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Effects of a Preschool Mathematics Curriculum:

Summary Research on the *Building Blocks* Project

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Abstract

This study evaluated the efficacy of a preschool mathematics program based on a comprehensive model of developing research-based software and print curricula. *Building Blocks*, funded by the National Science Foundation, is a curriculum development project focused on creating research-based, technology-enhanced mathematics materials for PreK through grade 2. In this article, we describe the underlying principles, development, and initial summary evaluation of the first set of resulting materials, as they were implemented in classrooms teaching children at risk for later school failure. Experimental and comparison classrooms included two principal types of public preschool programs serving low-income families, state funded and Head Start pre-kindergarten programs. Children in all classrooms were pre- and post-tested with an individual assessment based on the curriculum's hypothesized learning trajectories. The experimental treatment group score increased significantly more than the comparison group score. Effect sizes comparing posttest scores of the experiment group to those of the comparison group were .85 for number and 1.44 for geometry, and effect sizes comparing the experimental group's pretest and posttest scores were 1.71 for number and 2.12 for geometry. Thus, achievement gains of the experimental group were comparable to the sought-after 2-sigma effect of individual tutoring. This study contributes to research showing that focused early mathematical interventions help young children develop a foundation of informal mathematics knowledge, especially for children at risk for later school failure.

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Government agencies have recently emphasized the importance of evidence-based instructional materials (e.g., Feuer, Towne, & Shavelson, 2002; Reeves, 2002). However, the ubiquity and multifariousness of publishers' claims of research-based curricula, in conjunction with the ambiguous nature of the phrase "research-based," discourages scientific approaches to curriculum development (and allows the continued dominance of non-scientific "market research") and undermines attempts to create a shared research foundation for the creation of classroom curricula (Battista & Clements, 2000; Clements, 2002; Clements & Battista, 2000). Once produced, curricula are rarely evaluated scientifically. Less than 2% of research studies concerned the effects of textbooks (Senk & Thompson, 2003), even though these books predominate mathematics curriculum materials in U.S. classrooms and to a great extent determine teaching practices (Goodlad, 1984), even in the context of reform efforts (Grant, Peterson, & Shojgreen-Downer, 1996). This study is one of several coordinated efforts to assess the efficacy of a curriculum that was designed and evaluated according to specific criteria for both the development and evaluation of a scientifically based curriculum (Clements, 2002; Clements & Battista, 2000).

Building Blocks is a NSF-funded PreK to grade 2 mathematics curriculum development project, designed to comprehensively address recent standards for early mathematics education for all children (e.g., Clements, Sarama, & DiBiase, 2004; NCTM, 2000). Previous articles describe the design principles behind a set of research-based software microworlds included in the *Building Blocks* program and the research-based design model that guided its development (Clements, 2002, 2003). This article presents initial summary research on the first set of resulting materials, a research-based, technology-enhanced preschool mathematics curriculum. There have been only a few rigorous tests of the effects of preschool curricula. Some evidence indicates that curriculum can strengthen the development of young students' knowledge of number or geometry (Clements, 1984; Sharon Griffin & Case, 1997; Razel & Eylon, 1991), but no studies of which we are aware have studied the effects of a complete preschool mathematics curriculum, especially on low-income children who are at serious risk for later failure in mathematics (Bowman, Donovan, & Burns, 2001; Campbell & Silver, 1999; Denton & West, 2002; Mullis et al., 2000; Natriello, McDill, & Pallas, 1990; Secada, 1992; Starkey & Klein, 1992). These children possess less mathematical knowledge even before first grade (Denton & West, 2002; Ginsburg & Russell, 1981; Sharon Griffin, Case, & Capodilupo, 1995; Jordan, Huttenlocher, & Levine, 1992; Klein & Starkey, 2004). They receive less support for mathematics learning in the home and school environments, including preschool (Blevins-Knabe & Musun-Miller, 1996; Bryant, Burchinal, Lau, & Sparling, 1994; Farran, Silveri, & Culp, 1991; Holloway, Rambaud, Fuller, & Eggers-Pierola, 1995; Saxe, Guberman, & Gearhart, 1987; Starkey et al., 1999).

Rationale for the Building Blocks *Project*

Many curriculum and software publishers claim a research basis for their materials, but the bases of these claims are often dubitable (Clements, 2002). The *Building Blocks* project is based on the assumption that research-based curriculum development efforts can contribute to (a) more effective curriculum materials because the research reveals critical issues for instruction and contributes information on characteristics of effective curricula to the knowledge base, (b) better understanding of students' mathematical thinking, and (c) research-based change in mathematics curriculum (Clements, Battista, Sarama, & Swaminathan, 1997; Schoenfeld, 1999). Indeed, along with our colleagues, we believe that education will not improve substantially without a system-wide commitment to research-based curriculum and software development (Battista & Clements, 2000; Clements, 2003; Clements & Battista, 2000).

Our theoretical framework of research-based curriculum development and evaluation includes three categories and ten methods (Clements, 2002; 2003, see Table 1). Category I, *A Priori Foundations,* includes three variants of the research-to-practice model, in which extant research is reviewed and implications for the nascent curriculum development effort drawn. (1.) In *General A Priori Foundation*, broad philosophies, theories, and empirical results on learning and teaching are considered when creating curriculum. (2.) In *Subject Matter A Priori Foundation,* research is used to identify mathematics that makes a substantive contribution to students' mathematical development, is generative in students' development of future mathematical understanding, and is interesting to students. (3.) In *Pedagogical A Priori Foundation,* empirical findings on making activities educationally effective—motivating and efficacious—serve as general guidelines for the generation of activities.

In Category II, *Learning Model,* activities are structured in accordance with empirically-based models of children's thinking in the targeted subject-matter domain. This method, (4) *Structure According to Specific Learning Model,* involves creation of research-based learning trajectories, which we define as "descriptions of children's thinking and learning…and a related, conjectured route through a set of instructional tasks" (Clements & Sarama, 2004c, p. 83).

In Category III, *Evaluation*, empirical evidence is collected to evaluate the curriculum, realized in some form. The goal is to evaluate the appeal, usability, and effectiveness of an instantiation of the curriculum. (5.) *Market Research* is commercially-oriented, gathering information about the customer's needs and preferences. (6.) In *Formative Research: Small Group*, pilot testing with individuals or small groups of students is conducted on components (e.g., a particular activity, game, or software environment) or on small or large sections of the curriculum. Although teachers are ideally involved in all phases of research and development, the process of curricular enactment is emphasized in the next two methods. Research with a teacher who participated in the development of the materials in method (7) *Formative Research: Single Classroom*, and then teachers newly introduced to the materials in method (8) *Formative Research: Multiple Classrooms*, provide information about the usability of the curriculum and requirements for professional development and support materials. Finally, the last two methods, (9) *Summative Research: Small Scale* and (10) *Summative Research: Large Scale*, evaluate what can actually be achieved with typical teachers under realistic circumstances in larger contexts with teachers of diverse backgrounds. They use experiments, which provide the most efficient and least biased designs to assess causal relationships (Cook, 2002). Method 9 is employed before 10, due to the need to measure effectiveness in a controlled setting and to the large expense and effort involved in method 10; only effective curricula should be scaled up. Therefore, we employed method 9, *Summative Research: Small Scale,* in the present study.

Given this comprehensive framework of methods, claims that a curriculum is based on research should be questioned to reveal the exact nature between the curriculum and the research used or generated. Unfortunately, there is little documentation of the methods used for most curricula. Often, there is only a hint of *A Priori Foundations* methods, sometimes non-scientific market research, and minimal formative research with small groups. For example, "beta testing" of educational software is often merely polling of easily accessible peers, conducted late in the process, so that that changes are minimal, given the time and resources dedicated to the project already and the limited budget and pressing deadlines that remain (Char, 1989; Clements &

Battista, 2000). In contrast, we designed the *Building Blocks* approach to incorporate as many of the methods as possible. The next section describes this design.

Design of the Building Blocks *Materials*

Previous publications provide detailed descriptions of how we applied these research methods in our design process model (Clements, 2002; Sarama, 2004; Sarama & Clements, 2002); here we provide an overview using the framework previously described. *A Priori Foundation* methods were used to determine the curriculum's goals and pedagogy. Based on theory and research on early childhood learning and teaching (Bowman et al., 2001; Clements, 2001), we determined that *Building Blocks'* basic approach would be *finding the mathematics in, and developing mathematics from, children's activity*. The materials are designed to help children extend and mathematize their everyday activities, from building blocks (the first meaning of the project's name) to art and stories to puzzles. Activities are designed based on children's experiences and interests, with an emphasis on supporting the development of *mathematical* activity. To do so, the materials integrate three types of media: computers, manipulatives (and everyday objects), and print. Pedagogical foundations were similarly established; for example, we reviewed research on making computer software for young children motivating and educationally effective (Clements, Nastasi, & Swaminathan, 1993; Clements & Swaminathan, 1995; Steffe & Wiegel, 1994).

The method of *Subject Matter A Priori Foundation* was used to determine subject matter content by considering what mathematics is culturally valued (e.g., NCTM, 2000) and empirical research on what constituted the core ideas and skill areas of mathematics for young children (Baroody, 2004; Clements & Battista, 1992; Fuson, 1997), with an emphasis on topics that were mathematical foundational, generative for, and interesting to young children (Clements, Sarama et al., 2004). One of the reasons underlying the name we gave to our project was our desire that the materials emphasize the development of basic *mathematical building blocks* (the second meaning of the project's name)—ways of knowing the world mathematically— organized into two areas: (a) spatial and geometric competencies and concepts and (b) numeric and quantitative concepts, based on the considerable research in that domain. Research shows that young children are endowed with intuitive and informal capabilities in both these areas (Baroody, 2004; Bransford, Brown, & Cocking, 1999; Clements, 1999a; Clements, Sarama et al., 2004). For example, research shows that preschoolers know a considerable amount about shapes (Clements, Swaminathan, Hannibal, & Sarama, 1999; Lehrer, Jenkins, & Osana, 1998), and they can do more than we assume, especially working with computers (Sarama, Clements, & Vukelic, 1996). In the broad area of geometry and space, they can do the following: recognize, name, build, draw, describe, compare, and sort two- and three-dimensional shapes, investigate putting shapes together and taking them apart, recognize and use slides and turns, describe spatial locations such as "above" and "behind," and describe, and use ideas of direction and distance in getting around in their environment (Clements, 1999a). In the area of number, preschoolers can learn to count with understanding (Baroody, 2004; Baroody & Wilkins, 1999; Fuson, 1988; Gelman, 1994), recognize "how many" in small sets of objects (Clements, 1999b; Reich, Subrahmanyam, & Gelman, 1999), and compare numbers (Sharon Griffin et al., 1995). They can count higher and generally participate in a much more exciting and varied mathematics than usually considered (Ginsburg, Inoue, & Seo, 1999; Trafton & Hartman, 1997). Challenging number activities do not just develop children's number sense; they can also develop children's competencies in such logical competencies as sorting and ordering (Clements, 1984). Three mathematical themes are woven through both these main areas: (a) patterns, (b) data, and (c) sorting and sequencing.

Perhaps the most critical method for *Building Blocks* was *Structure According to Specific Learning Model*. All components of the *Building Blocks* project are based on learning trajectories for each core topic. First, empirically-based models of children's thinking and learning are synthesized to create a developmental progression of levels of thinking in the goal domain (Clements & Sarama, 2004b; Clements, Sarama et al., 2004; Cobb & McClain, 2002; Gravemeijer, 1999; Simon, 1995). Second, sets of activities are designed to engender those mental processes or actions hypothesized to move children through a developmental progression. We present two examples, one in each of the main domains of number and geometry.

The example for number involves addition. Many preschool curricula and practitioners consider addition as inappropriate before elementary school (Clements & Sarama, in press; Heuvel-Panhuizen, 1990; Sarama, 2002; Sarama & DiBiase, 2004). However, research shows that children as young as toddlers can learn simple ideas of addition and subtraction (Aubrey, 1997; Carpenter & Moser, 1984; Clements, 1984; Fuson, 1992a; Groen & Resnick, 1977; Siegler, 1996; Steffe & Cobb, 1988). As long as the situation makes sense to them (Hughes, 1986), preschool children can directly model different types of problems using concrete objects, fingers, and other strategies (Carpenter, Ansell, Franke, Fennema, & Weisbeck, 1993). These child-invented methods, usually using concrete objects and based on subitizing and counting, play a critical developmental role, as the sophisticated counting and composition strategies that develop later are all abbreviations or curtailments of these early solution strategies (Carpenter & Moser, 1984; Fuson, 1992a).

Most important for our purpose, reviews of research provide a consistent developmental sequence of the types of problems and solutions in which children can construct solutions (Carpenter & Moser, 1984; for the syntheses most directly related to our work, see Clements &

Conference Working Group, 2004; Clements & Sarama, in press; Fuson, 1992a). Selected levels of the resulting addition learning trajectory are presented in Figure 1. The left column, *Level,* briefly describes each level and the research supporting it. The middle column, *Example,* provides a behavioral example illustrating that level of thinking. Thus, *Non-Verbal Addition* is defined as reproducing small sums shown the joining of two groups. For example, many 3-year-olds, after watching 2, then 1 more, button placed under a cloth, can make a collection of 3 to show how many are under the cloth. The following rows describe subsequent developmental levels. The learning trajectory continues past Figure 1 through levels of *Counting Strategies*, *Derived Combinations*, and beyond.

The next step of building the learning trajectory is to design materials and activities that embody actions-on-objects in a way that mirrors what research has identified as critical mental concepts and processes—children's *cognitive building blocks* (the third meaning of the name). These cognitive building blocks are instantiated in on- and off-computer activity as actions (processes) on objects (concepts); for example, processes of creating, copying, and combining discrete objects, numbers, or shapes as representations of mathematical ideas. Offering students such objects and actions to be performed on these objects is consistent with the Vygotskian theory that mediation by tools and signs is critical in the development of human cognition (Steffe & Tzur, 1994). Further, designs based on objects and actions force the developer to focus on explicit actions or processes and what they will mean to the students.

For the addition trajectory, we designed three off- and on-computer activity sets, each with multiple levels: Double Trouble, Dinosaur Shop, and Number Pictures. These sets have the advantage of authenticity as well as serving as a way for children to mathematize these activities (e.g., in setting tables, using different mathematical actions such as establishing one-to-one correspondence, counting and using numerals to represent and generate quantities in the solution of variations of the task). At the *Non-Verbal Addition* level, the Double Trouble character might place 3 chocolate chips, then 1 more, on a cookie under a napkin. Children put the same number of chips on the other cookie (see the third column in Fig. 1). The teacher conducts similar activity with children using colored paper cookies and brown buttons for "chips." Similarly, the Dinosaur Shop scenario is used in several contexts. The teacher introduces a dinosaur shop in the socio-dramatic play area and encourages children to count and add during their play. The *Small Number Addition* row in Figure 1 illustrates a task in which children must move dinosaurs in two boxes into a third and label the sum. Thus, the objects in these and other tasks for the levels described in Figure 1 are single items, groups of items, and numerals. The actions include creating, duplicating, moving, combining, separately, counting, and labeling these objects and groups to solve tasks corresponding to the levels.

The example for geometry involves shape composition. We determined that a basic, often neglected, domain of children's learning of geometry was the composition and decomposition of two-dimensional geometric figures (other domains in geometry include shapes and their properties, transformations/congruence, and measurement). The geometric composition domain was determined to be significant for students in two ways. First, it is a basic geometric competence from building with geometric shapes in the preschool years to sophisticated interpretation and analysis of geometric situations in high school mathematics and above. Second, the concepts and actions of creating and then iterating units and higher-order units in the context of constructing patterns, measuring, and computing are established bases for mathematical understanding and analysis (Clements et al., 1997; Reynolds & Wheatley, 1996;

Steffe & Cobb, 1988). The domain is significant to research and theory in that there is a paucity of research on the trajectories students might follow in learning this content.

The basic structure of our model of students' knowledge of shape composition was determined by observations made in the context of early research (Sarama et al., 1996). This was refined through a research review (Mansfield & Scott, 1990; Sales, 1994) and a series of clinical interviews and focused observations by research staff and teachers (Clements, 2001 #1686, leading to the learning trajectory summarized in Figure 2, adapted from Clements, Wilson, & Sarama, 2004). From a lack of competence in composing geometric shapes (*Pre-Composer)*, children gain abilities to combine shapes—initially through trial and error (e.g., *Picture Maker)* and gradually by attributes—into pictures, and finally synthesize combinations of shapes into new shapes (composite shapes). For example, consider the *Picture Maker* level in Figure 2. Unlike earlier levels, children concatenate shapes to form a component of a picture. In the top picture in that row, a child made arms and legs from several contiguous rhombi. However, children do not conceptualize the new shapes created (parallelograms) qua geometric shapes. The puzzle task pictured at the bottom of the middle column for that row illustrates a child incorrectly choosing a square because the child is using only one component of the shape, in this case, side length. The child eventually finds this does not work, and completes the puzzle, but only by trial and error.

One main instructional task requires children to solve outline puzzles with shapes off and on the computer. Research shows this type of activity to be motivating for young children (Sales, 1994; Sarama et al., 1996). On the computer they play "Shape Puzzles," illustrated in the third column in Figure 2. The *objects* are shapes and composite shapes and the *actions* include creating, duplicating, positioning (with geometric motions), combining, and decomposing both individual shapes (units) and composite shapes (units of units). The characteristics of the tasks require actions on these objects corresponding to each level in the learning trajectory. Note that tasks in these tables are intended to support the developing of the *subsequent* level of thinking. That is, the instructional task in the *Pre-Composer* row is assigned to a child operating at the *Pre-Composer* level and is intended to facilitate the child's development of competencies at the *Piece Assembler* level.

Ample opportunity for student-led, student designed, open-ended projects are included in each set of activities. Problem posing on the part of students appears to be an effective way for students to express their creativity and integrate their learning (Brown & Walter, 1990; Kilpatrick, 1987; van Oers, 1994), although few empirical studies have been conducted, especially on young children. The computer can offer support for such projects (Clements, 2000). For Shape Puzzles, students design their own puzzles with the shapes; when they click on a "Play" button, their design is transformed into a shape puzzle that either they or their friends can solve. In the addition scenarios, children can make up their own problems with cookies and chips, or dinosaurs and boxes. As another example of a different activity, children design their own "Number Pictures" with shapes and see the resulting combination (Fig. 3; as always, it is also conducted off computer). This activity also illustrates the integration of counting, addition, geometry, and processes such as representation.

Our application of formative evaluation methods 5-8 is described in previous publications (Sarama, 2004; Sarama & Clements, 2002). In brief, we tested components of the curriculum and software using clinical interviews and observations of a small number of students to ascertain how children interpreted and understood the objects, actions, and screen design. Next, we tested whether children's actions-on-objects substantiated the actions of the researchers' model of children's mathematical activity and we determined effective prompts to incorporate into each level of each activity. We found that, following any incorrect answer, effective prompts first ask children to try again, and then provide one or more increasingly specific hints, and eventually demonstrate an effective strategy and the correct answer. Asking children to go slowly and try again was successful in a large number of cases; throughout, we strove to give "just enough help" and encourage the child to succeed as independently as possible. However, the specific hints had to be fine-tuned for each activity. (If explicit hints, such as strategy demonstrations, were provided, no assumptions were made about the child's learning or competence; they were given new problems at the same, or eventually, earlier, levels of the learning trajectory.) Given the activities and prompts, students employed the thinking strategies we had desired. Although teachers were involved in all phases of the design, in methods 7-8 we focused on the process of curricular enactment (Ball & Cohen, 1996), using classroom-based teaching experiments and observing the entire class for information concerning the usability and effectiveness of the software and curriculum.

Finally, a content analyses and critical review of the materials at each stage of development was conducted by the advisory board for the project. The following experts studied the materials and provided critiques in meetings twice per year: Arthur J. Baroody, University Of Illinois at Urbana-Champaign; Carol Copple, National Association for the Education of Young Children; Richard Lehrer, Vanderbilt University,; Mary Lindquist, Columbus College; Les Steffe, University of Georgia, and Chuck Thompson, University of Louisville.

In summary, we designed the *Building Blocks* materials upon research in what we consider a well-defined, rigorous, and complete fashion. The main purpose of this study was to evaluate whether materials created according to that model are effective in developing the mathematical knowledge of disadvantaged 4-year-old children and the size of that effect. A secondary purpose was to describe the degree to which the materials developed specific mathematics concepts and skills. To accomplish these two purposes, we used method 9, *Summative Research: Small Scale*.

Method

Design and Participants

Summary research was conducted at two sites, involving the two principal types of public preschool programs serving low-income families, state funded (site 1) and Head Start (site 2) pre-kindergarten programs. State funded programs are urban programs in which children are on 63% free, 11% reduced lunch and are 58% African American, 11% Hispanic, 28% White nonhispanic, and 3% other. Head Start programs are urban programs in which children are on 97% free, 2% reduced lunch and are 47% African American, 13% Hispanic, 30% White nonhispanic, and 10% other. At each site, one classroom was randomly assigned as experimental, one comparison. Both site 1 teachers had worked with us on the early development of the materials and were considered excellent teachers by their principal and peers. They agreed to have one selected to teach the *Building Blocks* materials and the other to continue using the school's curriculum until the following year. The experimental teacher at site 2 was inexperienced, but she had an experienced aide; the comparison teacher had taught several years in the Head Start program. None of these site 2 teachers had worked with us previously. The experimental teachers spent a half-day with us viewing and discussing the materials and were asked to teach the curriculum as they would any program.

All children in all four classes returned human subjects review forms. However, a total of nine children moved out of the school during the year, one from the site 1 and eight from site 2, leaving the following breakdown of children who participated in the pretest and completed at least one full section of the posttest: experimental—site 1, 6 boys, 11 girls, site 2, 7 boys, 6 girls; comparison—site 1, 9 boys, 7 girls, site 2, 13 boys, 9 girls. The average age of the 68 children at the time of pretesting was 49.9 months (*SD* = 6.2; range 34.8 to 57.8).

Mathematics knowledge of all participating children was assessed at the beginning and again at the end of the school year. The *Building Blocks* preschool curriculum was implemented in both experimental classes following the pretesting. This study is a component of the larger evaluation, which includes case studies of two students in each experimental classroom and observations of the teacher. A caveat is that the presence of research staff influences the classroom, although the classrooms often had adult helpers coming in and out, and the teacher said that children quickly adapted to all the study's components (e.g., note taking and videotaping). An advantage is that observations of the class assured a close evaluation of, and, moreover, a moderate (site 2) or high (site 1) degree of, implementation fidelity. That is, one or more staff members were easily available and the teacher occasionally asked them questions about the curriculum; also, if any aspect of the implementation was faulty, staff discussed the aspect with the teacher. The moderate implementation at site 2 was due to the availability of about 3 days per week for mathematics and the resulting use of most, but not all, of the curriculum's components. That is, there was little use of "every day" mathematics, such as discussion of mathematics during play. However, this was an optional component and, because all required activities were conducted, the overall implementation was judged to be adequate.

Instrument

The *Building Blocks Pre-K Assessment* uses an individual interview format, with explicit protocol, coding, and scoring procedures. It assesses children's thinking and learning along research-based developmental progressions within areas of mathematics considered significant for preschoolers, as determined by a consensus of participants in a national conference on early childhood mathematics standards (Clements, Sarama et al., 2004), rather than mirroring the experimental curriculum's objectives or activities. The assessment was refined in three full pilot tests. Content validity was assessed via expert panel review (1-2 days each from advisors to the project: Les Steffe, Mary Lindquist, Rene Parmar, Chuck Thompson); concurrent validity was established with a .86 correlation with Starkey and Klein's Child Math Assessment. The assessment is administered in two sections, each of which takes 20 to 30 minutes per child to complete. They were conducted by doctoral students who had been previously trained and evaluated until they achieved a perfect evaluation of their administration on three consecutive administrations. All assessments are videotaped and subsequently coded by independent coders, also previously trained and evaluated. Codes included correct/incorrect evaluations and separate codes for children's strategies in cases where those strategies were intrinsically related to the learning trajectory level the item was designed to measure. Assessors and coders were naïve as to treatment group of the teacher and children. Results were accumulated and analyzed by an independent professor of Educational Psychology who is expert in research design and statistics.

The number section measures eight learning trajectories. Verbal counting includes 7 items assessing counting forward, backward, up from a given number, before/after/between, and identifying mistakes in counting. Object counting includes 15 items assessing counting groups in array and scattered arrangements, producing groups, and identifying mistakes (e.g., "This doll is just learning to count. Watch her count some blocks. If she makes a mistake, tell me." Doll skips fifth of seven cubes.). Number recognition and subitizing include 7 items assessing recognition of small groups (name the number for a group of two, untimed) and subitizing (e.g., "I'm going to show you some cards, just for a quick moment! Try to tell me how many dots on each one.”). Number comparison includes 19 items assessing nonverbal comparison (e.g., shown cards with ••• and ∴; the child is asked, “Do these cards have the same number of dots?”) and verbal comparison (e.g., “Which is bigger: 7 or 9?”). Number sequencing includes 3 items assessing sequencing of groups (e.g., cards with 1-5 dots). Numerals includes 5 items assessing the child’s ability to connect written numerals to quantities. Number composition and decomposition includes 6 items (e.g., “Look. I am putting 5 blocks on this paper? Count with me. Now, I’m going to hide some.” Cover the blocks with cloth, then secretly hide 3, then remove the cover to show the remaining two. “How many am I hiding?”). Adding and subtracting includes 23 items assessing arithmetic competence, including concrete situations, story problems, and mental arithmetic. Place value includes 4 items assessing knowledge of the relative size of numbers above 20. The assessment proceeds along research-based trajectories (Clements, Sarama et al., 2004) for each of these topics until the child makes three consecutive errors. The final items measure skills typically achieved at eight years of age. The maximum score is 97; for this sample, children reached items associated with 6.5 years of age (i.e., all children missed three in a row before reaching items for ages 7-8); therefore, the maximum these preschoolers could have reached was 72 (coefficient alpha reliability, r = .89; interrater reliability of data coders, 98%).

The geometry test measures seven learning trajectories. Shape identification includes 4 master items assessing knowledge of squares, rectangles, triangles, and rhombuses. The child must choose all exemplars of the stated shape from a large array of manipulatable figures (Fig. 4a shows the master for the array). Shape composition and decomposition includes 5 items assessing the ability to physically or mentally compose or decompose shapes, such as choosing a pair of shapes that would result if a shape were cut along a diagonal (see Fig 4b). Congruence includes 2 items assessing the ability to match shapes of the same shape and same size. Construction of shapes includes 2 items assessing the ability to accurately build a shape from its components (e.g., the child is given 6 straws of each of 3 different lengths and asked, “We're going to use these straws to make shapes. Can you make a *triangle* using some of the straws?”). Turns included 1 item assessing recognition of a 90° rotation and orientation included 1 item on horizontal and vertical lines. The test also includes 1 item on geometric measurement (Which of these strings is about the same length as 4 cubes?) and 1 item on patterning (copy, then extend, an ABAB pattern). As with the number section, difficulties for some items on the geometry assessment were designed to measure abilities at 8 years of age. Children complete all 17 items (with several having multiple parts), for a maximum score of 30 (coefficient alpha reliability, r = . 71; interrater reliability of data coders, 97%). (Contact the authors for further information.)

Curricula

This study used the first curriculum produced by the *Building Blocks* project, a preschool curriculum that is a component of the *DLM Early Childhood Express* (Schiller, Clements, Sarama, & Lara-Alecio, 2003). The curriculum consists of daily activities in four teacher’s editions, and a *DLM Express Math Resource Package* (Clements & Sarama, 2003) including computer software, correlated games, activities, and centers, and ideas for integrating mathematics throughout the school day. The software includes 11 activity scenarios, each including between two and six activities. For example, the Double Trouble scenario includes activities on recognizing and comparing number, counting, and arithmetic. In the first three activities, children match cookies with the same number of chocolate chips (early number recognition and comparing), create a cookie with the same number of chips as a given cookie (counting to produce a set), and create a cookie that has a given number of chips given only a numeral (counting to produce a set that matches a numeral). Later activities in that setting involve addition (see the third column for *Non-Verbal Addition* and *Find Change* in Fig. 1). The software's management system presents tasks, contingent on success, along research-based learning trajectories. Activities within various scenarios are introduced according to the trajectory's sequences. Figure 1 illustrates, for example, how two activities from the Dinosaur Shop scenario are sequenced between two illustrated activities from the Double Trouble scenario. Off-computer activities, such as center activities, involve corresponding activities. For example, corresponding to the first Double Trouble activity, the teacher sets out a learning center by hiding paper "cookies" with different numbers of chips under several opaque containers and placing one such cookie with 3 chips in plain view. Children lift each container and count the chips until they find the matching cookie. They then show the teacher or other adult.

All participating teachers maintained their typical schedule, including circle (whole group) time, work at centers, snack, outdoor play, and so forth. The experimental teachers merely inserted the *Building Blocks* activities at the appropriate point of the day. For example, circle time might include a finger play that involved counting and a brief introduction to a new center or game. Center time would include individual work at the curriculum's software or learning centers, guided by the teacher or aide as they circulated throughout the room. As a specific example, children might be introduced to new puzzles such as those at the level of the *Picture Maker* level of Figure 2, then engage in physical puzzles with pattern blocks and tangrams in a learning center, or similar puzzles in the *Shape Puzzles* software activity. Teachers guided children by discussing the task, eliciting children's strategies, and, when necessary, modeling successful strategies.

Summarizing, children worked on the following number of activities, classified by their major goal (many activities addressed multiple goals; activities that were conducted on and off the computer are counted once): number—counting, 32 (site 1), 49 (site 2); comparing, 9, 7; numerals, 8, 6; sequencing, 4, 6; subitizing, 6, 6; adding/subtracting and composing number, 17, 15; and geometry—shape identification, 18, 14; composition, 9, 9; congruence, 2, 2; construction, 2, 1; spatial orientation, 2, 2; turn, 1, 1; measurement, 1, 1; and patterning, 4, 6. The site 1 comparison teacher agreed to continue using her school's mathematics activities, which were based on New York State and local district standards. The site 2 comparison teacher used Creative Curriculum (Teaching Strategies Inc., 2001) as well as additional "home-grown" curricular activities for mathematics. Visits to those classrooms indicated that each was following the curricula as written.

Analyses

Factorial repeated measures analyses, with time as the within group factor, and two between-group factors, site and treatment, were conducted to test for a difference in achievement from pre- to post test on both tests to assess the effectiveness of the curriculum. (Children did work in the same class, but the software and center activities were engaged in individually, so the child was used as the unit of analysis. Teachers did conduct short, whole-class activities, such as finger plays, however, so a caveat regarding this aspect of the design is appropriate, a point to which we return.) In addition, two effect sizes were computed for each test. We compared experimental posttest (E2) to the comparison posttest (C2) scores as an estimate of differential treatment effect. We also compared experimental posttest to experimental pretest (E2 to E1) scores as an estimate of the achievement gain within the experimental curriculum. Effect sizes were computed using adjusted pooled standard deviations (Rosnow & Rosenthal, 1996). We used the accepted benchmarks of .25 or greater as an effect size that has practical significance (i.e., is educationally meaningful), .5 for an effect size of moderate strength, and .8 as a large effect size (Cohen, 1977).

Results

Table 2 presents the raw data for the number and geometry tests. A factorial repeated measures analysis was conducted to test for a difference in achievement from pre- to post test on both tests to assess the effectiveness of the curriculum. We computed all factorial analyses including gender as well, but no main effects or interactions were significant; we thus present here only the more parsimonious model.

For the number test, there were significant main effects of time, $F(1, 57) = 183.19$, $MSE = 33.95$ $p < .0005$; treatment, $F(1, 57) = 6.02$, $MSE = 145.67$, $p < .05$; and site $F(1, 57) = 33.48$, $MSE = 145.67$, $p < .0005$; as well as significant interactions of time by treatment, $F(1, 57) = 20.10$, $MSE = 33.95$, $p < .0005$; and time by site, $F(1, 57) = 15.19$, $MSE = 33.95$, $p < .0005$. Inspection of the means indicates that the site 1 scores increased more than those of site 2, and that the experimental treatment group score increased more than the comparison group score. The effect size comparing E2 to C2 was .85, and the effect size comparing E2 to E1 was 1.61.

Likewise, for geometry, there were significant main effects of time, $F(1 , 49) = 139.08$, $MSE = 3.40$, $p < .0005$; treatment, $F(1 ,49) = 17.623$, $MSE = 4.44$, $p < .0005$; and site $F(1, 49) = 27.94$, $MSE = 4.44$, $p < .0005$; as well as significant interactions of time by treatment, $F(1, 49) = 43.57$, $MSE = 3.40$, $p < .0005$; and time by site, $F(1, 49) = 7.95$, $MSE = 3.40$, $p < .01$. Inspection of the means indicates that the site 1 scores increased more than those of site 2, and that the experimental treatment group score increased more than the comparison group score. The effect size comparing E2 to C2 was 1.47, and the effect size comparing E2 to E1 was 2.26.

To illuminate which specific topics were affected by the curriculum, Table 3 presents means and standard deviations for number and geometry subtests. Because we did not wish to increase alpha error and some subtests had a small number of items, we did not perform additional interferential statistics; however, an inspection of the means indicates that the effects were more pronounced on some topics, although positive effects were found for every topic except one. In the realm of number, the smallest relative effects were on object counting and comparing number; for both topics, the experimental group gained more points, but both groups nearly doubled their pretest scores. Both groups made large gains in verbal counting and connecting numerals to groups, with the experimental group's gain the larger. The experimental group's gains were even larger, relative to the comparison group, for the related topics of adding/subtracting and composing number. The largest relative gains in number were achieved in subitizing (tell how many dots are on a card with 5 to 10 dots, shown for 2 seconds) and sequencing (e.g., placing cards with groups of 1 to 5 dots in order from fewest to most).

In geometry, the relative effect on the turn item was small (and higher for the comparison group) and the effect on congruence was positive, but small. Effects on construction of shapes and spatial orientation were large. The largest relative gains in geometry were achieved on shape identification and composition of shapes. Effects on measurement and patterning were moderate.

Several items were coded to describe the strategies children employed (see Table 4) for those cases in which the research indicated that a level of sophistication in solution strategies was intrinsically related to the development of each subsequent level of the trajectory (Clements & Sarama, 2004c; Clements, Sarama et al., 2004; Clements, Wilson et al., 2004). Results on strategies support the scored results and provide additional description of the different abilities of the two groups. For the first object counting item, about 2/3 of both experimental and comparison children could provide a verbal response at pretest. At posttest, all experimental children did so, however 1/6 of the comparison children reproduced the set but could not give the verbal responses and between 1/5 and 1/4 gave no response. On a number comparison item, experimental children increased their use of a counting strategy more than comparison children, over half of whom did not respond (note the infrequent use of matching). On items in which children counted scrambled arrangements of objects, the experimental group increased their use of strategies more than the comparison group, especially systematic strategies such as progressing top to bottom, left to right. On the arithmetic items, more children used objects, and fewer used verbal strategies (a small minority on the comparison item, “how many dogs wouldn’t get a bone”).

Examining the addition learning trajectory reveals the curriculum’s positive effects in more detail. Increases in percentage correct from pretest to posttest for four illustrative items were as follows: 2 + 1 (increase of 37 for experimental vs. 23 for comparison), 3 + 2 (47 vs. 13) 5 + 3 (23 vs. 16), 6 - 4 (how many dogs wouldn’t get a bone?, 23 vs. 10). The curriculum follows the learning trajectory described in Figure 1. On average, children worked on Non-Verbal Addition activities 4 times, half on computer (illustrated in Fig. 1 and previously described in the section on “Design of the *Building Blocks* Materials”) and half off computer (similar tasks). Teachers modeled non-verbal strategies, but also encouraged post hoc verbal reflection. Children worked on Small Number Addition activities 6 times, 2 on and 4 off computer (similarly illustrated in Fig. 1 and previously described). Teachers focused on the meaning of addition as combining two disjoint sets, expressed informally. Children worked on Find Result activities 6 times, 2 on and 2 off computer. Use of a child’s invented counting strategies to solve join, result unknown problems was emphasized. Finally, children worked on Find Change problems 2 times, half on and half off computer. Both on- and off-computer activities emphasized counting on from a given number. The results of these activities is shown in the greater than double increase in correctness by the experimental group, as well as their greater use of solution strategies overall and greater use of more sophisticated strategies, such as verbal counting strategies, for most tasks. For example, on 5 + 3, a third of the experimental children, compared to a fifth of the comparison children, used objects, and a almost a fourth of the experimental children, compared to a fifteenth of the comparison children, used verbal counting strategies. These results are particularly striking when considering that such tasks are normally part of the first grade curriculum.

Table 4 also shows four strategy codes for geometry describe children's strategy on a composition task. By posttest, experimental children were far more likely to combine shapes without leaving gaps, turn shapes into correct orientation prior to placing them on the puzzle, search for a correct shape, and solve the puzzle immediately, systematically and confidently. This increased use of more sophisticated shape composition strategies suggests the development of mental imagery.

This development of more sophisticated strategies in the experimental group, along with the large relative gains on the subtest score, more than four times as large as those made by the comparison group (Table 3), substantiate the curriculum's positive effect on geometric composition. The curriculum engages children in several activities to develop this competence, including creating free-form pictures with a variety of shape sets, such as pattern blocks and tangrams, and solving outline puzzles with those same shape sets. Informal work with three-piece foam puzzles and playdough cutouts of conducted for several weeks during mid-Fall. In April, the outline puzzles, which provided the most guidance along the learning trajectories, were introduced. Most children in the present classrooms worked about two days on the puzzles designed for children at the *Pre-Composer* level (see the third column in Fig. 2), three days at the *Piece Assembler* level, and two at the *Picture Maker* level. Only about a third of the children completed all those puzzles, and thus could be confidently classified as operating at the *Shape Composer* level or above. Four children appeared to operate at best at the *Piece Assembler*. Based on evidence that the developmental sequence of this learning trajectory is valid (Clements, Wilson et al., 2004), we guided individual children to work on puzzles at level they had not mastered on the off-computer puzzles. The computer automatically monitored their progress on the "Shape Puzzles" software. The combination of off- and on-computer activities at an appropriate, progressive developmental level appeared to facilitate children's development of the mental actions on objects that engendered thinking at each subsequent level. This is shown in the strategies they employed (Table 4). Almost 90% of the children placed shapes together without leaving gaps, an indication of thinking at the Picture Maker level or above. About 67% of the children turned shapes into the correct orientation *prior* to physically placing them within the puzzle outline. The same percentage appeared to search for "just the right shape" that they "knew would fit." These behaviors are criteria for the Shape Composer level. Only about 56% however, showed immediate, confident, systematic completion of puzzles. Children's strategies therefore suggest that on the assessment roughly 10% were at the Piece Assembler level, 23% were operating at the Picture Maker level, 11% were in transition to the Shape Composer level, and 56% were at the Shape Composer level (or above; subsequent levels were not assessed). In contrast, the comparison group had roughly 77% at the Pre-Composer or Piece Assembler levels, 15% at the Picture Maker level, and 8% in transition to the Shape Composer level, consistent with developmental averages for this age group (Clements, Wilson et al., 2004).

Discussion

The main purpose of this research was to measure the efficacy of a preschool mathematics program based on a comprehensive model of developing research-based software and print curricula, on a small scale under controlled conditions (*Summative Research: Small Scale*). Scores at site 1, the state-funded preschool, increased more than those of site 2, the Head Start school. Site 1 teachers were more experienced, and site 2 children had lower mathematics scores at the start. Qualitative observations confirm that site 2 children entered preschool with fewer cognitive resources (attention, metacognition, disposition to learn mathematics, etc.; research reports are under preparation). This is a concern for those involved with Head Start. However, there was no evidence that the curriculum was differentially effective at the two sites. At both sites, the experimental treatment group score increased more than the comparison group score (using an assessment that measured board developmental sequences and thus offered a conservative estimates of the effects). The effect sizes comparing the posttest scores for the experiment group to those of the comparison group was .85 for number and 1.44 for geometry, and the effect size comparing the experimental group's posttest to their pretest scores was 1.71 for number and 2.12 for geometry. Average achievement gains of the experimental group doubled those considered large (Cohen, 1977) and approached or exceeded the sought-after 2-sigma effect of individual tutoring (Bloom, 1984).

Inspection of means for individual topics (subtests) substantiates our conversations with the comparison teachers that they emphasized object counting, comparing numbers, and, to a lesser extent, shapes. The experimental group still outperformed the comparison group even on these subtests. The relative gain of the experimental group was much greater on most other topics, with the exception of the concept of turn, which was not emphasized in any of the curricula. Small relative gains were made by the experimental group on congruence, moderate gains on measurement and patterning, and large gains on verbal counting, connecting numbers to groups, adding and subtracting, composing number, construction of shapes, and spatial orientation. The *Building Blocks* curriculum seems to have made a special contribution, with quite large relative gains, to children's learning of the topics of subitizing, sequencing, shape identification, and composition of shapes. Thus, even a moderate number of experiences (e.g., 4 to 6 for sequencing and subitizing) was sufficient to enhance children's learning of certain oft-ignored topics.

In addition, some items yielded information on children's use of strategies. The experimental group showed a greater increase in the use of more sophisticated numerical strategies and the development of spatial imagery.

Conclusions and Implications

Many have called for more research on the effects of curriculum materials (e.g., Senk & Thompson, 2003), especially because such materials have a large influence on teaching practices (Goodlad, 1984; Grouws & Cebulla, 2000; Woodward & Elliot, 1990). Results of this study indicate strong positive effects of the *Building Blocks* materials, with achievement gains near or approximately equal to those recorded for individual tutoring. This provides support for the efficacy of curricula built on comprehensive research-based principles. The *Building Blocks* materials include research-based computer tools that stand at the base, providing computer analogs to critical mathematical ideas and processes. These are used, or implemented, with activities that guide children through research-based learning trajectories (developed over years of synthesizing our own and others' empirical work). These activities-through-trajectories connect children's informal knowledge to more formal school mathematics. The result is a package that is motivating for children but, unlike “edu-tainment,” results in significant assessed learning gains. We believe these features lead to *Building Blocks’* substantial impact, although the present design does not allow attributing the effect to any particular feature or set of features (we are analyzing the qualitative data from these same classrooms that will provide insights relevant to this issue). One practical implication is that, when implemented with at least a moderate degree of fidelity, such materials are highly efficacious in helping preschoolers learn fundamental mathematics concepts and skills.

The results also provide initial, “proof of concept” support for our framework for “research-based curricula,” including three categories, *A Priori Foundations, Learning Model, and Evaluation,* and ten methods described previously (see also Clements, 2003), which extends and particularizes theories of curriculum research (Walker, 1992). Our own use of the framework emphasizes the actions-on-objects that should mirror the hypothesized mathematical activity of students and how that activity develops along learning trajectories (Clements, 2002; Clements & Battista, 2000). An implication is that such synthesis of curriculum/technology development as a scientific enterprise and mathematics education research may help reduce the separation of research and practice in mathematics and technology education and produce results that are immediately applicable by practitioners (parents, teachers, and teacher educators), administrators and policy makers, and curriculum and software developers. Of course, multiple studies, including comparisons, would need to be conducted to support any claims about the efficacy of the model per se.

Even a small number of experiences for certain topics, such as sequencing number and subitizing, were sufficient to produce large relative learning gains. We believe these topics may often be ignored in most early childhood classrooms. The experimental activities assessed here were efficacious, but other approaches to these topics might also be studied. In contrast, results on such topics as congruence, turn, and measurement indicate that future research should ascertain whether the small number of experiences (1-2) or the nature of the activities dedicated to these topics accounted for the small gains and whether changes to either or both could increase children's achievement.

This study contributes to extant research showing that organized experiences result in greater school readiness upon entry into kindergarten (Bowman et al., 2001; Shonkoff & Phillips, 2000) and that focused early mathematical interventions help young children develop a foundation of informal mathematics knowledge (Clements, 1984), especially for children living in poverty (Campbell & Silver, 1999; Fuson, Smith, & Lo Cicero, 1997; Sharon Griffin, 2004; Sharon Griffin et al., 1995; Ramey & Ramey, 1998). It extends this research by indicating that a comprehensive mathematics curriculum following NCTM's standards (2000) can increase knowledge of multiple essential mathematical concepts and skills. Unfortunately, most American children are not in such high-quality programs (Hinkle, 2000). We recommend that preschool programs adopt research-based curricula as described here (e.g., see Clements, 2002). With its emphasis on low-income children, this study also extends the research on standards-based mathematics curricula, most of which does not address social class ore cultural influences (cf. Senk & Thompson, 2003).

An important caveat is that this study represents method 9, *Summative Research: Small Scale* (Clements, 2003). As stated, this was justified because it provides an estimate of effect size under controlled conditions. However, the small number of classrooms, the use of the child as the unit of analysis inside classrooms, the presence of project staff, and our resultant ability to guarantee at least moderate fidelity limits generalizability. The results justify the subsequent use of method 10, *Summative Research: Large Scale*, which we are implementing in the 2003-2005 school years. Finally, the quantitative results reported here will be complemented and extended in corresponding studies of the same classrooms involving four qualitative case studies of children learning in the context of the curriculum. The focus of these analyses was on the children's development through the learning trajectories for the various mathematical topics.

References

Aubrey, C. (1997). Children's early learning of number in school and out. In I. Thompson (Ed.), *Teaching and learning early number* (pp. 20-29). Philadelphia, PA: Open University Press.

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is—or might be—the role of curriculum materials in teacher learning and instructional reform? *Educational Researcher, 16*(2), 6-8; 14.

Baroody, A. J. (1987). The development of counting strategies for single-digit addition. *Journal for Research in Mathematics Education, 18*, 141-157.

Baroody, A. J. (2004). The developmental bases for early childhood number and operations standards. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 173-219). Mahwah, NJ: Lawrence Erlbaum Associates.

Baroody, A. J., & Wilkins, J. L. M. (1999). The development of informal counting, number, and arithmetic skills and concepts. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 48-65). Reston, VA: National Council of Teachers of Mathematics.

Battista, M. T., & Clements, D. H. (2000). Mathematics curriculum development as a scientific endeavor. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 737-760). Mahwah, NJ: Lawrence Erlbaum Associates.

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting, 5*, 35-45.

Bloom, B. S. (1984). The 2-sigma problem: The search for methods of group instruction as effective as one-to-one tutoring. *Educational Researcher, 13*, 4-16.

Bowman, B. T., Donovan, M. S., & Burns, M. S. (Eds.). (2001). *Eager to learn: Educating our preschoolers*. Washington, DC: National Academy Press.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (1999). *How people learn*. Washington, DC: National Academy Press.

Brown, S. I., & Walter, M. I. (1990). *Art of problem posing*. Mahwah, NJ: Lawrence Erlbaum Associates.

Bryant, D. M., Burchinal, M. R., Lau, L. B., & Sparling, J. J. (1994). Family and classroom correlates of Head Start children's developmental outcomes. *Early Childhood Research Quarterly, 9*, 289-309.

Campbell, P. F., & Silver, E. A. (1999). *Teaching and learning mathematics in poor communities*. Reston, VA: National Council of Teachers of Mathematics.

Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E. H., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children’s problem-solving processes. *Journal for Research in Mathematics Education, 24*, 428-441.

Carpenter, T. P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15*, 179-202.

Char, C. A. (1989, March). *Formative research in the development of mathematics software for young children*. Paper presented at the meeting of the American Educational Research Association, San Francisco, CA.

Clements, D. H. (1984). Training effects on the development and generalization of Piagetian logical operations and knowledge of number. *Journal of Educational Psychology, 76*, 766-776.

Clements, D. H. (1999a). Geometric and spatial thinking in young children. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 66-79). Reston, VA: National Council of Teachers of Mathematics.

Clements, D. H. (1999b). Subitizing: What is it? Why teach it? *Teaching Children Mathematics, 5*, 400-405.

Clements, D. H. (2000). From exercises and tasks to problems and projects: Unique contributions of computers to innovative mathematics education. *Journal of Mathematical Behavior, 19*, 9-47.

Clements, D. H. (2001). Mathematics in the preschool. *Teaching Children Mathematics, 7*, 270-275.

Clements, D. H. (2002). Linking research and curriculum development. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education* (pp. 599-636). Mahwah, NJ: Lawrence Erlbaum Associates.

Clements, D. H. (2003). Curriculum research: Toward a framework for “research-based curricula”. *Submitted for publication.*

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-464). New York: Macmillan.

Clements, D. H., & Battista, M. T. (2000). Designing effective software. In A. E. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 761-776). Mahwah, NJ: Lawrence Erlbaum Associates.

Clements, D. H., Battista, M. T., Sarama, J., & Swaminathan, S. (1997). Development of students' spatial thinking in a unit on geometric motions and area. *The Elementary School Journal, 98*(2), 171-186.

Clements, D. H., & Conference Working Group. (2004). Part one: Major themes and recommendations. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 1-72). Mahwah, NJ: Lawrence Erlbaum Associates.

Clements, D. H., Nastasi, B. K., & Swaminathan, S. (1993). Young children and computers: Crossroads and directions from research. *Young Children, 48*(2), 56-64.

Clements, D. H., & Sarama, J. (2003). *DLM Early Childhood Express Math Resource Guide*. Columbus, OH: SRA/McGraw-Hill.

Clements, D. H., & Sarama, J. (2004a). *Building Blocks* for early childhood mathematics. *Early Childhood Research Quarterly, 19*, 181-189.

Clements, D. H., & Sarama, J. (Eds.). (2004b). Hypothetical learning trajectories [Special issue]. *Mathematical Thinking and Learning, 6*(2).

Clements, D. H., & Sarama, J. (2004c). Learning trajectories in mathematics education. *Mathematical Thinking and Learning, 6*, 81-89.

Clements, D. H., & Sarama, J. (in press). Early childhood mathematics learning. In F. K. Lester, Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning*. New York: Information Age Publishing.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (Eds.). (2004). *Engaging young children in mathematics: Standards for early childhood mathematics education*. Mahwah, NJ: Lawrence Erlbaum Associates.

Clements, D. H., Sarama, J., & Wilson, D. C. (2001). Composition of geometric figures. In M. v. d. Heuvel-Panhuizen (Ed.), *Proceedings of the 21st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 273-280). Utrecht, The Netherlands: Freudenthal Institute.

Clements, D. H., & Swaminathan, S. (1995). Technology and school change: New lamps for old? *Childhood Education, 71*, 275-281.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*, 192-212.

Clements, D. H., Wilson, D. C., & Sarama, J. (2004). Young children's composition of geometric figures: A learning trajectory. *Mathematical Thinking and Learning, 6*, 163-184.

Cobb, P., & McClain, K. (2002). Supporting students' learning of significant mathematical ideas. In G. Wells & G. Claxton (Eds.), *Learning for life in the 21st century: Sociocultural perspectives on the future of education* (pp. 154-166). Oxford, England: Blackwell.

Cohen, J. (1977). *Statistical power analysis for the behavioral sciences (rev. ed.)*. New York: Academic Press.

Cook, T. D. (2002). Randomized experiments in educational policy research: A critical examination of the reasons the educational evaluation community has offered for not doing them. *Educational Evaluation and Policy Analysis, 24*, 175-199.

Denton, K., & West, J. (2002). *Children's reading and mathematics achievement in kindergarten and first grade*, 2002, from http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2002125

Farran, D. C., Silveri, B., & Culp, A. (1991). Public preschools and the disadvantaged. In L. Rescorla, M. C. Hyson & K. Hirsh-Pase (Eds.), *Academic instruction in early childhood: Challenge or pressure? New directions for child development* (pp. 65-73).

Feuer, M. J., Towne, L., & Shavelson, R. J. (2002). Scientific culture and educational research. *Educational Researcher, 31*(8), 4-14.

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York: Springer-Verlag.

Fuson, K. C. (1992a). Research on learning and teaching addition and subtraction of whole numbers. In G. Leinhardt, R. Putman & R. A. Hattrup (Eds.), *Handbook of research on mathematics teaching and learning* (pp. 53-187). Mahwah, NJ: Lawrence Erlbaum Associates.

Fuson, K. C. (1992b). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243-275). New York: Macmillan.

Fuson, K. C. (1997). Research-based mathematics curricula: New educational goals require programs of four interacting levels of research. *Issues in Education, 3*(1), 67-79.

Fuson, K. C., Smith, S. T., & Lo Cicero, A. (1997). Supporting Latino first graders' ten-structured thinking in urban classrooms. *Journal for Research in Mathematics Education, 28*, 738-760.

Gelman, R. (1994). Constructivism and supporting environments. In D. Tirosh (Ed.), *Implicit and explicit knowledge: An educational approach* (Vol. 6, pp. 55-82). Norwood, NJ: Ablex.

Ginsburg, H. P., Inoue, N., & Seo, K.-H. (1999). Young children doing mathematics: Observations of everyday activities. In J. V. Copley (Ed.), *Mathematics in the early years* (pp. 88-99). Reston, VA: National Council of Teachers of Mathematics.

Ginsburg, H. P., & Russell, R. L. (1981). Social class and racial influences on early mathematical thinking. *Monographs of the Society for Research in Child Development, 46*(6, Serial No. 193).

Goodlad, J. I. (1984). *A place called school*. New York: McGraw-Hill.

Grant, S. G., Peterson, P. L., & Shojgreen-Downer, A. (1996). Learning to teach mathematics in the context of system reform. *American Educational Research Journal, 33*(2), 509-541.

Gravemeijer, K. P. E. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning, 1*, 155-177.

Griffin, S. (2004). Number Worlds: A research-based mathematics program for young children. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 325-342). Mahwah, NJ: Lawrence Erlbaum Associates.

Griffin, S., & Case, R. (1997). Re-thinking the primary school math curriculum: An approach based on cognitive science. *Issues in Education, 3*(1), 1-49.

Griffin, S., Case, R., & Capodilupo, A. (1995). Teaching for understanding: The importance of the Central Conceptual Structures in the elementary mathematics curriculum. In A. McKeough, J. Lupart & A. Marini (Eds.), *Teaching for transfer: Fostering generalization in learning* (pp. 121-151). Mahwah, NJ: Lawrence Erlbaum Associates.

Groen, G., & Resnick, L. B. (1977). Can preschool children invent addition algorithms? *Journal of Educational Psychology, 69*, 645-652.

Grouws, D. A., & Cebulla, K. J. (2000). Elementary and middle school mathematics at the crossroads. In T. L. Good (Ed.), *American education: Yesterday, today, and tomorrow (Volume II)* (pp. 209-255). Chicago: University of Chicago Press.

Heuvel-Panhuizen, M. v. d. (1990). Realistic arithmetic/mathematics instruction and tests. In K. P. E. Gravemeijer, M. van den Heuvel & L. Streefland (Eds.), *Contexts free productions tests and geometry in realistic mathematics education* (pp. 53-78). Utrecht, The Netherlands: OW&OC.

Hinkle, D. (2000). *School involvement in early childhood*: National Institute on Early Childhood Development and Education, U.S. Department of Education Office of Educational Research and Improvement.

Holloway, S. D., Rambaud, M. F., Fuller, B., & Eggers-Pierola, C. (1995). What is "appropriate practice" at home and in child care?: Low-income mothers' views on preparing their children for school. *Early Childhood Research Quarterly, 10*, 451-473.

Hughes, M. (1986). *Children and number: Difficulties in learning mathematics*. Oxford, U.K.: Basil Blackwell.

James, W. (1958). *Talks to teachers on psychology: And to students on some of life's ideas*. New York: Norton.

Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1992). Differential calculation abilities in young children from middle- and low-income families. *Developmental Psychology, 28*, 644-653.

Kilpatrick, J. (1987). Problem formulating: Where do good problems come from? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 123-147). Hillsdale, NJ: Lawrence Erlbaum Associates.

Klein, A., & Starkey, P. (2004). Fostering preschool children's mathematical development: Findings from the Berkeley Math Readiness Project. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 343-360). Mahwah, NJ: Lawrence Erlbaum Associates.

Lehrer, R., Jenkins, M., & Osana, H. (1998). Longitudinal study of children's reasoning about space and geometry. In R. Lehrer & D. Chazan (Eds.), *Designing learning environments for developing understanding of geometry and space* (pp. 137-167). Mahwah, NJ: Lawrence Erlbaum Associates.

Lester, F. K., Jr., & Wiliam, D. (2002). On the purpose of mathematics education research: Making productive contributions to policy and practice. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education* (pp. 489-506). Mahwah, NJ: Lawrence Erlbaum Associates.

Light, R. J., & Pillemer, D. B. (1984). *Summing up: The science of reviewing research*. Cambridge, MA: Harvard University Press.

Mansfield, H. M., & Scott, J. (1990). Young children solving spatial problems. In G. Booker, P. Cobb & T. N. deMendicuti (Eds.), *Proceedings of the 14th annual conference of the Internation Group for the Psychology of Mathematics Education* (Vol. 2, pp. 275-282). Oaxlepec, Mexico: Internation Group for the Psychology of Mathematics Education.

Mix, K. S., Huttenlocher, J., & Levine, S. C. (2002). *Quantitative development in infancy and early childhood*. New York: Oxford University Press.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., Gregory, K. D., Garden, R. A., O'Connor, K. M., et al. (2000). *TIMSS 1999 international mathematics report*. Boston, MA: The International Study Center, Boston College, Lynch School of Education.

Natriello, G., McDill, E. L., & Pallas, A. M. (1990). *Schooling disadvantaged children: Racing against catastrophe*. New York: Teachers College Press.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Ramey, C. T., & Ramey, S. L. (1998). Early intervention and early experience. *American Psychologist, 53*, 109-120.

Razel, M., & Eylon, B.-S. (1991, July). *Developing mathematics readiness in young children with the Agam Program*. Paper presented at the meeting of the Fifteenth Conference of the International Group for the Psychology of Mathematics Education, Genova, Italy.

Reeves, D. B. (2002, May 8). Galileo's dilemma. *Education Week,* p. 44.

Reich, S., Subrahmanyam, K., & Gelman, R. (1999, April). *Children's use of number and amount information when making judgments about quantity*. Paper presented at the meeting of the Society for Research in Child Development, Albuquerque, NM.

Reynolds, A., & Wheatley, G. H. (1996). Elementary students' construction and coordination of units in an area setting. *Journal for Research in Mathematics Education, 27*(5), 564-581.

Rosnow, R. L., & Rosenthal, R. (1996). Computing contrasts, effect sizes, and counternulls on other people's published data: General procedures for research consumers. *Psychological Methods, 1*, 331-340.

Sales, C. (1994). *A constructivist instructional project on developing geometric problem solving abilities using pattern blocks and tangrams with young children*. Unpublished Masters, University of Northern Iowa.

Sales, C., & Hildebrandt, C. (2002). A constructivist instructional project on developing geometric problem solving abilities using pattern blocks and tangrams with young children. In R. DeVries, B. Zan, C. Hildebrandt, R. Edmiaston & C. Sales (Eds.), *Developing constructivist early childhood curriculum* (pp. 165-180). New York: Teachers College.

Sarama, J. (2002). Listening to teachers: Planning for professional development. *Teaching Children Mathematics, 9*, 36-39.

Sarama, J. (2004). Technology in early childhood mathematics: *Building Blocks*™ as an innovative technology-based curriculum. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 361-375). Mahwah, NJ: Lawrence Erlbaum Associates.

Sarama, J., & Clements, D. H. (2002). *Building Blocks* for young children's mathematical development. *Journal of Educational Computing Research, 27*(1&2), 93-110.

Sarama, J., Clements, D. H., & Vukelic, E. B. (1996). The role of a computer manipulative in fostering specific psychological/mathematical processes. In E. Jakubowski, D. Watkins & H. Biske (Eds.), *Proceedings of the eighteenth annual meeting of the North America Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 567-572). Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Sarama, J., & DiBiase, A.-M. (2004). The professional development challenge in preschool mathematics. In D. H. Clements, J. Sarama & A.-M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 415-446). Mahwah, NJ: Lawrence Erlbaum Associates.

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development, 52*(2, Serial #216).

Schiller, P., Clements, D. H., Sarama, J., & Lara-Alecio, R. (2003). DLM Early Childhood Express. Columbus, OH: SRA/McGraw-Hill.

Schoenfeld, A. H. (1999). Looking toward the 21st century: Challenge of educational theory and practice. *Educational Researcher, 28*, 4-14.

Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623-660). New York: Macmillan.

Senk, S. L., & Thompson, D. R. (2003). *Standards-based school mathematics curricula. What are they? What do students learn?* Mahwah, NJ: Lawrence Erlbaum Associates.

Shonkoff, J. P., & Phillips, D. A. (Eds.). (2000). *From neurons to neighborhoods: The science of early childhood development*. Washington, DC: National Academy Press.

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Starkey, P., & Klein, A. (1992). Economic and cultural influence on early mathematical development. In F. L. Parker, R. Robinson, S. Sombrano, C. Piotrowski, J. Hagen, S. Randoph & A. Baker (Eds.), *New directions in child and family research: Shaping Head Start in the 90s* (pp. 440). New York: National Clouncil of Jewish Women.

Starkey, P., Klein, A., Chang, I., Qi, D., Lijuan, P., & Yang, Z. (1999, April). *Environmental supports for young children's mathematical development in China and the United States*. Paper presented at the meeting of the Society for Research in Child Development, Albuquerque, New Mexico.

Steffe, L. P., & Cobb, P. (1988). *Construction of arithmetical meanings and strategies*. New York: Springer-Verlag.

Steffe, L. P., & Tzur, R. (1994). Interaction and children's mathematics. *Journal of Research in Childhood Education, 8*(2), 99-116.

Steffe, L. P., & Wiegel, H. G. (1994). Cognitive play and mathematical learning in computer microworlds. *Journal of Research in Childhood Education, 8*(2), 117-131.

Teaching Strategies Inc. (2001). *A checklist for assessing your program's implementation of the Creative Curriculum for early childhood* (Third ed.). Washington, DC: Author.

Trafton, P. R., & Hartman, C. L. (1997). Developing number sense and computational strategies in problem-centered classrooms. *Teaching Children Mathematics, 4*(4), 230-233.

Tyler, R. W. (1949). *Basic principles of curriculum and instruction*. Chicago, IL: University of Chicago Press.

van Oers, B. (1994). Semiotic activity of young children in play: The construction and use of schematic representations. *European Early Childhood Education Research Journal, 2*, 19-33.

Walker, D. F. (1992). Methodological issues in curriculum research. In P. W. Jackson (Ed.), *Handbook of research on curriculum* (pp. 98-118). New York: Macmillan.

Woodward, A., & Elliot, D. L. (1990). Textbook use and teacher professionalism. In D. L. Elliot & A. Woodward (Eds.), *Textbooks and schooling in the United States (Eighty-ninth Yearbook of the National Society for the Student of Education, Part 1)* (pp. 178-193). Chicago: University of Chicago Press.

Table 1

Categories and Methods of Curriculum Research

Categories	*Questions Asked*	*Methods*
I. *A Priori Foundations*. In variants of the research-to-practice model, extant research is reviewed and implications for the nascent curriculum development effort drawn.	What is already known that can be applied to the anticipated curriculum?	Established review procedures (e.g., Light & Pillemer, 1984) are employed to garner knowledge concerning psychology, education, and systemic change in general (method 1, Clements, 2001); the specific subject matter content, including the role it would play in students' development (method 2, Clements, Sarama et al., 2004); and pedagogy, including the effectiveness of certain types of activities (method 3, Clements et al., 1993; Clements & Swaminathan, 1995).
II. *Learning Model.* Activities are structured in accordance with empirically-based models of children's thinking and learning in the targeted subject-matter domain	How might the curriculum be constructed to be consistent with models of students' thinking and learning (which are posited to have characteristics and developmental courses that are not arbitrary, and therefore not equally amenable to various instructional approaches or curricular routes)?	In *method 4,* the nature and content of activities is based on models of children's mathematical thinking and learning (Clements, Sarama et al., 2004; Clements, Wilson et al., 2004; cf. James, 1958; Tyler, 1949). In addition, a set of activities (the hypothetical mechanism of the research) may be sequenced according to specific learning trajectories (e.g., Sarama & Clements, 2002). What distinguishes method 4 from method 3, which concerns pedagogical a prior foundations, is not only the focus on the child's learning, rather than teaching strategies alone, but also the iterative nature of its application. That is, in practice, such models are usually applied and revised (or, not infrequently, created anew) dynamically, simultaneously with the development of instructional tasks, which is

		why it is classified separately from the a priori foundation methods.
III. *Evaluation*. In these methods, empirical evidence is collected to evaluate the curriculum, realized in some form. The goal is to evaluate the appeal, usability, and effectiveness of an instantiation of the curriculum.	How can market share for the curriculum be maximized?	*Method 5* focuses on marketability, using strategies such as gathering information about mandated educational objectives and opinions of consumers.
	Is the curriculum usable by, and effective with, various sizes of student groups and various teachers? How can it be improved in these areas?	Formative *methods 6 to 8* seek to understand the meaning that students and teachers give to the curriculum objects and activities in progressively expanding social contexts (Clements & Sarama, 2004a; Sarama & Clements, 2002) and the usability and effectiveness of specific components and characteristics of the curriculum as implemented by a teacher who is familiar with the materials (method 7, Sarama, 2004) and, later, by a diverse group of teachers (*method 8*). The curriculum is altered based on empirical results, with the focus expanding to include aspects of support for teachers.
	What is the effectiveness (e.g., in affecting teaching practices and ultimately student learning) of the curriculum, now in its complete form, as it is implemented in realistic contexts?	Summative *methods* 9 (present study) and *10* differ from each other most markedly on the characteristic of scale. That is, method 10 examines the fidelity and sustainability of the curriculum when implemented on a large scale, and the critical contextual and implementation variables that influence its effectiveness. Experimental or quasi-experimental designs are useful for generating political and public support, as well as for their research advantages. Calculation and reporting of effect sizes and confidence intervals are considered sine qua non for these methods. In addition,

qualitative approaches continue to be useful for dealing with the complexity and indeterminateness of educational activity (Lester & Wiliam, 2002).

Table 2

Means and Standard Deviations for Number and Geometry Tests by Site and Group

	Building Blocks						Comparison					
	Site 1		Site 2		Total		Site 1		Site 2		Total	
Test	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Number	12.38 (10.94)	36.55 (11.12)	6.13 (6.61)	20.17 (13.29)	9.67 (9.70)	29.46 (14.47)	14.07 (9.39)	26.86 (8.64)	3.17 (2.72)	9.56 (9.34)	8.44 (8.69)	17.93 (12.48)
Geometry	9.53 (2.31)	17.69 (2.64)	7.53 (1.86)	12.87 (3.64)	8.79 (2.26)	15.91 (3.81)	9.56 (1.48)	12.12 (2.10)	7.37 (1.40)	8.62 (3.76)	8.63 (1.89)	10.64 (3.35)

Note. These were the data used for the factorial analyses, so they represent data on those children who took all subtests at both pretest and posttest.

Table 3

Means and Standard Deviations for Number and Geometry Subtests by Treatment Group

	Building Blocks		Comparison		
Subtest	Pre	Post	Pre	Post	Maximum
			Number		
Verbal Counting	1.13 (1.10)	2.88 (1.51)	0.84 (.93)	1.78 (1.39)	6
Object Counting	5.53 (4.71)	10.97 (4.20)	4.66 (3.89)	8.16 (4.71)	16
Comparing	1.10 (0.99)	2.13 (0.90)	0.89 (0.85)	1.58 (0.96)	5
Numerals	0.60 (1.59)	3.90 (1.86)	0.32 (1.16)	2.48 (2.38)	5
Sequencing	0.07 (0.25)	1.20 (1.24)	0.05 (0.23)	0.39 (0.72)	3
Subitizing	0.18 (0.35)	2.81 (2.63)	0.23 (0.72)	1.00 (1.27)	10
Adding/ Subtracting	0.93 (1.68)	4.20 (2.80)	0.68 (1.38)	2.23 (2.57)	12
Composing	0.13 (0.51)	1.37 (2.31)	0.16 (0.72)	0.32 (0.91)	15
Total	9.67 (9.70)	29.46 (17.95)	7.83 (8.28)	17.93 (12.48)	72

Table 3 (continued)

	Building Blocks		Comparison		
Subtest	Pre	Post	Pre	Post	Maximum
			Geometry, Measurement, Patterning		
Shape Identification	5.42 (0.92)	7.34 (1.16)	5.66 (0.93)	5.89 (1.42)	10
Composition	1.07 (1.10)	4.47 (1.92)	1.23 (1.36)	2.01 (1.65)	11
Congruence	1.02 (0.38)	1.32 (0.35)	1.05 (0.39)	1.20 (0.52)	2
Construction	0.09 (0.23)	0.61 (0.56)	0.09 (0.29)	0.38 (0.48)	2
Orientation	0.15 (0.21)	0.31 (0.28)	0.08 (0.15)	0.08 (0.14)	1
Turns	0.38 (0.49)	0.41 (0.50)	0.21 (0.42)	0.27 (0.45)	1
Measurement	0.10 (0.31)	0.19 (0.40)	0.08 (0.18)	0.08 (0.23)	1
Patterning	0.50 (0.55)	1.26 (0.78)	0.23 (0.42)	0.73 (0.67)	2
Total	8.79 (2.26)	15.91 (3.81)	8.63 (1.89)	10.64 (3.35)	30

Note. These data are from all 68 children; 7 children missed some subtests. Therefore, the average totals differ slightly from those in Table 2 in some cases.

Table 4

Percentage of Children Using Strategies by Treatment Group

Number

	Experimental		Comparison	
	Pre	Post	Pre	Post
[Show 2 cubes and ask] How many?				
Reproduced the set but could not give the number name	13.3	0.0	5.3	16.1
Gave the number	66.7	100.0	65.8	61.3
No response	36.8	0.0	28.9	22.6
[Set out 4 cubes and 5 marbles, cubes physically larger and ask] Are there more blocks or more chips or are they the same?				
Does not match or count (in a reliability observable manner)	16.7	36.6	10.5	22.6
Uses matching	0.0	3.3	0.0	3.2
Counts	6.7	36.7	0.0	16.1
No response	76.7	23.3	89.5	58.1
Counting scrambled arrangements of objects				
"Reading order" – left to right, top to bottom	3.3	10.0	2.6	16.1
Similar strategy but different directions (e.g., top to bottom)	0.0	40.0	2.6	12.9
Around the perimeter then moving in to the middle	0.0	10.0	0.0	3.2
Other path through the objects	0.0	3.3	2.6	3.2
No response	96.7	36.7	92.1	64.5
Adding 5 + 3 with objects suggested				
Uses objects	3.3	33.3	2.6	19.4
No objects; solves with verbal counting strategy	3.3	23.3	0.0	6.5
No response	93.3	43.3	97.4	74.2
Solving 3 + 2 with objects nearby but not suggested				
Uses objects	0.0	23.3	0.0	9.7
No objects; solves with verbal counting strategy	0.0	13.3	2.6	6.5
No response	100.0	63.3	97.4	83.9
Solving: 6 dogs and 4 bones, how many dogs wouldn't get a bone? with objects suggested				
Uses objects	3.3	30.0	0.0	12.9
No objects; solves with verbal counting strategy	0.0	3.3	2.6	3.2
No response	96.7	66.7	97.4	83.9

Table 4 (con't)

Geometry

	Experimental		Comparison	
	Pre	Post	Pre	Post
Using pattern blocks to fill a puzzle (outline)				
Placing pieces randomly on puzzle that are not connected	79.3	11.1	90.9	76.9
Putting shapes together without leaving gaps	20.7	88.9	9.1	23.1
No response	0.0	0.0	0.0	0.0
Turning shapes after placing them on the puzzle in an attempt to get them to fit	6.9	22.2	6.1	23.1
Turning them into correct orientation prior to placing them	13.8	66.7	3.0	0.0
No response	79.3	11.1	90.9	76.9
Trying out shapes by picking them seemingly at random, then putting them back if they don't look right, so seemingly trial and error	6.9	22.2	6.1	15.4
Appearing to search for "just the right shape" that they "know will fit" and then finding and placing it	13.8	66.7	3.0	7.7
No response	79.3	11.1	90.9	76.9
Hesitant and not systematic	13.8	33.3	6.1	15.4
Overall, solving the puzzle immediately, systematically and confidently	6.9	55.6	3.0	7.7
No response	79.3	11.1	90.9	76.9

Note. *Experimental N's: pre-test, 29; post-test, 27. Comparison N's: pre-test, 33; post-test 26.*

Figure Captions

Figure 1. Hypothesized learning trajectory for addition, including developmental levels, examples of thinking at these levels, and correlated Building Blocks *activities*

Figure 2. Hypothesized learning trajectory for shape composition (Clements, Wilson et al., 2004) including developmental levels, examples of children's work, and correlated Building Blocks *activities*

Figure 3. Building Blocks *activity "Number Pictures"*

Figure 4. Sample geometry items from the Building Blocks Pre-K Assessment. *(a) Given the illustrated cut-out shapes, the child is asked to "Put only the triangles on this paper." (b) The children is asked, "Pretend you cut this pentagon from one corner to the other. Which shows the two cut pieces?"*

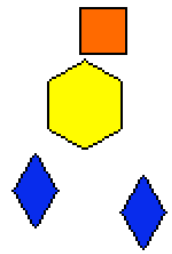
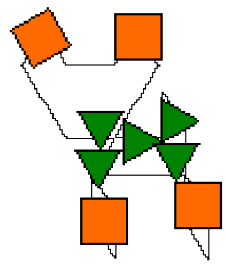
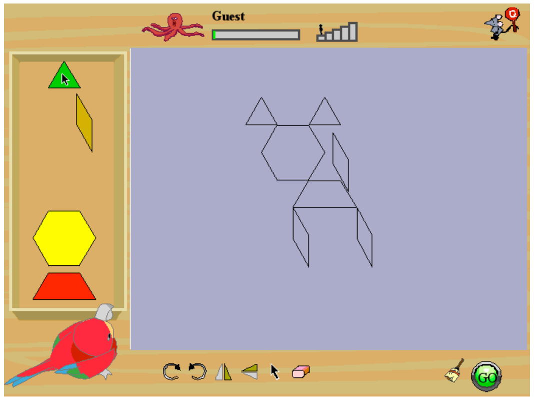
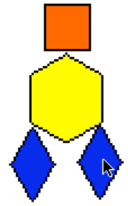
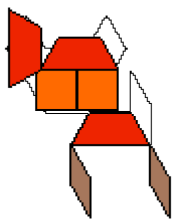
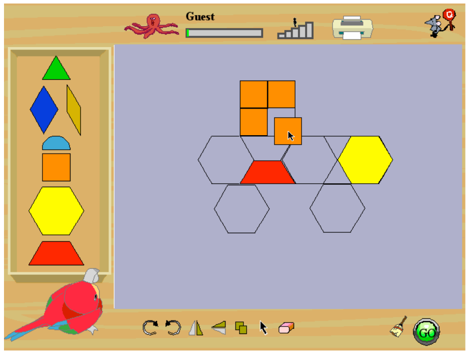
Figure 1

Level	*Behavioral Example*	*Instructional Task*
Non-Verbal Addition. Children reproduce small (< 5) sums when shown the addition of subtraction of groups of objects (Mix, Huttenlocher, & Levine, 2002).	After watching 2 objects, then 1 more placed under a cloth, children choose or make collections of 3 to show how many are hidden in all.	“Mrs. Double” puts 3 chips, then 1 more, on a cookie under a napkin. Children put the same number of chips on the other cookie. 
Small Number Addition. Children solve simple “join, “result unknown” word problems with sums to 5, usually by subitizing (instant identification of small collections) or using a “counting all” strategy (Baroody, 1987; Fuson, 1988).	“You have 2 balls and get 1 more. How many in all?” Child counts out 2, then counts out 1 more, then counts all 3.	The customer wants his order in one box; what should the label for that (rightmost) box be? 

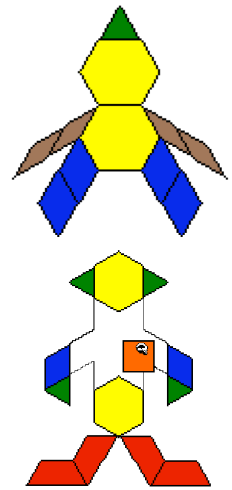
Find Result. Children solve "join, result unknown" problems by direct modeling—"separating from" for subtraction or counting all for addition, with sums to 10 (Carpenter et al., 1993; Clements & Conference Working Group, 2004; Fuson, 1992a).	"You have 3 red balls and 3 blue balls. How many in all? Child counts out 3 red, then counts out 3 blue, then counts all 6.	Children play with toy dinosaurs on a background scene. For example, they might place 4 tyrannosaurus rexes and 5 apatosauruses on the paper and then count all 9 to see how many dinosaurs they have in all. 
Find Change. Children solve "change unknown" word problems by direct modeling. For example, they might "add on" to answer how many more blocks they would have to get if they had 4 blocks and needed 6 blocks in all (Clements & Conference Working Group, 2004).	"You have 5 balls and then get some more. Now you have 7 in all. How many did you get? Child counts out 5, then counts those 5 again starting at one, then adds more, counting "6, 7," then counts the balls added to find the answer, 2.	Mrs. Double tells children the cookie has 5 chips, but should have 8. She asks them to "make it 8." 
Counting On. Children continue	"How much is 4 and 3 more?"	Children use cutout "cookies" and brown disks for

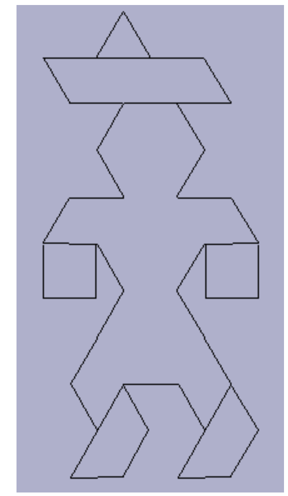
developing their counting methods even further, often using objects to keep track. Such counting requires conceptually embedding the 3 inside the total, 5 (Baroody, 2004; Carpenter & Moser, 1984; Fuson, 1992b).	"Fourrrrr…five [putting up one finger], six [putting up a second finger], seven [putting up a third finger]. Seven!"	chocolate chips. The teacher asks them to put 5 chips on their cookies, and then asks how many they would have in all if they put on 3 more. They count on to answer, then actually put the chips on to check.

Figure 2

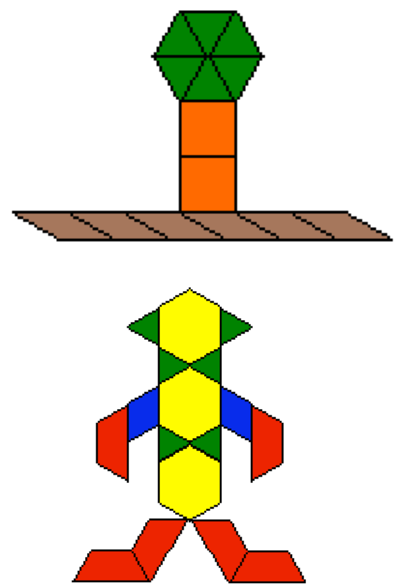
Level	*Examples (above, free-form pictures; below, puzzles)*	*Instructional Task*
Pre-Composer. Children manipulate shapes as individuals, but are unable to combine them to compose a larger shape. In free form—"make a picture"—tasks, shapes often do not touch (upper picture in middle column). In puzzle tasks, shapes do not match simple outlines (lower picture in middle column). The instructional task (illustrated on the computer in the last column; similar tasks are presented with manipulatives and paper outlines or wooden form puzzles) uses outlines in which children can simply match shapes without turn or flip motions. (This and subsequent levels emerged from the same body of research, Clements, Wilson et al., 2004; Mansfield & Scott, 1990; Sales & Hildebrandt, 2002; Sarama et al., 1996.)	 	
Piece Assembler. Children can place shapes contiguously to form pictures. In free-form tasks, each shape used represents a unique role, or function in the picture (e.g., one shape for one leg). Children can fill only simple frames, in which shape is outlined, although their use of turns and flips is limited.	 	

Picture Maker. In free-form tasks, children can concatenate shapes to form pictures in which several shapes play a single role, but use trial and error and do not anticipate creation of new geometric shapes. For puzzle tasks, children can match by a side length and use trial-and-error (a “pick and discard” strategy). Instructional tasks have “open” areas in which shape selection is ambiguous.





Shape Composer. Children combine shapes to make new shapes or fill puzzles, with growing intentionality and anticipation. Shapes are chosen using angles as well as side lengths. Eventually, the child considers several alternative shapes with angles equal to the existing arrangement. Instructional tasks (here, solving similar problems multiple ways) encourage higher levels in the hierarchy not described here, involve substitutions (three higher levels are described in Clements, Sarama, & Wilson, 2001).



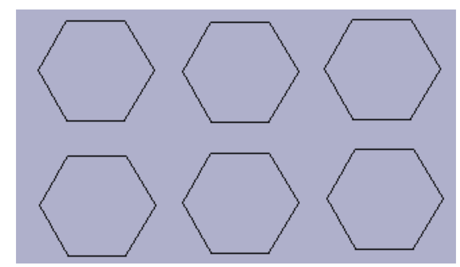


Figure 3

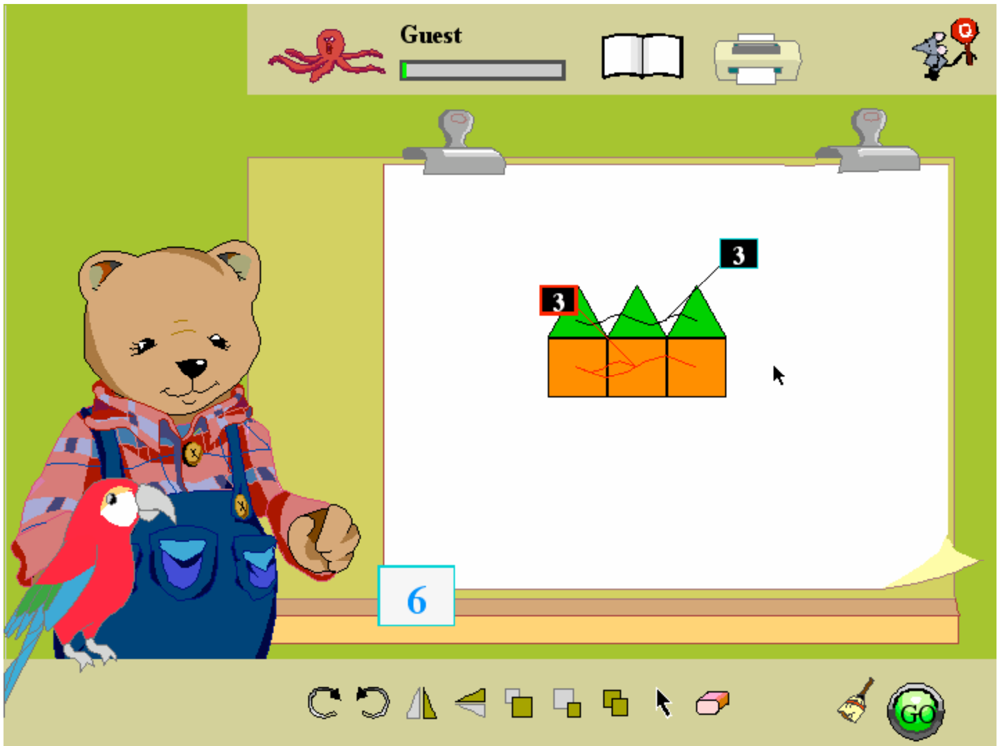
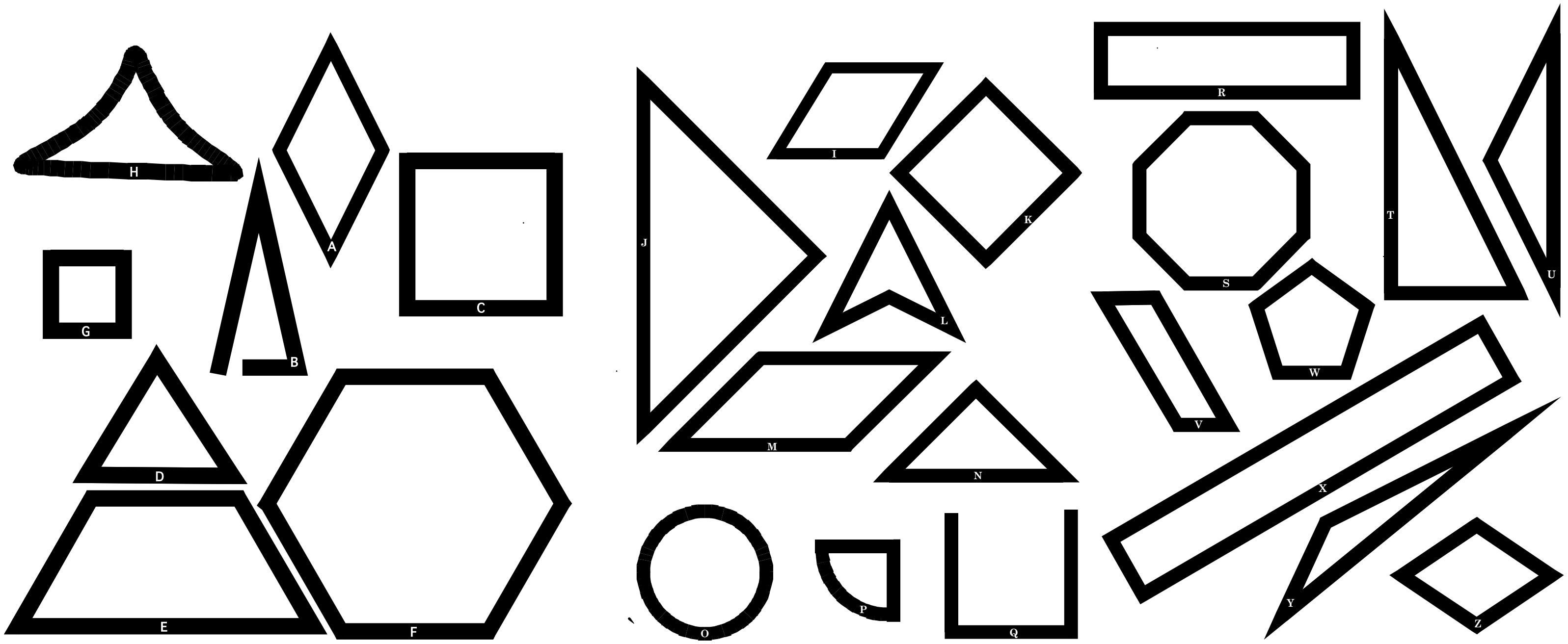


Figure 4

a.



b.