The Research Base for

Algebra Readiness

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The Research Base for *Algebra Readiness*

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Overview

The guide for the development of Glencoe/McGraw-Hill's *Algebra Readiness* program states that the purpose of the program is to reach the estimated 50% of entering high school students who are not Algebra ready. The goal is to provide them with the skills to learn successfully and efficiently so that they can achieve in Algebra. The writing guide list ways that the Algebra Readiness program can help students:

- connections between concepts that reveal big ideas;
- truly differentiated instruction, not just differentiated examples;
- vocabulary instruction and English language support that goes beyond a mere list;
- presentation of small amounts of content;
- numerous examples with different strategies;
- step-by-step exercises to walk through processes;
- communication practice – peer reviews, explanations, presentations, etc.; and,
- experiences that are engaging and motivating including hands-on activities and assessment.

The sections that follow provide a literature base for the two programs with specific examples as well as how the programs meet California Algebra Readiness Standards.

Mathematical Proficiency for All Learners

Struggling Learners

Before examining the results of studies themselves, it is useful to note that in the research regarding math intervention and Algebra Readiness, Seethaler and Fuchs (2005) analyze the literature in terms of the efficacy of studies completed. They found that randomized, controlled designs were clearly underrepresented in the literature. They conclude that to truly assess efficacy, study methodology may need to improve. In a related article, Augustyniak, Murphy, and Phillips (2005) argue that the research on the definition of a math disability is lacking with respect to identification of core deficits. They identify the core areas needing further explanation as numerical skills, visual/spatial deficits, cognitive skill development (memory retrieval, working memory, speed of processing, attention regulation, problem solving) and social cognition. Mazzocco (2005) reviewed research regarding practices of early identification and intervention for students with math diffciulties. The commentary discusses the criteria and nature of math difficulties and notes the need for additional research.

The above being said, Butler, Beckingham, and Lauscher (2005) report on three case studies regarding the support of students with math learning challenges. Three eighth grade students were given assistance in self-regulating their learning. General strategies found to be successful included:

- engaging the students in constructive conversation;
- supporting students reflection on their learning; and,
- the need for teachers to engage in dynamic, curriculum-based forms of assessment

Fuchs, Fuchs, and Hamlett (2006) report on the validation of an intervention to improve math problem-solving at the third grade. The intervention (Hot Math) involved explicit instructions, self-regulation strategies and tutoring. Results indicated positive, short term results for problem-solving skills.

Stinson (2006) suggested that a focus on the discourse of achievment in mathematics rather than the discourses of deficiency and rejection could prove beneficial in reducing the well known achievment gap between white and black students. He suggests that the limited amount of research shows that enrichment activities, mentoring competent teachers, and black students identifying with the "good kids group" (p. 496) may enhance math achievement in African-American males.

Research also suggests a variety of instructional strategies that are effective to meet the needs of students with special needs—including those with physical disabilities, mental impairments, and/or learning disabilities; English language learners (ELL); and low-performing students who require some special attention to bring out the best of their capabilities. Effective instruction for special-needs students, the research has found, includes:

- setting clear goals for students (Bray & Turner, 1986; Cherkes-Julkowski & Gertner, 1989; Ferritti, 1989; Ferritti & Cavalier 1991, as cited by Baroody, 1996; Schunk, 1985, as cited by Mastropieri, Scraggs, & Shinh, 1991);

- using a "big ideas" structure for concepts (Kameenui & Carnine, 1998, as cited by Fuson, 2003, p. 88);
- teaching content that is not too difficult (Bray & Turner, 1986, Cherkes-Julkowski & Gertner, 1989; Ferritti, 1989; Ferritti & Cavalier 1991, as cited by Baroody, 1996; Baroody, 1996) and presented within meaningful contexts (Miller & Mercer, 1997, as cited by Allsopp, Lovin, Green, & Savage-Davis, 2003);
- laying ample groundwork by providing background knowledge (Bray & Turner, 1986; Cherkes-Julkowski & Gertner, 1989, Ferritti, 1989, Ferritti & Cavalier 1991, as cited by Baroody, 1996; Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- modeling by teachers (Allsopp et al., 2003; Baroody, 1996; Blankenship, 1978, as cited by Mastropieri et al., 1991);
- sequencing instruction to go from the concrete to the abstract (Miller & Mercer, 1997, as cited by Allsopp et al., 2003);
- using mediated scaffolding (e.g., visual supports with cues, teachers' feedback on thinking, peer tutoring) (Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- discussing mathematics using language (Miller & Mercer, 1997, as cited by Allsopp et al., 2003);
- building in multiple practice opportunities (Miller & Mercer, 1997, as cited by Allsopp et al., 2003) and time for review by students (Kameenui & Carnine, 1998, as cited by Fuson, 2003);
- using reinforcement (e.g., earning verbal praise) (Mastropieri et al., 1991); and,
- providing continual feedback (Miller & Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Blankenship, 1978 and Schunk & Cox, 1986, as cited by Mastropieri et al., 1991).

Three of these elements of effective special needs instruction—modeling, mediated scaffolding, and feedback—are discussed in further detail below.

Modeling

Directly modeling both general problem-solving strategies and specific learning strategies using multisensory techniques has been shown to be useful with students having attention problems, cognitive-processing problems, memory problems, and metacognitive deficits, notes a summary of relevant research (Allsopp et al., 2003). A comparative study of 30 students suggests that direct modeling may be advantageous for students with slight mental retardation as well. One group of students who received direct modeling help (e.g., used blocks as physical manipulatives) and extra opportunities for purposeful practice employed "substantially fewer" inappropriate learning strategies than another group who didn't receive such support (Baroody, 1996, pp. 81-82). Furthermore, it was found that one or two direct-modeling demonstrations enabled such students to correct basic arithmetic procedural strategies and improve their proficiency (Baroody, 1996).

In addition, a review and synthesis of 30 studies on mathematics instruction for learning-disabled students found that modeling and demonstration with corrective feedback improved problem-solving accuracy and generalization skills by the students (Blankenship, 1978, as cited by Mastropieri et al., 1991). For example, an instructional model in which teachers solved a problem, verbalized how they did it, and left the problem as a reference model improved learning-disabled students' computational skills in seven different experiments (Smith & Lovitt, 1975, Rivera & Smith, 1987, 1988, as cited by Mastropieri et al., 1991).

Mediated Scaffolding

A review of 30 studies found several types of scaffolding strategies to be effective in improving learning-disabled students' mathematical achievement in grades K-6 (Mastropieri et al., 1991):

- use of manipulatives teamed with pictorial representations (Peterson, Mercer, & O'Shea, 1988, as cited by Mastropieri et al., 1991);
- multisensory approaches—mixing visual, auditory, and/or kinesthetic methods. Verbalization (a specific practice in which teachers and students repeat aloud problems, instructions, and solution steps) also improved students' mathematical performance, found a comparative study of 90 such students in grades 6 through 8 (Schunk & Cox, 1986, as cited by Mastropieri et al., 1991); and,
- use of pre-organizers (e.g., read problem, underline numbers, decide on the operation sign and problem type) and/or post-organizers (e.g., read problem, check operation, check math statement, check calculations, write labels) to support students when solving word problems (Mastropieri et al., 1991).

Feedback

Ongoing feedback is crucial with special-needs students. Such students require continual monitoring and feedback on their efforts to be successful, several studies and meta-analyses have found (Miller & Mercer, 1997, as cited by Allsopp et al., 2003; Fuson, 2003; Schunk & Cox, 1986, as cited by Mastropieri et al., 1991).

Addressing Specific Mathematics Disabilities

A synopsis of relevant research noted that four different kinds of mathematics disabilities have been identified (Geary, 1994, as cited by Fuson, 2003). They, and what the research suggested as useful strategies to address them, are as follows:

- Semantic memory disabilities: Students experience trouble with verbal and phonetic memory but may have normal visuospatial skills. Instruction that employs visual clues is most effective for these learners (Fuson, 2003).
- Procedural deficits: Students use less advanced methods overall. Conceptually based instruction is especially helpful for these students (Fuson, 2003);
- Visuospatial disabilities: Students struggle with concepts that use spatial relations, (e.g., place value). Instruction most helpful for these students includes extra cues to support visual processing and focuses on methods that can be carried out in either direction (Fuson, 2003); and,
- Problem-solving deficits: Such students benefit from problem-drawing supports, including visual representations and manipulatives (Fuson, 2003).

English Language Learners (ELL) and Special Needs Students

In his review of the research on how race, ethnicity, social class and language might affect student achievement in mathematics, Secada (1992) found a relationship between the amount of proficiency in a given language and mathematics achievement (Fernandez & Nielson, 1986, Duran, 1988, Secada, 1991b, as cited by Secada, 1992). To support academic achievement for non-native speakers of English and other diverse learners, Secada recommended:

- intervening early;
- providing ongoing extra support materials and strategies;
- using a student's native language for instruction;
- using a structured curriculum or focus teaching on basic skills;

- using small-group instruction, preferably in cooperative learning settings; and
- carefully grouping students by specific ability, if necessary (Secada, 1992).

Ample research has concluded that students find more success and satisfaction in school if they are taught in ways that are responsive to their readiness levels (e.g., Vygotsky, 1986), interests (e.g., Csikszentmihalyi, 1997), and learning profiles (e.g., Sternberg, Torff, & Grigorenko, 1998) (as cited by Tomlinson, 2000). Differentiated instruction is how this translates into classroom practice.

> Every elementary classroom holds a wide range of learners: In most elementary classrooms, some students struggle with learning, others perform well beyond grade-level expectations, and the rest fit somewhere in between. Within each of these categories of students, individuals also learn in a variety of ways and have different interests. To meet the needs of a diverse student population, many teachers differentiate instruction (Tomlinson, 2000).

Differentiated instruction involves varying ones' teaching according to each learner in either (1) content, (2) instructional process, (3) students' products (e.g., papers, projects, computer models), and/or (4) learning environment (e.g., cooperative learning in small groups, grouped by ability) (Tomlinson, 2000). By definition, differentiated instruction always involves ongoing assessment linked to instructional decisions and planning (Tomlinson, 2000). Because differentiated instruction focuses on each learner's varying needs, it is especially well suited for special-needs students.

The quality of the curriculum and instruction used during differentiation is crucial. High quality curriculum focuses on what experts deem the most essential mathematical concepts and skills. High quality instruction incorporates lessons, tasks, and materials designed to ensure that students (1) grapple with essential concepts and skills, (2) find the learning experiences relevant and interesting, and (3) are engaged in active learning experiences (Tomlinson, 2000).

Research has also shown that flexible groupings can improve the mathematical achievement of special-needs students (Slavin, Madden, & Leavey, 1984, as cited by Mastropieri et al., 1991; Mastropieri et al., 1991; Secada, 1992; Slavin, Madden, Karweit, Livermon, & Dolan, 1990, as cited by Secada, 1992). Teachers can use flexible grouping to deliver a variety of differentiated learning environments in their classrooms, including small workgroups, cooperative learning groups, cross-grade groups, between-grade groups, grouping by ability for guided or independent practice, as well as whole class, and individual practice settings (Tomlinson, 2000).

Furthermore, Burris, Heubert, & Levin (2006) found that students who have completed advanced math courses increase in all heterogeneous grouped students including minority and low SES students. The same conclusion was reached for all students at whatever initial achievement level. Initial high achievers performed the same as counterparts in homogeneous groups. Rates of participation and test scores improved in all groups.

According to an article by McElroy (2005), teachers need to expand their teaching tools to assist ELL students in content areas such as math. The article describes a website Colorin Colorado that teachers can use to work with ELL students. Material specific to ELL are presented along with teaching tips. While focused more on the language learning of ELL, several recommendations are made by Goldenberg (2006). The instructional practices seen as having a positive impact specific to math include:

- clear instructions and expectations
- additional opportunities for practice
- extended explanations

In a well presented case, Abedi (2004) reports on the difficulty of assessment of math and reading for ELL, especially as related to the No Child Left Behind (NCLB) act. Factors are presented as issues including the sparse ELL populations in some states, subgroup lack of stability, linguistic complexity of assessment tools and lower ELL baselines requiring greater gains. The implications are that unless these issues are considered, schools with large ELL populations face unfair and undue pressure under NCLB. In a similar vein, the American Federation of Teachers (AFT, 2004) point out that NCLB challenges faced by ELL students in math and reading include defining ELL subgroups and the paucity of natural language assessment. The AFT identified four changes needed:

- appropriate tests for ELL students
- relevant and valid testing of ELL students in English

- clarifying assessment of proficiency versus math and reading skills
- clarifying existing policies regarding ELL immigrant and non-immigrant groups

In yet another piece (Zehr, 2006), the changes brought by NCLB are debated as to their positive versus negative effects. The major point made is that many schools may be responding to NCLB in math and reading by narrowing the curriculum to focus only on test scores. With respect to the No Child Left Behind legislation, Kim and Sunderman (2005) suggets that use of the mean proficiency score may have a disparate effect on schools with low income children. Accountability rules for the sub groups can over identify racially diverse schools as failing to meet proficiency levels. The use of multiple indicators of performance may produce a more accurate assessment of student proficiency. Multiple measures could include a measure of improvement.

Much debate has surrounded the achievement issues in public versus private education. Charter schools, educational vouchers, etc., have been the responses of individuals and groups concerned about public school performance issues. Utilizing mathematics data from the 2003 NAEP data, Lubienski and Lubienski (2006) analyzed differences among students from public, charter and other types of private schools. They found that once demographic information was statistically controlled (ethnicity, gender and SES), no significant differences favoring private over public entities were found. In fact, public school fourth graders scored significantly higher than their private school counterparts. Their findings validate the importance of other in-school factors such as curriculum, teacher competence, etc.

How Algebra Readiness Reflects the Research on Mathematical Proficiency for All Learners

The guides for developing *Algebra Readiness* are quite explicit and accurately reflect the research base in terms of use by those developing the material. A summary of the strategies identified in the research include:

- clear goals;
- vocabulary support;
- ELL methods;
- word problems;

- sequencing;
- graphics and visuals;
- student reflection;
- cooperative learning;
- math conversation and discourse;
- enrichment;
- scaffolded questions;
- tiered questions;
- writing about mathematics;
- feedback; and,
- dynamic diagnostic and prescriptive assessment.

Each chapter in *Algebra Readiness* begins with clearly stated goals. Goals are expressed in 'The What' and 'The Why' sections of each chapter. Vocabulary in both English and Spanish are included in a 'Key Vocabulary' box. For example, in Chapter 1 (Lesson 1.2, A Plan for Problem Solving), 'The What' is learning how to approach problems and solve them. "The Why' is so students can solve a real-life problem concerning how many hours of work will be needed to buy a computer.

Word problems are given in a sequenced manner with graphic and visual support for all materials. In Lesson 8.2 of *Algebra Readiness,* the method is repeated in a task to make two congruent triangles using geoboards (visually first, then with manipulatives) and rubber bands. A sequenced approach is demonstrated via drawings.

Student reflection, cooperative learning, conversation, and discourse are encouraged. In *Algebra Readiness,* a "Talk Math" box appears. For example, in Chapter 8, students are paired to discuss similarities and differences between shapes.

Enrichment activities are given for all materials. In the book, online activity is encouraged (ca.algebrareadiness.com).

Tiering and scaffolding of questions appear in all materials. Teachers are shown alternative pedagogical strategies to explain concepts. For example, in Chapter 1, a 'Mental Math Minute' has students (orally, in writing, or in pairs) describe patterns within a series of numbers. In a 'Scaffolding Questions' box, students take turns writing responses to questions such as "write three numbers that have a sum of 25."

Finally, diagnostic and prescriptive assessment and feedback are used extensively. A readiness quiz begins each chapter. As lessons are taught practice questions are given to assess understanding. These questions cover math concepts, vocabulary and data-driven decision making. A 'Common Error Alert' is included in the Teacher Edition to assist in instruction. In addition, a 'Spiral Review' section assesses learning along with a concluding progress check. For all lessons, additional examples provide alternative ideas for concept presentation.

In sum, the development of *Algebra Readiness* is based, to a large extent, on the relevant and current literature in the area of mathematics instruction.

California Mathematics Content Standards

The California State Board of Education (California, 2006) developed the mathematics content standards to establish what they believe all students in California need to know with respect to mathematics. They were established to achieve six goals:

- Develop fluency in basic computational skills
- Develop an understanding of mathematical concepts
- Become mathematical problem solvers who can recognize and solve routine problems readily and develop ways to reach a solution or goal where no routine path is apparent
- Communicate precisely about quantities, logical relationships and unknown values through the use of symbols, models, graphs and mathematical terms
- Reason mathematically by gathering data, analyzing evidence and building arguments to support hypotheses
- Make connections among mathematical ideas and between mathematics and other disciplines

Algebra Readiness Program Standards

The standards developed for Algebra readiness curricula were put in place to help students master prealgebraic skills and concepts before they enroll in a course for Algebra I.

Topic 1 Whole Numbers
FOUNDATIONAL SKILLS AND CONCEPTS
• Concept of place value in whole numbers (Grade 3 Number Sense 1.3)
• Expanded form of whole numbers (Grade 3 Number Sense 1.5)
Topic 2 Operations on Whole Numbers
FOUNDATIONAL SKILLS AND CONCEPTS
• Standard algorithms for addition and subtraction (Grade 4 Number Sense 3.1)
• Standard algorithms for multiplication and division (Grade 4 Number Sense 3.2)
• Associative and commutative rules (Grade 2 Algebra and Functions 1.1; Grade 3 Algebra and Functions 1.5)
• Distributive rule (Grade 5 Algebra and Functions 1.3)
• Complete fluency with operations on whole numbers
Topic 3 Rational Numbers
FOUNDATIONAL SKILLS AND CONCEPTS
Definition of positive and negative fractions; number line representation
• Concept of a whole and its parts
• Concept of prime factorization and common denominators (Grade 5 Number Sense 1.4)
• Equivalence and ordering of positive and negative fractions (Grade 6 Number Sense 1.1)
• Expanded form of decimals using powers of ten
• Complete fluency with representing fractions, mixed numbers, decimals, and percentage
Topic 4 Operations on Rational Numbers
FOUNDATIONAL SKILLS AND CONCEPTS
• Definition of operations on fractions
• Mathematical reasoning with fractions in a structured, defined environment
• Understanding of why the standard algorithms work
• Complete fluency with operations on positive fractions (Grade 6 Number Sense 1.4, 2.0, 2.1, and 2.2)

The following five standards are to be included in the instructional materials:
Number Sense (Grade Seven)
1.2 Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
1.3 Convert fractions to decimals and percents and use these representations in estimations, computations, and applications.
1.5 Know that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions.
2.1 Understand negative whole-number exponents. Multiply and divide expressions involving exponents with a common base.
Algebra and Functions (Grade Seven)
2.1 Interpret positive whole-number powers as repeated multiplication and negative whole-number powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.
Topic 5 Symbolic Notation
FOUNDATIONAL SKILLS AND CONCEPTS
• Evaluating expressions with parentheses (Grade 4 Algebra and Functions 1.2)
• Writing equations using parentheses (Grade 4 Algebra and Functions 1.3)
• Using a "variable" to represent a number (Grade 5 Algebra and Functions 1.0; Grade 6 Algebra and Functions 1.1)
• Complete fluency with the use of symbols to express verbal information (Grade 6 Algebra and Functions 1.0)
Topic 6 Equations and Functions
FOUNDATIONAL SKILLS AND CONCEPTS
• The concept of an equation as a "prescription" (Grade 4 Algebra and Functions 1.5)
• The concept that equals added to equals are equal (Grade 4 Algebra and Functions 2.1)
• The concept that equals multiplied by equals are equal (Grade 4 Algebra and Functions 2.2)
• Basic techniques for manipulating symbols in an equation (Grade 4 Algebra and Functions 2.0)
• Complete fluency in writing and solving simple linear equations

The following four standards are to be included in the instructional materials:
Algebra and Functions (Grade Seven)
1.1 Use variables and appropriate operations to write an expression, an equation, an inequality, or a system of equations or inequalities that represents a verbal description (e.g., three less than a number, half as large as area A).
1.3 Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse, distributive, associative, commutative) and justify the process used.
4.1 Solve two-step linear equations and inequalities in one variable over the rational numbers, interpret the solution or solutions in the context from which they arose, and verify the reasonableness of the results.
4.2 Solve multistep problems involving rate, average speed, distance, and time or direct variation.
Topic 7 The Coordinate Plane
FOUNDATIONAL SKILLS AND CONCEPTS
• Plotting and interpreting points (ordered pairs) on the coordinate plane (Grade 4 Measurement and Geometry 2.0; Grade 5 Algebra and Functions 1.4)
• Plotting lines and simple polygons based on a "recipe" or set of instructions
• Graphing lines corresponding to simple linear equations, as a "prescription" (reference Grade 4 Measurement and Geometry 2.1)
• The concept that a graph is a collection of *all* the ordered pairs satisfying a defined condition
• Complete fluency in plotting points, interpreting ordered pairs from a graph, and interpreting lengths of horizontal and vertical line segments on a coordinate plane (reference Grade 4 Measurement and Geometry 2.2 and 2.3)
The following standard is to be included in the instructional materials:
Measurement and Geometry (Grade Seven)
3.3 Know and understand the Pythagorean theorem and its converse and use it to find the length of the missing side of a right triangle and the lengths of other line segments and, in some situations, empirically verify the Pythagorean theorem by direct measurement.
Topic 8 Graphing Proportional Relationships
FOUNDATIONAL SKILLS AND CONCEPTS
• Ratio and proportion; drawing and reading graphs of lines passing through the origin
• The geometric context for ratio and proportion; similar right triangles on a graph
• The concept of the slope of a line
• Complete fluency in graphing and interpreting relationships of the form $y = mx$

The following three standards are to be included in the instructional materials:
Algebra and Functions (Grade Seven)
3.3 Graph linear functions, noting that the vertical change (change in y-value) per unit of horizontal change (change in x-value) is always the same and know that the ratio ("rise over run") is called the slope of a graph.
3.4 Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the ratio of the quantities.
Measurement and Geometry (Grade Seven)
1.3 Use measures expressed as rates (e.g., speed, density) and measures expressed as products (e.g., person-days) to solve problems; check the units of the solutions; and use dimensional analysis to check the reasonableness of the answer.
Topic 9 Algebra (Introductory Examples)
Algebra I (Grades Eight Through Twelve)
2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents [*excluding fractional powers*].
4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as $3(2x - 5) + 4(x - 20) = 12$ [*excluding inequalities*].
5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step [*excluding inequalities*].

According to the California Department of Education (2006),

> The algebra readiness materials must also break these 16 standards into their component concepts and skills, with a primary focus on developing students' mastery of arithmetic. The instructional materials must provide diagnostic assessments on foundational concepts and skills and lessons that can be implemented in the classroom, as needed, to rebuild the missing foundational content. It is crucial that materials for an algebra readiness program include large numbers of exercises and problems with a wide range of difficulty, starting with simple one-step problems and progressing to multistep problems for which students have become prepared. The program should be based on a set of highly focused instructional materials that break each standard into a series of small conceptual steps and embedded skills and should be designed to prepare students to complete a course in algebra successfully in the following year. Programs should provide support for a variety of instructional strategies, including various ways to explain and develop a concept. (p. 366)

How Algebra Readiness Relates to the California Algebra Readiness Standards

The linkage of Glencoe *Algebra Readiness* to the California Algebra Readiness Standards is both clear and complete. Both the Student and Teacher Editions are explicit as to what standards are being met.

In *Algebra Readiness*, each lesson begins with key concepts and the California State-shaped icon in blue and gold identifying which standards are being addressed. Both the Student and Teacher Editions are explicit as to what standards are being met. For example, in Chapter 1, a plan for problem solving is introduced. The California icon identifies three standards to be covered, all from grade 7 (Algebra Functions 1.1 and Mathematical Reasoning 1.2 and 3.3). 'The What' is presented … "I will learn how to approach problems and solve them." As well, 'The Why' relates to a real life situation concerning the number of hours needed to work to buy a computer.

An outstanding feature in the program, which is strongly supported in the literature, is the opportunity for practice questions directly related to the standard covered. This assures that students become familiar with the format utilized for subsequent testing.

In summary, the Student and Teacher Editions of each chapter of *Algebra Readiness* provide a section on California content standards to be covered. There is no doubt as to what is being addressed as *Algebra Readiness* is ubiquitously tied to the California Standards.

References

Abedi, J. (2004, January/February). The no child left behind act and English language learners: Assessment and accountability issues. *Educational Researcher. 33* (1), 4-14.

Allsopp, D., Lovin, L., Green, G., & Savage-Davis, E. (2003). Why students with special needs have difficulty learning mathematics and what teachers can do to help. *Mathematics Teaching in the Middle School, 8,* 308-314.

American Federation of Teachers. (2004, March). English language learners and adequate yearly progress calculations: A catch-22. Washington, D.C.

Baroody, A.J. (1996). Self-invented addition strategies by children with mental retardation. *American Journal on Mental Retardation*, *10,* 72-89.

Bray, N.W. & Turner, L.A. (1986). The rehearsal deficit hypothesis. In R. Ellis & N.W. Bray (Eds.), *International review of research in mental retardation* (Vol. 14, pp. 55-111). New York: Academic Press.

Burris, C.C., Heubert, J.P., & Levin, H.M. (Spring, 2006). Accelerating mathematics achievement using heterogeneous grouping. *American Educational Research Journal. 43* (1), 105- 136.

Butler, D.L., Beckingham, B., & Lauscher, H.J. (2005). Promoting strategic learning by eighth grade students struggling in mathematics: A report of three cases studies. *Learning Disabilities Research and Practice, 20,* 3, 156-174.

California Department of Education. (2006). *Mathematics Framework for California public schools*. CDE Press.

Cherkes-Julkowski, M. & Gertner, N. (1989). *Spontaneous cognitive processes in handicapped children*. New York: Springer-Verlag.

Csikszentmihalyi, M. (1997). *Finding flow: The psychology of engagement with everyday life*. New York: Basic Books.

Ferritti, R.P. (1989). Problem solving and strategy production in mentally retarded persons. *Research in Developmental Disabilities, 10,* 19-31.

Fuchs, L.S., Fuchs, D., & Hamlett, C.L. (2006). Extending responsiveness to intervention to math problem solving at third grade. *Teaching Exceptional Children, 39,* 4, 59-63.

Fuson, K.C. (2003). Developing mathematical power in whole number operations. In J. Kilpatrick, W.G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 68-94). Reston, VA: National Council of Teachers of Mathematics.

Goldenberg, C. (2006, April). *Improving achievement for English learners: conclusions from 2 research reviews*. Retrieved on September 23, 2006 from: www.colorincolorado.org/articles/cgoldenberg_april06.php

Kim, J.S. & Sunderman, G.L. (November, 2005). Measuring academic proficiency under the No Child Left Behind Act: Implications for educational equity. *Educational Researcher, 34* (8), 3-13.

Lubienski, S.T. & Lubienski, C. (2006). School sector and academic achievement: A multilevel analysis of NAEP mathematics data. *American Educational Research Journal, 43*, 4, 651-698.

Mastropieri, M.A., Scraggs, T.E., & Shinh, S. (1991). Mathematics instruction for learning disabled students: A review of research. *Learning Disabilities Research and Practice*, 6 (2), 89-98.

Mazzocco, M.M. (2005). Challenges in identifying target skills for math disability screening and intervention. *Journal of Learning Disabilities, 38*, 4, 318-323.

McElroy, E.J. (2005, November/December). Teacher to teacher: Supporting English language learners. *Teaching pre K-8*, 36, 3, 8-9.

Secada, W.G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 623-660). New York: Macmillan.

Seethaler, P.M. & Fuchs, L.S. (2005, May). A drop in the bucket: Randomized control trials testing reading and math interventions. *Learning Disabilities Research and Practice, 20*, 2, *98-102. standards for school mathematics* (pp. 289-303). Reston, VA: National Council of Teachers of Mathematics.

Smith, D.D. & Lovitt, T.C. (1975). The use of modeling techniques to influence the acquisition of computational arithmetic skills in learning disabled children. In E. Ramp & G. Senf (Eds.), *Behavior analysis: Areas of research and application* (pp. 283-308). Hillsdale, NJ: Erlbaum.

Stinson, D.W. (2006). African American male adolescents, schooling (and mathematics): Deficiency, rejection and achievement. *Review of Educational Research, 76*, 4, 477-506.

Tomlinson, C.A. (2000). *Differentiation of instruction in the elementary grades*. ERIC Digest. Champaign, IL: ERIC Clearinghouse on Elementary and Early Childhood Education (ERIC Document Reproduction Service No. ED443572).

Vygotsky, L. (1986). *Thought and language*. Cambridge, MA: MIT Press.

Zehr, M.A. (2006, March). "No child" effect on English-learners mulled: Teachers welcome attention, fault focus on test scores. *Education week, 25*, 25, 14-15.