Subtract Integers

LESSON GOAL

Students will solve problems subtracting integers.

1 LAUNCH

Launch the lesson with a warm up and an introduction.

2 EXPLORE AND DEVELOP

Explore: Use Algebra Tiles to Subtract Integers

Learn: Subtract Integers
Example 1: Subtract Integers
Example 2: Subtract Integers
Example 3: Subtract Expressions

Explore: Find Distance on a Number Line

Learn: Find the Distance Between Integers
Example 4: Find the Distance Between Integers
Example 5: Find the Distance Between Integers
Apply: The Solar System

Have your students complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

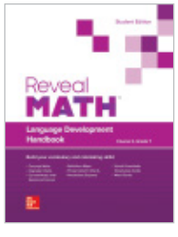
View reports of student progress of the **Checks** after each example to differentiate instruction.

Resources	AL	OL	BL
Remediation: Review Resources	●	●	
ArriveMATH Take Another Look	●		
Collaboration Strategies	●	●	●

Language Development Support

Assign page 15 of the *Language Development Handbook* to help your students build mathematical language related to subtraction of integers.

ELL You can use the tips and suggestions on page T15 of the handbook to support students who are building English proficiency.



Suggested Pacing

90 min	1.5 days
45 min	3 days

Focus

Domain: The Number System
Major Cluster(s): In this lesson, students address major cluster **7.NS.A** by subtracting integers.
Standards for Mathematical Content: **7.NS.A.1, 7.NS.A.1.C, 7.NS.A.1.D,** *Also addresses 7.NS.A.3, 7.EE.B.3*
Standards for Mathematical Practice: **MP1, MP2, MP3, MP4, MP5, MP6**

Coherence

Vertical Alignment

Previous
Students solved problems involving adding integers.
7.NS.A.1, 7.NS.A.1.B, 7.NS.A.1.D

Now
Students solve problems involving subtracting integers.
7.NS.A.1.C

Next
Students will solve problems involving multiplying integers.
7.NS.A.2, 7.NS.A.2.A, 7.NS.A.2.C

Rigor

The Three Pillars of Rigor

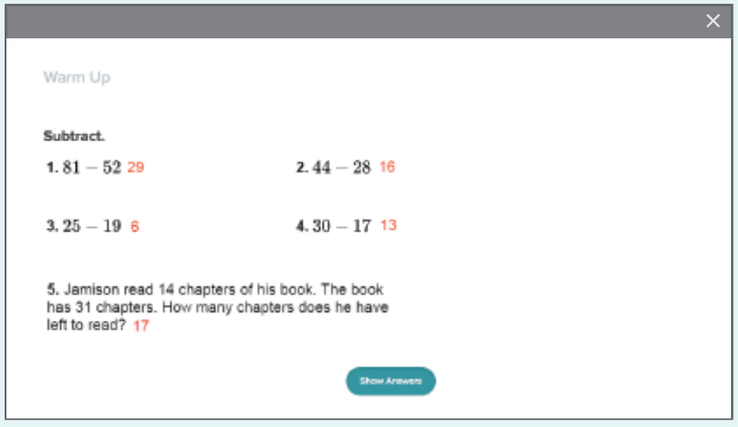
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students draw on their knowledge of integers and subtraction to develop *understanding* of and build *fluency* in subtraction of integers. They will gain an understanding of finding the distance between between two integers. They *apply* their knowledge of finding the distance to real-world problems.

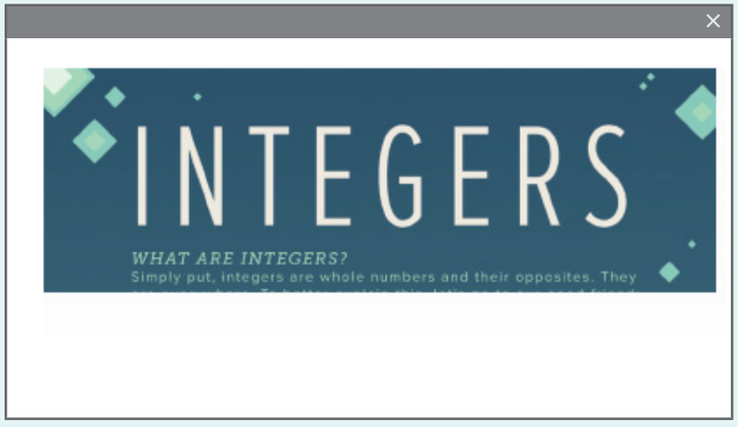
Mathematical Background

Addition and subtraction are inverse operations. To subtract an integer, add its additive inverse (opposite). To find the distance between two integers on a number line, find the absolute value of the difference between the two integers.

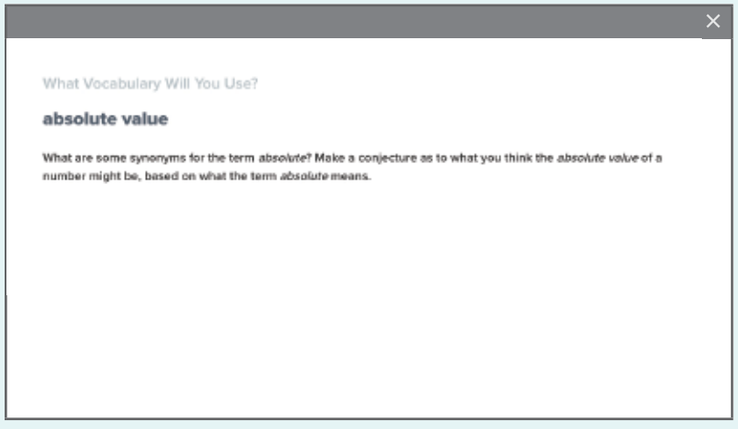
Interactive Presentation



Warm Up



Launch the Lesson, Slide 1 of 1



What Vocabulary Will You Use?

Warm Up

Prerequisite Skills

The Warm-Up exercises address the following prerequisite skill for this lesson:

- subtracting whole numbers (Exercises 1–5)

Answers

1. 29
2. 16
3. 6
4. 13
5. 17

Launch The Lesson

The Launch the Lesson feature is designed to engage students with real-world situations that reflect the mathematics of the lesson. This lesson launches with a discussion about integers using an infographic.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will be addressing these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

What Vocabulary Will You Use?

Use the following question to engage students and facilitate a class discussion.

Ask:

- What are some synonyms for the term *absolute*? Make a conjecture as to what you think the *absolute value* of a number might be, based on what the term *absolute* means. Sample answer: Some synonyms are total, complete, universal, not in relation to other things. The absolute value of a number might mean the total value of the number.

Explore Use Algebra Tiles to Subtract Integers

Objective

Students will use algebra tiles to explore how to subtract integers.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will be presented with algebra tiles representing 1 and −1. Throughout this activity, students will use the algebra tiles to subtract integers with the same sign and integers with different signs. They will see how the algebra tiles illustrate why subtracting integers is the same as adding the additive inverse.

Inquiry Question

How can you use algebra tiles to model integer subtraction? Sample answer: By using tiles to represent positive and negative integers, integer subtraction can be modeled by taking away the number of tiles that represent the integer being subtracted. Sometimes it is necessary to add zero pairs to the workspace before taking away tiles.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 4 is shown.

Talk About It!

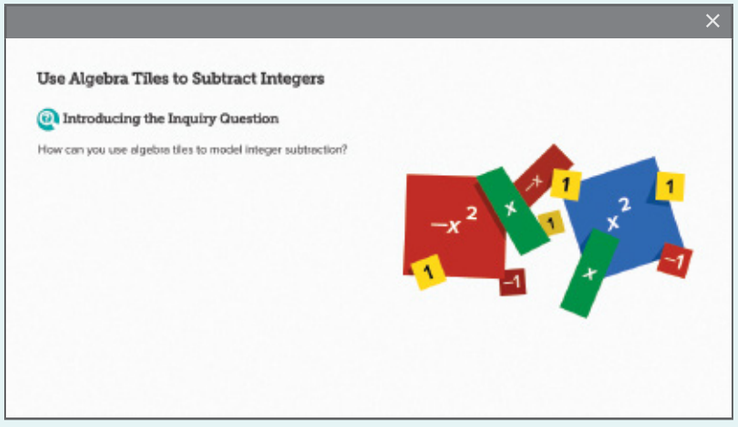
SLIDE 4

Mathematical Discourse

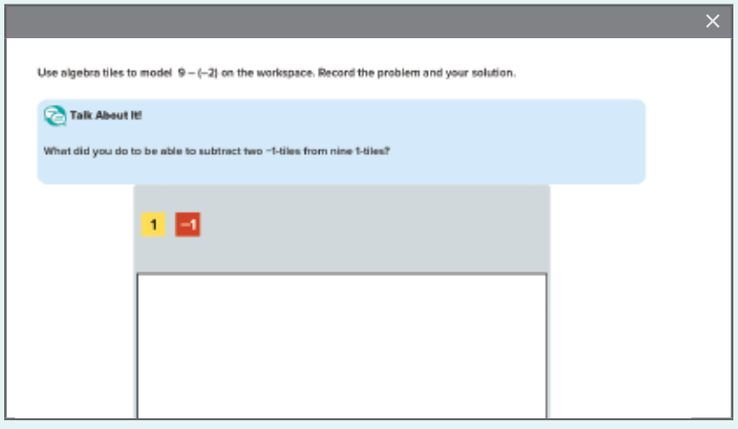
What did you do to be able to subtract two −1-tiles from nine 1-tiles? Sample answer: I added enough zero pairs so that there were two negative 1-tiles to take away.

(continued on next page)

Interactive Presentation



Explore, Slide 1 of 9



Explore, Slide 4 of 9

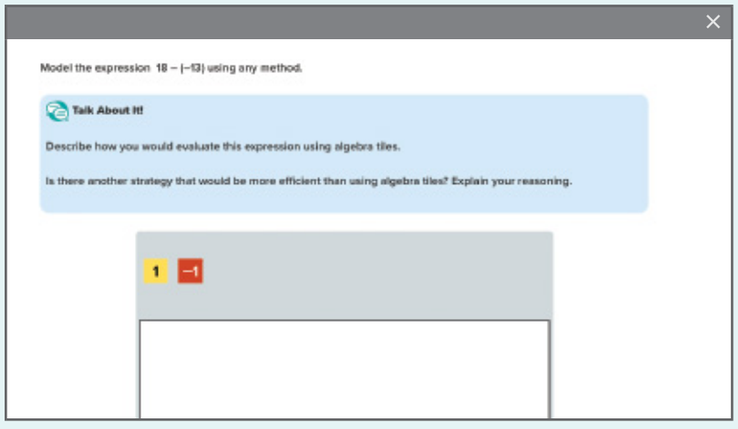
DRAG & DROP

Throughout the Explore, students drag algebra tiles to model subtracting integers.

WATCH

On Slide 3, students watch a video that explains how to subtract integers with algebra tiles.

Interactive Presentation



Explore, Slide 8 of 9

TYPE

On Slide 7, students describe the patterns they have observed.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.

Explore Use Algebra Tiles to Subtract Integers *(continued)*

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use algebra tiles to explore integer subtraction. The strategy of using algebra tiles helps build conceptual understanding for why and how subtraction of integers can be represented as addition of the additive inverse.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. Sample responses for the *Talk About It!* questions on Slide 8 are shown.

Talk About It!

SLIDE 8

Mathematical Discourse

Describe how you would evaluate this expression using algebra tiles. Sample answer: Model 18 by placing 18 positive 1-tiles on the workspace. I need to subtract 13 negative tiles, however there are no negative tiles on the workspace. Add 13 zero pairs to the workspace. Then I can remove 13 negative tiles from the workspace.

Is there another strategy that would be more efficient than using algebra tiles? Explain your reasoning. Sample answer: Yes, using algebra tiles is not necessarily efficient because there are so many tiles to add and subtract. It would be more efficient to evaluate the expression by adding the additive inverse.

Learn Subtract Integers

Objective

Students will understand that they can use a number line to subtract integers.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question on Slide 4, encourage them to draw number lines and use mathematical reasoning to justify why the Commutative Property does not hold true for subtraction.

Go Online to find additional teaching notes.

Talk About It!

SLIDE 4

Mathematical Discourse

The Commutative Property is true for addition. For example, $7 + 2 = 2 + 7$. Is the Commutative Property true for subtraction? Does $7 - 2 = 2 - 7$? Explain your reasoning using a number line. No, the Commutative Property does not hold true for subtraction. $7 - 2 = 5$, but $2 - 7 = -5$. See students' number lines.

DIFFERENTIATE

Reteaching Activity AL

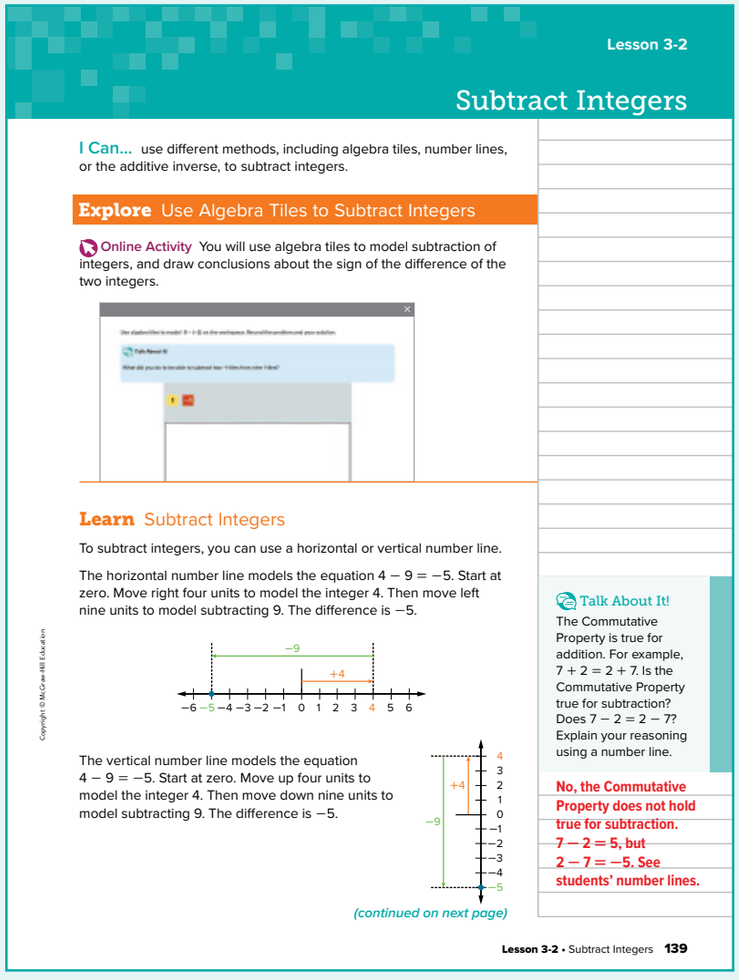
To help students better understand how to subtract integers, have them write each of the following subtraction expressions as an addition expression using an additive inverse.

$9 - (-6)$ $9 + 6$

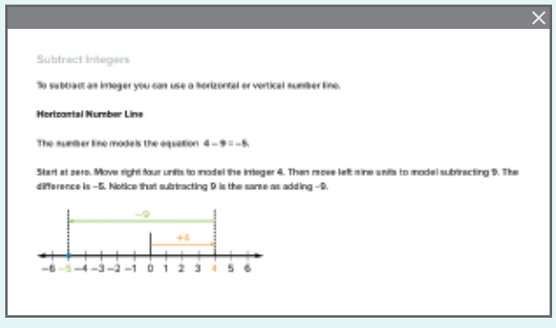
$-6 - 3$ $-6 + (-3)$

$5 - 21$ $5 + (-21)$

$-4 - (-1)$ $-4 + 1$



Interactive Presentation



Learn, Subtract Integers, Slide 1 of 4

FLASHCARDS

On Slide 3, students use Flashcards to learn about rules for subtracting two integers.

CLICK

On Slide 3, students move through slides to see an example using the additive inverse.

Your Notes

The number lines illustrate the rules for subtracting two integers.

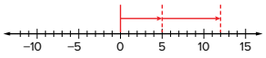
Words	Symbols	Example
To subtract an integer, add the additive inverse of the integer.	$p - q = p + (-q)$	$4 - 9 = 4 + (-9) = -5$

Example 1 Subtract Integers

Find 5 − (−7).

Method 1 Use a number line.

Go Online You can use the Web Sketchpad number line.



Start at zero. Move right 5 units to model the integer 5. Then move right 7 units to model subtracting −7, which is the same as adding the additive inverse, 7. The sum is 12.

So, 5 − (−7) = 12.

Method 2 Use the additive inverse.

$5 - (-7) = 5 + 7$ To subtract −7, add the additive inverse of −7.

$= 12$ Add 5 + 7.

So, 5 − (−7) = 12.

Check

Find 11 − (−15). 26

Show your work here

Think About It!
Will you use a number line or will you add the additive inverse to solve this problem?

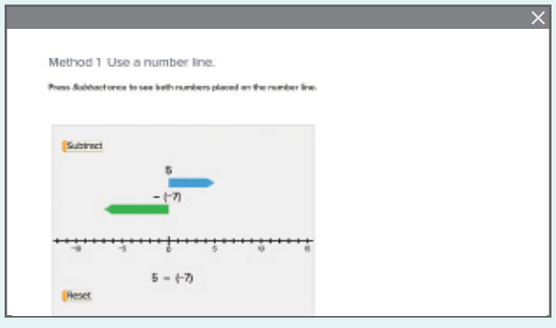
See students' responses.

Talk About It!
Compare and contrast Method 1 and Method 2.

Sample answer: Method 1 uses the number line to determine if the answer will be negative or positive. Method 2 uses the additive property which also determines if the answer will be negative or positive

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 1, Subtract Integers, Slide 2 of 5

WEB SKETCHPAD

On Slide 2, students use Web Sketchpad to find the difference with a number line (Method 1).

CLICK

On Slide 3, students use the additive inverse to find the difference (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 1 Subtract Integers

Objective

Students will subtract a negative integer from a positive integer.

Teaching the Mathematical Practices

5 Use Appropriate Tools Strategically Encourage students to use either method, a number line or the additive inverse, when subtracting two integers. As students discuss the *Talk About It!* question on Slide 4, encourage them to understand the benefits of each method and how they are related.

Questions for Mathematical Discourse

SLIDE 2

AL Are we subtracting a positive integer from 5, or a negative integer? We are subtracting a negative integer, −7, from 5.

OL After pressing *Subtract*, how does the number line illustrate how to subtract a negative integer? Sample answer: The number line shows that subtracting a negative integer is the same as adding the integer's additive inverse.

BL How would the number line change if the expression was 5 + (−7)? Sample answer: Instead of subtracting a negative number, we would add a negative number. Adding a negative number would move to the left on the number line, instead of to the right.

SLIDE 3

AL Of which integer do we find the additive inverse? Explain. We find the additive inverse of the second integer, −7, because that is the integer that is being subtracted.

OL Explain why it makes sense that the answer is positive. Sample answer: If I modeled this expression using algebra tiles, I would need to subtract 7 negative tiles from 5 positive tiles. To do so, I would need to add 7 zero pairs. After removing all 7 negative tiles, only positive tiles remain.

BL Compare and contrast the expressions 5 − (−7), 5 + (−7), and 5 − 7. Sample answer: Two of the expressions are subtraction expressions, 5 − (−7) and 5 − 7. The other expression, 5 + (−7) is an addition expression. In the first two expressions, the second integer is negative. In the last expression, the second integer is positive.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Example 2 Subtract Integers

Objective

Students will subtract a negative integer from a negative integer.

Questions for Mathematical Discourse

SLIDE 2

AL What is the additive inverse of −17? 17

AL Rewrite the subtraction expression as an addition expression. −24 + 17

OL What other method could you use to find the difference? Sample answer: Use a number line.

BL If the first integer remained the same, what would the second integer need to be in order for the difference to be the least positive integer possible? −25

Example 3 Subtract Expressions

Objective

Students will evaluate an algebraic expression that involves subtracting integers.

Questions for Mathematical Discourse

SLIDE 1

AL What integer should replace x in the expression? −23

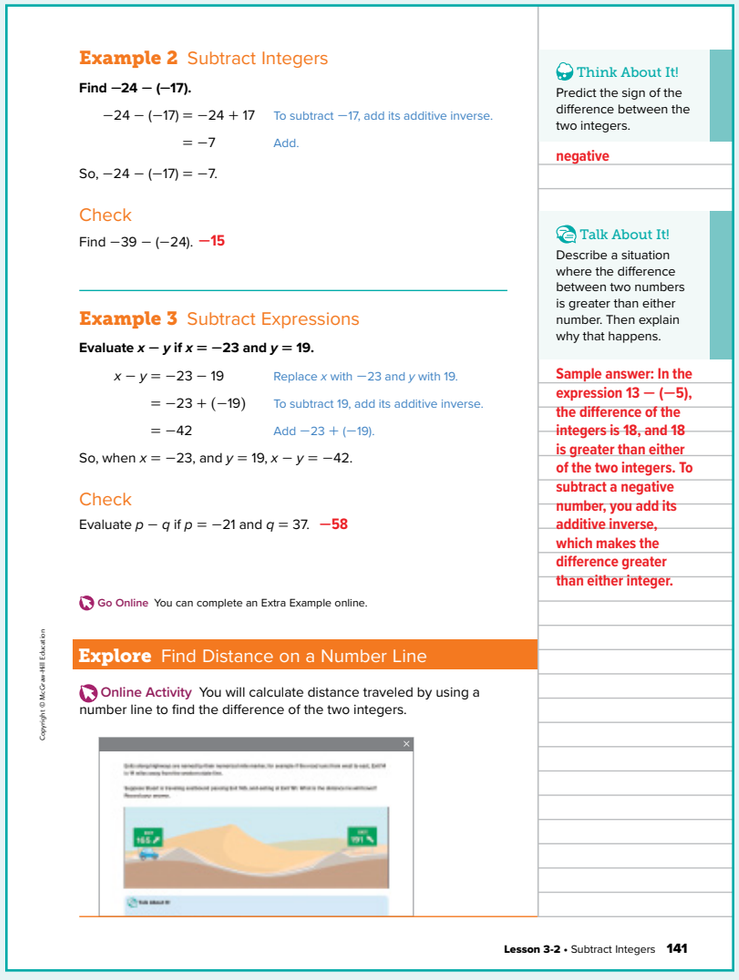
AL What integer should replace y in the expression? 19

OL Suppose a classmate substituted the values and wrote the expression −23 − (−19). How can you explain to them their error? Sample answer: The second integer is positive 19, not negative 19.

BL How would the answer change if the original expression was $y - x$? The answer would be 19 − (−23), which equals 42.

Go Online

- Find additional teaching notes, Teaching the Mathematical Practices, and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present the Extra Examples.



Example 2 Subtract Integers

Find −24 − (−17).

−24 − (−17) = −24 + 17 To subtract −17, add its additive inverse.
= −7 Add.

So, −24 − (−17) = −7.

Check

Find −39 − (−24). **−15**

Example 3 Subtract Expressions

Evaluate $x - y$ if $x = -23$ and $y = 19$.

$x - y = -23 - 19$ Replace x with −23 and y with 19.
$= -23 + (-19)$ To subtract 19, add its additive inverse.
$= -42$ Add −23 + (−19).

So, when $x = -23$, and $y = 19$, $x - y = -42$.

Check

Evaluate $p - q$ if $p = -21$ and $q = 37$. **−58**

Go Online You can complete an Extra Example online.

Explore Find Distance on a Number Line

Online Activity You will calculate distance traveled by using a number line to find the difference of the two integers.

Think About It!
Predict the sign of the difference between the two integers.

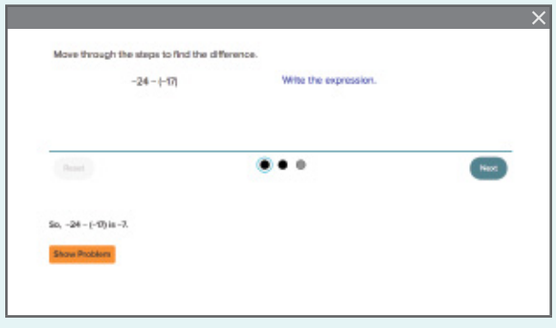
negative

Talk About It!
Describe a situation where the difference between two numbers is greater than either number. Then explain why that happens.

Sample answer: In the expression 13 − (−5), the difference of the integers is 18, and 18 is greater than either of the two integers. To subtract a negative number, you add its additive inverse, which makes the difference greater than either integer.

Lesson 3-2 • Subtract Integers 141

Interactive Presentation



Example 2, Subtract Integers, Slide 2 of 4

CLICK

On Slide 2 of Example 2, students move through the steps to find the difference.

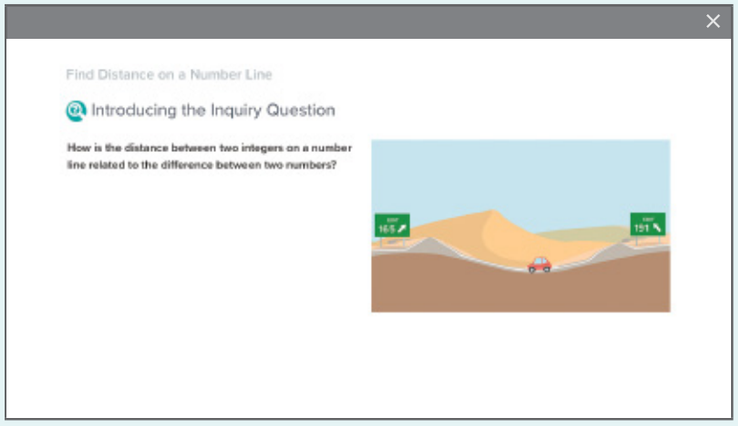
TYPE

On Slide 1 of Example 3, students evaluate the expression.

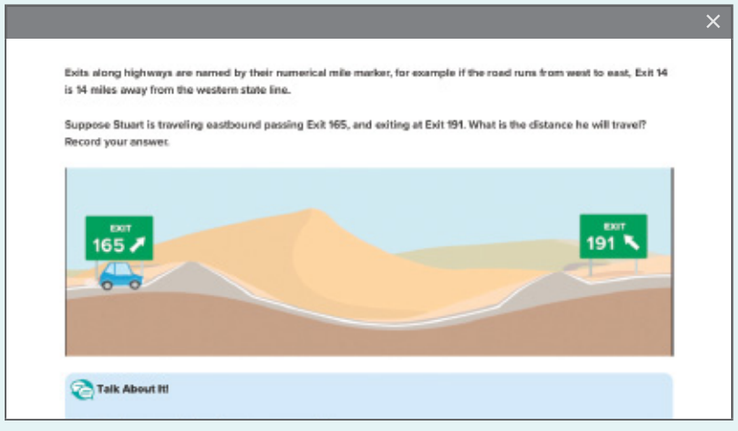
CHECK

Students complete the Check exercises online to determine if they are ready to move on.

Interactive Presentation



Explore, Slide 1 of 9



Explore, Slide 2 of 9

Explore Find Distance on a Number Line

Objective

Students will explore how the distance between integers on a number line is related to their difference.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the *Talk About It!* questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of Activity

Students will use a number line to find the distance that a car travels from one exit to another. Throughout this activity, students will write subtraction expressions to find the difference between two integers on a number line. They will compare these differences to the actual distances between the two numbers on the number line. They should note that, while the difference of a subtraction expression might be negative, the *distance* between those integers is always positive.

Inquiry Question

How is the distance between two integers on a number line related to the difference between the two numbers? Sample answer: The distance between two rational numbers is the absolute value of their difference. For example, the distance between −88 and −11 is 77 units.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 2 is shown.

Talk About It!

SLIDE 2

Mathematical Discourse

Describe how you calculated the distance traveled. Sample answer: I found the difference 191 − 165. Stuart traveled 26 miles.

(continued on next page)

Explore Find Distance on a Number Line *(continued)*

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to explore the distance between two integers on a number line, and analyze how the distance compares to the difference of the subtraction expression.

Go Online to find additional teaching notes and sample answers for the *Talk About It!* questions. A sample response for the *Talk About It!* question on Slide 6 is shown.

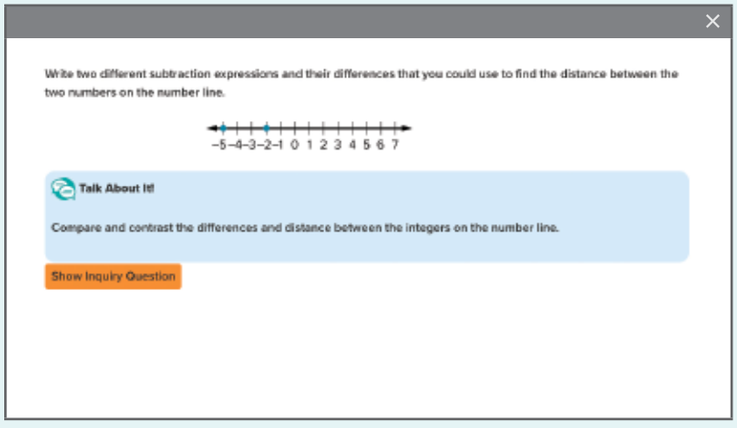
Talk About It!

SLIDE 6

Mathematical Discourse

Compare and contrast the differences and distance of the integers on the number line. Sample answer: The differences in the values of the expressions are opposites; $-2-(-5)=3$ and $-5-(-2)=-3$. But the distance between the two integers is the same, 3 units.

Interactive Presentation



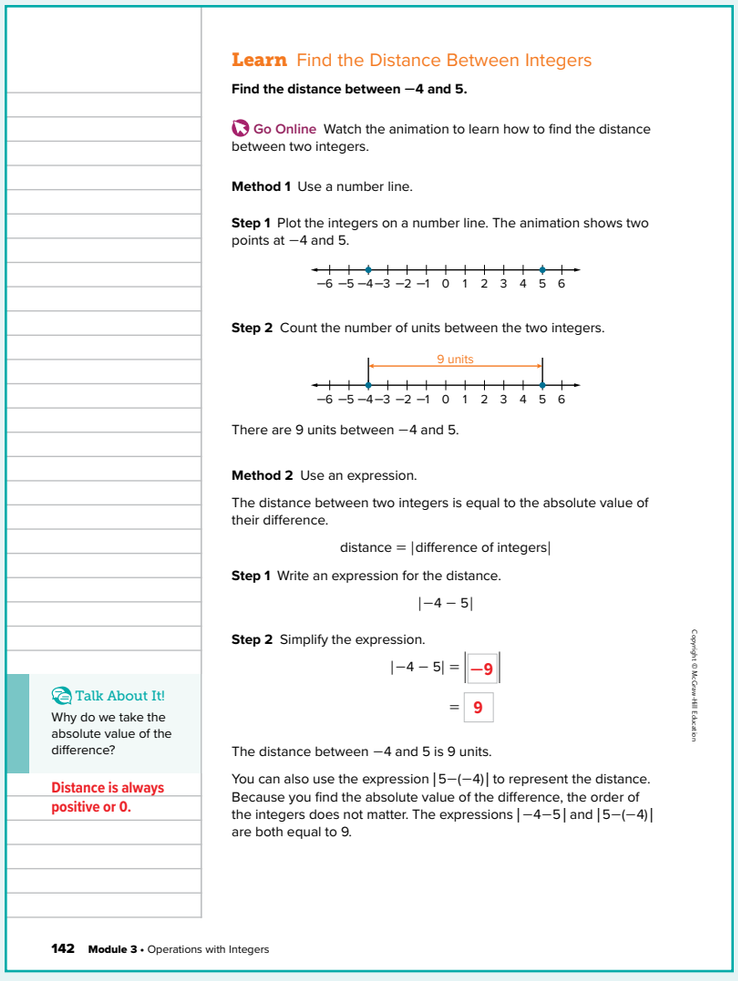
Explore, Slide 6 of 9

TYPE

On Slide 7, students type to make a conjecture about how the distance between two integers on a number line is related to their difference.

TYPE

On Slide 9, students respond to the Inquiry Question and view a sample answer.



Learn Find the Distance Between Integers

Find the distance between −4 and 5.

Go Online Watch the animation to learn how to find the distance between two integers.

Method 1 Use a number line.

Step 1 Plot the integers on a number line. The animation shows two points at −4 and 5.

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

Step 2 Count the number of units between the two integers.

9 units

−6 −5 −4 −3 −2 −1 0 1 2 3 4 5 6

There are 9 units between −4 and 5.

Method 2 Use an expression.

The distance between two integers is equal to the absolute value of their difference.

distance = |difference of integers|

Step 1 Write an expression for the distance.

|−4 − 5|

Step 2 Simplify the expression.

|−4 − 5| = |−9|

= 9

The distance between −4 and 5 is 9 units.

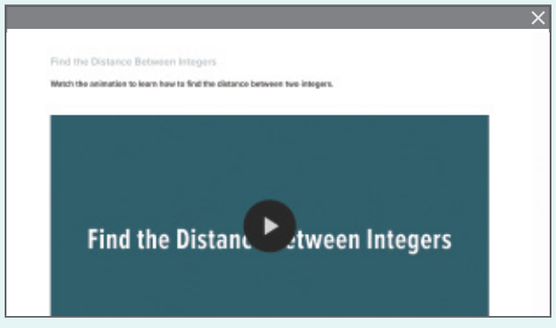
You can also use the expression |5−(−4)| to represent the distance. Because you find the absolute value of the difference, the order of the integers does not matter. The expressions |−4−5| and |5−(−4)| are both equal to 9.

Talk About It!

Why do we take the absolute value of the difference?

Distance is always positive or 0.

Interactive Presentation



Learn, Find the Distance Between Integers, Slide 1 of 2

WATCH

On Slide 1, students watch an animation that explains how to find the distance between two integers on a number line.

Learn Find the Distance Between Integers

Objective

Students will learn how to find the distance between two integers on a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively As students discuss the *Talk About It!* question on Slide 2, encourage them to make sense of quantities and their relationships in problem situations. Students should recognize that distance is always positive.

Go Online

- Find additional teaching notes.
- Have students watch the animation on Slide 1. The animation illustrates how to find the distance between two integers on a number line.

Talk About It!

SLIDE 2

Mathematical Discourse

Why do we take the absolute value of the difference? Distance is always positive or 0.

DIFFERENTIATE

Enrichment Activity BL

To challenge students' understanding of distance between integers, have them find the integer(s) that satisfy each of the following descriptions.

8 units from 3 −5, 11

6 units from −2 −8, 4

10 units from −22 −32, −12

Example 4 Find the Distance Between Integers

Objective

Students will find the distance between two integers on a number line.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to make sense of the integers given in the example and the distance between them, whether they use a number line to find the distance or absolute value.

6 Attend to Precision As students discuss the *Talk About It!* question on Slide 4, encourage them to communicate precisely the similarities and differences of the two methods.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the distance between –9 and 8

OL How many units are between the integers? 17 units

BL What is the difference of the expression –9 – 8? How does this compare to the distance between the integers? The difference is –17, but the distance between the integers is positive.

SLIDE 3

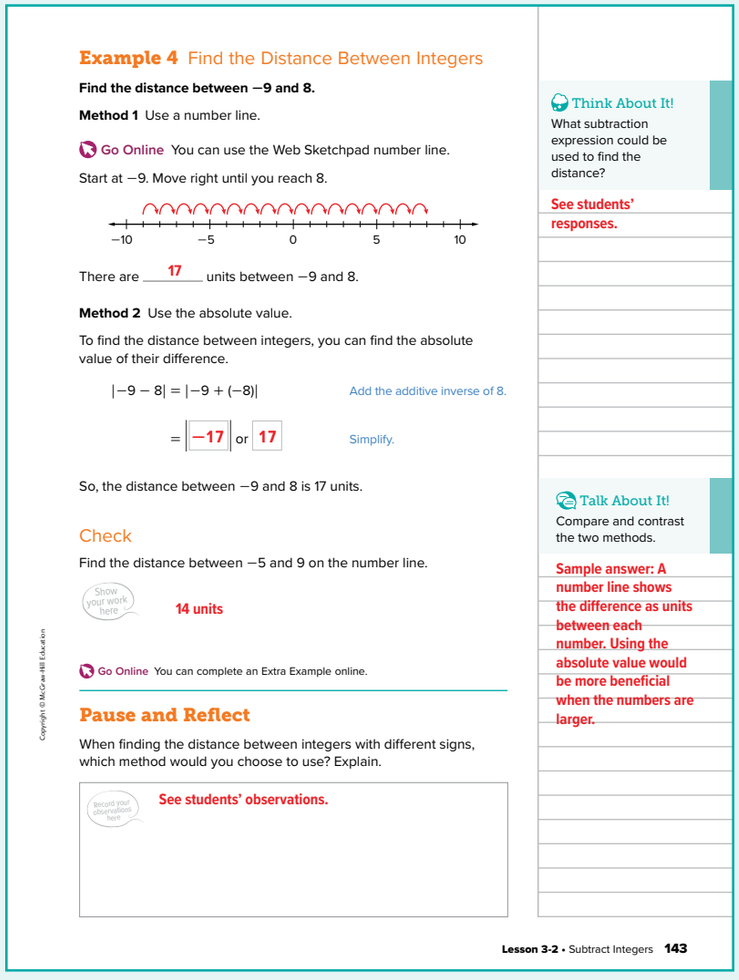
AL What is the absolute value of each integer? The absolute value of –9 is 9. The absolute value of 8 is 8.

OL Why do you need to find the absolute value of the difference? Distance cannot be negative.

BL Give an example of two integers, on opposite sides of zero, where the distance between them is 25? Sample answer: 15 and –10

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Example 4 Find the Distance Between Integers

Find the distance between −9 and 8.

Method 1 Use a number line.

Go Online You can use the Web Sketchpad number line.

Start at −9. Move right until you reach 8.

−10 −5 0 5 10

There are 17 units between −9 and 8.

Method 2 Use the absolute value.

To find the distance between integers, you can find the absolute value of their difference.

$|-9 - 8| = |-9 + (-8)|$ Add the additive inverse of 8.

$= |-17|$ or 17 Simplify.

So, the distance between −9 and 8 is 17 units.

Check

Find the distance between −5 and 9 on the number line.

Show your work here

14 units

Go Online You can complete an Extra Example online.

Pause and Reflect

When finding the distance between integers with different signs, which method would you choose to use? Explain.

Record your observations here

See students' observations.

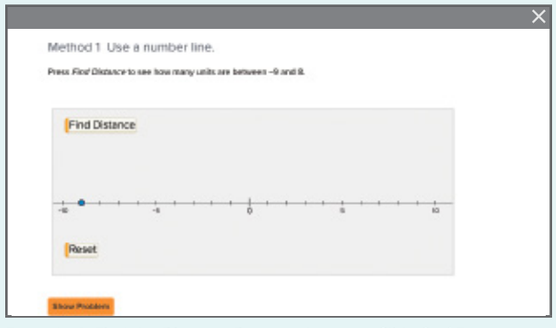
Think About It!
What subtraction expression could be used to find the distance?

See students' responses.

Talk About It!
Compare and contrast the two methods.

Sample answer: A number line shows the difference as units between each number. Using the absolute value would be more beneficial when the numbers are larger.

Interactive Presentation



Example 4, Find the Distance Between Integers, Slide 2 of 5

WEB SKETCHPAD

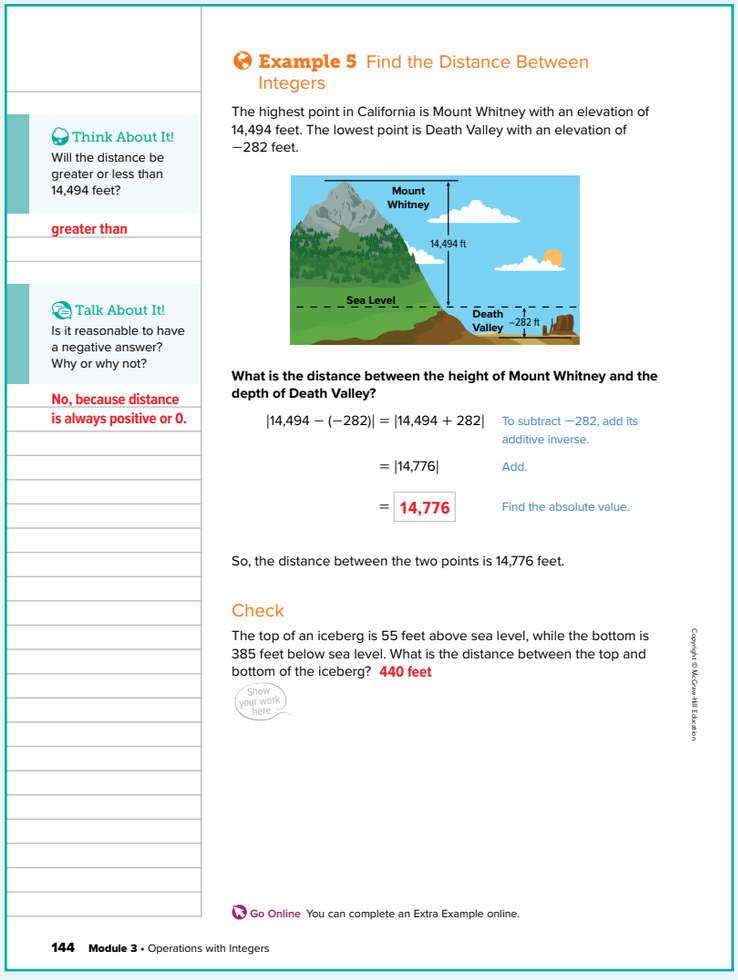
On Slide 2, students use Web Sketchpad to find the distance with a number line (Method 1).

TYPE

On Slide 3, students use absolute value to find the distance (Method 2).

CHECK

Students complete the Check exercise online to determine if they are ready to move on.



Example 5 Find the Distance Between Integers

The highest point in California is Mount Whitney with an elevation of 14,494 feet. The lowest point is Death Valley with an elevation of −282 feet.

Think About It!
Will the distance be greater or less than 14,494 feet?

greater than

Talk About It!
Is it reasonable to have a negative answer? Why or why not?

No, because distance is always positive or 0.

What is the distance between the height of Mount Whitney and the depth of Death Valley?

$|14{,}494 - (-282)| = |14{,}494 + 282|$ To subtract −282, add its additive inverse.

$= |14{,}776|$ Add.

$= 14{,}776$ Find the absolute value.

So, the distance between the two points is 14,776 feet.

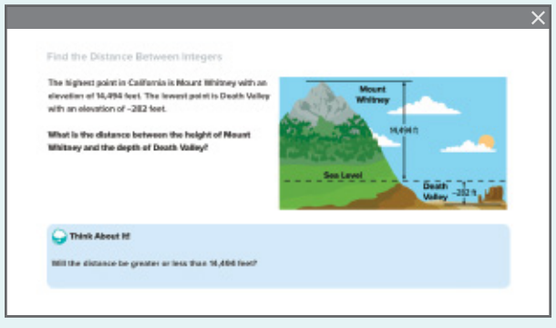
Check

The top of an iceberg is 55 feet above sea level, while the bottom is 385 feet below sea level. What is the distance between the top and bottom of the iceberg? **440 feet**

Show your work here

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 5, Find the Distance Between Integers, Slide 1 of 4

CLICK

On Slide 2, students move through the steps to find the absolute value of the difference between two integers.

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Example 5 Find the Distance Between Integers

Objective

Students will find the distance between two integers to solve a real-world problem.

Teaching the Mathematical Practices

2 Reason Abstractly and Quantitatively Encourage students to use the mathematics they know, finding the distance between two integers, to solve the real-world problem and to make sure their answer makes sense in the context of the problem.

3 Construct Viable Arguments and Critique the Reasoning of Others As students discuss the *Talk About It!* question, encourage them to think logically as they reason about whether a negative answer makes sense.

Questions for Mathematical Discourse

SLIDE 2

AL What do you need to find? the distance between the elevations of Mount Whitney and Death Valley

AL What does it mean that the elevation of Death Valley is a negative integer? The elevation of Death Valley is below sea level.

OL Why do we find the absolute value of the difference? Distance cannot be negative.

BL Suppose a classmate stated that the distance between the elevations is 14,212 feet. How can you explain to them that their answer is not reasonable? Sample answer: Mount Whitney is above sea level and Death Valley is below sea level. The distance between them must be greater than either elevation.

Go Online

- Find additional teaching notes and the *Talk About It!* question to promote mathematical discourse.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Apply The Solar System

Objective

Students will come up with their own strategy to solve an application problem involving temperature of celestial objects.

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change directions, if necessary.

3 Construct Viable Arguments and Critique the Reasoning of Others As students respond to the Write About It! prompt, have them make sure their argument uses correct mathematical reasoning. If you choose to have them share their responses with others, encourage the listeners to ask clarifying questions to verify that the reasoning is correct.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

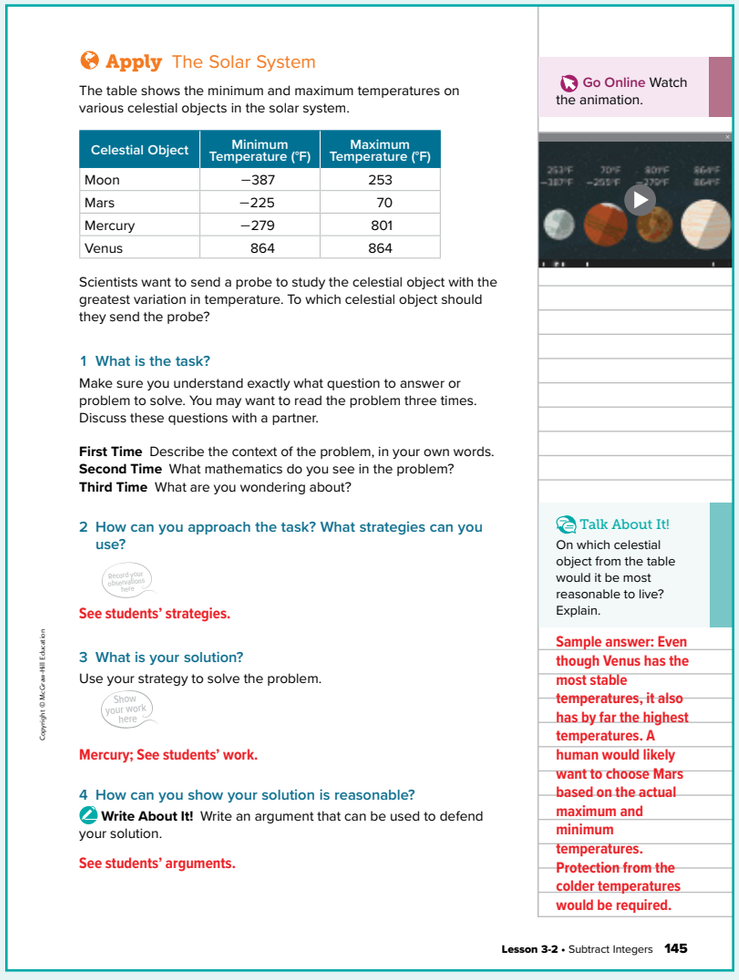
Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustration, or disengagement, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- What does variation mean?
- What do you notice about Venus' temperatures?
- How might thinking about 0°F help you?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.



Apply The Solar System

The table shows the minimum and maximum temperatures on various celestial objects in the solar system.

Celestial Object	Minimum Temperature (°F)	Maximum Temperature (°F)
Moon	−387	253
Mars	−225	70
Mercury	−279	801
Venus	864	864

Scientists want to send a probe to study the celestial object with the greatest variation in temperature. To which celestial object should they send the probe?

1 What is the task?

Make sure you understand exactly what question to answer or problem to solve. You may want to read the problem three times. Discuss these questions with a partner.

First Time Describe the context of the problem, in your own words.
Second Time What mathematics do you see in the problem?
Third Time What are you wondering about?

2 How can you approach the task? What strategies can you use?

Record your observations here

See students' strategies.

3 What is your solution?

Use your strategy to solve the problem.

Show your work here

Mercury; See students' work.

4 How can you show your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

See students' arguments.

Go Online Watch the animation.

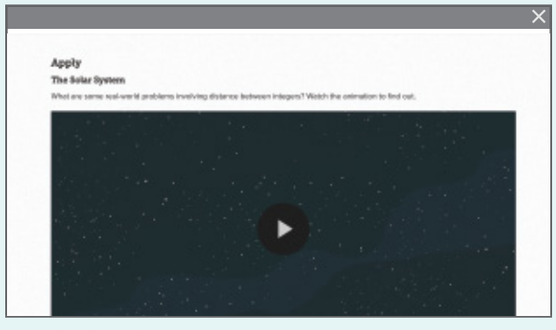
Talk About It!

On which celestial object from the table would it be most reasonable to live? Explain.

Sample answer: Even though Venus has the most stable temperatures, it also has by far the highest temperatures. A human would likely want to choose Mars based on the actual maximum and minimum temperatures. Protection from the colder temperatures would be required.

Lesson 3-2 • Subtract Integers 145

Interactive Presentation



Apply, The Solar System

CHECK

Students complete the Check exercise online to determine if they are ready to move on.

Check

The table shows the highest and lowest points of elevation, in relation to sea level, in four countries. Which country in this list has the greatest variation in elevation? the least?

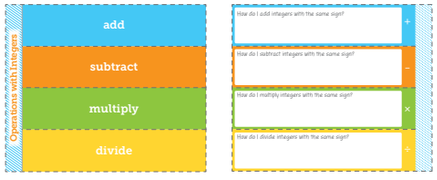
Country	Highest Point (ft)	Lowest Point (ft)
Jordan	6,083	−1,404
United Kingdom	4,406	−13
Sweden	6,903	−8
Ireland	3,406	−10

Show your work here

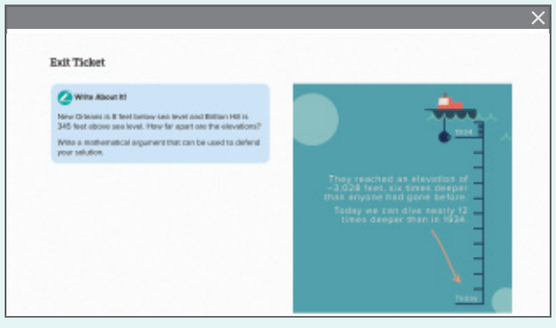
Jordan; Ireland

Go Online You can complete an Extra Example online.

Foldables It's time to update your Foldable, located in the Module Review, based on what you learned in this lesson. If you haven't already assembled your Foldable, you can find the instructions on page FL1.



Interactive Presentation



Exit Ticket

Foldables

Have students update their Foldables based on what they learned in this lesson. For this lesson, students could record examples of subtracting integers. You may wish to have students share their Foldables with a partner to compare the information they recorded, discussing and resolving any differences.

Essential Question Follow-Up

How are operations with integers related to operations with whole numbers?

In this lesson, students learned how to subtract integers by adding the additive inverse. Encourage them to work with a partner to compare and contrast subtracting integers to subtracting whole numbers. For example, have them compare and contrast how they would simplify each of the expressions −15 − 7, −15 − (−7), 15 − (−7), and 15 − 7.

Exit Ticket

Refer to the Exit Ticket slide. New Orleans is 8 feet below sea level and Britton Hill is 345 feet above sea level. How far apart are the elevations? Explain how to find the distance between the elevations. Write a mathematical argument that can be used to defend your solution.

353 feet; Sample answer: Find the absolute value of the difference of the elevations; $|345 - (-8)| = 353$.

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or above on the Checks, **THEN** assign: BL

- Practice, Exercises 15, 17, 18–21
- ALEKS® Addition and Subtraction with Integers

IF students score 66–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–14, 16, 19
- Remediation: Review Resources
- Personal Tutor
- Extra Examples 1–5
- ALEKS® Plotting and Comparing Integers

IF students score 65% or below on the Checks, **THEN** assign: AL

- Remediation: Review Resources
- ArriveMATH Take Another Look
- ALEKS® Plotting and Comparing Integers

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their *Interactive Student Edition*.

The following online homework options are available for you to assign to your students. These assignments include technology-enhanced questions that are auto-scored, as well as essay questions. Many of the Practice exercises on these pages are found in the online assignments, as well as additional exercises.

AL Practice Form B
OL Practice Form A
BL Practice Form C

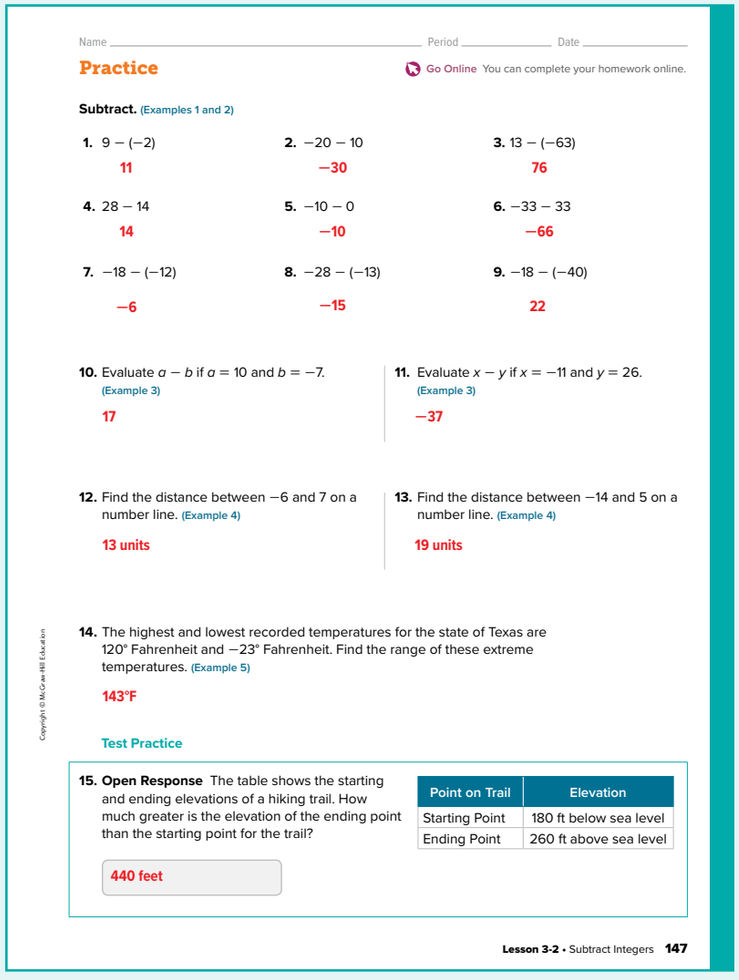
Suggested Assignments

Use the table below to select appropriate exercises for your students' needs.

DOK	Topic	Exercises
1	subtract integers	1–9
1	evaluate algebraic expressions involving subtraction	10, 11
1	find the distance between two integers on a number line	12, 13
2	find the distance between two integers to solve a real-world problem	14
2	extend concepts learned in class to apply them in new contexts	15
3	solve application problems that involve subtracting integers	16, 17
3	higher-order and critical thinking skills	18–21

Common Misconception

Students may have trouble identifying the sign of the difference when subtracting negative integers. In Exercise 7, students may recognize that the distance between –18 and –12 is 6, but fail to realize that subtracting –12 from –18 results in –6, not 6.



Name _____ Period _____ Date _____

Practice

Go Online You can complete your homework online.

Subtract. (Examples 1 and 2)

1. 9 − (−2)
11

2. −20 − 10
−30

3. 13 − (−63)
76

4. 28 − 14
14

5. −10 − 0
−10

6. −33 − 33
−66

7. −18 − (−12)
−6

8. −28 − (−13)
−15

9. −18 − (−40)
22

10. Evaluate $a - b$ if $a = 10$ and $b = -7$. (Example 3)
17

11. Evaluate $x - y$ if $x = -11$ and $y = 26$. (Example 3)
−37

12. Find the distance between −6 and 7 on a number line. (Example 4)
13 units

13. Find the distance between −14 and 5 on a number line. (Example 4)
19 units

14. The highest and lowest recorded temperatures for the state of Texas are 120° Fahrenheit and −23° Fahrenheit. Find the range of these extreme temperatures. (Example 5)
143°F

Test Practice

15. Open Response The table shows the starting and ending elevations of a hiking trail. How much greater is the elevation of the ending point than the starting point for the trail?

Point on Trail	Elevation
Starting Point	180 ft below sea level
Ending Point	260 ft above sea level

440 feet

Lesson 3-2 • Subtract Integers 147

Apply *indicates multi-step problem

*16. The table shows the maximum and minimum account balances for three college students for one month. Giovanni claimed that he had the least variation (from maximum to minimum) in his account balance that month. Is he correct? Write a mathematical argument to justify your solution.

Student	Maximum Balance ($)	Minimum Balance ($)
Jordan	145	−25
Giovanni	168	15
Elisa	152	−10

Giovanni is correct; Sample answer: Giovanni's variation is $168 − $15, or $153. Jordan's variation is $145 − (−$25), or $170. Elisa's variation is $152 − (−$10), or $162.

*17. The table shows the record high and record low temperatures for certain U.S. states. Which state in the list had the greatest variation in temperature? the least?

State	Record High Temperature (°F)	Record Low Temperature (°F)
Alaska	100	−80
Idaho	118	−60
Nevada	125	−50
Utah	117	−69

Utah; Nevada

Higher-Order Thinking Problems

18. **Use a Counterexample** Determine if each statement is *true* or *false*. If false, provide a counterexample.

a. Distance is always positive.

true

b. Change is always positive.

false; Change can be positive or negative. For example, the temperature dropping 2°F would be represented by a −2.

19. **Find the Error** A student is finding 4 − (−2). Find the student's mistake and correct it.

$4 - (-2) = 4 - 2$

$= 2$

The student incorrectly wrote 4 − 2 instead of 4 + 2. The correct solution is 6.

20. Create Write a subtraction expression with a positive and negative integer whose difference is negative. Then find the difference.

Sample answer: −3 − 2; −5

21. If you subtract two negative integers, will the difference *always*, *sometimes*, or *never* be negative? Explain using examples to justify your solution.

sometimes; Sample answer: For example, −10 − (−40) = 30 and −28 − (−13) = −15.

Teaching the Mathematical Practices

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 18, students use a counterexample if a statement is false.

3 Construct Viable Arguments and Critique the Reasoning of Others In Exercise 19, students find the error in another student's reasoning and correct it.

Collaborative Practice

Have students work in pairs or small groups to complete the following exercises.

Interview a student.

Use with Exercise 16–17 Have pairs of students interview each other as they complete these application problems. Students take turns being the interviewer and interviewee for each problem. Interview questions should include asking the interviewee to think aloud through their solution process. An example of a good interview question for Exercise 17 might be "How do you find the variation in temperature?"

Listen and ask clarifying questions.

Use with Exercises 20–21 Have students work in pairs. Have students individually read Exercise 20 and formulate their strategy to solve the problem. Assign one student as the coach. The other student should talk through their strategy, while the coach listens, asks clarifying questions, and offers encouragement and/or redirection. Have students switch roles to complete Exercise 21.