# Reveal MATH

K-12 Academic Research

McGraw Hill

# Built on the Science of Learning

The development of *Reveal Math*, a K-12 core mathematics program, draws from a range of academic research in learning science. In addition to academic research, several other critical inputs contributed to the development of the program, including extensive in-classroom testing, user testing, and direct feedback from hundreds of educators across the country.

*Reveal Math* is based on proven classroom practices and research from our expert advisory team, as well as current academic research brought forward by McGraw Hill's Learning Scientists. This collective team played a critical role in the design of the program's instructional model. This document provides an overview of the key research areas that *Reveal Math* was built on and demonstrates the application of each within the program.

## *Key research areas:*

- Rigor
- Learning Targets
- Formative Assessment
- Rich Tasks
- Productive Struggle
- Mathematical Discourse
- Collaborative Learning
- Misconceptions, Error Analysis and Perception

# Rigor

Regardless of the content standards being implemented, states and districts have understood the benefits of key shifts in standards development and the importance of putting them into practice in the classroom. These shifts include: greater focus on key topics, coherence within and across grades and topics, and rigor. While *rigor* is used by many education researchers in different contexts, it is generally understood as a balance between conceptual understanding, procedural skills/fluency, and application or successful transfer to a variety of new problem contexts.

> " Rigorous lessons build on and extend prior knowledge. They encourage productive struggling. Although the objective of a lesson should be clear in the teacher's mind, the lesson should not focus on one correct path to a solution or even one correct answer. A rigorous lesson embraces the messiness of a good mathematics task and the deep learning that it has the potential to achieve."
>
> (Linda M. Gojak, President's Messages, NCTM, 2013)

*Citation reference resources found on p.11*

## *Where it appears in the program*

### THE INSTRUCTIONAL MODEL

- **Conceptual Understanding** is deliberately developed with an inquiry task that provides students with opportunities to make sense of a mathematical idea through exploration using representations and models to build understanding. Exploration is followed by classroom discourse in which students share and clarify their thinking.
- **Procedural Skills and Fluency** are explicitly linked to conceptual understanding through clear examples. Students build skill and fluency through practice in small groups, independent practice, homework, and through the use of the adaptive technology of *Redbird Mathematics* and *ALEKS*.
- **Applications** take place through a variety of problem solving opportunities within the Develop and Practice parts of a lesson. Students build deeper understanding by learning the mathematics in real-world contexts. In addition, students develop problem solving strategies using rich tasks and performance tasks in every module. K-5 students also apply their knowledge and skills through hands-on workshops.

# Learning Targets

Learning targets are the foundational critical aspect of formative assessment. They provide a way for teachers to share with students the learning that is intended to happen and indicators that it is taking place. Learning targets help students understand and own the mathematical ideas in a lesson. Using the learning targets as touchstones throughout the lesson provides the opportunity for students to reflect on their learning trajectory process. This reflective process helps students see their growth while teachers are able to use the success criteria for formative assessment questioning and gain insight to the students' perceptions of their learning.

> " Teachers need better ways of determining where their students are in their thinking and understanding prior to and throughout the instructional process. Students need to be actively involved in the assessment process, so that they are learning through assessment as well as providing useful feedback to the teacher. "
>
> (Keeley & Tobey, 2011, p. 10)

> " At some point, we help students make sense of their discussions within the context of the mathematical goals of the lesson. To close the lesson, our responsibility is to make sure that students know what mathematics they have learned. "
>
> (Seeley, 2016, p. 33)

*Citation reference resources found on p.11*

## *Where it appears in the program*

**"I CAN" STATEMENTS**

Learning targets are presented to the students at the beginning of each lesson as "I can" statements that communicate the mathematical idea and success criteria in student-friendly language. Each lesson accomplishes this by including both procedural and analytical learning targets. Students can self-assess their comfort level in achieving these targets throughout each lesson and at the end in the form of metacognitive checks. Learning targets are the foundation by which all formative assessment questions are derived in *Reveal Math*.

# Formative Assessment

The key to reaching all learners is to adjust instruction based on students' understanding. Using formative assessments lets teachers know when to provide additional guidance or additional challenges to keep students on track and engaged in learning. Based on student data, the teacher can create more refined and targeted differentiation.

> " The shorter the time interval between eliciting the evidence and using it to improve instruction, the bigger the likely impact on learning. "
>
> (Wiliam, 2011)

*Citation reference resources found on p.11*

## *Where it appears in the program*

Embedded formative assessment questioning and advice on what to watch for is provided to the teacher throughout every lesson. Formal check point opportunities found in *Reveal Math* are designed to provide teachers with actionable data for differentiation and skills gap identification. Actionable reports provide key information to quickly inform instruction and differentiation.

### LESSON CHECKS AND EXIT TICKETS

Formative Lesson Checks and Exit Tickets are questions designed to be completed by students in a short amount of time. Teachers can use this information to decide how each student should proceed with independent practice, homework, and differentiation.

- K-5 students are assessed in each lesson at a consolidated point of formative assessment. The easy-to-use results improve classroom management of daily, multi-modal differentiation.
- 6-12 students have formative assessment checks after one or more examples. Results from the checks are displayed in easy-to-read reports that can be viewed in real-time to adjust instruction on the spot or at a later time, depending on the needs of the teacher.

### MATH PROBES

- Each module contains a math probe that poses a problem, set of problems, or task that elicits information about student misconceptions. Teachers can use a rubric to evaluate student responses and modify upcoming curriculum, as needed. The probe can also be used for student self-reflection at the end of the module.

# Rich Tasks

Rich tasks provide opportunities for students to dive deeply into mathematics and the freedom to try different ways of thinking about solving problems. Rich tasks can take the form of a contextualized word problem or a task with only numbers focusing on student thinking and not just the answer. It is not essential for a rich task to appear complex but rather promote a high level of cognitive demand and deeper conceptual thinking for the learner.

According to former NCTM president Linda Gojak, rich tasks can take many forms but they are generally distinguished by seven key characteristics.

Rich tasks:

- Are engaging
- Require substantive mathematics to reach a solution
- Are accessible for all students in the class
- Have multiple solution paths
- Can have multiple solutions
- Encourage student discourse and questions
- Treat the solution process as equally important as the solution

> " Tasks must provide entry points for all students, offer them well-defined opportunities to make connections to other mathematics, and include both opportunities and expectations for them to develop deeper understanding. The focus and coherence of the Common Core State Standards lead the way to rigorous instruction. "
>
> (Linda M. Gojak, President's Messages, NCTM, 2013)

*Citation reference resources found on p.11*

## *Where it appears in the program*

**EXPLORE ACTIVITIES**

Prior to formal instruction of a mathematical topic, students have the opportunity to explore a rich task using and extending knowledge from previous experiences. The Explore Activities are intentionally implemented in collaborative groups so students can use inquiry to approach a high-cognitive demand task using representations that make sense to them, and share ideas and approaches with their peers. It is not necessary for each student to arrive at a solution or full understanding of the mathematical idea in the Explore Activity. Rather, these activities set the stage for building connections to conceptual understanding through classroom discourse and similar tasks.

**K-5 EXPLORE AND DEVELOP**
Problem solving is embedded in every lesson in *Reveal Math*. Students use problem contexts to make sense of mathematical ideas rather than learn procedures in isolation. Every lesson focuses on helping students build a toolbox of specific problem solving strategies and determine how and when to use those tools in both every day problem situations and higher order thinking problems.

# Productive Struggle

Providing opportunities for students to grapple with challenging tasks supports the goal of all students becoming successful problem solvers. Through purposeful questions, the teacher guides student thinking without showing and telling what to do, rather encouraging productive approaches and perseverance so that students own their thinking.

> " Effective mathematics teaching supports students in struggling productively as they learn mathematics... Teaching that embraces and uses productive struggle leads to long-term benefits, with students more able to apply their learning to new problem situations. "
>
> (NCTM 2014, p. 48)

*Citation reference resources found on p.11*

Many leading researchers identify productive struggle as an essential component of effective mathematics classrooms. To foster productive struggle, teachers are urged to include opportunities for students to attempt solving problems that target new concepts instead of limiting opportunities to tasks with familiar/known skills. The role of the teacher is to guide student thinking with open, yet focused questioning which is instrumental in developing this effective classroom environment. Allowing for productive struggle encourages perseverance and a growth mindset, and also supports rich discourse in the classroom.

## *Where it appears in the program*

**EXPLORE**
While student perseverance is key throughout the program, rich tasks in *Reveal Math* immerse students in the opportunity to work through difficulties and try multiple options. In the Explore activity, it is not important that each student arrives at the correct solution to the problem/ task at this phase of the lesson – rather, the aim is to elicit students' intuitions and existing knowledge, and to give each an opportunity to engage in reasoning about the nature of the problem.

**TEACHER QUESTIONING**
Precisely crafted questioning in the teacher materials are found throughout the lesson. These give the teacher a framework to guide student thinking, encourage perseverance, and promote critical thinking.

# Mathematical Discourse

Mathematical discourse helps students expand their mathematical thinking and consider new strategies. Defending reasoning requires a deep understanding of processes and outcomes, and helps to solidify conceptual understanding. When students are asked to articulate their understanding and listen as others do the same, they deepen and expand their own comprehension of mathematics.

Teachers play a pivotal role in mathematical discourse. When teachers use focused questions, they are also modeling how to ask clarifying questions in a way that will serve students better in later phases of learning.

> " Working at their best, questions that probe and explore meaning and relationships press students to explain the why of their thinking and, in so doing, help them discover the methods of mathematical reasoning as well as the relationships at the heart of the central ideas of the discipline. "
>
> (Smith & Stein, 2011, p. 73)

*Citation reference resources found on p.11*

## *Where it appears in the program*

**STRONG QUESTIONING TECHNIQUES FOR THE TEACHER**
The teacher materials provide strong questioning examples to help teachers focus discussions so students develop efficient approaches to a problem.

**K-5 TALK MATH ACTIVITIES**
Following small group work in the Explore Activities, teachers facilitate student discourse where students explain and clarify their reasoning. Teacher questions are carefully constructed to help students connect their work to the mathematical ideas that develop from their exploration. This "conceptual bridge" explicitly links conceptual understanding to developing procedural skills and fluency in the next section of a lesson.

**6-8 TALK ABOUT IT! PROMPTS**
The Talk About It! questions throughout the Explore, Learn, and Example sections of each lesson offer opportunities for students to build meaning, to reason and explain their thinking, and ultimately to work toward building conceptual understanding of the math concepts covered in that lesson.

**COLLABORATE**
Students are encouraged to work together at various points in each lesson, listening to others and discussing their approach to the mathematics.

# Collaborative Learning

There are tremendous benefits to be gained by student collaboration. Not only do students learn from the thinking of others, but they also learn to appreciate diverse viewpoints. In explaining their own understanding, students grow as individual learners. Research suggests that collaborative learning activities boost student engagement. The way in which collaborative learning activities promote student learning outcomes compliments the benefits of meaningful discourse, productive struggle, and rich tasks described earlier. Both whole-class and small-group collaboration provide opportunities for these rich experiences.

> " Social interaction provides us with the opportunity to use others as resources, to share our ideas with others, and to participate in the joint construction of knowledge. In mathematics classrooms, high-quality discussions support students' learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in a mathematically sound direction, and encouraging students to evaluate their own and each other's mathematical ideas. These are all important features of what it means to be 'mathematically literate.' "
>
> (Smith & Stein, 2011 p. 1)

*Citation reference resources found on p.11*

## *Where it appears in the program*

### K-5 WORK TOGETHER

During Work Together, students can work together to solve problems by applying strategies and skills learned in the lesson. Students are encouraged to explain and justify their thinking to their peers with the support of teacher prompts and questioning as needed. Throughout *Reveal Math* lessons, many problems allow for multiple approaches and/or models and strategies to be applied, such that the conceptual connections between strategies emerges through the collaborative small group work.

### 6-8 COLLABORATE STRATEGIES

Each lesson includes collaborative learning strategies that reinforce the main objectives and vocabulary learned in the lesson.

### 6-12 COLLABORATIVE PRACTICE

Collaborative Practice tips in the Teacher Edition offer suggestions on how students can work collaboratively to write their own problems, make sense of problems, explain their reasoning to a peer, and/or discuss how they can solve the same problem using more than one strategy. These problems can be assigned for in-class practice or homework.

# Misconceptions, Error Analysis, and Perception

Research suggests that learning occurs optimally in "mistake friendly" environments, in which taking risks and making errors are considered a natural part of the learning process rather than evidence of teacher or student failure. Promoting such an environment requires a shift in thinking for both the teacher and the student so that errors are considered opportunities for meaningful classroom discourse centered around the learners' thinking about the connection between concepts and procedures. When the teacher recognizes misconceptions, they shed light on how to best provide support so that learners move to a deeper and more accurate understanding of a concept. Less emphasis is on getting the right answer. Rather, instruction focuses on using mistakes, misconceptions, and opportunities to learn. This type of mistake-friendly environment is closely related to the notions of productive struggle and rich tasks, as it allows students to engage with content in a way that is learner-centered and learner-driven. This type of environment helps students engage with mathematics more deeply and without anxiety about immediate correct answers impeding their learning processes.

> " Combined with what we know about the contribution of mistakes to growing intelligence, it seems clear that a classroom environment where mistakes are welcomed could create many opportunities for learning mathematics. Lightening our grip on getting correct answers quickly may allow us to view the structure of teaching differently. "
>
> (Seeley, 2016 pp. 22–23)

*Citation reference resources found on p.11*

## *Where it appears in the program*

**COMMON ERRORS AND MISCONCEPTIONS**

Teachers are equipped with advance knowledge of common errors and misconceptions related to the lesson content. This allows teachers to anticipate the emergence of those errors in students' discussion, and provide scaffolding accordingly.

To help teachers scaffold an activity, point-of-use differentiation adjustments are provided in the teacher materials.

**MATH PROBES**

In each module, assessments probe on students' thinking and current background knowledge to equip teachers with advanced knowledge of misconceptions related to the module content. This allows teachers to uncover incorrect generalizations and address them during the instruction in the module..

**References**


ASCD. (2014). Instruction that sticks: How to combine rigor with engagement. *Educational Leadership, 72(2).*

Boaler, J., & Dweck, C. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages, and innovative teaching.* San Francisco, CA: Jossey-Bass; a Wiley Brand.

Boaler, J. / YouCubed.org. *Mistakes Grow Your Brain.* Retrieved April 26, 2017, from https://www.youcubed.org/think-it-up/mistakes-grow-brain/

Brunner, J. (2013). Academic rigor: The core of the core. *Principal Leadership, 13*(6), 24–28.

Donovan, S., & Bransford, J. D. (Eds.). (2005). How students learn: *Mathematics in the classroom.* Washington, D.C: National Academies Press.

Gojak, L. / NCTM.org. *What's All This Talk about Rigor*? Retrieved May 12, 2016, from http://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/What_s-All-This-Talk-about-Rigor_/

Hattie, J. (2017). Visible learning for mathematics, grades K-12: *What works best to optimize student learning.* Thousand Oaks, California: Corwin Mathematics.

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science, 38*(5), 1008–1022.

Keeley, P., & Tobey, C. R. (2011). *Mathematics formative assessment: 75 practical strategies for linking assessment, instruction, and learning.* Thousand Oaks, CA: Corwin Press.

Langer-Osuna, J. M. (2017). Authority, identity, and collaborative mathematics. *Journal for Research in Mathematics Education*, 48(3), 237–247.

Moss, C. M., Brookhart, S. M., & Long, B. A. (2011). Knowing Your Learning Target. *Educational Leadership*, 68(6), 66–69.

National Council of Teachers of Mathematics (NCTM). (2013). What Does Research Say the Benefits of Formative Assessment Are? (Research Brief). Reston, VA.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: National Council of Teachers of Mathematics (NCTM).

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, D.C.

Pasquale, M. (2015). Productive Struggle in Mathematics (Research Brief). Education Development Center, Inc. Retrieved from http://interactivestem.org/wp-content/uploads/2015/08/EDC-RPC-Brief-Productive-Struggle.pdf

Seeley, C. (2015). Constructive struggling. In Faster isn't smarter: Messages about math, teaching, and learning in the 21st century. Sausalito, CA: Math Solutions.

Seeley, C. L. (2016). Making sense of math: *How to help every student become a mathematical thinker and problem solver.* Alexandria, Virginia, USA: ASCD.

Seeley, C. L. (2016). Making sense of math: *How to help every student become a mathematical thinker and problem solver.* Alexandria, Virginia, USA: ASCD.

Smith, M. S., Stein, M.K, (2011). *5 Tasks for Orchestrating Productive Mathematics Discussions*. Reston, Virginia, USA: NCTM.

Steuer, G., Rosentritt-Brunn, G., & Dresel, M. (2013). Dealing with errors in mathematics classrooms: Structure and relevance of perceived error climate. *Contemporary Educational* Psychology, 38(3), 196–210.

Whitenack, J., & Yackel, E. (2002). Making mathematical arguments in the primary grades: The importance of explaining and justifying ideas. *Teaching Children Mathematics, 8*(9), 524.

Wiliam, D. (2011). *Embedded Formative Assessment - practical strategies and tools for K-12 teachers.* Bloomington, IN. Solution Tree Press.

Reveal

MATH

McGraw Hill