{(x-1)}$ for $x \neq 1$.			545–548, 553–554	82
F-BF.4b (+) Verify by composition that one function is the inverse of another.				82–83, 88
F-BF.4c (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.			538–548, 552 #14	88
F-BF.4d (+) Produce an invertible function from a non- invertible function by restricting the domain.			544, 579–581, 584	88, 299
F-BF.5 (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.*			559–562, 572 #24, 573 #25	340–348, 367–370, 375–386
Linear, Quadratic, and Exponential Models, F-LE				
Construct and compare linear, quadratic, and exp	onential mod	els and solve	e problems.	
F-LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.	Units 3 and 5			
F-LE.1a Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	175 #16, 303 STM a		55–56	
F-LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	150–183	5, 11, 100 #23, 391, #32, 508 #14	128–142	
F-LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	27–32, 36–44, 290–303, 307–319, 322–332, 338–350, 355–359	100 #23, 382–383, 390 #27, 391 #32		
F-LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	26–45, 157– 183, 290–303, 307–319, 322–332, 338–350, 355–359	5, 11, 100 #23, 382–383, 391 #32, 508 #14, 577 #9, #12	481–489, 503 #10	
F-LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	310 #9, 311, #11, 314 #21, 353 #40b, 483 #12, 487 #22, 455 #34	321 #35, 515 #36	124 #21, 281 #28	

🚥 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
F-LE.4 For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where <i>a</i> , <i>c</i> , and <i>d</i> are numbers and the base <i>b</i> is 2, 10, or <i>e</i> ; evaluate the logarithm using technology.			56, 559–564, 568–574, 602–604	340–359, 367–370, 389–390
Interpret expressions for functions in terms of the	e situation th	ey model.		
F-LE.5 Interpret the parameters in a linear or exponential function in terms of a context.	26–45, 47–55, 153–183, 307–319, 322–332, 338–343, 348–350, 355–359	30–33, 49– 66, 280–285, 286 CYU b, 305–308, 316 #18, 357 #46, 382–385, 486 #38	128–142	2–10, 14–17, 26–64, 340–359, 367–370
Trigonometric Functions, F-TF				
Extend the domain of trigonometric functions using the second sec	ng the unit ci	rcle.		
F-TF.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.			427–432, 444 #16, 445 #18, 446 #23	162–168, 173 #9
F-TF.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.			121 #13, 427-432, 425	
F-TF.3 (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for <i>x</i> , where <i>x</i> is any real number.			427–432, 441 #9 b, 445 #18	270–275, 279–286
F-TF.4 (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.				290 #15
Model periodic phenomena with trigonometric fu	nctions.			
F-TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*			425–427, 432–437, 441–442, 446 #24, 449 #30, 577–583, 589–594	56, 59–73, 270, 278, 287–292, 300–302, 305–310, 476–477
F-TF.6 (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.			577–589	299
F-TF.7 (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*		471–473	585, 589–599	296-311

🔞 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Prove and apply trigonometric identities.				
F-TF.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ) , cos (θ) , or tan (θ) given sin (θ) , cos (θ) , or tan (θ) and the quadrant of the angle.		479–480	68 #20	272
F-TF.9 (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.				273–280, 287–294

Geometry

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Congruence, G-CO				
Experiment with transformations in the plane.				
G-CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	363 TATS, 386, 391	170–180	29–39, 401–403, 415	
G-CO.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	370–373	195–229, 231–250, 254–256	13–16, 26 #27, 178, 192, 208–213, 225	313–327
G-CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	398-403			
G-CO.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.			13–16, 20, 208–211	
G-C0.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.		162–163, 195–229, 232–250, 254–256	193 #33, 209, 210 #2, 208–215, 219, 225	
Understand congruence in terms of rigid motions	•			
G-CO.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	370–373, 454 #27	195–229, 254, 487 #39	13–16, 20, 103 #3, 193 #33, 208–215, 218, 224, 445 #19	

🚥 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
G-C0.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.		487 #39	208–215	
G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	370–371	195–222	208–215	
Prove geometric theorems.				
G-C0.9 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	374–377, 385 #7, 387, 391 #18, 392 #21e, 405 #3		13–16, 29–49, 198 #7	
G-C0.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	369–382, 391 #19	192 #32	20, 190 #22, 195–225	
G-C0.11 Prove theorems about parallelograms. <i>Theorems include:</i> opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	387 #10, 390 #17, 391 #18	184 #10	204–208, 217 #8, 221, #19 & #20, 222 #24, 233 #6	
Make geometric constructions.				
G-C0.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	386 #8, #9, 401 #2, #4	192 #32	14, 15, 28, 33–34, 39–40, 43, 191, 201 #3, 202 #7, 220 #18, 221 #20, 223 #29, 224 #30, 399 #4, 402 #3a, 416 #29	
G-CO.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.		46	221, 414–415	
Similarity, Right Triangles, and Trigonometry, G-S	RT			
Understand similarity in terms of similarity trans	formations.			
G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor.		Unit 3	Unit 3	
G-SRT.1a A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.		206–209, 221 #10	176–177, 188 #18, 230 #1	

瓍 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
G-SRT.1b The dilation of a line segment is longer or shorter in the ratio given by the scale factor.		205–210, 220 #9, 223 #15	173–179, 183 #7, 188 #18	
G-SRT.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.		216, 222, 254, 487 #39	177–179, 213–214	
G-SRT.3 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.			208, 213–215	
Prove theorems involving similarity.				
G-SRT.4 Prove theorems about triangles. <i>Theorems</i> include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.		184 #9	163–192, 230 #2	
G-SRT.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	372–377, 384–391	184 #9, #10, 197–222	162–192, 195–226, 230–234, 385 #20, 400, 401, 412	
Define trigonometric ratios and solve problems in	volving right	triangles.		
G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.		461-462	383 #13	
G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles.		481 #20	445 #18	276–280
G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*		458–460, 464, 467–477, 481 #18, 482–484		
Apply trigonometry to general triangles.				
G-SRT.9 (+) Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.		506 #9		
G-SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.		488–513, 518–520	60–61, 168–169	92 #26, 113, 121–125, 291 #18
G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).		488–513, 518–520	168–169	92 #26, 113, 121–125, 291 #18

🚥 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Circles, G-C				
Understand and apply theorems about circles.				
G-C.1 Prove that all circles are similar.			190 #23, 397	
G-C.2 Identify and describe relationships among inscribed angles, radii, and chords. <i>Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.</i>			397–417, 454–456	
G-C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.			201–203, 410 #10	
G-C.4 (+) Construct a tangent line from a point outside a given circle to the circle.			399 #4	
Find arc lengths and areas of sectors of circles.				
G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.			415, 427–431, 431 STM b, 444	
Expressing Geometric Properties with Equations,	G-GPE			
Translate between the geometric description and	the equation	for a conic s	ection.	
G-GPE.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.		175–180, 184 #11, 187 #17, 351 #22	359 #10	250, 407, 411, 421
G-GPE.2 Derive the equation of a parabola given a focus and directrix.				407–409, 421, 464
G-GPE.3 (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.				409–414, 421, 464
Use coordinates to prove simple geometric theore	ems algebraio	cally.		
G-GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.		166, 169, 173–174, 178–179, 184 #9 & #10, 191 #30, 354 #36, 356 #42	51 #34, 187 #16, 191 #28, 221 #16, 221 #18, 223 #24, 411 #12, 413 #14	

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
G-GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	177 #22, 180 #30	170–172, 186, 190, 251 #23	46 #17, 187, 191	
G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.		186 #15		
G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*		181 #2, 208, 210, 218, 221, 376 #30, 514 #31	314 #29	
Geometric Measurement and Dimension, G-GMD				
Explain volume formulas and use them to solve pr	oblems.			
G-GMD.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	175 #14, 447 #12, 448 #13, 449–450			454-455
G-GMD.2 (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	448 #14, 449–450, 453 #26			454–455
G-GMD.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	447–448, 453 #26	24 #27, 376 #28, 380 #1, 391 #29, 481 #18		8 #5, 16, 17, 19, 21, 231, 382, 434, 454–455
Visualize relationships between two-dimensional	and three-di	mensional ob	ojects.	
G-GMD.4 Identify the shapes of two-dimensional cross- sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two- dimensional objects.	430 #8, 453, 484			394–405, 415–422, 437–458
Modeling with Geometry, G-MG				
Apply geometric concepts in modeling situations.				
G-MG.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*	366–369, 383 #2, #3, 388– 395, 407–422, 424–453	458–459, 464 #8, 474 #1 & #2, 481 #18, 498– 502, 505 #6, 512–513, 519 #4	421–424	138–176, 145 #1, 146 #4, 147 #6, 201 #12, 231 CYU, 238 #3, 394–405, 446–449, 453 #10, 454 #14

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
G-MG.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*	112 CYU, 137 #16, 452 #24, 454 #28, 567–570	18 #6, 23 #20, 321 #34, 376 #28	93 #3	5 #2 & #3, 373 #5
G-MG.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*	237–278, 282 #16, 283 #17, 286–288, 366–369, 383 #2, #3, 390 #17, 393 #27, 415–416, 444 #6, 457 #4	176 #1, 182 #6, 188 #19, 220 #9, 231–250, 252–256, 399–456, 498–502, 505 #6, 513 #27, 519 #4	42 #6, 43 #8, 173–176, 183 #7, 198 #6, 216, 217, 218 #10, 219 #12, 229–234, 404, 413–414, 513 #33	104, 109–120, 138–176, 145 #1, 146 #4, 147 #6, 201 #12, 231 CYU, 238 #3, 417 #4, 453 #10, 454 #14
Interpreting Categorical and Quantitative Data, S	-ID			
Summarize, represent, and interpret data on a sin	gle count or	measuremen	t variable.	
S-ID.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).	67, 73–101, 106, 108–142, 144–148, 231 #29, 454 #31, 554–556, 558, 560– 562, 564, 571–575, 587	48 #34, 155 #26, 277 #18, 433 #30, 560–569	81–99, 104– 105, 228 #45, 236–239, 283–293, 297–302, 502 #10	
S-ID.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	73–101, 103–142, 144–148, 397 #39, 490 #31	131 #31, 155 #26, 277 #18, 279 #22, 433 #30	81–88, 157 #41, 227 #44, 228 #45, 241–244, 253 #9, 255 #17, 256 #21, 262 #5, 274 #7, 285–293, 297–302, 306, #8	
S-ID.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	73–101, 103– 142, 144–148, 454 #31, 561	155 #26, 277 #18, 279 #22	81–83, 241–244, 259–280, 316–318	
S-ID.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.			240–257, 259–280, 287–289, 294–302, 316–318, 479 #21, 511 #27, 601 #41	

🔞 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Summarize, represent, and interpret data on two	categorical a	nd quantitati	ive variables.	
S-ID.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	139–140, 539, 543, 544	392 #33, 521–542, 543 #26, 584 #25		
S-ID.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	Units 1, 3, 5, and 7	Units 1, 4, and 5	Units 2, 5, 6, 7, and 8	Unit 5
S-ID.6a Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.	5-8, 11-18, 20-24, 27-28, 44, 70-72, 150-154, 156, 159, 161-174, 176, 178, 180, 181-183, 232-234, 236, 290-303, 307-319, 322-332, 338-343, 348-350, 355-359, 462-467, 499-502, 579-580	2-6, 11, 14, 19 #8, 20 #10, 21#12, 22 #17, 23 #19, 100 #23, 250 #22, 279 #26, 280-298, 305-310, 314-315, 317 #21, 390 #27, 391, #32, 508 #14, 577 #9 & #12	482–489, 495–498, 505, 509, 593 #17	361–386, 391
S-ID.6b Informally assess the fit of a function by plotting and analyzing residuals.		279–286, 305–306, 308 #4, 313–314, 318 b, 317 #22, 322–323		361–386, 391
S-ID.6c Fit a linear function for a scatter plot that suggests a linear association.	150–154, 156, 159, 161–174, 181–183, 206, 232–234, 236	280–291, 305–308, 310 #6 312 #8 b., 315 # 15, 486 #38		361–383, 386, 391
Interpret linear models.				
S-ID.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	1–7, 60, 150–155, 156– 159, 161–174, 176, 181–183, 354 #46	279 #26, 280– 285, 286 CYU b, 305–308, 316 #18, 280– 286, 357 #46, 486 #38	101 #23, 126 #32. 257 #42	

📧 Common Core State Standards	Course 1	Course 2	Course 3	Course 4	
S-ID.8. Compute (using technology) and interpret the correlation coefficient of a linear fit.		258–277, 286–297, 309–310, 313–316, 319 #26, 486 #38, 584 #25	126 #32, 254 #14		
S-ID.9. Distinguish between correlation and causation.	44	299–304, 310–311, 452 #26	92 #2		
Making Inferences and Justifying Conclusions, S-	C				
Understand and evaluate random processes unde	rlying statist	ical experim	ents.		
S-IC.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.			237–239, 266–280, 283–296, 511 #27, 601 #41		
S-IC.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?	553–564	554 #8, 560–565, 573 #1, 574 #3			
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.					
S-IC.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.			74–80, 89–91 STM b, 92, 95, 99 #17		
S-IC.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.			279 #17, 280 #20		
S-IC.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.			81–88, 92–99, 104–106		
S-IC.6 Evaluate reports based on data.		299–304, 310–313	79–80, 95, 99, 278 #16		
Conditional Probability and the Rules of Probability, S-CP					
Understand independence and conditional probability and use them to interpret data.					
S-CP1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	531–539, 540 #6, 546 #11, 547 #16, 548 #20, 553, 557–558, 571	522–542		547, 550, 564	

🚥 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
S-CP.2 Understand that two events <i>A</i> and <i>B</i> are independent if the probability of <i>A</i> and <i>B</i> occurring together is the product of their probabilities, and use this characterization to determine if they are independent.		526–528, 532–542, 586		553
S-CP.3 Understand the conditional probability of <i>A</i> given <i>B</i> as $\frac{P(A \text{ and } B)}{P(B)}$, and interpret independence of <i>A</i> and <i>B</i> as saying that the conditional probability of <i>A</i> given <i>B</i> is the same as the probability of <i>A</i> , and the conditional probability of <i>B</i> given <i>A</i> is the same as the probability of <i>B</i> .		528–542, 586		579–580, 588 #7, 617
S-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.		521–535, 536 #2, 538 #6, 542, 586		542, 547
S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. <i>For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</i>		528–542, 586		
Use the rules of probability to compute probabilities of compound events in a uniform probability model.				
S-CP.6 Find the conditional probability of <i>A</i> given <i>B</i> as the fraction of <i>B</i> 's outcomes that also belong to <i>A</i> , and interpret the answer in terms of the model.		528-531		579–580, 588, 617
S-CP.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	24, 43, 212 #38, 396 #32, 531–544, 586	358 #52, 392 #33, 515 #34, 534–535	51 #30, 73 #40, 295–296, 304–305, 308	
S-CP.8 (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)$, and interpret the answer in terms of the model.		358 #52, 392 #33, 532–542, 586	51 #30, 73 #40, 258, 266 #1, 278 #15, 280 #23, 295–296, 304–305, 308, 310	579–580, 588, 617
S-CP.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	143, 355–356 #2, 547, 581 #22		96 #8, 311 #21, 312 #23	575–592, 614

🙃 Common Core State Standards	Course 1	Course 2	Course 3	Course 4
Using Probability to Make Decisions, S-MD				
Calculate expected values and use them to solve p	oroblems.			
S-MD.1 (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	533, 542, 544–547, 554, 560–562	549–551, 555 #9, 561–569, 571, 573, 579, 580	502–503	
S-MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	8–11, 544, 554	545–558, 562–564, 570–581, 587–589	259–280, 502–503	
S-MD.3 (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.	533–536, 542, 544–547, 564, 580, 586	545–558, 563–570, 581, 587–589	259–280, 503 #10	
S-MD.4 (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?	553–555, 557–558, 559–567, 571–576, 581–582, 587–588	562	73 #40, 259–280	
Use probability to evaluate outcomes of decisions.				
S-MD.5 (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.		Unit 8	Unit 4	
S-MD.5a Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.		545-558	280 #19	
S-MD.5b Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.		545-558		614 #2
S-MD.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).		545-558		
S-MD.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).		557 #16 & #17	280 #19, 283–313	

Core-Plus Mathematics CCSS Edition Common Core State Standards Pathway

Introduction

This section is a guide for your planning of CCSS-aligned *Core-Plus Mathematics* courses. If you complete the investigations and problems as outlined in this section, students will be able to complete subsequent units in *Core-Plus Mathematics* and will develop proficiency in the topics and mathematical practices specified in the CCSS.

It is advisable to collaborate with middle school mathematics teachers from your feeder school(s) to plan articulation between the Grades 6–8 CCSS and those for high school. Some topics in the CCSS are introduced in middle school and used or extended in high school. For these standards you may find that some material in *Core-Plus Mathematics* will need fewer days to complete than is suggested in the Unit Planning Guides in the *Teacher's Editions*. Typically, the cognitive demand of the problems in *Core-Plus Mathematics* Course 1 exceeds that of similar content in middle school programs. Thus, it is in students' best interest to complete the CCSS Courses 1–3 Pathway outlined in this section with careful consideration of their prior knowledge.

Course 1 Pathway

Unit 1 Patterns of Change

Lesson 1 Cause and Effect

Investigation 1 Physics and Business at Five Star Amusement ParkInvestigation 2 Taking ChancesInvestigation 3 Trying to Get Rich Quick

Lesson 2 Change Over Time

- Investigation 1 Predicting Population Change
- Investigation 2 Tracking Change with Spreadsheets

Lesson 3 Tools for Studying Patterns of Change

- Investigation 1 Communicating with Symbols
- Investigation 2 Quick Tables, Graphs, and Solutions
- Investigation 3 The Shapes of Algebra

Unit 2 Patterns of Data

Lesson 1 Exploring Distributions

Investigation 1 Shapes of Distributions Investigation 2 Measures of Center

Lesson 2 Measuring Variability

Investigation 1 Measuring Position

Investigation 2 Measuring and Displaying Variability: The Five-

Number Summary and Box Plots

Investigation 3 Identifying Outliers

Investigation 4 Measuring Variability: The Standard Deviation

Investigation 5 Transforming Measurements

Unit 3 Linear Functions

Lesson 1 Modeling Linear Relationships

Investigation 1 Getting Credit

Investigation 2 Symbolize It

Investigation 3 Fitting Lines

Lesson 2 Linear Equations and Inequalities

Investigation 1 Who Will Be the Doctor?Investigation 2 Using Your HeadInvestigation 3 Using Your Head ... More or Less

Investigation 4 Making Comparisons

Lesson 3 Equivalent Expressions

Investigation 1 Different, Yet the Same Investigation 2 The Same, Yet Different

Unit 4 Discrete Mathematical Modeling

Lesson 1 Euler Circuits: Finding the Best Path

Investigation 1 Planning Efficient RoutesInvestigation 2 Making the CircuitInvestigation 3 Graphs and Matrices

Unit 5 Exponential Functions

Lesson 1 Exponential Growth

Investigation 1 Counting in Tree Graphs
Investigation 2 Getting Started
Investigation 3 Compound Interest
Investigation 4 Modeling Data Patterns
Investigation 5 Properties of Exponents I

Lesson 2 Exponential Decay

Investigation 1 More Bounce to the Ounce
Investigation 2 Medicine and Mathematics
Investigation 3 Modeling Decay
Investigation 4 Properties of Exponents II
Investigation 5 Square Roots and Radicals

Unit 6 Patterns in Shape

Lesson 1 Triangles, Quadrilaterals, and Their Properties

- Investigation 1 Form and Function
- Investigation 2 Congruent Shapes
- Investigation 3 Reasoning with Shapes
- Investigation 4 Getting the Right Angle

Lesson 2 Polygons and Their Properties

- Investigation 1 Patterns in Polygons
- Investigation 2 The Triangle Connection
- **Investigation 3** Patterns with Polygons

Lesson 3 Polyhedra and Their Properties

- Investigation 1 Recognizing and Constructing Three-Dimensional Shapes
- Investigation 2 Visualizing and Sketching Three-Dimensional Shapes
- Investigation 3 Patterns in Polyhedra

Unit7 Quadratic Functions

Lesson 1 Quadratic Patterns

Investigation 1 Pumpkins in FlightInvestigation 2 Golden Gate QuadraticsInvestigation 3 Patterns in Tables, Graphs, and Rules

Lesson 2 Equivalent Quadratic Expressions

Investigation 1 Finding Expressions for Quadratic Patterns Investigation 2 Reasoning to Equivalent Expressions

Lesson 3 Solving Quadratic Equations

Investigation 1 Solving $ax^2 + c = d$ and $ax^2 + bx = 0$ **Investigation 2** The Quadratic Formula

Unit 8 Patterns in Chance

Lesson 1 Calculating Probabilities

Investigation 1 Probability DistributionsInvestigation 2 The Addition Rule

Lesson 2 Modeling Chance Situations

Investigation 1 When It's a 50-50 Chance

Investigation 2 Simulation Using Random Digits

Course 2 Pathway

Unit 1 Functions, Equations, and Systems

Lesson 1 Direct and Inverse Variation

Investigation 1 On a Roll Investigation 2 Power Models

Lesson 2 Multivariable Functions

Investigation 1 Combining Direct and Inverse Variation Investigation 2 Linear Functions and Equations

Lesson 3 Systems of Linear Equations

Investigation 1 Solving with Graphs and SubstitutionInvestigation 2 Solving by EliminationInvestigation 3 Systems with Zero and Infinitely Many Solutions

Unit 2 Matrix Methods

Lesson 1 Constructing, Interpreting, and Operating on Matrices

Investigation 1 There's No Business Like Shoe Business Investigation 3 Combining Matrices

Lesson 2 Multiplying Matrices

Investigation 1 Brand Switching

Investigation 2 More Matrix Multiplication

Lesson 3 Matrices and Systems of Linear Equations

Investigation 1 Properties of Matrices

Unit 3 Coordinate Methods

Lesson 1 A Coordinate Model of a Plane

Investigation 1Representing Geometric Ideas with CoordinatesInvestigation 2Reasoning with Slopes and Lengths

 $Investigation \ 3 \ \ Representing \ and \ Reasoning \ with \ Circles$

Lesson 2 Coordinate Models of Transformations

Investigation 1 Modeling Rigid Transformations Investigation 2 Modeling Size Transformations

Investigation 3 Combining Transformations

Unit 4 Regression and Correlation

Lesson 1 Bivariate Relationships

Investigation 1 Rank Correlation

Investigation 2 Shapes of Clouds of Points

Lesson 2 Least Squares Regression and Correlation

Investigation 1 How Good Is the Fit?

Investigation 2 Behavior of the Regression Line

Investigation 3 How Strong Is the Association?

Investigation 4 Association and Causation

Unit 5 Nonlinear Functions and Equations

Lesson 1 Quadratic Functions, Expressions, and Equations

- Investigation 1 Functions and Function Notation
- Investigation 2 Designing Parabolas
- Investigation 3 Expanding and Factoring
- Investigation 4 Solving Quadratic Equations

Lesson 2 Nonlinear Systems of Equations

- Investigation 1 Supply and Demand
- Investigation 2 Making More by Charging Less

Lesson 3 Common Logarithms and Exponential Equations Investigation 1 How Loud Is Too Loud? Investigation 2 Solving for Exponents

Unit 6 Modeling and Optimization

Lesson 2 Scheduling Projects Using Critical Paths

Investigation 1 Building a Model

Investigation 2 Critical Paths and the Earliest Finish Time

Unit 7 Trigonometric Methods

Lesson 1 Trigonometric Functions

Investigation 1 Connecting Angle Measures and Linear MeasuresInvestigation 2 Measuring Without MeasuringInvestigation 3 What's the Angle?

Lesson 2 Using Trigonometry in Any Triangle

Investigation 1 The Law of Sines Investigation 2 The Law of Cosines

Unit 8 Probability Distributions

Lesson 1 Probability Models

- Investigation 1 The Multiplication Rule for Independent Events
- Investigation 2 Conditional Probability
- Investigation 3 The Multiplication Rule When Events Are Not Independent

Lesson 2 Expected Value

Investigation 1 What's a Fair Price?

Investigation 2 Expected Value of a Probability Distribution

Course 3 Pathway

Unit 1 Reasoning and Proof

Lesson 1 Reasoning Strategies

Investigation 1 Reasoned Arguments Investigation 2 If-Then Statements

Lesson 2 Geometric Reasoning and Proof

Investigation 1 Reasoning about Intersecting Lines and AnglesInvestigation 2 Reasoning about Parallel Lines and Angles

Lesson 3 Algebraic Reasoning and Proof

Investigation 1 Reasoning with Algebraic ExpressionsInvestigation 2 Reasoning with Algebraic Equations

Lesson 4 Statistical Reasoning

Investigation 1 Design of ExperimentsInvestigation 2 By Chance or from Cause?Investigation 3 Statistical Studies

Unit 2 Inequalities and Linear Programming

Lesson 1 Inequalities in One Variable

Investigation 1 Getting the Picture

Investigation 2 Quadratic Inequalities

Investigation 3 Complex Inequalities

Lesson 2 Inequalities with Two Variables

Investigation 1 Solving InequalitiesInvestigation 2 Linear Programming—A Graphic ApproachInvestigation 3 Linear Programming—Algebraic Methods

Unit 3 Similarity and Congruence

Lesson 1 Reasoning about Similar Figures

Investigation 1 When Are Two Polygons Similar?Investigation 2 Sufficient Conditions for Similarity of TrianglesInvestigation 3 Reasoning with Similarity Conditions

Lesson 2 Reasoning about Congruent Figures

Investigation 1 Congruence of Triangles Revisited

Investigation 2 Congruence in Triangles

Investigation 3 Congruence in Quadrilaterals

Investigation 4 Congruence and Similarity: A Transformation Approach

Unit 4 Samples and Variation

Lesson 1 Normal Distributions

Investigation 1 Characteristics of a Normal Distribution

Investigation 2 Standardized Values

Investigation 3 Using Standardized Values to Find Percentiles

Lesson 2 Binomial Distributions

Investigation 1 Shapes, Center, and Spread

Investigation 2 Binomial Distributions and Making Decisions

Unit 5 Polynomial and Rational Functions

Lesson 1 Polynomial Expressions and Functions

- Investigation 1 Modeling with Polynomial Functions
- Investigation 2 Addition, Subtraction, and Zeroes
- Investigation 3 Zeroes and Products of Polynomials

Lesson 2 Quadratic Polynomials

Investigation 1 Completing the Square

Investigation 2 The Quadratic Formula and Complex Numbers

Lesson 3 Rational Expressions and Functions

- Investigation 1 Domains and Graphs of Rational Functions
- Investigation 2 Simplifying Rational Expressions
- Investigation 3 Adding and Subtracting Rational Expressions
- Investigation 4 Multiplying and Dividing Rational Expressions

Unit 6 Circles and Circular Functions

Lesson 1 Circles and Their Properties

- Investigation 1 Tangents to a Circle Investigation 2 Chords, Arcs, and Central Angles
- Investigation 3 Angles Inscribed in a Circle

Lesson 2 Circular Motion and Periodic Functions

- Investigation 1 Angular and Linear Velocity
- Investigation 2 Modeling Circular Motion
- Investigation 3 Revolutions, Degrees, and Radians
- Investigation 4 Patterns of Periodic Change

Unit 7 Recursion and Iteration

Lesson 1 Modeling Sequential Change

- Investigation 1 Modeling Population Change
- Investigation 2 The Power of Notation and Technology

Lesson 2 A Recursive View of Functions

Investigation 1 Arithmetic and Geometric Sequences

Unit 8 Inverse Functions

Lesson 1 What Is an Inverse Function?

Investigation 1 Coding and Decoding Messages

Investigation 2 Finding and Using Inverse Functions

Lesson 2 Common Logarithms and Their Properties

Investigation 1 Common Logarithms Revisited

Investigation 2 Covering All the Bases

Investigation 3 Properties of Logarithms