Module 4
Linear and Nonlinear Functions

Module Goals

- Students graph linear, piecewise-defined, step, and absolute value functions.
- Students find and interpret the rate of change and slope of lines.
- Students identify the effects of transformations on the graphs of linear and absolute value functions.

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Also addresses A.CED.2, A.REI.10, F.IF.4, F.IF.6, F.BF.1a, F.BF.2, F.LE.1a, F.LE.2, and F.LE.5.

Standards for Mathematical Practice:

All Standards for Mathematical Practice will be addressed in this module.

Be Sure to Cover

To completely cover F.LE.1a, go online to assign the following activity:

- Linear Growth Patterns (Expand 4-3)

Coherence

Vertical Alignment

Previous
Students graphed functions and interpreted key features in graphs of functions.
F.IF.1, F.IF.4

Now
Students write and graph linear and nonlinear equations.
F.IF.7a, F.IF.7b, F.BF.3

Next
Students will create linear equations and analyze data to make predictions.
A.CED.2

Rigor

The Three Pillars of Rigor

To help students meet standards, they need to illustrate their ability to use the three pillars of rigor. Students gain conceptual understanding as they move from the Explore to Learn sections within a lesson. Once they understand the concept, they practice procedural skills and fluency and apply their mathematical knowledge as they go through the Examples and Practice.

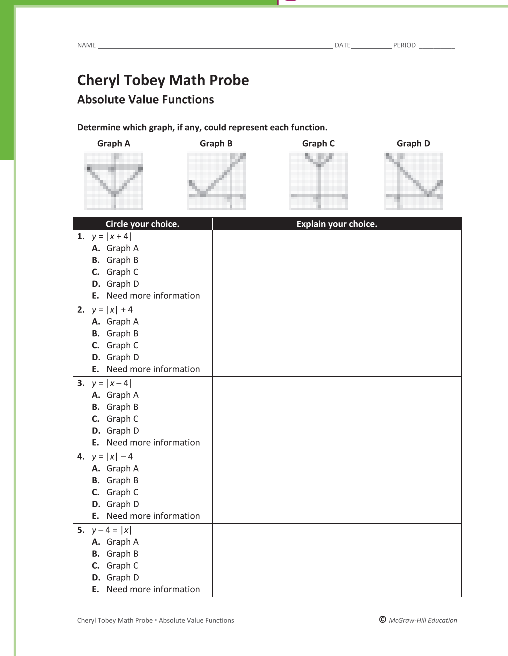
1 CONCEPTUAL UNDERSTANDING		2 FLUENCY	3 APPLICATION
EXPLORE	LEARN	EXAMPLE & PRACTICE	

Suggested Pacing

Lessons	Standards	45-min classes	90-min classes
Module Pretest and Launch the Module Video		1	0.5
4-1 Graphing Linear Functions	A.REI.10, F.IF.7a, F.LE.5	1	0.5
4-2 Rate of Change and Slope	F.IF.6, F.LE.5	1	0.5
4-3 Slope-Intercept Form	A.CED.2, F.IF.7a, F.LE.5	2	1
4-3 Expand Linear Growth Patterns	F.LE.1a	1	0.5
4-4 Transformations of Linear Functions	F.IF.7a, F.BF.3	2	1
4-5 Arithmetic Sequences	F.BF.1a, F.BF.2, F.LE.2	1	0.5
4-6 Piecewise and Step Functions	F.IF.4, F.IF.7b	1	0.5
4-7 Absolute Value Functions	F.IF.7b, F.BF.3	2	1
Put It All Together: Lessons 4-6 through 4-7		1	0.5
Module Review		1	0.5
Module Assessment		1	0.5
	Total Days	**15**	**7.5**

Formative Assessment Math Probe
Absolute Value Functions

Module Resource


NAME ______ DATE______ PERIOD ______

Cheryl Tobey Math Probe
Absolute Value Functions

Determine which graph, if any, could represent each function.

Graph A, Graph B, Graph C, Graph D

Circle your choice.	Explain your choice.
1. $y = \lvert x + 4 \rvert$ **A.** Graph A **B.** Graph B **C.** Graph C **D.** Graph D **E.** Need more information	
2. $y = \lvert x \rvert + 4$ **A.** Graph A **B.** Graph B **C.** Graph C **D.** Graph D **E.** Need more information	
3. $y = \lvert x - 4 \rvert$ **A.** Graph A **B.** Graph B **C.** Graph C **D.** Graph D **E.** Need more information	
4. $y = \lvert x \rvert - 4$ **A.** Graph A **B.** Graph B **C.** Graph C **D.** Graph D **E.** Need more information	
5. $y - 4 = \lvert x \rvert$ **A.** Graph A **B.** Graph B **C.** Graph C **D.** Graph D **E.** Need more information	

Cheryl Tobey Math Probe • Absolute Value Functions © McGraw-Hill Education

Correct Answers: 1. B 2. C 3. D 4. A 5. C

Analyze the Probe

Review the probe prior to assigning it to your students.

In this probe, students will determine which graph matches the correct function and explain their choices.

Targeted Concepts Certain modifications to the parent function of an absolute value function will result in predictable transformations of the graph.

Targeted Misconceptions

- Students may not recognize a horizontal transformation and/or predict an incorrect direction of a horizontal transformation.
- Students may not recognize a vertical transformation and/or predict an incorrect direction of a vertical transformation.

Use the Probe after Lesson 4-7.

Collect and Assess Student Answers

If the student selects these responses...	**Then** the student likely...
1. D **3.** B	recognizes the horizontal shift but fails to use the opposite value of the number associated with *x* to determine the direction of the shift. **Example:** For Item 1, the student recognizes that positive 4 is associated with the *x*-value (horizontal shift) but moves the graph to the right.
2. A **4.** C	recognizes the vertical shift but fails to use the same value of the number associated with *y* to determine the direction of the shift. **Example:** For Item 2, the student recognizes that positive 4 is associated with the *y*-value (vertical shift) but moves the graph down.
5. A	recognizes the vertical shift but is confused with the direction of the shift when the number is placed on the same side as *y*. **Example:** For Item 5, the student recognizes that negative 4 is associated with the *y*-value (vertical shift) but does not solve for *y* before using the “rules” of transformation and moves the graph down.
1. A **2.** D **3.** C **4.** B	confuses a horizontal shift with a vertical shift **Example:** For Item 3, the student incorrectly moves the graph up 4 units instead of to the right 4 units.

Take Action

After the Probe Design a plan to address any possible misconceptions. You may wish to assign the following resources.

- ALEKS® Absolute Value Functions
- Lesson 4-7, all Learns, all Examples

Revisit the probe at the end of the module to be sure that your students no longer carry these misconceptions.

IGNITE!

The Ignite! activities, created by Dr. Raj Shah, cultivate curiosity and engage and challenge students. Use these open-ended, collaborative activities, located online in the module Launch section, to encourage your students to develop a growth mindset towards mathematics and problem solving. Use the teacher notes for implementation suggestions and support for encouraging productive struggle.

Essential Question

At the end of this module, students should be able to answer the Essential Question.

What can a function tell you about the relationship that it represents?
Sample answer: It can tell you about the rate of change, whether the relationship is positive or negative, the locations of the x- and y-intercepts, and what points fall on the graph.

What Will You Learn?

Prior to beginning this module, have your students rate their knowledge of each item listed. Then, at the end of the module, you will be reminded to have your students return to these pages to rate their knowledge again. They should see that their knowledge and skills have increased.

DINAH ZIKE FOLDABLES

Focus As students read and study this module, they should show examples and write notes about linear functions and relations.

Teach Have students make and label their Foldables as illustrated. Students should label the front of each half page with the lesson title. On the back of each of these pages, they can record concepts and notes from that particular lesson.

When to Use It Encourage students to add to their Foldables as they work through the module and to use them to review for the module test.

Launch the Module

For this module, the Launch the Module video uses real-world scenarios to illustrate how functions and their graphs can be used to model both linear and nonlinear relationships. Students learn about using graphs to model the change in altitude of an airplane and the change in strength of a Wi-Fi signal.

Module 4

Linear and Nonlinear Functions

Essential Question
What can a function tell you about the relationship that it represents?

What Will You Learn?

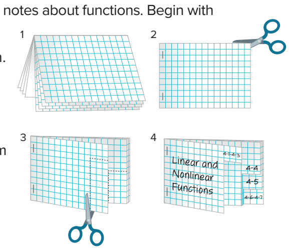
How much do you already know about each topic **before** starting this module?

KEY
— I don't know. — I've heard of it. — I know it!

	Before			After		
	I don't know	I've heard of it	I know it	I don't know	I've heard of it	I know it
graph linear equations by using a table						
graph linear equations by using intercepts						
find rates of change						
determine slopes of linear equations						
write linear equations in slope-intercept form						
graph linear functions in slope-intercept form						
translate, dilate, and reflect linear functions						
identify and find missing terms in arithmetic sequences						
write arithmetic sequences as linear functions						
model and use piecewise functions, step functions, and absolute value functions						
translate absolute value functions						

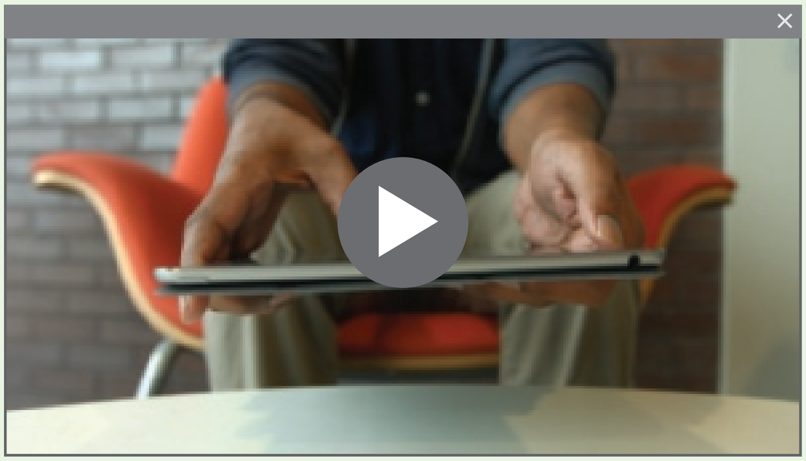
Foldables Make this Foldable to help you organize your notes about functions. Begin with five sheets of grid paper.

1. **Fold** five sheets of grid paper in half from top to bottom.
2. **Cut** along fold. Staple the eight half-sheets together to form a booklet.
3. **Cut** tabs into margin. The top tab is 4 lines wide, the next tab is 8 lines wide, and so on. When you reach the bottom of a sheet, start the next tab at the top of the page.
4. **Label** each tab with a lesson number. Use the extra pages for vocabulary.



Module 4 • Linear and Nonlinear Functions 207

Interactive Presentation



What Vocabulary Will You Learn?

- absolute value function
- arithmetic sequence
- common difference
- constant function
- dilation
- family of graphs
- greatest integer function
- interval
- identity function
- *n*th term of an arithmetic sequence
- parameter
- parent function
- piecewise-defined function
- piecewise-linear function
- rate of change
- reflection
- sequence
- slope
- step function
- term of a sequence
- transformation
- translation
- vertex

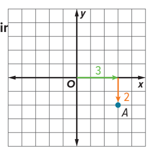
Are You Ready?

Complete the Quick Review to see if you are ready to start this module. Then complete the Quick Check.

Quick Review

Example 1

Graph $A(3, -2)$ on a coordir



Start at the origin. Since the x-coordinate is positive, move 3 units to the right. Then move 2 units down since the y-coordinate is negative. Draw a dot and label it A.

Example 2

Solve $x - 2y = 8$ for y.

$x - 2y = 8$	Original expression
$x - x - 2y = 8 - x$	Subtract x from each side.
$-2y = 8 - x$	Simplify.
$\frac{-2y}{-2} = \frac{8 - x}{-2}$	Divide each side by -2.
$y = \frac{1}{2}x - 4$	Simplify.

Quick Check

Graph and label each point on the coordinate plane.

1. $B(-3, 3)$ **2.** $C(-2, 1)$ **3.** $D(3, 0)$

4. $E(-5, -4)$ **5.** $F(0, -3)$ **6.** $G(2, -1)$

1-6. See margin.

Solve each equation for y.

7. $3x + y = 1$ **8.** $8 - y = x$

9. $5x - 2y = 12$ **10.** $3x + 4y = 10$

11. $3 - \frac{1}{2}y = 5x$ **12.** $\frac{y+1}{3} = x + 2$

7. $y = -3x + 1$ **8.** $y = -x + 8$

9. $y = \frac{5}{2}x - 6$ **10.** $y = -\frac{3}{4}x + \frac{5}{2}$

11. $y = -10x + 6$ **12.** $y = 3x + 5$

How did you do?

Which exercises did you answer correctly in the Quick Check?

What Vocabulary Will You Learn?

ELL As you proceed through the module, introduce the key vocabulary by using the following routine.

Define The **slope** of a line is the rate of change in the y-coordinates (rise) for the corresponding change in the x-coordinates (run) for points on the line.

Example A line passes through the points (1, 4) and (3, 8).

Ask What is the slope of the line? 2

Are You Ready?

Students may need to review the following prerequisite skills to succeed in this module.

- identifying domain and range
- identifying slopes
- translating and reflecting geometric figures
- finding the next terms in patterns
- graphing linear functions
- evaluating absolute value expressions

ALEKS®

ALEKS is an adaptive, personalized learning environment that identifies precisely what each student knows and is ready to learn, ensuring student success at all levels.

You can use the ALEKS pie report to see which students know the topics in the **Functions and Lines** module—who is ready to learn these topics and who isn't quite ready to learn them yet—in order to adjust your instruction as appropriate.

Mindset Matters

Collaborative Risk Taking

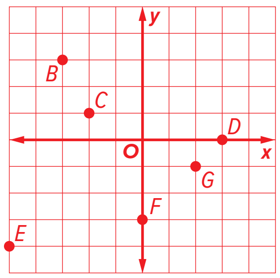
Some students may be averse to taking risks during math class, like sharing an idea, strategy, or solution. They may worry about their grades or scores on tests, or some might feel less confident solving math problems, especially in front of their peers.

How Can I Apply It?

Assign the **Practice** problems of each lesson and encourage students to take risks as they solve problems, try new paths, and discuss their strategies with their partner or group.

Answer

1-6.



Graphing Linear Functions

LESSON GOAL

Students graph linear functions by using tables and intercepts.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Points on a Line

Develop:

Graphing Linear Functions by Using Tables
- Graph by Making a Table
- Choose Appropriate Domain Values
- Graph $y = a$
- Graph $x = a$

Explore: Lines Through Two Points

Develop:

Graphing Linear Functions by Using the Intercepts
- Graph by Using Intercepts
- Use Intercepts

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Proportional Relationships and Slope	●	●		●
Extension: Graphing Equations in Three Dimensions		●	●	●

Language Development Handbook

Assign page 20 of the *Language Development Handbook* to help your students build mathematical language related to graphing linear functions.

ELL You can use the tips and suggestions on page T20 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Algebra, Functions

Standards for Mathematical Content:

A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous
Students sketched graphs and compared graphs of functions.
F.IF.4, F.IF.9

Now
Students graph linear functions using tables and intercepts.
A.REI.10, F.IF.7a, F.LE.5

Next
Students will investigate rate of change and slope.
F.IF.6, F.LE.5

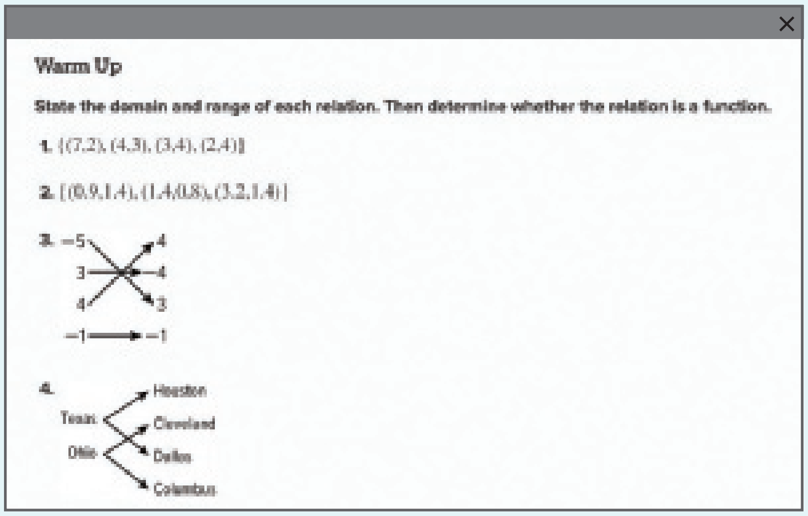
Rigor

The Three Pillars of Rigor

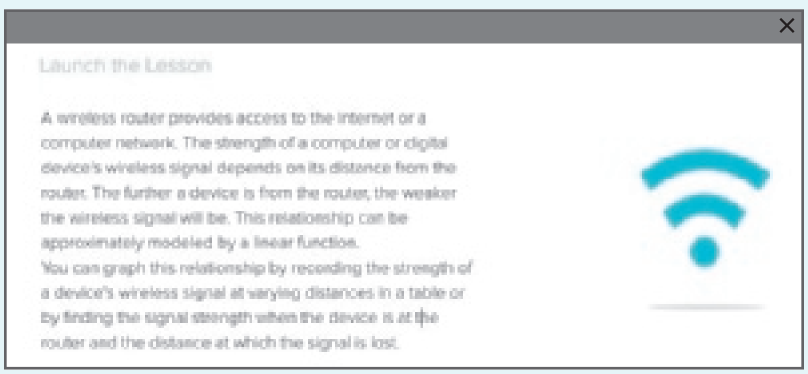
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of and fluency with linear functions (first studied in Grade 8) to graphing linear functions by using a table and by using intercepts. They apply their understanding of linear functions by solving real-world problems.

Interactive Presentation



Warm Up



Launch the Lesson

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying domain and range

Answers:

1. D: {2, 3, 4, 7}, R: {2, 3, 4}; yes
2. D: {0.9, 1.4, 3.2}, R: {0.8, 1.4}; yes
3. D: {–5, –1, 3, 4}, R: {–4, –1, 3, 4}; yes
4. D: {Ohio, Texas}, R: {Cleveland, Columbus, Dallas, Houston}; no
5. D: {dog, fish, cat, other, bird, rabbit, hamster, horse, snake}, R: {5, 6, 20, 21, 22, 24, 42, 60, 71}; yes

Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain how the verbal description of the relationship between the device's strength and the distance from the router can be modeled by a function, which can be used to create a table of values and a graph.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Mathematical Background

The graph of a linear function is a line. The coordinates of the points on the line are the solutions of the related linear equation. If you know at least two solutions of the equation, you can use them to graph the line. You can also use the x- and y-intercepts to graph the line. The intercepts can be found by alternately replacing x and y with 0. The line that connects the intercepts is the graph of the linear equation.

Explore Points on a Line

Objective

Students explore the relationship between graphs of linear equations and their solutions.

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in this Explore.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

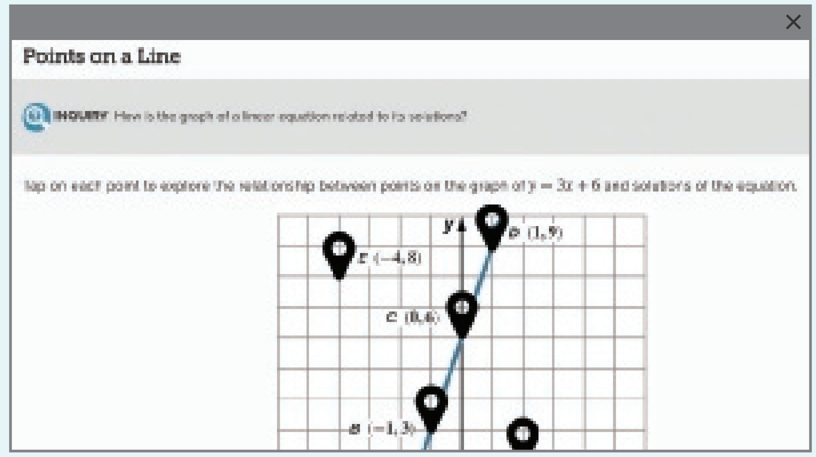
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

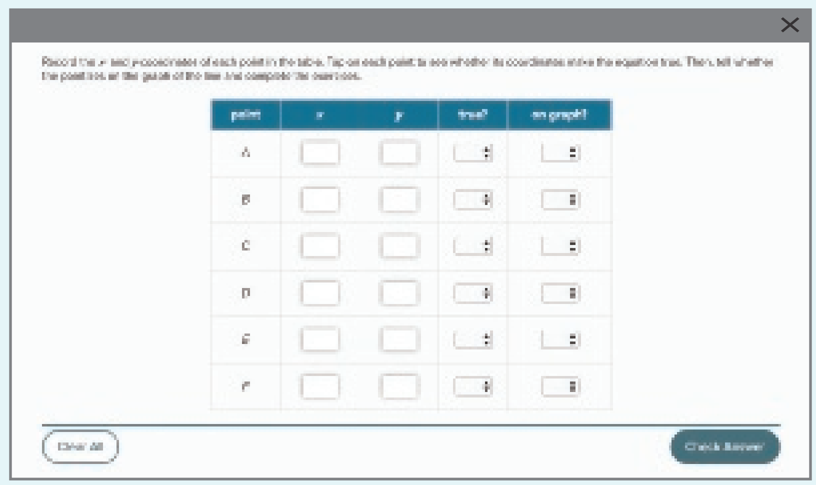
Students will complete guiding exercises throughout the Explore activity. Students will be presented with a linear equation and its graph. Several points on the coordinate plane are marked and labeled, some on the graph, and some not on the graph. Students will record the coordinates of the marked points, and determine whether each pair of coordinates makes the equation true. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



Explore



Explore

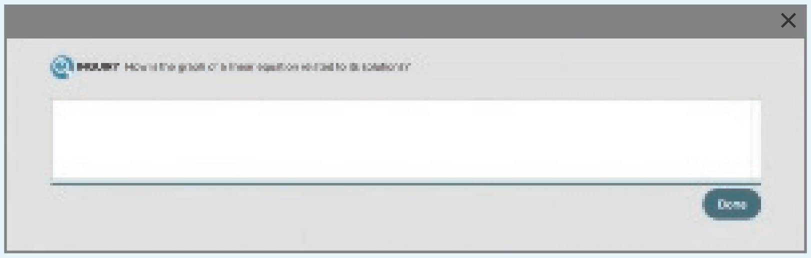
TAP

Students tap on each point to explore the relationship between points on a graph and solutions of an equation.

TYPE

Students complete a table and answer questions about the points that make an equation true.

Interactive Presentation



Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Points on a Line (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is it important to know if coordinates make an equation true? Sample answer: It is important to know when the substituted values make both sides of the equation equal. The coordinates that make the equation true are solutions of the equation.
- Given a graph of a linear function, how could you find a solution of the related equation? Sample answer: I could look for coordinates on the line because any point on the line is a solution of the related equation.

Inquiry

How is the graph of a linear equation related to its solutions?
Sample answer: The graph of a line is all of the solutions of its equation plotted on a coordinate plane.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Explore Lines Through Two Points

Objective

Students use a sketch to explore the number of lines that pass through two points.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this Explore, students will need to use a sketch. Work with students to explore and deepen their understanding of lines through two points.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

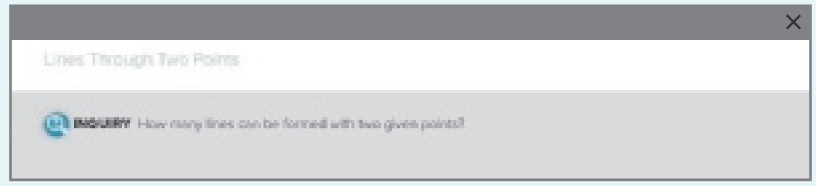
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

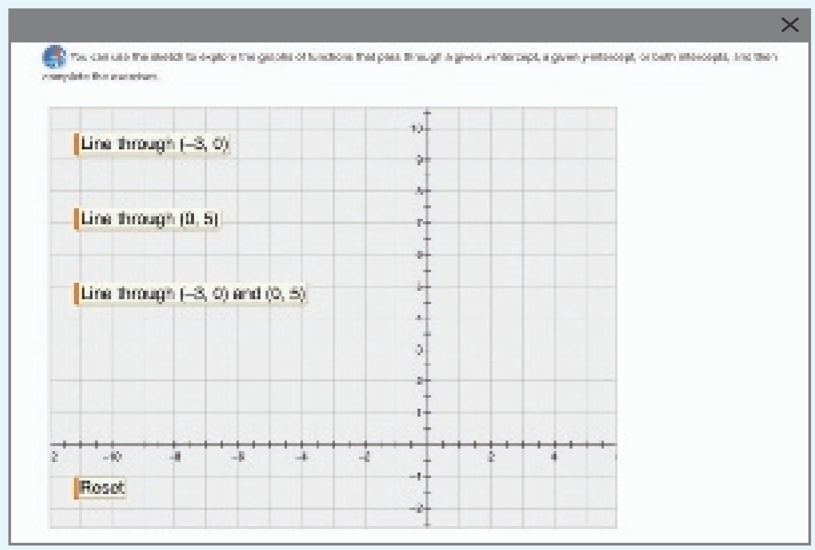
Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to explore the number of lines that can be drawn through a single point. They will then explore the number of lines that can be drawn through two points. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



Explore



Explore

WEB SKETCHPAD

Students use a sketch to explore the graphs of linear functions.

TYPE

Students answer questions about the graphed functions.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Lines Through Two Points (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Can you graph a function from a table that has only two points? Sample answer: As long as you know that the function is linear, it is okay for the table to only list two points.
- When graphing, do you think it would be better to use two points close together or farther apart? Sample answer: Farther apart would help you get a better idea of where the line should be drawn. If the points are too close together, you might not have your ruler or line tool lined up correctly.

Inquiry

How many lines can be formed with two given points? Sample answer: There is only one line that can be formed with two given points.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Graphing Linear Functions by Using Tables

Objective

Students graph linear functions by making a table of values.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the table, coordinates, equation, and graph of a linear function.

What Students Are Learning

Students come to understand that although a table of values can be used to construct the graph of a linear function, the graph represents all of the solutions of the equation. They learn that every point on the graph represents a pair of coordinates that is a solution of the equation.

Example 1 Graph by Making a Table

Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument.

Questions for Mathematical Discourse

AL What values are in the domain of the function? all real numbers

OL Why is it helpful to choose both positive and negative values? Sample answer: Choosing positive and negative values gives you a better idea of what the graph will look like and will show you where the graph crosses the y-axis.

BL What should you do if one of the points you graph is not on the same line as the others? Sample answer: Check your work to see if you miscalculated the y-value.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Lesson 4-1

Graphing Linear Functions

Explore Points on a Line

Online Activity Use an interactive tool to complete an Explore.

INQUIRY How is the graph of a linear equation related to its solutions?

Learn Graphing Linear Functions by Using Tables

A table of values can be used to graph a linear function. Every ordered pair that makes the equation true represents a point on its graph. So, a graph represents all the solutions of an equation.

Linear functions can be represented by equations in two variables.

Example 1 Graph by Making a Table

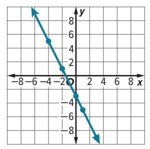
Graph $-2x - 3 = y$ by making a table.

Step 1 Choose any values of x from the domain and make a table.

Step 2 Substitute each x-value into the equation to find the corresponding y-value. Then, write the x- and y-values as an ordered pair.

x	$-2x - 3$	y	(x, y)
−4	−2(−4) −3	5	(−4, 5)
−2	−2(−2) −3	1	(−2, 1)
0	−2(0) −3	−3	(0, −3)
1	−2(1) −3	−5	(1, −5)
3	−2(3) −3	−9	(3, −9)

Step 3 Graph the ordered pairs in the table and connect them with a line.



Go Online You can complete an Extra Example online.

Today's Goals
- Graph linear functions by making tables of values.
- Graph linear functions by using the x- and y-intercepts.

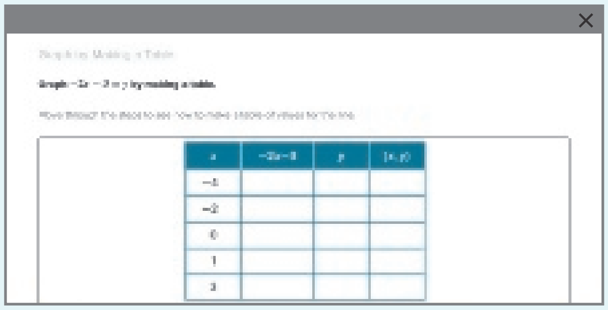
Talk About It! What values of x might be easiest to use when graphing a linear equation when the x-coefficient is a whole number? Justify your argument.

Sample answer: −2, −1, 0, 1, and 2 would be easiest to use because they are small and therefore easy to substitute into the equation to find the y-values.

Study Tip

Exactness Although only two points are needed to graph a linear function, choosing three to five x-values that are spaced out can verify that your graph is correct.

Interactive Presentation



Example 1

TAP

Students move through the steps to see how to make a table of values for a line.

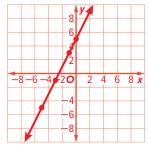
TYPE

Students discuss the x-values that would be easiest to use when graphing a linear equation if the coefficient of x is an integer.

Check

Graph $y = 2x + 5$ by using a table. Copy and complete the table. Then graph the function.

x	y
−5	−5
−3	−1
−1	3
0	5
2	9



Think About It!

What are some values of x that you might choose in order to graph $y = \frac{1}{7}x - 12$?

Sample answer: $\{-14, -7, 0, 7, 14\}$

Example 2 Choose Appropriate Domain Values

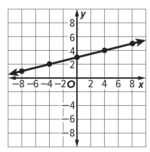
Graph $y = \frac{1}{4}x + 3$ by making a table.

Step 1 Make a table.

Step 2 Find the y-values.

Step 3 Graph the ordered pairs in the table and connect them with a line.

x	$\frac{1}{4}x + 3$	y	(x, y)
−8	$\frac{1}{4}(-8) + 3$	1	(−8, 1)
−4	$\frac{1}{4}(-4) + 3$	2	(−4, 2)
0	$\frac{1}{4}(0) + 3$	3	(0, 3)
4	$\frac{1}{4}(4) + 3$	4	(4, 4)
8	$\frac{1}{4}(8) + 3$	5	(8, 5)



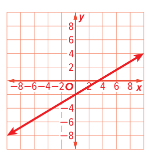
Watch Out!

Equivalent Equations Sometimes, the variables are on the same side of the equal sign. Rewrite these equations by solving for y to make it easier to find values for y.

Check

Graph $y = \frac{3}{5}x - 2$ by making a table. Copy and complete the table. Then graph the function.

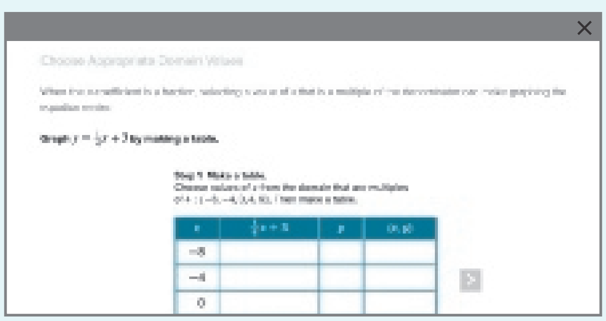
x	y
−10	−8
−5	−5
0	−2
5	1
10	4



Go Online You can complete an Extra Example online.

210 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Example 2

WEB SKETCHPAD

Students use a sketch to graph the ordered pairs from the table of values.

TYPE

Students give the possible domain values for a given linear equation with a rational slope.

Example 2 Choose Appropriate Domain Values

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the equation, table, and graph in this example.

Questions for Mathematical Discourse

AL What values are in the domain? all real numbers

OL Why were the values selected for x in the table −8, −4, 0, 4, and 8? Sample answer: They were all multiples of 4 and since the coefficient is $\frac{1}{4}$ this makes multiplication easier.

BL What would happen if you used multiples of 2 for x in the table? Sample answer: Multiples of 2 that are also multiples of 4 would cancel out the denominator, but others would reduce to have a denominator of 2.

Common Error

Some students may make calculation errors when working with a coefficient that is a fraction. Help them avoid this by suggesting that they write the integer that they are substituting for x as a fraction with a denominator of 1.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 3 Graph $y = a$

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL How is this equation different from other linear equations that you have worked with? Sample answer: The coefficient of x is zero.

OL What would the table look like for other values of x? Sample answer: The y-values would all be 5.

BL Is the graph a function? Explain Yes; sample answer: This is a function because it passes the vertical line test.

Common Error

Some students may interpret an equation such as $y = 5$ as a point, not a line. Help them to see that although the equation specifies that $y = 5$, x could be infinitely many values. Use a table to show how this leads to the graph of $y = 5$ consisting of more than one point.

Example 4 Graph $x = a$

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this example, students will need to use a sketch. Work with students to explore and deepen their understanding of graphs of horizontal lines.

Questions for Mathematical Discourse

AL How is this equation different from other linear equations that you have worked with? Sample answer: There is only one variable, x.

OL What is the x-intercept for the graph of an equation of the form $x = a$? $(a, 0)$

BL Why does every point of the form $(-2, y)$ satisfy the equation? Sample answer: Because the equation has no y-variable, substituting any point $(-2, y)$ into the equation will result in the true statement $-2 = -2$.

DIFFERENTIATE

Language Development Activity AL BL ELL

IF students are having difficulty remembering which equations represent horizontal lines and which represent vertical lines, **THEN** have them use the acronyms HOY and VUX to remember which is which. HOY stands for "Horizontal, 0 slope, $y =$," and VUX stands for "Vertical, Undefined slope, $x =$."

Example 3 Graph $y = a$

Graph $y = 5$ by making a table.

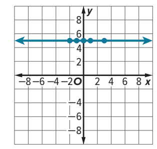
Step 1 Rewrite the equation.
$y = 0x + 5$

Step 2 Make a table.

x	$0x + 5$	y	(x, y)
−2	0(−2) + 5	5	(−2, 5)
−1	0(−1) + 5	5	(−1, 5)
0	0(0) + 5	5	(0, 5)
1	0(1) + 5	5	(1, 5)
2	0(2) + 5	5	(2, 5)

Step 3 Graph the line.

The graph of $y = 5$ is a horizontal line through $(x, 5)$ for all values of x in the domain.



Think About It! In general, what does the graph of an equation of the form $y = a$, where a is any real number, look like?

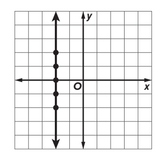
Sample answer: a horizontal line through (x, a) for all values of x in the domain.

Example 4 Graph $x = a$

Graph $x = -2$.

You learned in the previous example that equations of the form $y = a$ have graphs that are horizontal lines. Equations of the form $x = a$ have graphs that are vertical lines.

The graph of $x = -2$ is a vertical line through $(-2, y)$ for all real values of y. Graph ordered pairs that have x-coordinates of -2 and connect them with a vertical line.

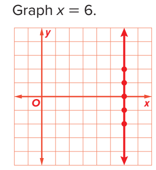


Think About It! Is the graph of $x = a$ a function? Why or why not?

No; sample answer: The element a in the domain is paired with more than one element of the range.

Check

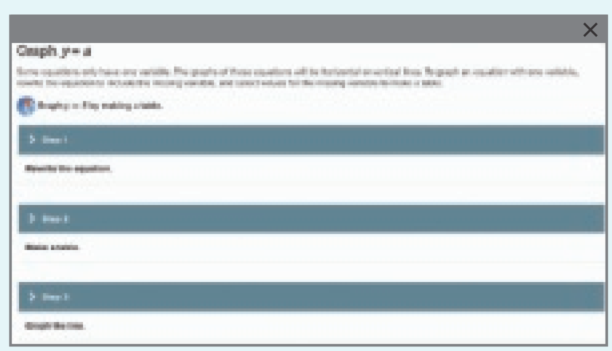
Graph $x = 6$.



Go Online You can complete an Extra Example online.

Lesson 4-1 • Graphing Linear Functions 211

Interactive Presentation



Example 3

TAP Students move through the steps to graph a line in the form $y = a$.

TYPE Students explain what the graph of an equation in the form $y = a$ will look like.

CHECK Students complete the Check online to determine whether they are ready to move on.

Go Online
You can watch a video to see how to graph linear functions.

Think About It!
Why are the x- and y-intercepts easy to find?

Sample answer: Because either the x- or y-value of an intercept is 0.

Think About It!
What does a line that only has an x-intercept look like? a line that only has a y-intercept?

Sample answer: A line that only has an x-intercept is a vertical line. A line that only has a y-intercept is a horizontal line.

Study Tip
Tools When drawing lines by hand, it is helpful to use a straightedge or a ruler.

Explore Lines Through Two Points

Online Activity Use graphing technology to complete an Explore.

INQUIRY How many lines can be formed with two given points?

Learn Graphing Linear Functions by Using the Intercepts

You can graph a linear function given only two points on the line. Using the x- and y-intercepts is common because they are easy to find. The intercepts provide the ordered pairs of two points through which the graph of the linear function passes.

Example 5 Graph by Using Intercepts

Graph $-x + 2y = 8$ by using the x- and y-intercepts.

To find the x-intercept, let $y = 0$.

$-x + 2y = 8$	Original equation
$-x + 2(0) = 8$	Replace y with 0.
$-x = 8$	Simplify.
$x = -8$	Divide.

This means that the graph intersects the x-axis at $(-8, 0)$.

To find the y-intercept, let $x = 0$.

$-x + 2y = 8$	Original equation
$-0 + 2y = 8$	Replace x with 0.
$2y = 8$	Simplify.
$y = 4$	Divide.

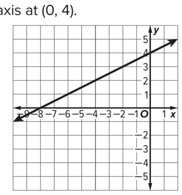
This means that the graph intersects the y-axis at $(0, 4)$.

Graph the equation.

Step 1 Graph the x-intercept.

Step 2 Graph the y-intercept.

Step 3 Draw a line through the points.



Go Online You can complete an Extra Example online.

212 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation

Graph by Using Intercepts

Graph $-x + 2y = 8$ by using the x- and y-intercepts.

Find the x- and y-intercepts.

To find the x-intercept, let $y = 0$.

$-x + 2y = 8$	Original equation
$-x + 2(0) = 8$	Replace y with 0.
$-x = 8$	Simplify.
$x = -8$	Divide.

Example 5

WEB SKETCHPAD

Students use a sketch to graph a linear function.

TYPE

Students describe what a line looks like that only has an x– or y– intercept.

Learn Graphing Linear Functions by Using the Intercepts

Objective

Students graph linear functions by using the x- and y-intercepts.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Common Misconception

Some students may think that the x- and y-intercepts are the coefficients of x and y. Use an example such as $3x + 2y = 12$ to review the process of finding intercepts and show that neither coefficient is an intercept.

Example 5 Graph by Using Intercepts

Teaching the Mathematical Practices

5 Decide When to Use Tools Mathematically proficient students can make sound decisions about when to use mathematical tools such as a straightedge. Help them see why using these tools will help to solve problems and what the limitations are of using the tool.

Questions for Mathematical Discourse

AL What are the intercepts of the graph of a linear function? the points where the line crosses the x- and y-axes

OL How does finding the x- and y-intercepts help you to graph the function? Sample answer: Two points make a line, so a line can be drawn using the intercepts as the two points.

BL When finding the x-intercept, why do you substitute 0 for y in the equation? Sample answer: The y-coordinate of any point on the x-axis is 0, so substituting 0 for y in the equation tells you the value of x when $y = 0$, which is the x-intercept of the graph of the function.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 6 Use Intercepts

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about graphing linear functions to solving a real-world problem.

Questions for Mathematical Discourse

AL What information is given in the problem? Angelina starts with 60 cups of dog food and feeds her dog $\frac{5}{2}$ cups per day.

OL What does each variable represent, and what does this tell you about the intercepts? Sample answer: x represents days, and y represents cups of food. So the x-intercept represents the number of days when there are 0 cups of food left, and the y-intercept represents the amount of food when 0 days have passed.

BL Explain what the intercepts mean in the context of the problem. Sample answer: At 24 days, there is no food left. The bag started with 60 cups of food and after 24 days, the bag was empty.

Common Error

Some students may interchange the intercepts, thinking that when they let $x = 0$, they are finding the x-intercept or vice versa. Help students avoid this error by having them write the ordered pairs with the zeros in place before they solve algebraically. Then have them fill in the values they find, and plot the points from the ordered pairs.

Essential Question Follow-Up

Students have used a variety of methods to graph linear equations.

Ask:

Why is it helpful to have different ways to graph linear functions? Sample answer: Some methods of graphing are easier in different contexts. For instance, graphing by finding the x- and y-intercepts might be obvious from inspecting the particular equation. For a function that represents a real-world situation, it might be easier to create a table of values for the situation.

DIFFERENTIATE

Enrichment Activity BL

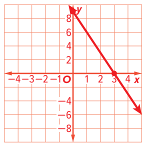
Have students work in pairs to create a poster about graphing linear equations. Have them include information about tables of values, intercepts, and the solutions of the equations in their display.

Check

Graph $4y = -12x + 36$ by using the x- and y- intercepts.

x-intercept: ___?___ 3

y-intercept: ___?___ 9



Example 6 Use Intercepts

PETS Angelina bought a 15-pound bag of food for her dog. The bag contains about 60 cups of food, and she feeds her dog $2\frac{1}{2}$ or $\frac{5}{2}$ cups of food per day. The function $y + \frac{5}{2}x = 60$ represents the amount of food left in the bag y after x days. Graph the amount of dog food left in the bag as a function of time.

Go Online You can watch a video to see how to use a graphing calculator with this example.

Part A

Find the x- and y-intercepts and interpret their meaning in the context of the situation.

To find the x-intercept, let $y = 0$.

$y + \frac{5}{2}x = 60$ Original equation

$0 + \frac{5}{2}x = 60$ Replace y with 0.

$\frac{5}{2}x = 60$ Simplify.

$x = 24$ Multiply each side by $\frac{2}{5}$.

The x-intercept is 24. This means that the graph intersects the x-axis at (24, 0). So, after 24 days, there is no dog food left in the bag.

To find the y-intercept, let $x = 0$.

$y + \frac{5}{2}x = 60$ Original equation

$y + \frac{5}{2}(0) = 60$ Replace x with 0.

$y = 60$ Simplify.

The y-intercept is 60. This means that the graph intersects the y-axis at (0, 60). So, after 0 days, there are 60 cups of food in the bag.

Think About It! Find another point on the graph. What does it mean in the context of the problem?

Sample answer: (10, 35); After 10 days, there are 35 cups of dog food left in the bag.

(continued on the next page)

Interactive Presentation

Example 6

WEB SKETCHPAD

Students use a sketch to plot the intercepts and graph the line.

TYPE

Students explain the meaning of another point in context and identify the assumptions made.

WATCH

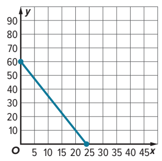
Students can watch a video to review how to graph a linear function using a graphing calculator.

Think About It!
What assumptions did you make about the amount of food Angelina feeds her dog each day?

Sample answer: I assumed that the bag of food contains exactly 60 cups and that Angelina feeds her dog the exact same amount each day.

Go Online
You can watch a video to see how to graph a linear function using a graphing calculator.

Part B
Graph the equation by using the intercepts.



Check

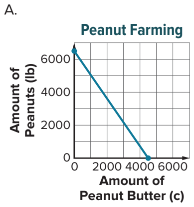
PEANUTS A farm produces about 4362 pounds of peanuts per acre. One cup of peanut butter requires about $\frac{2}{3}$ pound of peanuts. If one acre of peanuts is harvested to make peanut butter, the function $y = -\frac{2}{3}x + 4362$ represents the pounds of peanuts remaining y after x cups of peanut butter are made.

x-intercept: ? 6543

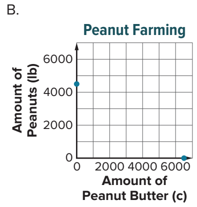
y-intercept: ? 4362

Which graph uses the x- and y-intercepts to correctly graph the equation? C

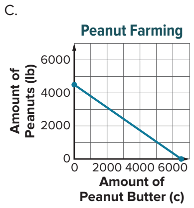
A.



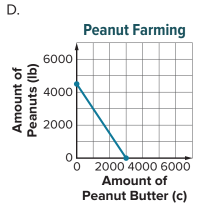
B.



C.

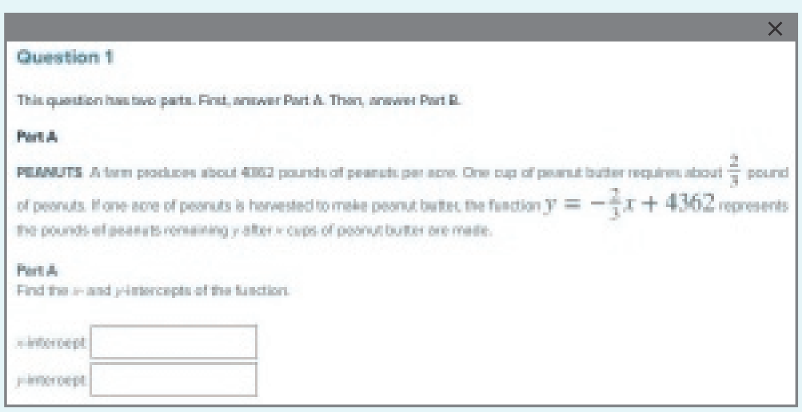


D.



Go Online You can complete an Extra Example online.

Interactive Presentation



CHECK

Students complete the Check online to determine whether they are ready to move on.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–16
2	exercises that use a variety of skills from this lesson	17–25
2	exercises that extend concepts learned in this lesson to new contexts	26–29
3	exercises that emphasize higher-order and critical-thinking skills	30–37

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–25 odd, 30–37
- Extension: Graphing Equations in Three Dimensions
- ALEKS® Ordered Pairs; Graphing Lines

IF students score 66%–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–37 odd
- Remediation, Review Resources: Proportional Relationships and Slope
- Personal Tutors
- Extra Examples 1–6
- ALEKS® Proportional Relationships; Slope

IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice, Exercises 1–15 odd
- Remediation, Review Resources: Proportional Relationships and Slope
- *Quick Review Math Handbook*: Linear Functions
- Arrive**MATH** Take Another Look
- ALEKS® Proportional Relationships; Slope

Practice

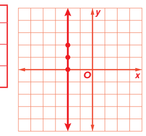
Go Online You can complete your homework online.

Examples 1 through 4

Graph each equation by making a table.

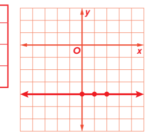
1. $x = -2$

x	y
−2	0
−2	1
−2	2



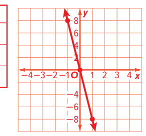
2. $y = -4$

x	y
0	−4
1	−4
2	−4



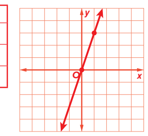
3. $y = -8x$

x	y
−1	8
0	0
1	−8



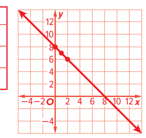
4. $3x = y$

x	y
0	0
1	3
2	6



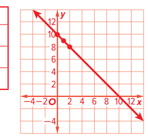
5. $y - 8 = -x$

x	y
0	8
1	7
2	6



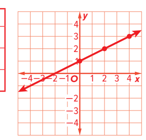
6. $x = 10 - y$

x	y
0	10
1	9
2	8



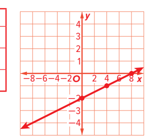
7. $y = \frac{1}{2}x + 1$

x	y
0	1
2	2
4	3



8. $y + 2 = \frac{1}{4}x$

x	y
0	−2
4	−1
8	0

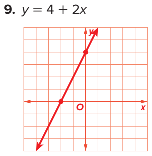


Lesson 4-1 • Graphing Linear Functions **215**

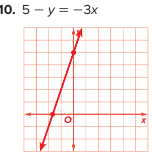
Example 5

Graph each equation by using the *x*-and *y*-intercepts.

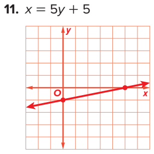
9. $y = 4 + 2x$



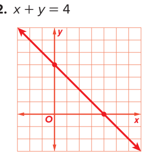
10. $5 - y = -3x$



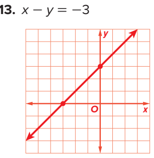
11. $x = 5y + 5$



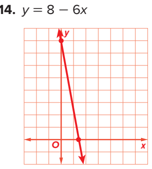
12. $x + y = 4$



13. $x - y = -3$



14. $y = 8 - 6x$



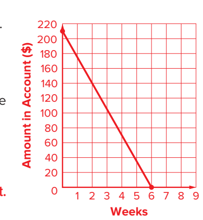
Example 6

15. SCHOOL LUNCH Amanda has $210 in her school lunch account. She spends $35 each week on school lunches. The equation $y = 210 - 35x$ represents the total amount in Amanda's school lunch account y for x weeks of purchasing lunches.

a. Find the *x*- and *y*-intercepts and interpret their meaning in the context of the situation.

b. Graph the equation by using the intercepts.

15a. The *x*-intercept is 6. This means that after 6 weeks, Amanda will have $0 in her school lunch account. The *y*-intercept is 210. This means that there was initially $210 in Amanda's school lunch account.

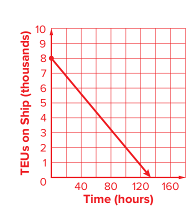


16. SHIPPING The *OOCL Shenzhen,* one of the world's largest container ships, carries 8063 TEUs (1280-cubic-feet containers). Workers can unload a ship at a rate of 1 TEU every minute. The equation $y = 8063 - 60x$ represents the number of TEUs on the ship y after x hours of the workers unloading the containers from the *Shenzhen*.

a. Find the *x*- and *y*-intercepts and interpret their meaning in the context of the situation.

b. Graph the equation by using the intercepts.

16a. The *x*-intercept is about 134. This means that after about 134 hours, there are 0 TEUs on the ship. The *y*-intercept is 8063. This means that there were initially 8063 TEUs on the ship.

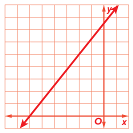


216 **Module 4** • Linear and Nonlinear Functions

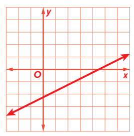
Mixed Exercises

Graph each equation.

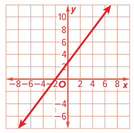
17. $1.25x + 7.5 = y$



18. $2x - 3 = 4y + 6$



19. $3y - 7 = 4x + 1$



Find the *x*-intercept and *y*-intercept of the graph of each equation.

20. $5x + 3y = 15$
x-int: 3; y-int: 5

21. $2x - 7y = 14$
x-int: 7; y-int: −2

22. $2x - 3y = 5$
x-int: $2\frac{1}{2}$; y-int: $-1\frac{2}{3}$

23. $6x + 2y = 8$
x-int: $1\frac{1}{3}$; y-int: 4

24. $y = \frac{1}{4}x - 3$
x-int: 12; y-int: −3

25. $y = \frac{2}{3}x + 1$
x-int: $-1\frac{1}{2}$; y-int: 1

26. HEIGHT The height of a woman can be predicted by the equation $h = 81.2 + 3.34r$, where h is her height in centimeters and r is the length of her radius bone in centimeters.

a. Is this a linear function? Explain. Yes; the equation can be written in standard form where $A = 3.34$, $B = -1$, and $C = -81.2$.

b. What are the r- and h-intercepts of the equation? Do they make sense in the situation? Explain. h-int: 81.2; r-int: about −24.3; no, we would expect a woman 81.2-cm tall to have a radius bone of some length, and a negative radius bone length has no real meaning.

c. Graph the equation by using the intercepts. See margin.

d. Use the graph to find the approximate height of a woman whose radius bone is 25 centimeters long. 165 cm

27. TOWING Pick-M-Up Towing Company charges \$40 to hook a car and \$1.70 for each mile that it is towed. Write an equation that represents the total cost y for x miles towed. Graph the equation. Find the y-intercept, and interpret its meaning in the context of the situation. See margin.

28. USE A MODEL Elias has \$18 to spend on peanuts and pretzels for a party. Peanuts cost \$3 per pound and pretzels cost \$2 per pound. Write an equation that relates the number of pounds of pretzels y and the number of pounds of peanuts x. Graph the equation. Find the x- and y-intercepts. What does each intercept represent in terms of context? See margin.

29. REASONING One football season, a football team won 4 more games than they lost. The function $y = x + 4$ represents the number of games won y and the number of games lost x. Find the x- and y-intercepts. Are the x- and y-intercepts reasonable in this situation? Explain. Sample answer: The x-intercept is −4. The x-intercept is not reasonable because the football team cannot lose −4 games. The y-intercept is 4. The y-intercept is reasonable because the y-intercept means that if the football team won 4 games, they lost 0 games.

Higher-Order Thinking Skills

30. WRITE Consider real-world situations that can be modeled by linear functions.

a. Write a real-world situation that can be modeled by a linear function. Sample answer: Keith is climbing down a 28-foot cliff. Keith descends 7 feet per minute.

b. Write an equation to model your real-world situation. Be sure to define variables. Then find the x- and y-intercepts. What does each intercept represent in your context? Sample answer: $y = 28 - 7x$, where y is Keith's height in feet after x minutes. The x-intercept, 4, represents the number of minutes it takes Keith to reach the bottom of the cliff. The y-intercept, 28, represents Keith's initial height above the bottom of the cliff.

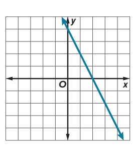
c. Graph your equation by making a table. Include a title for the graph as well as labels and titles for each axis. Explain how you labeled the x- and y-axes. State a reasonable domain for this situation. What does the domain represent? See margin.

31. FIND THE ERROR Geroy claims that every line has both an x- and a y-intercept. Is he correct? Explain your reasoning. No; sample answer: A horizontal line only has a y-intercept and a vertical line only has an x-intercept.

32. WHICH ONE DOESN'T BELONG? Which equation does not belong with the other equations? Justify your conclusion. $y - 4 = 0$; When graphed, all of the other equations have x- and y-intercepts, but $y - 4 = 0$ only has a y-intercept.

$y = 2 - 3x$	$5x = y - 4$	$y = 2x + 5$	$y - 4 = 0$

33. ANALYZE Robert sketched a graph of a linear equation $2x + y = 4$. What are the x- and y-intercepts of the graph? Explain how Robert could have graphed this equation using the x- and y-intercepts. Sample answer: In the equation, let $y = 0$ to find the x-intercept; $2x + 0 = 4$. So the x-intercept is 2. In the equation, let $x = 0$ to find the y-intercept; $2(0) + y = 4$. So the y-intercept is 4. Robert graphed points at (2, 0) and (0, 4) and connected the points with a line.



34. ANALYZE Compare and contrast the graph of $y = 2x + 1$ with the domain $\{1, 2, 3, 4\}$ and $y = 2x + 1$ with the domain all real numbers. Sample answer: The first graph is a set of points that are not connected. The second graph is of a line. The points of the first graph are points on the line in the second graph.

CREATE **Give an example of a linear equation in the form $Ax + By = C$ for each condition. Then describe the graph of the equation.**

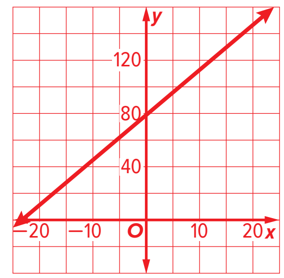
35. $A = 0$
Sample answer: $y = 8$; horizontal line

36. $B = 0$
Sample answer: $x = 5$; vertical line

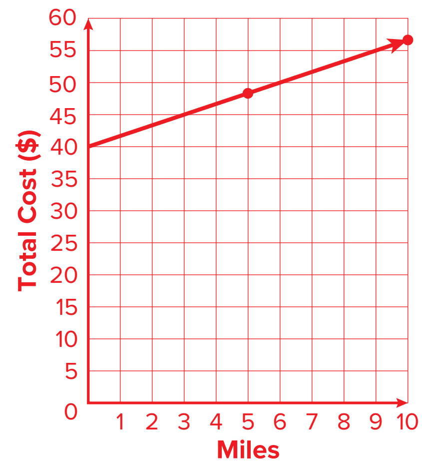
37. $C = 0$
Sample answer: $x - y = 0$; line through (0, 0)

Answers

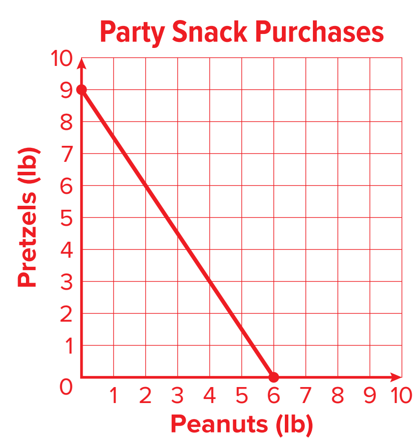
26c.



27. $y = 1.7x + 40$; The y-intercept is 40. This means that it would cost \$40 to hook up the car.

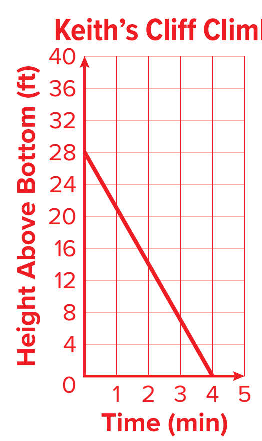


28. $3x + 2y = 18$; The x-intercept is 6. The x-intercept represents how many pounds of peanuts can be bought if no pretzels are bought. The y-intercept is 9. The y-intercept represents how many pounds of pretzels can be bought if no peanuts are bought.



30c.

Time (min) x	Height (ft) y
0	28
1	21
2	14
4	0



Rate of Change and Slope

LESSON GOAL

Students find and interpret the rate of change and slopes of lines.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Rate of Change of a Linear Function

- Find the Rate of Change
- Compare Rates of Change
- Constant Rate of Change
- Rate of Change

Explore: Investigating Slope

Develop:

Slope of a Line

- Positive Slope
- Negative Slope
- Slopes of Horizontal Lines
- Slopes of Vertical Lines
- Find Coordinates Given the Slope
- Use Slope

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Order of Integer Operations	●	●		●
Extension: Treasure Hunt with Slopes		●	●	●

Language Development Handbook

Assign page 21 of the *Language Development Handbook* to help your students build mathematical language related to rates of change and slopes.

ELL You can use the tips and suggestions on page T21 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

Coherence

Vertical Alignment

Previous
Students graphed linear functions using tables and intercepts.
A.REI.10, F.IF.7a, F.LE.5

Now
Students find and interpret the rate of change and slopes of lines.
F.IF.6, F.LE.5

Next
Students will graph equations in slope-intercept form.
A.CED.2, F.IF.7a, F.LE.5

Rigor

The Three Pillars of Rigor

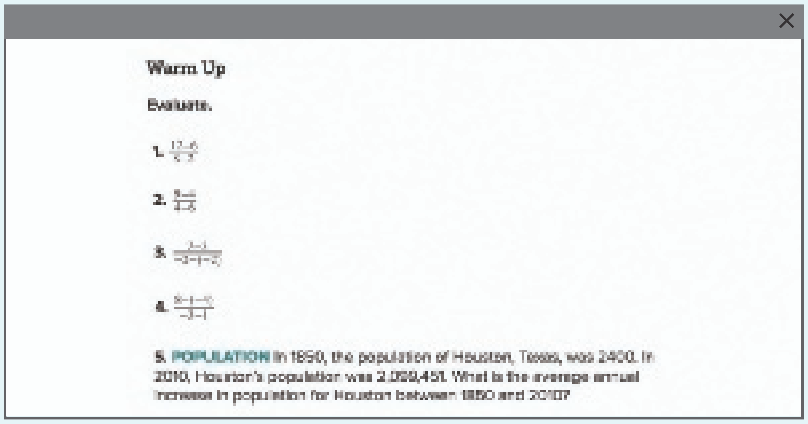
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of and fluency with slope and rate of change (first studied in Grade 8). They apply their understanding of slope and rate of change by solving real-world problems.

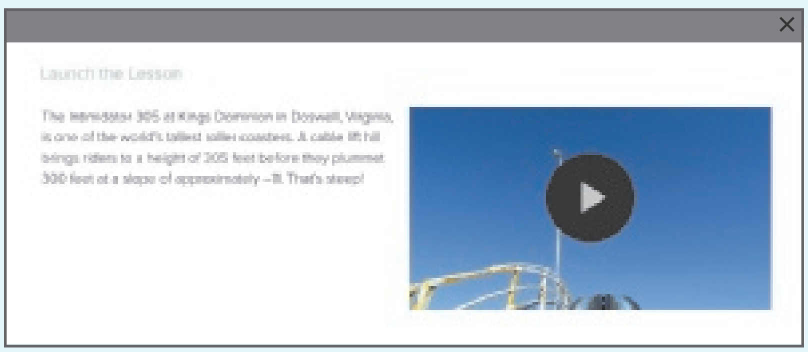
Mathematical Background

Rate of change is a ratio that describes, on average, how one quantity changes with respect to a change in another quantity. Slope can be used to describe rate of change. The slope of a line is the ratio of the vertical change (the rise) to the horizontal change (the run). The slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$, where (x_1, y_1) and (x_2, y_2) are two points that lie on the line, can be used to find the slope of a line without graphing.

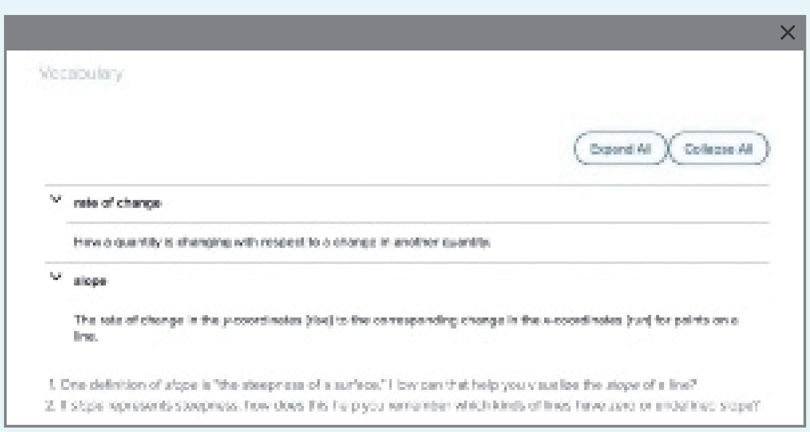
Interactive Presentation



Warm Up



Launch the Lesson



Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- subtracting integers in fractions

Answers:

1. 1
2. $-\frac{5}{2}$
3. undefined
4. -3
5. 13.1 thousands

Launch the Lesson

Teaching the Mathematical Practices

2 Make Sense of Quantities
Mathematically proficient students need to be able to make sense of quantities, such as slope and rate of change, and their relationships.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will be using these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Investigating Slope

Objective

Students use a sketch to explore how the slope of a line affects its graph.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

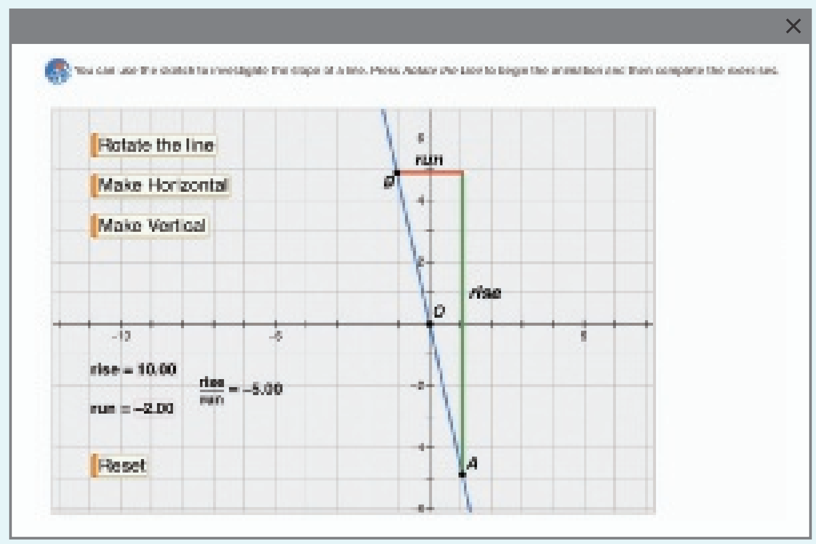
Summary of the Activity

Students will complete guiding exercises throughout the Explore activity. Students will use the sketch to see how the slope of a line changes as the line is rotated. They will observe how the rise and the run are affected as the line is rotated, and how that affects the calculation of the slope. They will explore lines with positive slopes and negative slopes and will investigate the slopes of horizontal and vertical lines. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

Explore



Explore

WEB SKETCHPAD

Students use a sketch to investigate the slope of a line.

TYPE

Students answer questions about the slope of a line.

Interactive Presentation

Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Investigating Slope (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How can the words “rise” and “run” remind you whether to look for a change in y or a change in x? Sample answer: You can think of rise as something going up or down, which goes along with a change in vertical distance along the y-axis. You can think of “run” as something you do on the ground, which is horizontal or along the x-axis.
- What does a slope of -3 tell you about the line? Sample answer: The negative sign tells me that the line will be decreasing as it moves from left to right. I also know that the line will go down three units for every one unit to the right.

Inquiry

How does slope help to describe a line? Sample answer: The slope of a line can tell you whether the graph of the line will slope up or down from left to right or if it will be a horizontal or vertical line.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Rate of Change of a Linear Function

Objective

Students calculate and interpret rate of change by identifying the change in the independent and dependent variables.

Teaching the Mathematical Practices

2 Make Sense of Quantities In this Learn, help students to notice the relationship between the variables when calculating rate of change.

Example 1 Find the Rate of Change

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

AL What are the quantities being compared in the table? the number of pancakes and the number of cups of flour

OL How do you know which is the independent variable and which is the dependent variable? Sample answer: The pancakes depend on the flour because the number of pancakes you can make depends on how much flour you use. So the number of pancakes is the dependent variable, and the amount of flour is the independent variable.

BL Would the ratio be different if you used the first and last pairs of values from the table to calculate the rate of change? Explain. No; sample answer: $\frac{36-12}{6-2} = \frac{24}{4}$ or 6.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Lesson 4-2

Rate of Change and Slope

Learn Rate of Change of a Linear Function

The **rate of change** is how a quantity is changing with respect to a change in another quantity.

If x is the independent variable and y is the dependent variable, then rate of change $= \frac{\text{change in } y}{\text{change in } x}$.

Example 1 Find the Rate of Change

COOKING **Find the rate of change of the function by using two points from the table.**

Amount of Flour x (cups)	Pancakes y
2	12
4	24
6	36

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{change in pancakes}}{\text{change in flour}} = \frac{24-12}{4-2} = \frac{12}{2} \text{ or } \frac{6}{1}$$

The rate is $\frac{6}{1}$ or 6. This means that you could make 6 pancakes for each cup of flour.

Check

Find the rate of change.

$\frac{?}{3.32}$ $\frac{\text{dollars}}{\text{gallons}}$

Amount of Gasoline Purchased (Gallons)	Cost (Dollars)
4.75	15.77
6	19.92
7.25	24.07
8.5	28.22

Today's Goals
- Calculate and interpret rate of change.
- Calculate and interpret slope.

Today's Vocabulary
rate of change
slope

Think About It!
Suppose you found a new recipe that makes 6 pancakes when using 2 cups of flour, 12 pancakes when using 4 cups of flour, and 18 pancakes when using 6 cups of flour. How does this change the rate you found for the original recipe?

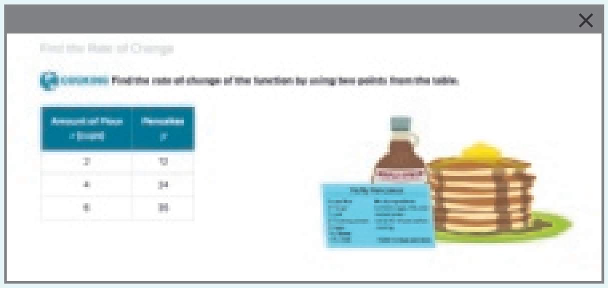
Sample answer: The new recipe makes half as many pancakes for the same amount of flour. So, the rate of the new recipe is half the rate of the original recipe and I could make only 3 pancakes per cup of flour.

Study Tip
Placement Be sure that the dependent variable is in the numerator and the independent variable is in the denominator. In this example, the number of pancakes you can make *depends* on the amount of flour you can use.

Go Online You can complete an Extra Example online.

Lesson 4-2 • Rate of Change and Slope 219

Interactive Presentation



Example 1

TYPE

Students explain how the rate of change can be used to find the number of pancakes for a given number of cups of flour.

Think About It!

How is a greater increase or decrease of funds represented graphically?

Sample answer: The line between points is steeper when there is a greater change in funds.

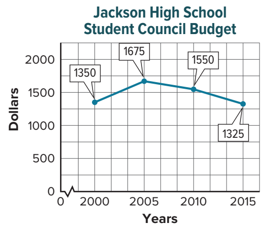
Study Tip

Assumptions In this example, we assumed that the rate of change for the budget was constant between each 5-year period. Although the budget might have varied from year to year, analyzing in larger periods of time allows us to see trends within data.

Example 2 Compare Rates of Change

STUDENT COUNCIL **The Jackson High School Student Council budget varies based on the fundraising of the previous year.**

Part A Find the rate of change for 2000–2005 and describe its meaning in the context of the situation.



$\frac{\text{change in budget}}{\text{change in time}} =$

$\frac{1675 - 1350}{2005 - 2000} = \frac{325}{5}$, or 65

This means that the student council's budget increased by \$325 over the 5-year period, with a rate of change of \$65 per year.

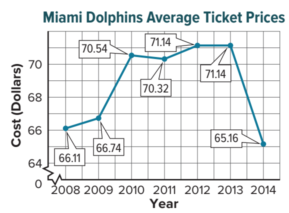
Part B Find the rate of change for 2010–2015 and describe its meaning in the context of the situation.

$\frac{\text{change in budget}}{\text{change in time}} = \frac{1325 - 1550}{2015 - 2010} = \frac{-225}{5}$, or $= -45$

This means that the student council's budget was reduced by \$225 over the 5-year period, with a rate of change of −\$45 per year.

Check

TICKETS The graph shows the average ticket prices for the Miami Dolphins football team.



Part A Find the rate of change in ticket prices between 2009–2010.

$\frac{?}{} \frac{\text{dollars}}{\text{year}}$ 3.80

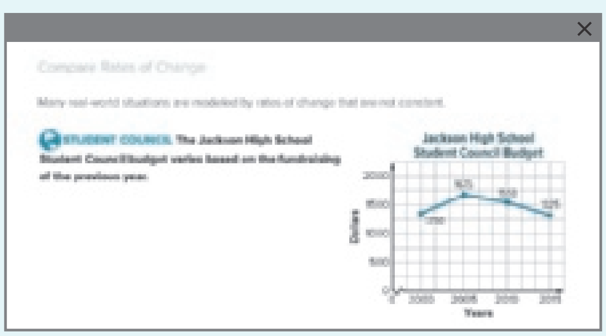
Part B The ticket prices have the greatest rate of change between ___?___ 2013–2014

Part C Between ___?___ 2010–2011 and ___?___ 2013–2014, the rate of change is negative.

Go Online You can complete an Extra Example online.

220 Module 4 • Linear and Nonlinear Functions

Interactive Presentation



Example 2

DRAG & DROP

Students complete the rate of change formula by dragging the values to the correct bins.

TYPE

Students answer a question about how a greater increase or decrease of funds is represented graphically.

Example 2 Compare Rates of Change

Teaching the Mathematical Practices

4 Make Assumptions In the Study Tip, have students point out where an assumption or approximation was made in the solution.

Questions for Mathematical Discourse

AL What are you trying to determine? the rate of change for two time periods: 2000–2005 and 2010–2015

OL What do x and y in the formula for the rate of change represent? y represents dollars, and x represents years

BL Can you find the rate of change by simply subtracting the numbers that are called out on the graph? Why or why not? No; sample answer: Subtracting those numbers will not produce a rate. Each listed point is at an interval of 5 years, so you need to divide by 5 to determine the rate of change per year.

Common Error

When interpreting a solution, some students may ignore the sign of a rate that is negative. Explain that in any real-world problem, the sign of a quantity has meaning. Help them to see that in this example, the negative rate of change means that the budget was reduced over that time period.

Example 3 Constant Rate of Change

Teaching the Mathematical Practices

8 Look for a Pattern Help students to see the pattern in this example.

Questions for Mathematical Discourse

AL What do you need to know in order to determine whether the function is linear? whether there is a constant rate of change

OL How does finding the differences between successive values in the table help you determine whether a function is linear? Sample answer: If the differences are the same, then I know that the rate of change is constant, and therefore the function is linear.

BL What is another way you can use the table to determine if the rate of change is constant? Sample answer: Because consecutive x-values decrease by 3, I can check to see if consecutive y-values increase or decrease by the same number. Because consecutive y-values increase by 2, I know there is a constant rate of change.

Example 4 Rate of Change

Teaching the Mathematical Practices

1 Explain Correspondences Use the Study Tip to encourage students to explain the relationship between the graph and rate of change of a linear function.

Questions for Mathematical Discourse

AL How will you determine whether the function is linear? Sample answer: I will find the changes in the x-values and the changes in the y-values, and see if those changes are constant.

OL Is it necessary to calculate the rate of change between every pair of points to determine linearity? Explain. No; sample answer: Once you have found two pairs that have different rates of change, you have shown that the function is not linear.

BL If you graphed the points from the table, would they lie on a straight line? How do you know? No; sample answer: Because the rates of change are not constant, the function is not linear, and therefore the graph of the points will not lie on a line.

Common Error

Some students may observe the pattern in the differences between the y–values (3, 2, 3, 2) and think that this regularity indicates that the function is linear. Correct this reasoning, and reinforce that when the differences in the x-values are the same, the differences in the y-values must also be the same for the function to be linear.

Example 3 Constant Rate of Change

Determine whether the function is linear. If it is, state the rate of change.

x	y
11	−5
8	−3
5	−1
2	1
−1	3

Find the changes in the x-values and the changes in the y-values.

Notice that the rate of change for each pair of points shown is $-\frac{2}{3}$.

The rates of change are constant, so the function is linear. The rate of change is $-\frac{2}{3}$.

Example 4 Rate of Change

Determine whether the function is linear. If it is, state the rate of change.

x	y
22	−4
29	−1
36	1
43	4
50	6

Find the changes in the x-values and the changes in the y-values.

The rates of change are not constant. Between some pairs of points the rate of change is $\frac{3}{7}$, and between the other pairs it is $\frac{2}{7}$. Therefore, this is not a linear function.

Study Tip

Linear Versus Not Linear Remember that the word *linear* means that the graph of the function is a straight line. For the graph of a function to be a line, it has to be increasing or decreasing at a constant rate.

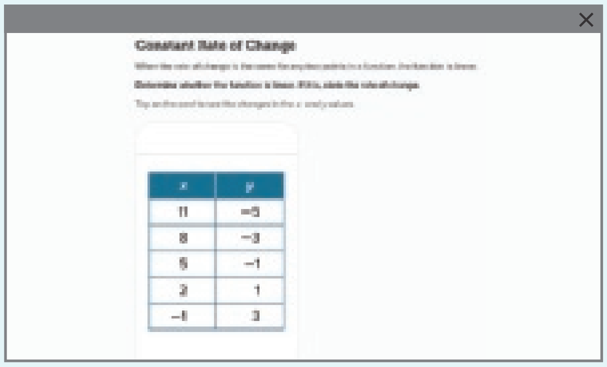
Check

Copy and complete the table so that the function is linear.

x	y
12	−2.25
11.5	1
11	4.25
10.5	7.5
10	10.75
9.5	14

Go Online You can complete an Extra Example online.

Interactive Presentation

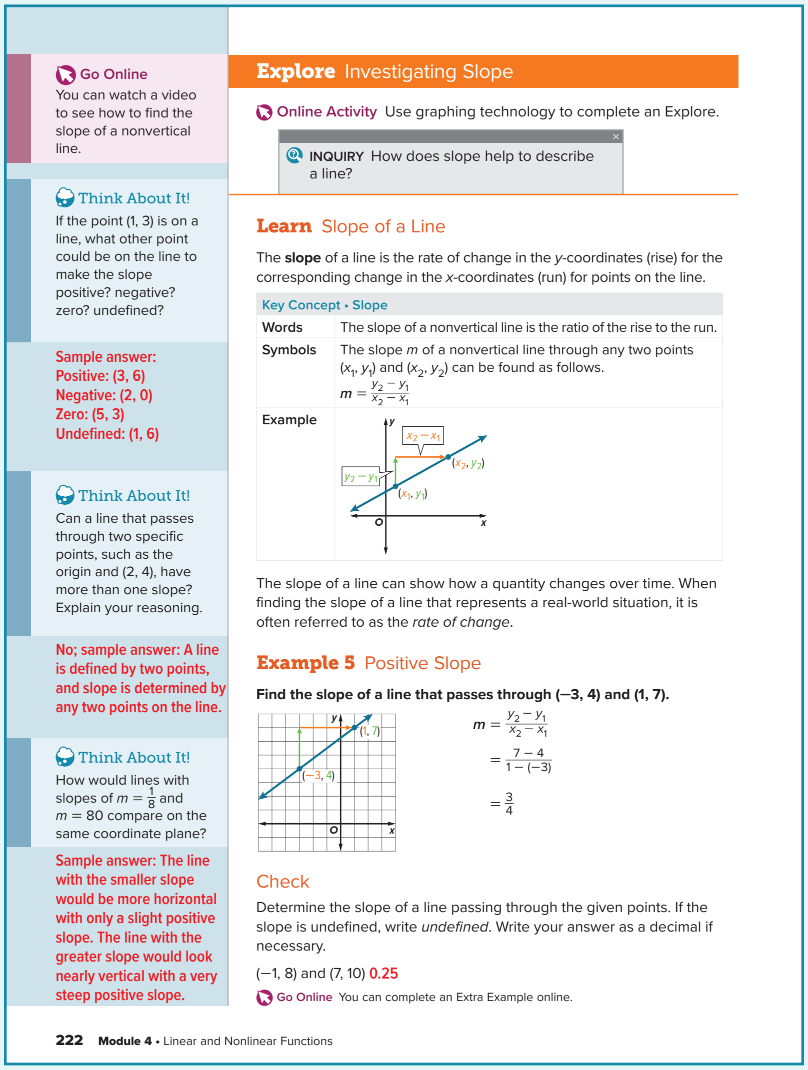


Example 3

TAP Students tap on the card to see the changes in the x- and y-values.

MULTI-SELECT Students select the correct word to complete the sentence.

CHECK Students complete the Check online to determine whether they are ready to move on.



Go Online You can watch a video to see how to find the slope of a nonvertical line.

Explore Investigating Slope

Online Activity Use graphing technology to complete an Explore.

INQUIRY How does slope help to describe a line?

Think About It! If the point (1, 3) is on a line, what other point could be on the line to make the slope positive? negative? zero? undefined?

Sample answer: Positive: (3, 6) Negative: (2, 0) Zero: (5, 3) Undefined: (1, 6)

Learn Slope of a Line

The **slope** of a line is the rate of change in the *y*-coordinates (rise) for the corresponding change in the *x*-coordinates (run) for points on the line.

Key Concept • Slope

Words	The slope of a nonvertical line is the ratio of the rise to the run.
Symbols	The slope m of a nonvertical line through any two points (x_1, y_1) and (x_2, y_2) can be found as follows. $m = \frac{y_2 - y_1}{x_2 - x_1}$
Example	

The slope of a line can show how a quantity changes over time. When finding the slope of a line that represents a real-world situation, it is often referred to as the *rate of change*.

Think About It! Can a line that passes through two specific points, such as the origin and (2, 4), have more than one slope? Explain your reasoning.

No; sample answer: A line is defined by two points, and slope is determined by any two points on the line.

Example 5 Positive Slope

Find the slope of a line that passes through (−3, 4) and (1, 7).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7 - 4}{1 - (-3)}$

$= \frac{3}{4}$

Think About It! How would lines with slopes of $m = \frac{1}{8}$ and $m = 80$ compare on the same coordinate plane?

Sample answer: The line with the smaller slope would be more horizontal with only a slight positive slope. The line with the greater slope would look nearly vertical with a very steep positive slope.

Check

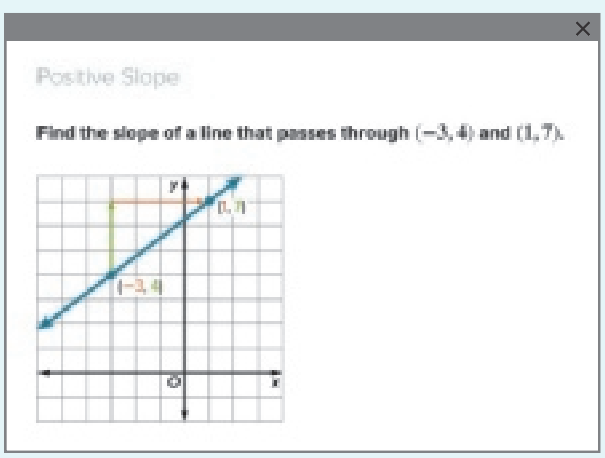
Determine the slope of a line passing through the given points. If the slope is undefined, write *undefined*. Write your answer as a decimal if necessary.

(−1, 8) and (7, 10) 0.25

Go Online You can complete an Extra Example online.

222 Module 4 • Linear and Nonlinear Functions

Interactive Presentation



Example 5

TYPE

Students describe and correct the error made when finding the slope of the line.

WATCH

Students can watch a video to see how find the slope of a nonvertical line.

Learn Slope of a Line

Objective

Students calculate and interpret slope by using the Slope Formula.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of slopes of lines in this Learn.

Example 5 Positive Slope

Questions for Mathematical Discourse

AL Finding the slope is the same as finding what other measure? the rate of change

OL Is the slope of this line positive, negative, or zero? How can you tell by looking at the graph? Positive; sample answer: The line slopes upward from left to right.

BL Does it matter which coordinates you use as x_2 and y_2? Explain. No; sample answer: You can use either of the *x*-coordinates as x_2, but the value for y_2 must then be the *y*-coordinate that corresponds with x_2.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students automatically assume that the left-most point has to be (x_1, y_1) and the point farther right is (x_2, y_2),

THEN explain that the designation of (x_1, y_1) and (x_2, y_2) is arbitrary. Write pairs of points on index cards. Give one card to each student. Have them find the slope both ways. Then ask which way made the subtraction easier.

DIFFERENTIATE

Language Development Activity ELL

Intermediate Instruct a small group of students to write a paragraph describing what is happening in the illustration of slope in the Key Concept box. Their paragraphs should describe all parts of the diagram in their own words. Ask for volunteers to read their paragraphs. Have students ask for clarification as needed.

Example 6 Negative Slope

Teaching the Mathematical Practices

8 Use Slope Help students to pay attention to the calculation of the slope of the line.

Questions for Mathematical Discourse

AL If $x_1 = -1$, what is the value of y_1? 3

OL Is the slope of this line positive, negative, or zero? How can you tell by looking at the graph? Negative; sample answer: The line slopes downward from left to right.

BL What would the value of the slope be if you used (4, 1) for (x_1, y_1) and (−1, 3) for (x_2, y_2)? It would still be $-\frac{2}{5}$.

Example 7 Slopes of Horizontal Lines

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationship between the graph, points, and slope in this example.

Questions for Mathematical Discourse

AL How would you describe what is meant by the slope of a line? Sample answer: It is the steepness of the line.

OL Is the slope positive, negative, or zero? zero

BL Why is the slope zero? Sample answer: The slope is zero because there is no change in *y*-values, so the numerator will be zero and zero divided by any number is zero.

Example 8 Slopes of Vertical Lines

Teaching the Mathematical Practices

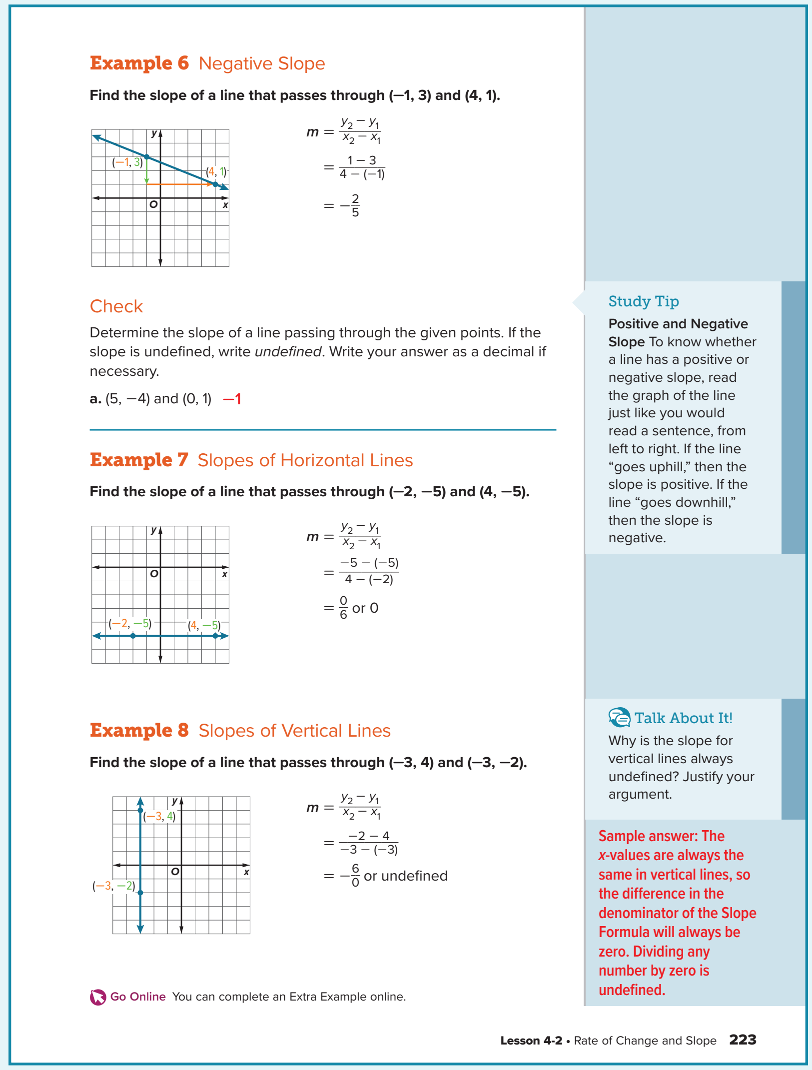
8 Use Slope Help students to pay attention to the calculation of slope for a vertical line.

Questions for Mathematical Discourse

AL Which values are the same? *x*-values: −3

OL Why is the slope undefined instead of zero? It is not possible to divide by 0. So, the slope of a vertical line is undefined.

BL Does the graph of a line with an undefined slope represent a function? Why or why not? No; sample answer: In a function, every *x*-value is paired with exactly one *y*-value. In a relation that is represented by a vertical line, there is one *x*-value paired with infinitely many *y*-values.



Example 6 Negative Slope

Find the slope of a line that passes through (−1, 3) and (4, 1).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{1 - 3}{4 - (-1)}$

$= -\frac{2}{5}$

Check

Determine the slope of a line passing through the given points. If the slope is undefined, write *undefined*. Write your answer as a decimal if necessary.

a. (5, −4) and (0, 1) −1

Example 7 Slopes of Horizontal Lines

Find the slope of a line that passes through (−2, −5) and (4, −5).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-5 - (-5)}{4 - (-2)}$

$= \frac{0}{6}$ or 0

Example 8 Slopes of Vertical Lines

Find the slope of a line that passes through (−3, 4) and (−3, −2).

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-2 - 4}{-3 - (-3)}$

$= -\frac{6}{0}$ or undefined

Go Online You can complete an Extra Example online.

Study Tip

Positive and Negative Slope To know whether a line has a positive or negative slope, read the graph of the line just like you would read a sentence, from left to right. If the line "goes uphill," then the slope is positive. If the line "goes downhill," then the slope is negative.

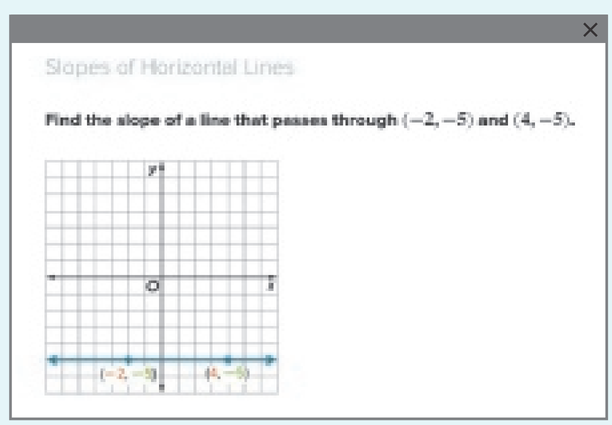
Talk About It!

Why is the slope for vertical lines always undefined? Justify your argument.

Sample answer: The *x*-values are always the same in vertical lines, so the difference in the denominator of the Slope Formula will always be zero. Dividing any number by zero is undefined.

Lesson 4-2 • Rate of Change and Slope 223

Interactive Presentation



Example 7

Study Tip

Converting Slope When solving for an unknown coordinate, like the previous example, converting a slope from a decimal or mixed number to an improper fraction might make the problem easier to solve. For example, a slope of $1.\overline{333}$ can be rewritten as $\frac{4}{3}$.

Example 9 Find Coordinates Given the Slope

Find the value of r so that the line passing through (−4, 5) and (4, r) has a slope of $\frac{3}{4}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$	Use the Slope Formula.
$\frac{3}{4} = \frac{r - 5}{4 - (-4)}$	$(-4, 5) = (x_1, y_1)$ and $(4, r) = (x_2, y_2)$
$\frac{3}{4} = \frac{r - 5}{8}$	Subtract.
$8\left(\frac{3}{4}\right) = \frac{8(r - 5)}{8}$	Multiply each side by 8.
$6 = r - 5$	Simplify.
$6 + 5 = r - 5 + 5$	Add 5 to each side.
$11 = r$	Simplify.

Check

Find the value of r so that the line passing through $(-3, r)$ and $(7, -6)$ has a slope of $2\frac{2}{5}$.

$r =$ ___?___ −30

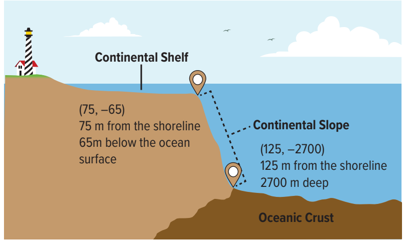
Think About It!

If a crab is walking along the ocean floor 112 meters away from the shoreline to 114 meters away from the shoreline, how far does it descend?

Sample answer: 105.4 meters

Example 10 Use Slope

OCEANS **What is the slope of the continental slope at Cape Hatteras?**



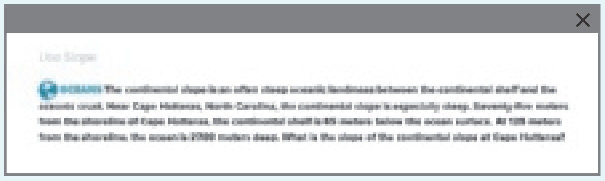
$m = \frac{y_2 - y_1}{x_2 - x_1}$	Use the Slope Formula
$= \frac{-2700 - (-65)}{125 - 75}$	$(75, -65) = (x_1, y_1)$ and $(125, -2700) = (x_2, y_2)$
$= \frac{-2635}{50}$ or -52.7	Simplify.

The continental slope at Cape Hatteras has a slope of −52.7.

Go Online You can complete an Extra Example online.

224 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Example 10

TAP

Students tap on each marker to find the points to use in the Slope Formula.

TYPE

Students use the slope to answer a question about another point along the continental slope.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 9 Find Coordinates Given the Slope

Teaching the Mathematical Practices

1 Check Answers Mathematically proficient students continually ask themselves, "Does this make sense?" In this example, encourage students to check their answer.

Questions for Mathematical Discourse

AL For what variable in the equation do you substitute $\frac{3}{4}$? m

OL How could a graph help determine the missing coordinate? Sample answer: I can plot the given point and then use the slope to move to the next point. I can continue using the slope until I get to the point with the x-coordinate of 4.

BL Name another point on the same line. Sample answers: (0, 8), (8, 14)

Example 10 Use Slope

Teaching the Mathematical Practices

4 Interpret Mathematical Results In this example, point out that to solve the problem, students should interpret their mathematical results in the context of the problem.

Questions for Mathematical Discourse

AL What are the two ordered pairs you can use to find the slope? (75, −65) and (125, −2700)

OL Interpret the value of the slope in the context of the problem. Sample answer: The slope means that the water gets 52.7 meters deeper for every meter you move farther from shore.

BL Do you think the continental slope is constant? Sample answer: No, there are probably places where the drop is less steep and places where it is more steep.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–49
2	exercises that use a variety of skills from this lesson	50–58
2	exercises that extend concepts learned in this lesson to new contexts	59–62
3	exercises that emphasize higher-order and critical-thinking skills	63–68

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: BL

- Practice, Exercises 1–61 odd, 63–68
- Extension: Treasure Hunt with Slopes
- ALEKS® Equations of Lines

IF students score 66%–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–67 odd
- Remediation, Review Resources: Order of Integer Operations
- Personal Tutors
- Extra Examples 1–10
- ALEKS® Multiplication and Division with Integers

IF students score 65% or less on the Checks, **THEN** assign: AL

- Practice, Exercises 1–49 odd
- Remediation, Review Resources: Order of Integer Operations
- *Quick Review Math Handbook*: Rate of Change and Slope
- Arrive**MATH** Take Another Look
- ALEKS® Multiplication and Division with Integers

Practice

Go Online You can complete your homework online.

Example 1

Find the rate of change of the function by using two points from the table.

1. $\frac{1}{5}$

x	y
5	2
10	3
15	4
20	5

2. -6

x	y
1	15
2	9
3	3
4	−3

3. POPULATION DENSITY The table shows the population density for the state of Texas in various years. Find the average annual rate of change in the population density from 2000 to 2009. increased about 1.9 people per square mile

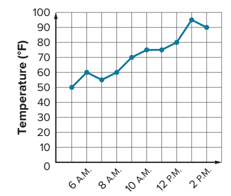
Population Density	
Year	People Per Square Mile
1930	22.1
1960	36.4
1980	54.3
2000	79.6
2009	96.7

Source: Bureau of the Census, U.S. Dept. of Commerce

4. BAND In 2012, there were approximately 275 students in the Delaware High School band. In 2018, that number increased to 305. Find the annual rate of change in the number of students in the band. increased by 5/yr

Example 2

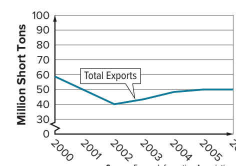
5. TEMPERATURE The graph shows the temperature in a city during different hours of one day.



a. Find the rate of change in temperature between 6 A.M. and 7 A.M. and describe its meaning in the context of the situation. −5; This means the temperature decreased 5°F per hour from 6 A.M. to 7 A.M.

b. Find the rate of change in temperature from 1 P.M. and 2 P.M. and describe its meaning in the context of the situation. −5; This means the temperature decreased 5°F per hour from 1 P.M. to 2 P.M.

6. COAL EXPORTS The graph shows the annual coal exports from U.S. mines in millions of short tons.



Source: Energy Information Association

a. Find the rate of change in coal exports between 2000 and 2002 and describe its meaning in the context of the situation. −10; This means the coal exports decreased 10 million tons per year between 2000 and 2002.

b. Find the rate of change in coal exports between 2005 and 2006 and describe its meaning in the context of the situation. 0; This means the coal exports did not change between 2005 and 2006.

Examples 3 and 4

Determine whether the function is linear. If it is, state the rate of change.

7.

x	4	2	0	−2	−4
y	−1	1	3	5	7

linear; $-\frac{1}{1}$ or -1

8.

x	−7	−5	−3	−1	0
y	11	14	17	20	23

not linear

9.

x	−0.2	0	0.2	0.4	0.6
y	0.7	0.4	0.1	0.3	0.6

not linear

10.

x	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

linear; $\frac{1}{2}$

Examples 5 through 8

Find the slope of the line that passes through each pair of points.

11. $(4, 3), (-1, 6)$ $-\frac{3}{5}$
12. $(8, -2), (1, 1)$ $-\frac{3}{7}$
13. $(2, 2), (-2, -2)$ 1
14. $(6, -10), (6, 14)$ undefined
15. $(5, -4), (9, -4)$ 0
16. $(11, 7), (-6, 2)$ $\frac{5}{17}$
17. $(-3, 5), (3, 6)$ $\frac{1}{6}$
18. $(-3, 2), (7, 2)$ 0
19. $(8, 10), (-4, -6)$ $\frac{4}{3}$
20. $(-12, 15), (18, -13)$ $-\frac{14}{15}$
21. $(-8, 6), (-8, 4)$ undefined
22. $(-8, -15), (-2, 5)$ $\frac{10}{3}$
23. $(2, 5), (3, 6)$ 1
24. $(6, 1), (-6, 1)$ 0
25. $(4, 6), (4, 8)$ undefined
26. $(-5, -8), (-8, 1)$ -3
27. $(2, 5), (-3, -5)$ 2
28. $(9, 8), (7, -8)$ 8
29. $(5, 2), (5, -2)$ undefined
30. $(10, 0), (-2, 4)$ $-\frac{1}{3}$
31. $(17, 18), (18, 17)$ -1
32. $(-6, -4), (4, 1)$ $\frac{1}{2}$
33. $(-3, 10), (-3, 7)$ undefined
34. $(2, -1), (-8, -2)$ $\frac{1}{10}$
35. $(5, -9), (3, -2)$ $-\frac{7}{2}$
36. $(12, 6), (3, -5)$ $\frac{11}{9}$
37. $(-4, 5), (-8, -5)$ $\frac{5}{2}$

Example 9

Find the value of *r* so the line that passes through each pair of points has the given slope.

38. $(12, 10), (-2, r), m = -4$ 66
39. $(r, -5), (3, 13), m = 8$ $\frac{3}{4}$
40. $(3, 5), (-3, r), m = \frac{3}{4}$ $\frac{1}{2}$
41. $(-2, 8), (r, 4), m = -\frac{1}{2}$ 6
42. $(r, 3), (5, 9), m = 2$ 2
43. $(5, 9), (r, -3), m = -4$ 8
44. $(r, 2), (6, 3), m = \frac{1}{2}$ 4
45. $(r, 4), (7, 1), m = \frac{3}{4}$ 11

Answer

64. Sample answer: Slope can be used to describe a rate of change. Rate of change is a ratio that describes how much one quantity changes with respect to a change in another quantity. The slope of a line is also a ratio and it is the ratio of the change in the *y*-coordinates to the change in the *x*-coordinates.

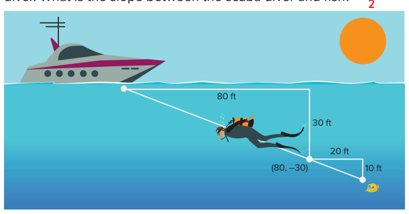
Example 10

46. ROAD SIGNS Roadway signs such as the one shown are used to warn drivers of an upcoming steep down grade. What is the grade, or slope, of the hill described on the sign? $\frac{2}{25}$

47. HOME MAINTENANCE Grading the soil around the foundation of a house can reduce interior home damage from water runoff. For every 6 inches in height, the soil should extend 10 feet from the foundation. What is the slope of the soil grade? $\frac{1}{20}$

48. USE A SOURCE Research the Americans with Disabilities Act (ADA) regulation for the slope of a wheelchair ramp? What is the slope of an ADA regulation ramp? Use the slope to determine the length and height of an ADA regulation ramp. $\frac{1}{12}$; Sample answer: length: 60 inches, height: 5 inches

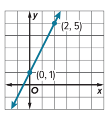
49. DIVERS A boat is located at sea level. A scuba diver is 80 feet along the surface of the water from the boat and 30 feet below the water surface. A fish is 20 feet along the horizontal plane from the scuba diver and 10 feet below the scuba diver. What is the slope between the scuba diver and fish? $-\frac{1}{2}$



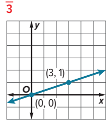
Mixed Exercises

STRUCTURE **Find the slope of the line that passes through each pair of points.**

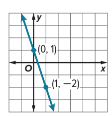
50. 2



51. $\frac{1}{3}$



52. -3



53. $(6, -7), (4, -8)$ $\frac{1}{2}$

54. $(0, 5), (5, 5)$ 0

55. $(-2, 6), (-5, 9)$ -1

56. $(5, 8), (-4, 6)$ $\frac{2}{9}$

57. $(9, 4), (5, -3)$ $\frac{7}{4}$

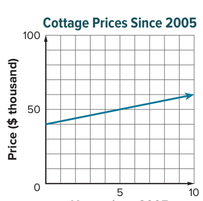
58. $(1, 4), (3, -1)$ $-\frac{5}{2}$

59. REASONING Find the value of *r* that gives the line passing through (3, 2) and $(r, -4)$ a slope that is undefined. 3

60. REASONING Find the value of *r* that gives the line that passing through $(-5, 2)$ and $(3, r)$ a slope of 0. 2

61. CREATE Draw a line on a coordinate plane so that you can determine at least two points on the graph. Describe how you would determine the slope of the graph and justify the slope you found. After drawing a graph, use the two points on the graph to determine the slope. This can be done by counting squares for the rise and run of the line or by using the coordinates of the points in the slope formula.

62. ARGUMENTS The graph shows median prices for small cottages on a lake since 2005. A real estate agent says that since 2005, the rate of change for house prices is $10,000 each year. Do you agree? Use the graph to justify your answer. No; The graph appears to show an increase in price of about $10,000 over 5 years or about $2000 per year.



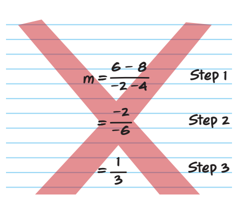
Higher-Order Thinking Skills

63. CREATE Use what you know about rate of change to describe the function represented by the table. The rate of change is $2\frac{1}{4}$ inches of growth per week.

Time (wk)	Height of Plant (in.)
4	9.0
6	13.5
8	18.0

64. WRITE Explain how the rate of change and slope are related and how to find the slope of a line. See margin.

65. FIND THE ERROR Fern is finding the slope of the line that passes through $(-2, 8)$ and $(4, 6)$. Determine in which step she made an error. Explain your reasoning. Step 1; she reversed the order of the *x*-coordinates in the formula.



66. PERSEVERE Find the value of *d* so that the line that passes through (a, b) and (c, d) has a slope of $\frac{1}{2}$. $\frac{c-a+2b}{2}$

67. ANALYZE Why is the slope undefined for vertical lines? Explain. The difference in the *x*-values is always 0, and division by 0 is undefined.

68. WRITE Tarak wants to find the value of *a* so that the line that passes through $(10, a)$ and $(-2, 8)$ has a slope of $\frac{1}{4}$. Explain how Tarak can find the value of *a*. Use the slope formula. Substitute $(10, a)$ for (x_1, y_1), $(-2, 8)$ for (x_2, y_2), and $\frac{1}{4}$ for *m*. Cross multiply and then solve the equation to find that $a = 11$.

Lesson 4-3

Slope-Intercept Form

A.CED.2, F.IF.7a, F.LE.5

LESSON GOAL

Students graph equations in slope-intercept form.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Writing Linear Equations in Slope-Intercept Form

- Write Linear Equations in Slope-Intercept Form
- Rewrite Linear Equations in Slope-Intercept Form
- Write Linear Equations

Explore: Graphing Linear Functions by Using the Slope-Intercept Form

Develop:

Graphing Linear Functions in Slope-Intercept Form

- Graph Linear Functions in Slope-Intercept Form
- Graph Linear Functions
- Graph Constant Functions
- Use Graphs of Linear Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Slope of a Line	●	●		●
Extension: Pencils of Lines		●	●	●

Language Development Handbook

Assign page 22 of the *Language Development Handbook* to help your students build mathematical language related to equations in slope-intercept form.

ELL You can use the tips and suggestions on page T22 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Algebra, Functions

Standards for Mathematical Content:

A.CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

4 Model with mathematics.

5 Use appropriate tools strategically.

Coherence

Vertical Alignment

Previous
Students found and interpreted the rate of change and slopes of lines.
F.IF.6, F.LE.5

Now
Students graph equations in slope-intercept form.
A.CED.2, F.IF.7a, F.LE.5

Next
Students will Identify the effects of transformations of the graphs of linear functions.
F.IF.7a, F.BF.3

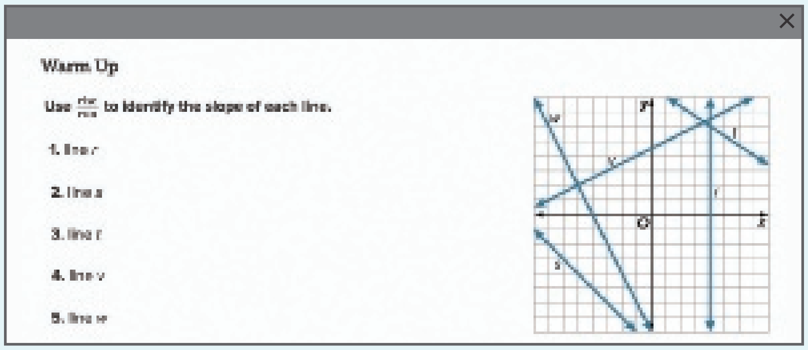
Rigor

The Three Pillars of Rigor

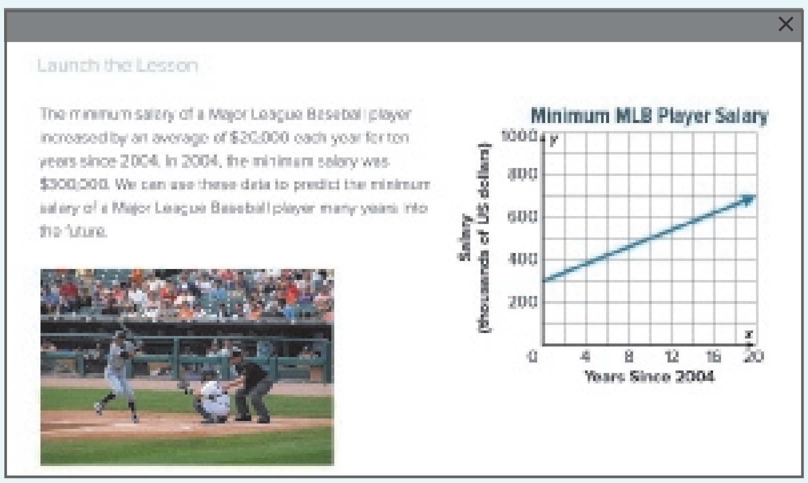
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of slope. They build fluency by rewriting equations in slope-intercept form to find the slope and *y*-intercept. They apply their understanding by solving real-world problems involving slope and *y*-intercept.

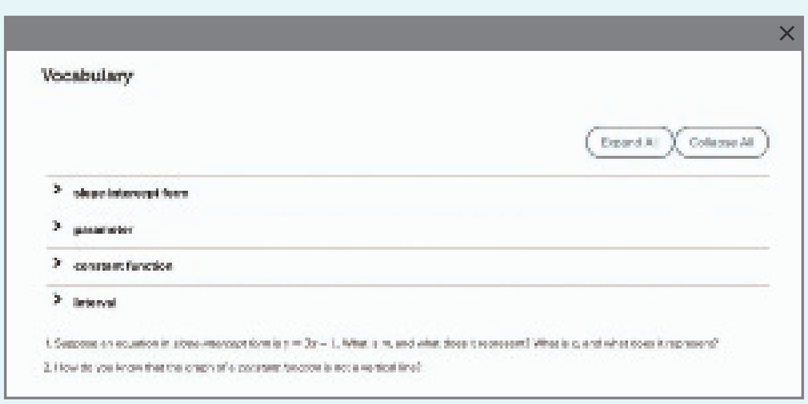
Interactive Presentation



Warm Up



Launch the Lesson



Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- identifying slopes

Answers:

1. undefined
2. -1
3. $-\frac{2}{3}$
4. $\frac{1}{2}$
5. -2

Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the verbal description and graphs representing the minimum salary for a Major League Baseball player.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope, and b is the y-intercept. Writing a linear equation in this form is helpful when you want to graph the function. There are two methods that can be used. The first is to select two values of x, substitute those values into the equation to calculate the corresponding values of y, plot the resulting ordered pairs, and draw the line that passes through the points. The second method is to plot the y-intercept, use it as a starting point, and then use the slope to determine another point on the line. The line can then be drawn through the two points.

Explore Graphing Linear Functions by Using the Slope-Intercept Form

Objective

Students use a sketch to explore how changing the slope and *y*-intercept changes the graph of the line.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in the Explore, students will need to use a sketch. Work with students to explore and deepen their understanding of slope-intercept form of a linear equation.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

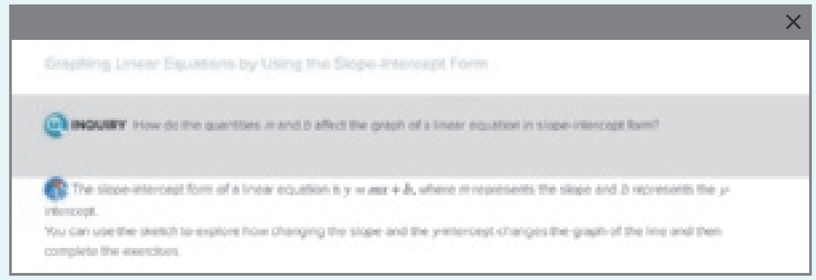
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

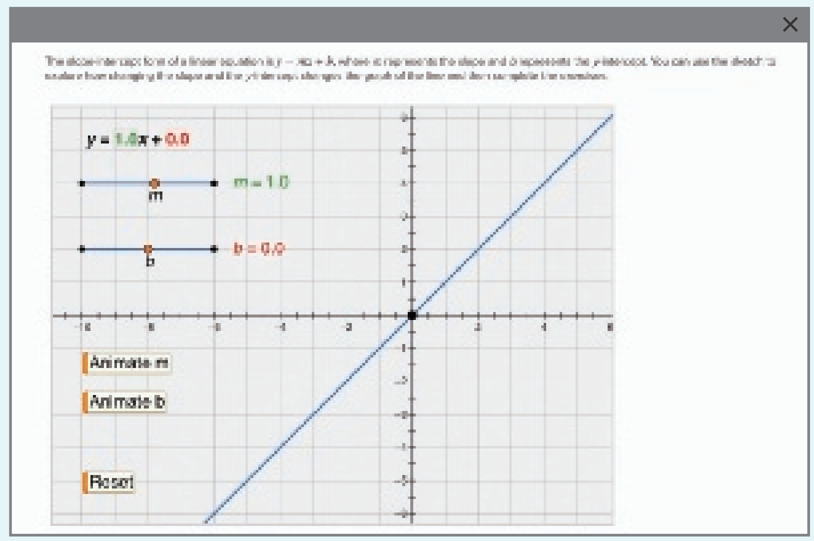
Students will complete guiding exercises throughout the Explore activity. Students will use the sketch to explore how changing the value of *m* and *b* in the equation of a line affects the graph of the function. They will use sliders and animations to change the values of *m* and/or *b* in a linear equation, and observe the change in orientation of the related line. Then, students will answer the Inquiry Question.

(*continued on the next page*)

Interactive Presentation



Explore



Explore

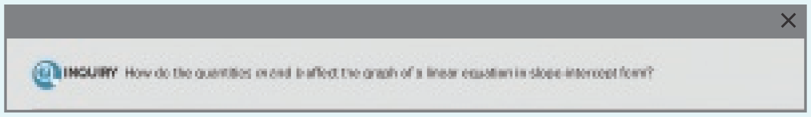
WEB SKETCHPAD

Students use a sketch to graph a line by changing the slope and *y*-intercept.

TYPE

Students answer questions about changing the parameters in the slope-intercept form of a line.

Interactive Presentation



Explore

TYPE

a|

Students respond to the Inquiry Question and can view a sample answer.

Explore Graphing Linear Functions by Using the Slope-Intercept Form (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Describe the graph when $0 < m < 1$. Sample answer: When the slope is a fraction between 0 and 1, the run is greater than the rise. This means that the slant of the line is more gradual.
- What are the slope and y-intercept of $y = \frac{2}{3}x - 4$? The slope is $\frac{2}{3}$ and the y-intercept is -4.

Inquiry

How do the quantities m and b affect the graph of a linear function in slope-intercept form? Sample answer: Changing the slope affects the steepness of the graph. Changing the y-intercept determines the distance and direction that the graph is shifted from the origin.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Writing Linear Equations in Slope-Intercept Form

Objective

Students rewrite equations in slope-intercept form by applying the properties of equality.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 1 Write Linear Equations in Slope-Intercept Form

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the verbal description and equation in this example.

Questions for Mathematical Discourse

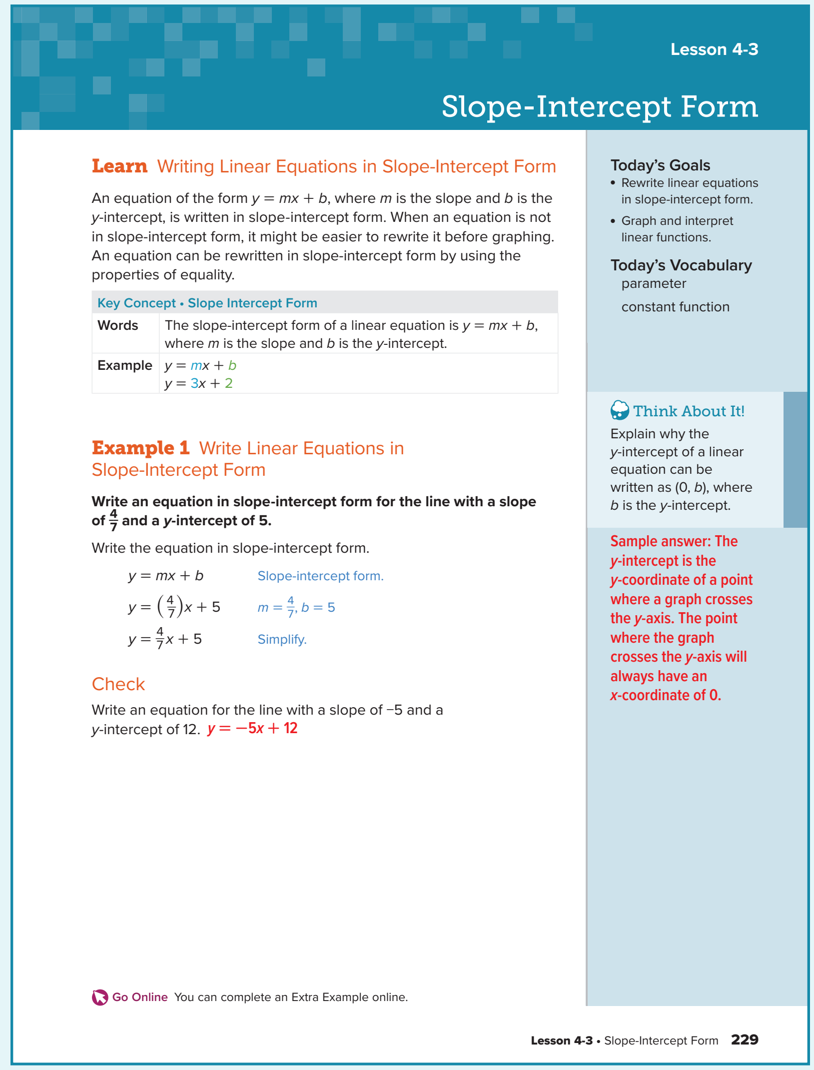
AL What is the slope of the line? $\frac{4}{7}$

OL Which variable represents the slope in $y = mx + b$? m

BL How would this equation have changed if the slope had been $-\frac{4}{7}$? It would have been $y = -\frac{4}{7}x + 5$.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.



Lesson 4-3

Slope-Intercept Form

Learn Writing Linear Equations in Slope-Intercept Form

An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is written in slope-intercept form. When an equation is not in slope-intercept form, it might be easier to rewrite it before graphing. An equation can be rewritten in slope-intercept form by using the properties of equality.

Key Concept • Slope Intercept Form	
Words	The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.
Example	$y = mx + b$ $y = 3x + 2$

Example 1 Write Linear Equations in Slope-Intercept Form

Write an equation in slope-intercept form for the line with a slope of $\frac{4}{7}$ and a y-intercept of 5.

Write the equation in slope-intercept form.

$y = mx + b$ Slope-intercept form.

$y = \left(\frac{4}{7}\right)x + 5$ $m = \frac{4}{7}$, $b = 5$

$y = \frac{4}{7}x + 5$ Simplify.

Check

Write an equation for the line with a slope of −5 and a y-intercept of 12. $y = -5x + 12$

Go Online You can complete an Extra Example online.

Today's Goals
- Rewrite linear equations in slope-intercept form.
- Graph and interpret linear functions.

Today's Vocabulary
parameter
constant function

Think About It!
Explain why the y-intercept of a linear equation can be written as $(0, b)$, where b is the y-intercept.

Sample answer: The y-intercept is the y-coordinate of a point where a graph crosses the y-axis. The point where the graph crosses the y-axis will always have an x-coordinate of 0.

Lesson 4-3 • Slope-Intercept Form 229

Interactive Presentation

Example 1

TYPE

Students explain how the equation would change if the y-intercept was negative.

Example 2 Rewrite Linear Equations in Slope-Intercept Form

Write $-22x + 8y = 4$ in slope-intercept form.

$-22x + 8y = 4$	Original equation
$-22x + 8y + 22x = 4 + 22x$	Add $22x$ to each side.
$8y = 22x + 4$	Simplify.
$\frac{8y}{8} = \frac{22x + 4}{8}$	Divide each side by 8.
$y = 2.75x + 0.5$	Simplify.

Think About It!
Can $x = 5$ be rewritten in slope-intercept form? Justify your argument.

No; sample answer: $x = 5$ is a vertical line, and vertical lines have no slope. So, $x = 5$ cannot be rewritten in slope-intercept form.

Check

What is the slope intercept form of $-16x - 4y = -56$? $y = -4x + 14$

Example 3 Write Linear Equations

JOBS **The number of job openings in the United States during a recent year increased by an average of 0.06 million per month since May. In May, there were about 4.61 million job openings in the United States. Write an equation in slope-intercept form to represent the number of job openings in the United States in the months since May.**

Use the given information to write an equation in slope-intercept form.

- You are given that there were 4.61 million job openings in May.
- Let x = the number of months since May and y = the number of job openings in millions.
- Because the number of job openings is 4.61 million when $x = 0$, $b = 4.61$, and because the number of job openings has increased by 0.06 million each month, $m = 0.06$.
- So, the equation $y = 0.06x + 4.61$ represents the number of job openings in the United States since May.

Think About It!
When $x = 2$, describe the meaning of the equation in the context of the situation.

Sample answer: When $x = 2$, the equation represents the number of job openings in July, or two months after May.

Check

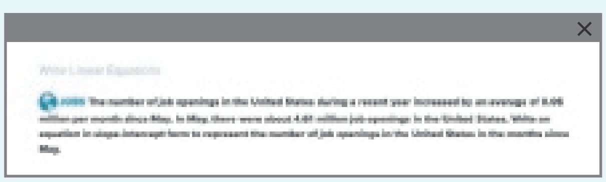
SOCIAL MEDIA In the first quarter of 2012, there were 183 million users of a popular social media site in North America. The number of users increased by an average of 9 million per year since 2012. Write an equation that represents the number of users in millions of the social media site in North America after 2012.

$y = 9x + 183$

Go Online You can complete an Extra Example online.

230 Module 4 • Linear and Nonlinear Functions

Interactive Presentation



Example 3

TYPE

Students explain the meaning of a certain x-value in context of the situation.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Rewrite Linear Equations in Slope-Intercept Form

Teaching the Mathematical Practices

3 Justify Conclusions Mathematically proficient students can explain the conclusions drawn when solving a problem. The Think About It! feature asks students to justify their conclusions.

Questions for Mathematical Discourse

AL Is this equation in slope-intercept form? Why? No; sample answer: Slope-intercept form is $y = mx + b$, and in this equation, the y-variable is not isolated.

OL How do you know if a linear equation is in slope-intercept form? Sample answer: The y-variable is isolated and it is in the form $y = mx + b$

BL How would this problem be different if the original equation had been $-22x - 8y = 4$? The last step would have involved dividing by -8 instead of 8, resulting in $y = -2.75x - 0.5$.

Example 3 Write Linear Equations

Teaching the Mathematical Practices

2 Attend to Quantities Point out that it is important to note the meaning of the quantities used in this problem.

Questions for Mathematical Discourse

AL Which number is the y-intercept? the slope? 4.61, 0.06

OL What do the slope and y-intercept represent in the context of this situation? the increase in the number of millions of job openings per month since May; 4.61 million job openings in May

BL What would it mean if the rate of change was -0.06 in the context of the situation? Sample answer: It would mean a decrease of 0.06 million job openings per month.

Learn Graphing Linear Functions in Slope-Intercept Form

Objective

Students graph and interpret linear functions by writing them in slope-intercept form.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between linear functions in slope-intercept form and their graphs.

Common Misconception

Some students may think that when the slope is negative, they should count down for the rise and left for the run to find additional points. Show students that this would lead to a line that is rising from left to right, not falling, as would be the orientation for a line with a negative slope. Tell them to count up and to the right for positive slopes, and down and to the right for negative slopes.

Example 4 Graph Linear Functions in Slope-Intercept Form

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL In slope-intercept form, which variable represents the slope? *m* the *y*-intercept? *b*

OL When graphing a line in slope-intercept form, why is *b* graphed first? Sample answer: In order to use the slope, you have to have a starting point.

BL Why do you find the next point by counting down 3 and to the right 2? Sample answer: The slope is negative, so instead of counting up and to the right, you count down and to the right.

Explore Graphing Linear Equations by Using the Slope-Intercept Form

Online Activity Use graphing technology to complete an Explore.

INQUIRY How do the quantities *m* and *b* affect the graph of a linear equation in slope-intercept form?

Learn Graphing Linear Functions in Slope-Intercept Form

The slope-intercept form of a linear equation is $y = mx + b$ where m is the slope and b is the y-intercept. The variables m and b are called **parameters** of the equation because changing either value changes the graph.

A **constant function** is a linear function of the form $y = b$. Constant functions where $b \neq 0$ do not cross the x-axis. The graphs of constant functions have a slope of 0. The domain of a constant function is all real numbers, and the range is b.

Example 4 Graph Linear Equations in Slope-Intercept Form

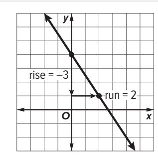
Graph a linear equation with a slope of $-\frac{3}{2}$ and a y-intercept of 4.

Write the equation in slope-intercept form and graph the equation.

$y = mx + b$

$y = \left(-\frac{3}{2}\right)x + 4$

$y = -\frac{3}{2}x + 4$



Study Tip

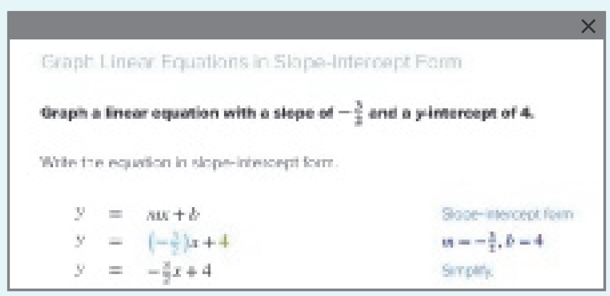
Negative Slope When counting rise and run, a negative sign may be associated with the value in the numerator or denominator. In this case, we associated the negative sign with the numerator. If we had associated it with the denominator, we would have moved up 3 and left 2 to the point $(-2, 7)$. Notice that this point is also on the line. The resulting line will be the same whether the negative sign is associated with the numerator or denominator.

Think About It!

Use the slope to find another point on the graph. Explain how you found the point.

Sample answer: $(4, -2)$; I moved down 3 units and right 2 units from the point $(2, 1)$

Interactive Presentation



Example 4

WEB SKETCHPAD

Students use a sketch to graph a linear function in slope-intercept form.

TYPE

Students explain how to find another point on the graph by using the slope.

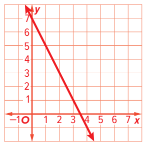
Talk About It!

Why is it useful to write an equation in slope-intercept form before graphing it?

Sample answer: When an equation is in slope-intercept form, you can easily determine the slope and *y*-intercept and use them to create the graph.

Check

Graph a linear function with a slope of −2 and a *y*-intercept of 7.



Example 5 Graph Linear Functions

Graph $12x - 3y = 18$.

Rewrite the equation in slope-intercept form.

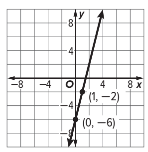
$12x - 3y = 18$	Original equation
$12x - 3y - 12x = 18 - 12x$	Subtract 12*x* from each side.
$-3y = -12x + 18$	Simplify.
$\frac{-3y}{-3} = \frac{-12x + 18}{-3}$	Divide each side by −3.
$y = 4x - 6$	Simplify.

Graph the equation.

Plot the *y*-intercept (0, −6).

The slope is $\frac{rise}{run} = 4$. From (0, −6), move up 4 units and right 1 unit. Plot the point (1, −2).

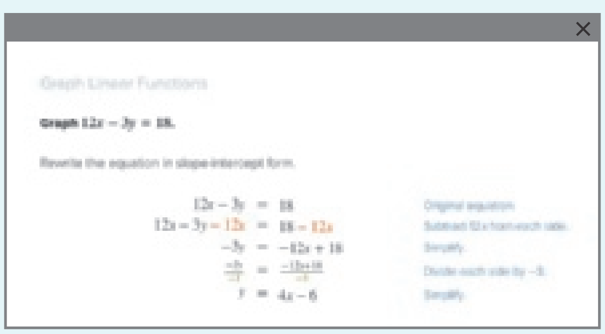
Draw a line through the points (0, −6) and (1, −2).



Go Online You can complete an Extra Example online.

232 Module 4 • Linear and Nonlinear Functions

Interactive Presentation



Example 5

TAP

Students move through the steps to graph a linear function.

TYPE

Students explain why it is useful to write an equation in slope-intercept form before graphing.

Example 5 Graph Linear Functions

Teaching the Mathematical Practices

1 Seek Information Mathematically proficient students must be able to transform algebraic expressions to reach solutions. Point out that gaining fluency in this skill is as important as learning their math facts was in the elementary grades.

Questions for Mathematical Discourse

AL What variable must you solve for in order to write the equation in slope-intercept form? *y*

OL What are the slope and the *y*-intercept of the line? The slope is 4. The *y*-intercept is −6.

BL How can the intercepts of the line be used to check your answer? Sample answer: Using the given form of the line, I know the *x*-intercept will be (1.5, 0) and the *y*-intercept will be (0, −6). My graph crosses at those points, so the graph is correct.

Common Error

For an equation such as $y = 4x - 6$, some students may state that $b = 6$. Review the general form of the slope-intercept form of a linear equation ($y = mx + b$), and highlight the plus sign. Help students to see that $y = 4x - 6$ is equivalent to $y = 4x + (-6)$, so $b = -6$. Therefore, the *y*-intercept is −6.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have difficulty distinguishing between the variables and the parameters in the equation,

THEN write several different equations on the board, each in slope-intercept form, and point out that in each case, the equation contains numbers where *m* and *b* would be the parameters while the variables *x* and *y* represent the coordinates of the solutions of the equation. Examining several equations side by side helps to strengthen understanding of the concept.

DIFFERENTIATE

Enrichment Activity BL

Write $3x + 2y = 8$ and $-3x + 2y = 8$ on the board. Ask students to tell how the equations are alike and how they are different. Then, ask students to tell how the graphs of the two functions are alike and how they are different without graphing them. Finally, have them graph the functions and check their answers.

Example 6 Graph Constant Functions

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to solve the problem in this example, students will need to use a sketch. Work with students to explore and deepen their understanding of slope-intercept form.

Questions for Mathematical Discourse

AL What is the y-intercept? x-intercept? 2; There is no x-intercept.

OL Why is the graph a horizontal line? Sample answer: Because the slope is 0, the graph will not rise, but can run left to right any amount.

BL What is the domain of this function? the range? D = all real numbers; R = 2

Common Error

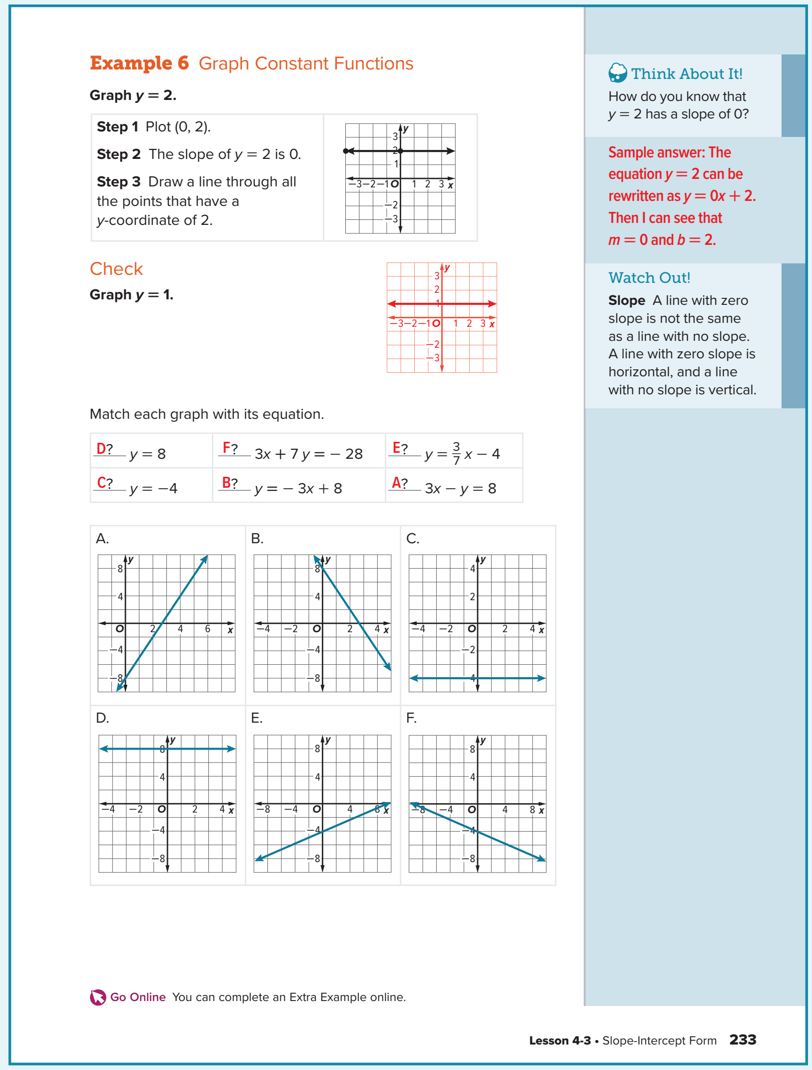
Some students may think that the slope is 2, since, when an equation is written in slope-intercept form, the slope is the number after the equal sign. Point out that if the slope were 2, the equation would be $y = 2x$. Since there is no x term, the slope is 0, and the equation is $y = 0x + 2$.

Essential Question Follow-Up

Students have explored the relationship between the parameters of a linear function and its graph.

Ask:

What can you learn about the graph of a linear function by analyzing its equation? Sample answer: If the equation is in slope-intercept form, I can tell where the graph intersects the y-axis and what the slope of the line is.



Example 6 Graph Constant Functions

Graph $y = 2$.

Step 1 Plot (0, 2).

Step 2 The slope of $y = 2$ is 0.

Step 3 Draw a line through all the points that have a y-coordinate of 2.

Check

Graph $y = 1$.

Match each graph with its equation.

D? $y = 8$	F? $3x + 7y = -28$	E? $y = \frac{3}{7}x - 4$
C? $y = -4$	B? $y = -3x + 8$	A? $3x - y = 8$

A. B. C. D. E. F.

Think About It!

How do you know that $y = 2$ has a slope of 0?

Sample answer: The equation $y = 2$ can be rewritten as $y = 0x + 2$. Then I can see that $m = 0$ and $b = 2$.

Watch Out!

Slope A line with zero slope is not the same as a line with no slope. A line with zero slope is horizontal, and a line with no slope is vertical.

Go Online You can complete an Extra Example online.

Lesson 4-3 • Slope-Intercept Form 233

Interactive Presentation

Example 6

TAP

Students move through the steps of graphing a constant function.

WEB SKETCHPAD

Students use a sketch to graph a constant function.

TYPE

Students explain why the graph has a slope of 0.

Apply Example 7 Use Graphs of Linear Functions

SHOPPING **The number of online shoppers in the United States can be modeled by the equation $-5.88x + y = 172.3$, where y represents the number of millions of online shoppers in the United States x years after 2010. Estimate the number of people shopping online in 2020.**

1. What is the task?
Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

Sample answer: I need to find the number of people who shop online in 2020.

2. How will you approach the task? What have you learned that you can use to help you complete the task?

Sample answer: I will graph the given equation. Then I can figure out from the graph how many people will be shopping online in 2020.

3. What is your solution?
Use your strategy to solve the problem. Graph the equation.

In 2020, there will be approximately 230 million online shoppers in the United States.

Think About It!
Estimate the year when the number of online shoppers in the United States will reach 271 million.

Sample answer: 17 years after 2010 or 2027

4. How can you know that your solution is reasonable?

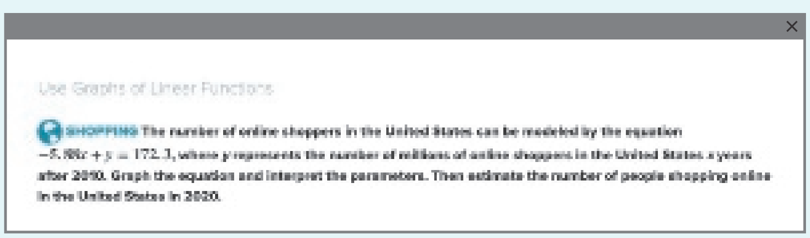
Write About It! Write an argument that can be used to defend your solution.

Sample answer: Rewriting the equation in slope-intercept form shows that $b = 172.3$ and $m = 5.88$. This means that there were 172.3 million online shoppers in 2010. The number of online shoppers is increasing at a rate of 5.88 million per year. The graph of this line shows that in 2020 the number of online shoppers is more than 225 million but less than 250 million. From the graph, there will be approximately 230 million online shoppers in 2020.

Go Online to learn about intervals in linear growth patterns in Expand 4-3.

Go Online You can complete an Extra Example online.

Interactive Presentation



Apply Example 7

TYPE

Students estimate the year when the online shoppers will reach a certain number.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Apply Example 7 Use Graphs of Linear Functions

Teaching the Mathematical Practices

1 Make Sense of Problems and Persevere in Solving Them, 4 Model with Mathematics Students will be presented with a task. They will first seek to understand the task, and then determine possible entry points to solving it. As students come up with their own strategies, they may propose mathematical models to aid them. As they work to solve the problem, encourage them to evaluate their model and/or progress, and change direction, if necessary.

Recommended Use

Have students work in pairs or small groups. You may wish to present the task, or have a volunteer read it aloud. Then allow students the time to make sure they understand the task, think of possible strategies, and work to solve the problem.

Encourage Productive Struggle

As students work, monitor their progress. Instead of instructing them on a particular strategy, encourage them to use their own strategies to solve the problem and to evaluate their progress along the way. They may or may not find that they need to change direction or try out several strategies.

Signs of Non-Productive Struggle

If students show signs of non-productive struggle, such as feeling overwhelmed, frustrated, or disengaged, intervene to encourage them to think of alternate approaches to the problem. Some sample questions are shown.

- How can you determine the domain?
- How can you use the graph to estimate how many people will be shopping online in 2020?

Write About It!

Have students share their responses with another pair/group of students or the entire class. Have them clearly state or describe the mathematical reasoning they can use to defend their solution.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–30
2	exercises that use a variety of skills from this lesson	31–39
2	exercises that extend concepts learned in this lesson to new contexts	40–43
3	exercises that emphasize higher-order and critical-thinking skills	44–47

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: BL

- Practice, Exercises 1–43 odd, 44–47
- Extension: Pencils of Lines
- ALEKS® Equations of Lines

IF students score 66%–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–47 odd
- Remediation, Review Resources: Slope of a Line
- BrainPOP Video: Slope and Intercepts
- Extra Examples 1–7
- ALEKS® Slope

IF students score 65% or less on the Checks, **THEN** assign: AL

- Practice, Exercises 1–29 odd
- Remediation, Review Resources: Slope of a Line
- *Quick Review Math Handbook*: Writing Equations in Slope-Intercept Form
- Arrive**MATH** Take Another Look
- ALEKS® Slope

Practice

Go Online You can complete your homework online.

Example 1

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

1. slope: 5, y-intercept: −3 $y = 5x - 3$
2. slope: −2, y-intercept: 7 $y = -2x + 7$
3. slope: −6, y-intercept: −2 $y = -6x - 2$
4. slope: 7, y-intercept: 1 $y = 7x + 1$
5. slope: 3, y-intercept: 2 $y = 3x + 2$
6. slope: −4, y-intercept: −9 $y = -4x - 9$
7. slope: 1, y-intercept: −12 $y = x - 12$
8. slope: 0, y-intercept: 8 $y = 8$

Example 2

Write each equation in slope-intercept form.

9. $-10x + 2y = 12$ $y = 5x + 6$
10. $4y + 12x = 16$ $y = -3x + 4$
11. $-5x + 15y = -30$ $y = \frac{1}{3}x - 2$
12. $6x - 3y = -18$ $y = 2x + 6$
13. $-2x - 8y = 24$ $y = -0.25x - 3$
14. $-4x - 10y = -7$ $y = -0.4x + 0.7$

Example 3

15. **SAVINGS** Wade's grandmother gave him $100 for his birthday. Wade wants to save his money to buy a portable game console that costs $275. Each month, he adds $25 to his savings. Write an equation in slope-intercept form to represent Wade's savings y after x months. $y = 25x + 100$

16. **FITNESS CLASSES** Toshelle wants to take strength training classes at the community center. She has to pay a one-time enrollment fee of $25 to join the community center, and then $45 for each class she wants to take. Write an equation in slope-intercept form for the cost of taking x classes. $y = 45x + 25$

17. **EARNINGS** Macario works part time at a clothing store in the mall. He is paid $9 per hour plus 12% commission on the items he sells in the store. Write an equation in slope-intercept form to represent Macario's hourly wage y. $y = 0.12x + 9$

18. **ENERGY** From 2002 to 2005, U.S. consumption of renewable energy increased an average of 0.17 quadrillion BTUs per year. About 6.07 quadrillion BTUs of renewable power were produced in the year 2002. Write an equation in slope-intercept form to find the amount of renewable power P in quadrillion BTUs produced in year y between 2002 and 2005. $P = 0.17y + 6.07$

Example 4

Graph a linear equation with the given slope and y-intercept. 19–22. See margin.

19. slope: 5, y-intercept: 8
20. slope: 3, y-intercept: 10
21. slope: −4, y-intercept: 6
22. slope: −2, y-intercept: 8

Lesson 4-3 • Slope-Intercept Form 235

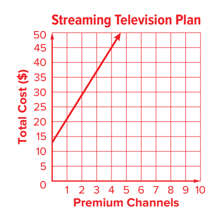
Examples 5 and 6

Graph each equation. 23–28. See margin.

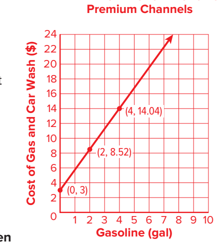
23. $5x + 2y = 8$
24. $4x + 9y = 27$
25. $y = 7$
26. $y = -\frac{2}{3}$
27. $21 = 7y$
28. $3y - 6 = 2x$

Example 7

29. **STREAMING** An online company charges $13 per month for the basic plan. They offer premium channels for an additional $8 per month.
    a. Write an equation in slope-intercept form for the total cost c of the basic plan with p premium channels in one month. $c = 13 + 8p$
    b. Graph the equation.
    c. What would the monthly cost be for a basic plan plus 3 premium channels? $37



30. **CAR CARE** Suppose regular gasoline costs $2.76 per gallon. You can purchase a car wash at the gas station for $3.
    a. Write an equation in slope-intercept form for the total cost y of purchasing a car wash and x gallons of gasoline. $y = 2.76x + 3$
    b. Graph the equation.
    c. Find the cost of purchasing a car wash and 8 gallons of gasoline. $25.08



Mixed Exercises

Write an equation of a line in slope-intercept form with the given slope and y-intercept.

31. slope: $\frac{1}{2}$, y-intercept: −3 $y = \frac{1}{2}x - 3$
32. slope: $\frac{2}{3}$, y-intercept: −5 $y = \frac{2}{3}x - 5$

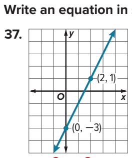
Graph an equation of a line with the given slope and y-intercept. 33–36. See Mod. 4 Answer Appendix.

33. slope: 3, y-intercept: −4
34. slope: 4, y-intercept: −6

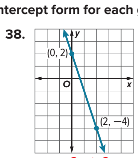
Graph each equation.

35. $-3x + y = 6$
36. $-5x + y = 1$

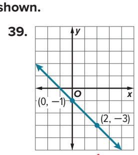
Write an equation in slope-intercept form for each graph shown.

37.


$y = 2x - 3$

38.


$y = -3x + 2$

39.


$y = -x - 1$

236 Module 4 • Linear and Nonlinear Functions

40. MOVIES MovieMania, an online movie rental Web site charges a one-time fee of \$6.85 and \$2.99 per movie rental. Let m represent the number of movies you watch and let C represent the total cost to watch the movies.

a. Write an equation that relates the total cost to the number of movies you watch from MovieMania. $C = 2.99m + 6.85$

b. Graph the equation. See Mod. 4 Answer Appendix.

c. Explain how to use the graph to estimate the cost of watching 13 movies at MovieMania. See Mod. 4 Answer Appendix.

d. SuperFlix has no sign-up fee, just a flat rate per movie. If renting 13 movies at MovieMania costs the same as renting 9 movies at SuperFlix, what does SuperFlix charge per movie? Explain your reasoning. See Mod. 4 Answer Appendix.

e. Write an equation that relates the total cost to the number of movies you watch from SuperFlix. Round to the nearest whole number. $C = 5m$

41. FACTORY A factory uses a heater in part of its manufacturing process. The product cannot be heated too quickly, nor can it be cooled too quickly after the heating portion of the process is complete.

a. The heater is digitally controlled to raise the temperature inside the chamber by 10°F each minute until it reaches the set temperature. Write a function to represent the temperature, T, inside the chamber after x minutes if the starting temperature is 80°F. $T = 10x + 80$

b. Graph the equation. See Mod. 4 Answer Appendix.

c. The heating process takes 22 minutes. Use your graph to find the temperature in the chamber at this point. 300°F

d. After the heater reaches the temperature determined in **part c**, the temperature is kept constant for 20 minutes before cooling begins. Fans within the heater control the cooling so that the temperature inside the chamber decreases by 5°F each minute. Write a function to represent the temperature, T, inside the chamber x minutes after the cooling begins. $T = -5x + 300$

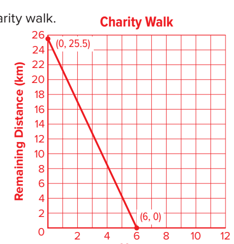
42. SAVINGS When Santo was born, his uncle started saving money to help pay for a car when Santo became a teenager. Santo's uncle initially saved \$2000. Each year, his uncle saved an additional \$200.

a. Write an equation that represents the amount, in dollars, Santo's uncle saved y after x years. $y = 200x + 2000$

b. Graph the equation. See Mod. 4 Answer Appendix.

c. Santo starts shopping for a car when he turns 16. The car he wants to buy costs \$6000. Does he have enough money in the account to buy the car? Explain. No; Santo only has \$5200 in his account. He needs to save an additional \$800 to buy the car he wants.

43. STRUCTURE Jazmin is participating in a 25.5-kilometer charity walk. She walks at a rate of 4.25 km per hour. Jazmin walks at the same pace for the entire event.



a. Write an equation in slope-intercept form for the remaining distance, y, in kilometers of walking for x hours. $y = -4.25x + 25.5$

b. Graph the equation.

c. What do the x- and y-intercepts represent in this situation? The x-intercept (6) represents the number of hours it will take Jazmin to complete the walk. The y-intercept (25.5) represents the length of the walk.

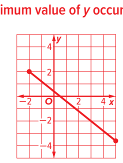
d. After Jazmin has walked 17 kilometers, how much longer will it take her to complete the walk? Explain how you can use your graph to answer the question. Using the graph, I can determine the value of x when y equals $-17 + 25.5$ or 8.5 km, and use the value of the x-intercept. The value of x is 4 when $y = 8.5$ and the x-intercept is 6. Therefore, Jazmin has $6 - 4$ or 2 hours more to walk.

Higher-Order Thinking Skills

For Exercises 44 and 45, refer to the equation $y = -\frac{4}{5}x + \frac{2}{5}$ where $-2 \le x \le 5$.

44. ANALYZE Copy and complete the table to help you graph the equation $y = -\frac{4}{5}x + \frac{2}{5}$ over the interval. Identify any values of x where maximum or minimum values of y occurs. Maximum value of y occurs when $x = -2$; Minimum value of y occurs when $x = 5$

x	$-\frac{4}{5}x + \frac{2}{5}$	y	(x, y)
−2	$-\frac{4}{5}(-2) + \frac{2}{5}$	2	$(-2, 2)$
0	$-\frac{4}{5}(0) + \frac{2}{5}$	$\frac{2}{5}$	$(0, \frac{2}{5})$
5	$-\frac{4}{5}(5) + \frac{2}{5}$	$-\frac{18}{5}$	$(5, -\frac{18}{5})$

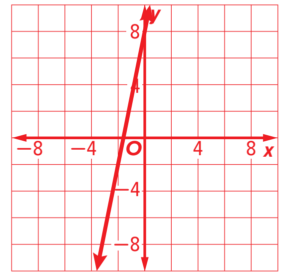


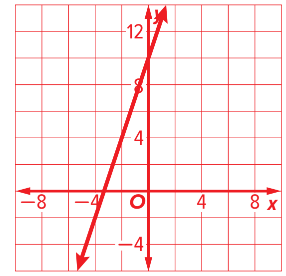
45. WRITE A student says you can find the solution to $-\frac{4}{5}x + \frac{2}{5} = 0$ using the graph. Do you agree? Explain your reasoning. Include the solution to the equation in your response. Yes; you can find the value of x on the graph when $y = 0$; $x = \frac{1}{2}$.

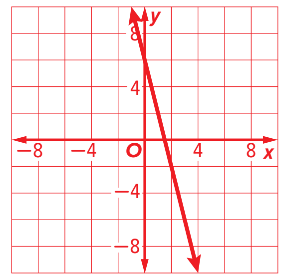
46. PERSEVERE Consider three points that lie on the same line (3, 7), (−6, 1), and (9, p). Find the value of p and explain your reasoning. 11; Use the first two points to find the equation of the line, then replace x and y with 9 and p, respectively, to solve for p.

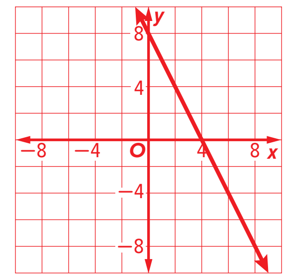
47. CREATE Linear equations are useful in predicting future events. Create a linear equation that models a real-world situation. Make a prediction from your equation. Sample answer: $y = 25x + 200$; I have \$200 in savings and will save \$25 per week until I have enough money to buy a new phone. I can predict how much money I'll have after x number of weeks.

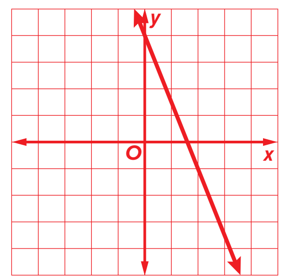
Answers

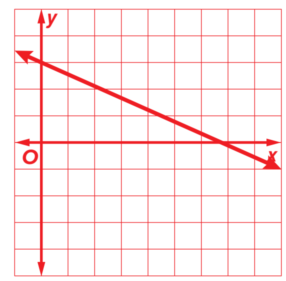
19.


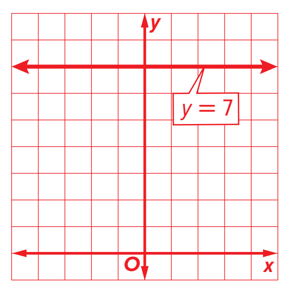
20.


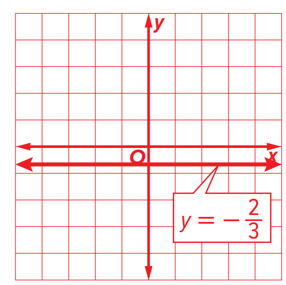
21.


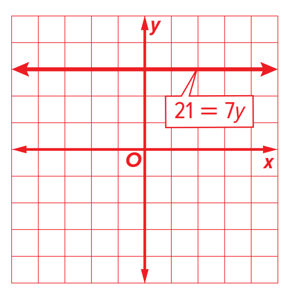
22.


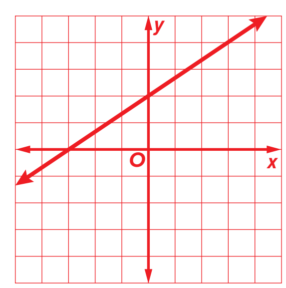
23.


24.


25.


26.


27.


28.


Lesson 4-4

Transformations of Linear Functions

F.IF.7a, F.BF.3

LESSON GOAL

Students identify the effects of transformations of the graphs of linear functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Transforming Linear Functions

Develop:

Translations of Linear Functions

- Vertical Translations of Linear Functions
- Horizontal Translations of Linear Functions
- Multiple Translations of Linear Functions
- Translations of Linear Functions

Dilations of Linear Functions

- Vertical Dilations of Linear Functions
- Horizontal Dilations of Linear Functions

Reflections of Linear Functions

- Reflections of Linear Functions Across the *x*-Axis
- Reflections of Linear Functions Across the *y*-Axis

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Reflections	●	●		●
Extension: Transformations of Other Families of Functions		●	●	●

Language Development Handbook

Assign page 23 of the *Language Development Handbook* to help your students build mathematical language related to transformations of the graphs of linear functions.

ELL You can use the tips and suggestions on page T23 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.

5 Use appropriate tools strategically.

7 Look for and make use of structure.

Coherence

Vertical Alignment

Previous
Students graphed equations in slope-intercept form.
A.CED.2, F.IF.7a, F.LE.5

Now
Students Identify the effects of transformations of the graphs of linear functions.
F.IF.7a, F.BF.3

Next
Students will write and graph equations of arithmetic sequences.
F.BF.1a, F.BF.2, F.LE.2

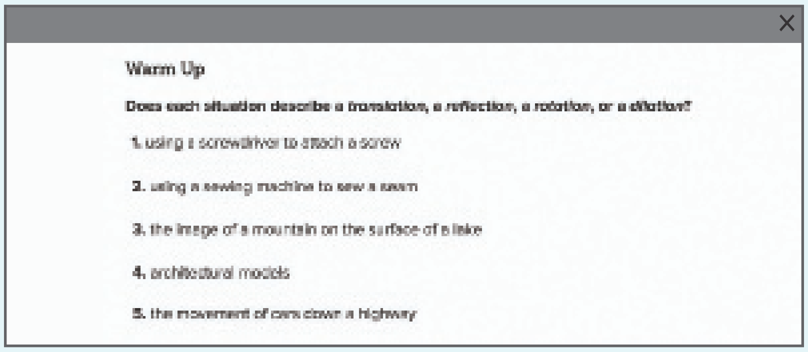
Rigor

The Three Pillars of Rigor

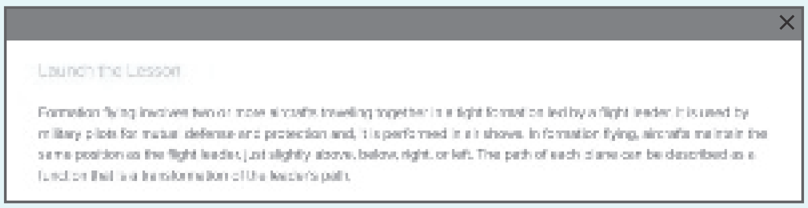
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students develop understanding of transformations of functions by examining the family of linear functions. They build fluency by describing transformations and identifying transformed functions. They apply their understanding by solving real-world problems.

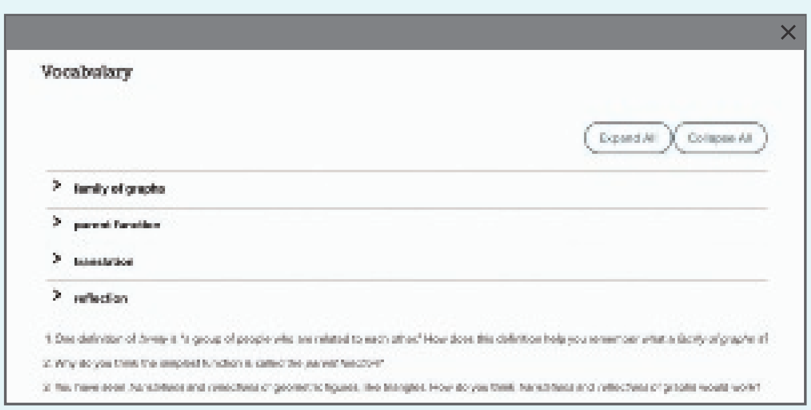
Interactive Presentation



Warm Up



Launch the Lesson



Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- translating and reflecting geometric figures

Answers:

1. rotation
2. translation
3. reflection
4. dilation
5. translation

Launch the Lesson

Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of a linear function to identify the effect on the graph when replacing $f(x)$ with $f(x) + k$, $k \cdot f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

The parent function of the family of linear functions is $f(x) = x$. Transformations of the parent graph occur when a constant is added to or subtracted from the function or the argument, or when the function or the argument is multiplied by a number. These transformations alter the graph, translating it in a particular direction, dilating it, or reflecting it. Recognizing the effect produced by each type of transformation allows for the new graph to be easily obtained from the graph of the parent function.

Explore Transforming Linear Functions

Objective

Students use a sketch to explore how changing the parameters changes the graphs of linear functions.

Teaching the Mathematical Practices

3 Construct Arguments In this Explore, students will use stated assumptions, definitions, and previously established results to construct arguments.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

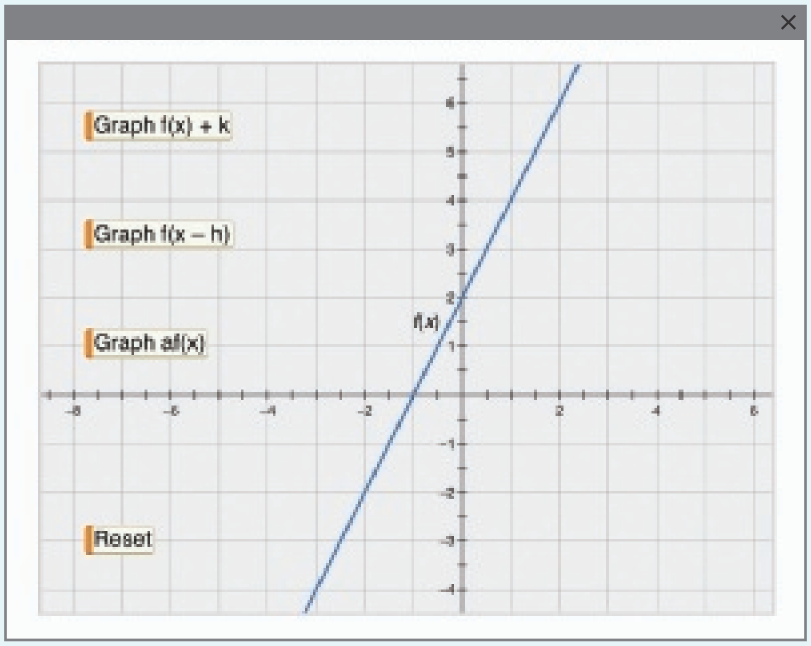
Summary of the Activity

Students complete guiding exercises throughout the Explore activity. Students use a sketch to explore how the graph of a function is affected when a number is added to the function, when a number is subtracted from the argument of the function, or when the function is multiplied by a number. They enter various values for the number and view the resulting graph. Then, students answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

Explore



Explore

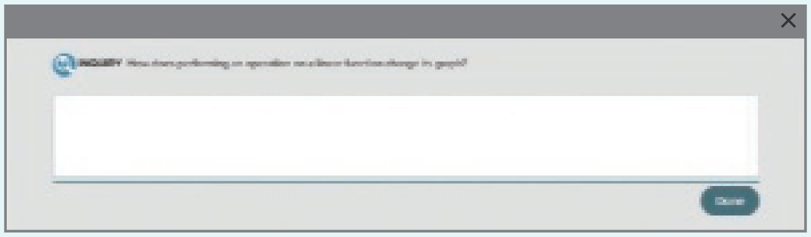
WEB SKETCHPAD

Students use a sketch to explore the effects of addition and multiplication on a function.

TYPE

Students answer questions about transformations of linear functions.

Interactive Presentation



Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Transforming Linear Functions *(continued)*

Questions

Have students complete the Explore activity.

Ask:

- Does adding or subtracting a value to a function change the slope or y-intercept? Sample answer: The line moves up/down or left/right when you add or subtract values to the function. This means that the y-intercept is changing, but not the slope.
- Why does multiplying a function by a value make it more or less steep? Sample answer: If we multiply every value in a function, then we are changing the value of y for every x-value. If we multiply by a value greater than one, then the difference between the y-values will be greater, resulting in a greater slope and a steeper line.

Inquiry

How does performing an operation on a linear function change its graph? Sample answer: Adding a value to the function moves the graph up or down. Subtracting a value from x moves the graph left or right. Multiplying the function by a value makes the graph more steep or less steep.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Translations of Linear Functions

Objective

Students identify the effects on the graphs of linear functions by replacing $f(x)$ with $f(x) + k$ and $f(x - h)$ for positive and negative values.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of translations in this Learn.

What Students Are Learning

The parent function of the family of linear functions is $f(x) = x$. Its graph is the line that passes through the origin and has a slope of 1. The graph of every other linear function is a transformation of this function. The first type of transformation students will learn about is translations. Under a translation, the graph of a line is slid to a new location.

Common Misconception

Students may believe that a translation will change the orientation of the figure. Help them to see that this is not the case. When a figure is slid in its entirety up, down, left, or right, its orientation remains the same. In the case of a line, its slope is not affected, so the new image has the same slope as the original graph.

Vertical Translations

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of vertical translations in this Learn.

About the Key Concept

When k is added to the function $f(x) = x$, the graph of the function is translated vertically. This is because adding k to the function increases the y-value that is associated with each x-value by k units. When k is negative, each y-value decreases, which translates the graph down $|k|$ units.

Common Misconception

Some students may think that adding k to a function increases (or decreases) the x-value in each ordered pair. Remind students that the notation $f(x)$ represents the y-value that is paired with x. Thus, $f(x) + k$ represents an increase (or decrease) in y-values, resulting in a vertical translation.

Lesson 4-4

Transformations of Linear Functions

Today's Goals
- Apply translations to linear functions.
- Apply dilations to linear functions.
- Apply reflections to linear functions.

Today's Vocabulary
family of graphs
parent function
identity function
transformation
translation
dilation
reflection

Explore Transforming Linear Functions

Online Activity Use graphing technology to complete an Explore.

INQUIRY How does performing an operation on a linear function change its graph?

Learn Translations of Linear Functions

A **family of graphs** includes graphs and equations of graphs that have at least one characteristic in common. The **parent function** is the simplest of the functions in a family.

The family of linear functions includes all lines, with the parent function $f(x) = x$, also called the **identity function**. A **transformation** moves the graph on the coordinate plane, which can create new linear functions.

One type of transformation is a translation. A **translation** is a transformation in which a figure is slid from one position to another without being turned. A linear function can be slid up, down, left, right, or in two directions.

Study Tip
Slope When translating a linear function, the graph of the function moves from one location to another, but the slope remains the same.

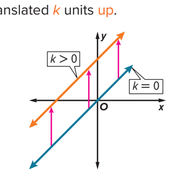
Vertical Translations

When a constant k is added to a linear function $f(x)$, the result is a vertical translation. The y-intercept of $f(x)$ is translated up or down.

Key Concept • Vertical Translations of Linear Functions

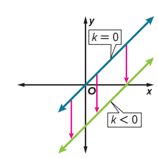
The graph of $g(x) = x + k$ is the graph of $f(x) = x$ translated vertically.

If $k > 0$, the graph of $f(x)$ is translated k units up.



Every point on the graph of $f(x)$ moves k units up.

If $k < 0$, the graph of $f(x)$ is translated $|k|$ units down.

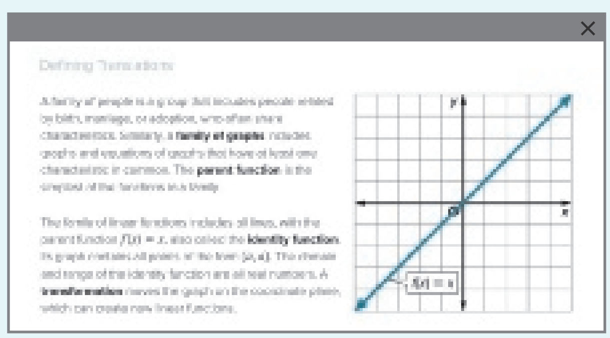


Every point on the graph of $f(x)$ moves $|k|$ units down.

Watch Out!
Translations of $f(x)$ When a translation is the only transformation performed on the identity function, adding a constant before or after evaluating the function has the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Lesson 4-4 • Transformations of Linear Functions 239

Interactive Presentation



Learn

TAP

Students tap each flash card to learn more about vertical translations.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Think About It!
What do you notice about the y-intercepts of vertically translated functions compared to the y-intercept of the parent function?

Sample answer: The y-intercepts move up k units or down $|k|$ units from the y-intercept of the parent function.

Example 1 Vertical Translations of Linear Functions

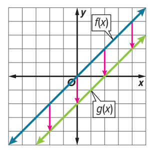
Describe the translation in $g(x) = x - 2$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$f(x)$	$f(x) - 2$	$(x, g(x))$
-2	-2	-4	$(-2, -4)$
0	0	-2	$(0, -2)$
1	1	-1	$(1, -1)$

Because $f(x) = x$, $g(x) = f(x) + k$ where $k = -2$.

$g(x) = x - 2 \rightarrow f(x) + (-2)$



The constant k is not grouped with x, so k affects the output, or y-values. The value of k is less than 0, so the graph of $f(x) = x$ is translated $|-2|$ units down, or 2 units down.

$g(x) = x - 2$ is the translation of the graph of the parent function 2 units down.

Check

Describe the translation in $g(x) = x - 1$ as it relates to the graph of the parent function.

The graph of $g(x) = x - 1$ is a translation of the graph of the parent function 1 unit ___?___.
down

Horizontal Translations

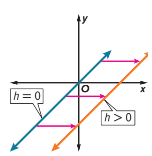
Go Online You can watch a video to see how to describe translations of functions.

When a constant h is subtracted from the x-value before the function $f(x)$ is performed, the result is a horizontal translation. The x-intercept of $f(x)$ is translated right or left.

Key Concept • Horizontal Translations of Linear Functions

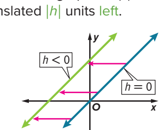
The graph of $g(x) = (x - h)$ is the graph of $f(x) = x$ translated horizontally.

If $h > 0$, the graph of $f(x)$ is translated h units right.



Every point on the graph of $f(x)$ moves h units right.

If $h < 0$, the graph of $f(x)$ is translated $|h|$ units left.



Every point on the graph of $f(x)$ moves $|h|$ units left.

Go Online You may want to complete the Concept Check to check your understanding.

Go Online You can complete an Extra Example online.

240 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation

Example 1

TAP

Students move through the steps to graph a vertical translation.

TYPE

Students describe how the y-intercept of the translated function compares to the parent function.

Example 1 Vertical Translations of Linear Functions

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL Looking at only the equation, how do you know the type of transformation? 2 is being subtracted from the parent function so this is a vertical translation.

OL How is the y-value of each ordered pair in the parent function affected? Each y-value decreases by 2.

BL How would you write this function as a vertical translation of the parent graph up 2 units? $g(x) = f(x) + 2$ or $g(x) = x + 2$

Horizontal Translations

Teaching the Mathematical Practices

3 Analyze Cases Guide students to examine the cases of different translations. Encourage students to familiarize themselves with all of the cases.

Common Misconception

Some students may think that the graph of $f(x + h)$, where h is a positive number, is a translation of the parent graph h units to the right. Point out that $f(x + h) = f(x - (-h))$, so the number being subtracted is a negative number. Thus, the shift is to the left, not to the right.

Example 2 Horizontal Translations of Linear Functions

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the translated function and the graph of the parent function used in this example.

Questions for Mathematical Discourse

AL Looking at only the equation, how do you know the type of transformation? The $+5$ is grouped with the x in the parentheses so this is a horizontal translation.

OL What are the coordinates when $g(x) = 0$? $(-5, 0)$

BL Write the function that shows a horizontal translation of the parent function 3 units right. $f(x - 3)$ 7 units left $f(x + 7)$

Example 3 Multiple Translations of Linear Functions

Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of the transformed function to identify the translations in the function.

Questions for Mathematical Discourse

AL Looking at only the equation, how many translations are there? 2

OL Looking at only the equation, how do you know that the horizontal translation is to the right? because the number being subtracted from x is positive 6

BL Write a function that represents a translation 6 units left and 3 units down.
$f(x) = (x + 6) - 3$

Example 2 Horizontal Translations of Linear Functions

Describe the translation in $g(x) = (x + 5)$ as it relates to the graph of the parent function.

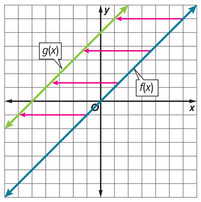
Graph the parent graph for linear functions.

Because $f(x) = x$, $g(x) = f(x-h)$ where $h = -5$.

$g(x) = (x + 5) \rightarrow g(x) = f(x - (-5))$

x	$x + 5$	$f(x + 5)$	$(x, g(x))$
−2	3	3	(−2, 3)
0	5	5	(0, 5)
1	6	6	(1, 6)

The constant h is grouped with x, so k affects the input, or x-values. The value of h is less than 0, so the graph of $f(x) = x$ is translated $|-5|$ units left, or 5 units left.



$g(x) = (x + 5)$ is the translation of the graph of the parent function 5 units left.

Check

Describe the translation in $g(x) = (x + 12)$ as it relates to the graph of the parent function.

The graph of $g(x) = (x + 12)$ is a translation of the graph of the parent function 12 units ___?___.
left

Think About It! What do you notice about the x-intercepts of horizontally translated functions compared to the x-intercept of the parent function?

Sample answer: The x-intercepts move right h units or left $|h|$ units from the x-intercept of the parent function.

Example 3 Multiple Translations of Linear Functions

Describe the translation in $g(x) = (x - 6) + 3$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

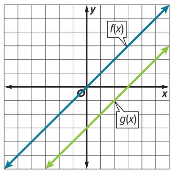
Because $f(x) = x$, $g(x) = f(x - h) + k$ where $h = 6$ and $k = 3$.

x	$x - 6$	$f(x - 6)$	$f(x - 6) + 3$	$(x, g(x))$
−2	−8	−8	−5	(−2, −5)
0	−6	−6	−3	(0, −3)
1	−5	−5	−2	(1, −2)

$g(x) = (x - 6) + 3 \rightarrow g(x) = f(x - 6) + 3$

The value of h is grouped with x and is greater than 0, so the graph of $f(x) = x$ is translated 6 units right.

The value of k is not grouped with x and is greater than 0, so the graph of $f(x) = x$ is translated 3 units up.



$g(x) = (x - 6) + 3$ is the translation of the graph of the parent function 6 units right and 3 units up.

Think About It! Eleni described the graph of $g(x) = (x - 6) + 3$ as the graph of the parent function translated down 3 units. Is she correct? Explain your reasoning.

Yes; sample answer: $g(x) = (x - 6) + 3$ can be simplified to $g(x) = x - 3$, which is the same as $g(x) = f(x) - 3$.

Go Online You can complete an Extra Example online.

Lesson 4-4 • Transformations of Linear Functions 241

Interactive Presentation

Example 2

TAP

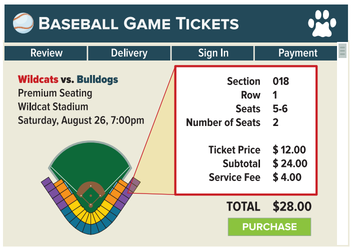
Students move through the steps to graph a horizontal translation.

TYPE

Students describe how the x-intercept of the translated function compares to the parent.

Example 4 Translations of Linear Functions

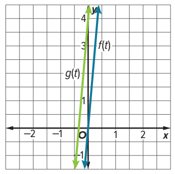
TICKETS **A Web site sells tickets to concerts and sporting events. The total price of the tickets to a certain game can be modeled by $f(t) = 12t$, where t represents the number of tickets purchased. The Web site then charges a standard service fee of \$4 per order. The total price of an order can be modeled by $g(t) = 12t + 4$. Describe the translation of $g(t)$ as it relates to $f(t)$.**



Complete the steps to describe the translation of $g(t)$ as it relates to $f(t)$.
Since $f(t) = 12t$, $g(t) = f(t) + k$, where $k = 4$. $g(t) = 12t + 4 \rightarrow f(t) + 4$

The constant k is added to $f(t)$ after the total price of the tickets has been evaluated and is greater than 0, so the graph of will be shifted 4 units up. $g(t) = 12t + 4$ is the translation of the graph of $f(t)$ 4 units up.

Graph the parent function and the translated function.

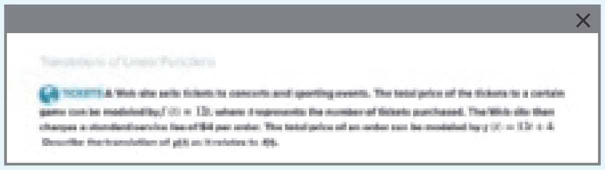


Check

RETAIL Jerome is buying paint for a mural. The total cost of the paint can be modeled by the function $f(p) = 6.99p$. He has a coupon for \$5.95 off his purchase at the art supply store, so the final cost of his purchase can be modeled by $g(p) = 6.99p - 5.95$. Describe the translation in $g(p)$ as it relates to $f(p)$. **The graph of $g(p) = 6.99p - 5.95$ is the translation of the graph of $f(p)$ 5.95 units down.**

Go Online You can complete an Extra Example online.

Interactive Presentation



Example 4

TYPE

Students complete the statements to identify the value for the translation.

WEB SKETCHPAD

Students use a sketch to graph the translated function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 4 Translations of Linear Functions

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about translations of linear functions to solving a real-world problem.

Questions for Mathematical Discourse

AL What does the 12 in the function represent? the cost per ticket
What does the t in the function represent? the number of tickets

OL What does the parent function represent in the context of the situation? cost of tickets without the online service fee

BL What would the function be if, in addition to the service fee, there was also a \$5 charge for tax? $g(t) = 12t + 4 + 5$ or $g(t) = 12t + 9$

Common Error

Some students may try to work the 12 into the translation. Remind these students that translations occur when numbers are added or subtracted, not multiplied.

Essential Question Follow-Up

Students have observed how a function that models a real-world situation can be a transformation of another function.

Ask:

Why is it important to understand how the structure of a function models a situation? Sample answer: The structure helps you understand how the different quantities in the situation affect the function.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students are having difficulty determining the direction of a translation,

THEN have them create four examples of functions that represent each type of translation, and write each one on an index card. Have them sketch the transformation on a coordinate plane on the back of the card, and write the description. Then have them use the flash cards (in both directions) to practice what they have learned.

Learn Dilations of Linear Functions

Objective

Students identify the effects on the graphs of linear functions by replacing $f(x)$ with $af(x)$ and by replacing $f(x)$ with $f(ax)$.

Teaching the Mathematical Practices

7 Use Structure Help students to explore the structure of vertical and horizontal dilations in this Learn.

About the Key Concept

When the function $f(x) = x$ is multiplied by a number a, the graph of the function is dilated vertically. This is because multiplying the function by a number affects the y-value that is associated with each x-value. When $|a| > 1$, the graph is stretched vertically, making it steeper. When $|a| < 1$, the graph is compressed vertically, making it less steep.

Common Misconception

Some students may think that when a is positive, the dilation stretches the graph, and when it is negative, the dilation compresses the graph. Use a table of values for several functions to show students the error in this reasoning. Sample functions: $f(x) = x$, $g(x) = 2f(x)$, $g(x) = -2f(x)$, $g(x) = 0.5f(x)$, $g(x) = -0.5f(x)$

About the Key Concept

When the argument of the function $f(x) = x$ is multiplied by a number a, the graph of the function is dilated horizontally. This is because multiplying the argument by a number affects the x-value that is associated with each y-value. When $|a| > 1$, the graph is compressed horizontally, making it steeper. When $|a| < 1$, the graph is stretched horizontally, making it less steep.

Learn Dilations of Linear Functions

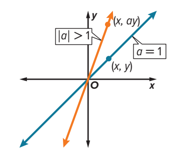
A **dilation** stretches or compresses the graph of a function.

When a linear function $f(x)$ is multiplied by a positive constant a, the result $a \cdot f(x)$ is a vertical dilation.

Key Concept • Vertical Dilations of Linear Functions

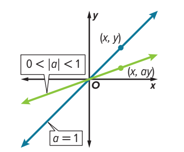
The graph of $g(x) = ax$ is the graph of $f(x) = x$ stretched or compressed vertically.

If $|a| > 1$, the graph of $f(x)$ is stretched vertically away from the x-axis.



The slope of the graph of $a \cdot f(x)$ is steeper than that of the graph of $f(x)$.

If $0 < |a| < 1$, the graph of $f(x)$ is compressed vertically toward the x-axis.



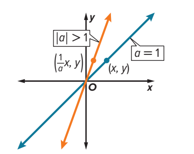
The slope of the graph of $a \cdot f(x)$ is less steep than that of the graph of $f(x)$.

When x is multiplied by a positive constant a before a linear function $f(a \cdot x)$ is evaluated, the result is a horizontal dilation.

Key Concept • Horizontal Dilations of Linear Functions

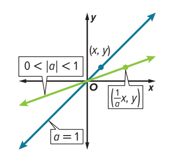
The graph of $g(x) = (a \cdot x)$ is the graph of $f(x) = x$ stretched or compressed horizontally.

If $|a| > 1$, the graph of $f(x)$ is compressed horizontally toward the y-axis.



The slope of the graph of $f(a \cdot x)$ is steeper than that of the graph of $f(x)$.

If $0 < |a| < 1$, the graph of $f(x)$ is stretched horizontally away from the y-axis.



The slope of the graph of $f(a \cdot x)$ is less steep than that of the graph of $f(x)$.

Watch Out!

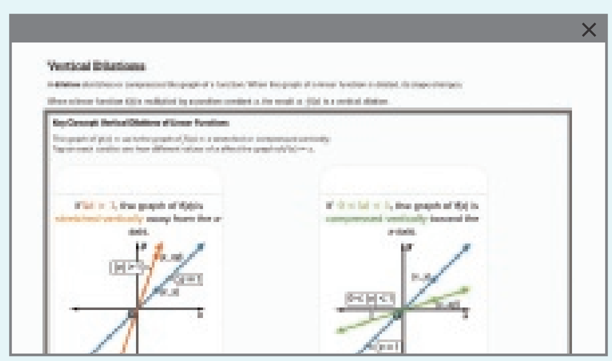
Dilations of $f(x) = x$ When a dilation is the only transformation performed on the identity function, multiplying by a constant before or after evaluating the function has the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Go Online You can watch a video to see how to describe dilations of functions.

Go Online You can complete an Extra Example online.

Lesson 4-4 • Transformations of Linear Functions 243

Interactive Presentation



Learn

TAP

Students tap on each flash card to learn more about vertical and horizontal dilations.

DRAG & DROP

Students drag the correct term to identify vertical and horizontal dilations.

TYPE

Students explain the effect of dilations on the intercepts of the parent function.

Think About It! What do you notice about the slope of the vertical dilation $g(x)$ compared to the slope of $f(x)$?

Sample answer: The slope of $g(x)$ is twice the slope of $f(x)$.

How does this relate to the constant a in the vertical dilation?

Sample answer: $a = 2$, so the slope of $g(x)$ is the slope of $f(x)$ multiplied by a.

Example 5 Vertical Dilations of Linear Functions

Describe the dilation in $g(x) = 2(x)$ as it relates to the graph of the parent function.

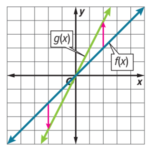
Graph the parent graph for linear functions.

x	$f(x)$	$2f(x)$	$(x, g(x))$
−2	−2	−4	(−2, −4)
0	0	0	(0, 0)
1	1	2	(1, 2)

Since $f(x) = x$, $g(x) = a \cdot f(x)$ where $a = 2$.

$g(x) = 2(x) \rightarrow g(x) = 2f(x)$

The positive constant a is not grouped with x, and $|a|$ is greater than 1, so the graph of $f(x) = x$ is stretched vertically by a factor of a, or 2.



$g(x) = 2(x)$ is a vertical stretch of the graph of the parent function. The slope of the graph of $g(x)$ is steeper than that of $f(x)$.

Check

Describe the transformation in $g(x) = 6(x)$ as it relates to the graph of the parent function.

The graph of $g(x) = 6(x)$ is a __vertical stretch__ of the graph of the parent function.

The slope of the graph $g(x)$ is __steeper__ than that of the parent function.

Think About It! What do you notice about the slope of the horizontal dilation $g(x)$ compared to the slope of $f(x)$?

Sample answer: The slope of $g(x)$ is one fourth the slope of $f(x)$.

How does this relate to the constant a in the horizontal dilation?

Sample answer: $a = \frac{1}{4}$, so, the slope of $g(x)$ is the slope of $f(x)$ multiplied by a.

Example 6 Horizontal Dilations of Linear Functions

Describe the dilation in $g(x) = \left(\frac{1}{4}x\right)$ as it relates to the graph of the parent function.

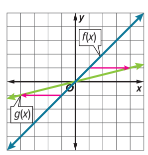
Graph the parent graph for linear functions.

x	$\frac{1}{4}x$	$f\left(\frac{1}{4}x\right)$	$(x, g(x))$
−4	−1	−1	(−4, −1)
0	0	0	(0, 0)
4	1	1	(4, 1)

Since $f(x) = x$, $g(x) = f(a \cdot x)$ where $a = \frac{1}{4}$.

$g(x) = \left(\frac{1}{4}x\right) \rightarrow g(x) = f\left(\frac{1}{4}x\right)$

The positive constant a is grouped with x, and $|a|$ is between 0 and 1, so the graph of $f(x) = x$ is stretched horizontally by a factor of $\frac{1}{a}$, or 4.



$g(x) = \left(\frac{1}{4}x\right)$ is a horizontal stretch of the graph of the parent function. The slope of the graph of $g(x)$ is less steep than that of $f(x)$.

Go Online You can complete an Extra Example online.

244 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation

Example 5

TAP

Students move through the steps to graph a vertical dilation.

TYPE

Students compare the slope and y-intercept of the vertical dilation to the parent graph.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 5 Vertical Dilations of Linear Functions

Teaching the Mathematical Practices

7 Look for a Pattern Help students to see the pattern in calculating the coordinates for $g(x)$ in this example.

Questions for Mathematical Discourse

AL Will the placement of the 2 cause a change to the x-value or to the y-value of each ordered pair of the parent function? y-value; sample answer: Because the 2 is not grouped with the x-variable, it will change the y-value.

OL Looking at only the equation, what kind of dilation is this? a vertical stretch of 2

BL How would the transformation be different if the function had been $g(x) = \frac{1}{2}f(x)$? There would be a vertical compression instead of a vertical stretch.

Example 6 Horizontal Dilations of Linear Functions

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graphs and equations of the functions in this example.

Questions for Mathematical Discourse

AL What value is grouped with the x? $\frac{1}{4}$ Will this cause a change to the x-value or to the y-value of each ordered pair of the parent function? the x-value

OL Looking at only the equation, what kind of dilation is this? a horizontal stretch by a factor of 4

BL How is a horizontal stretch by a factor of 4 related to a vertical compression by a factor of 4? They result in the same line.

Learn Reflections of Linear Functions

Objective

Students identify the effects on the graphs of linear functions by replacing $f(x)$ with $-af(x)$ and $f(-ax)$.

Teaching the Mathematical Practices

1 Explain Correspondence Encourage students to explain the relationships between the coordinates, equations, and graphs of reflected functions and the parent function.

Example 7 Reflections of Linear Functions Across the *x*-Axis

Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument in the Talk About It! feature.

Questions for Mathematical Discourse

AL Looking at only the equation, is this function a reflection? yes What other type of transformation is it? a dilation

OL How do you know $-\frac{1}{2}$ is not grouped with x? Sample answer: It is not inside the parentheses with x.

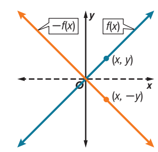
BL The point (2, 2) lies on the graph of the parent function. To what point does this correspond on the graph of $g(x)$? $(2, -1)$

Learn Reflections of Linear Functions

A **reflection** is a transformation in which a figure, line, or curve, is flipped across a line. When a linear function $f(x)$ is multiplied by -1 before or after the function has been evaluated, the result is a reflection across the x- or y-axis. Every x- or y-coordinate of $f(x)$ is multiplied by -1.

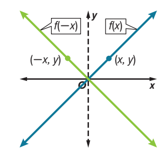
Key Concept • Reflections of Linear Functions

The graph of $-f(x)$ is the reflection of the graph of $f(x) = x$ across the x-axis.



Every y-coordinate of $-f(x)$ is the corresponding y-coordinate of $f(x)$ multiplied by -1.

The graph of $f(-x)$ is the reflection of the graph of $f(x) = x$ across the y-axis.



Every x-coordinate of $f(-x)$ is the corresponding x-coordinate of $f(x)$ multiplied by -1.

Example 7 Reflections of Linear Functions Across the x-Axis

Describe the dilation in $g(x) = -\frac{1}{2}(x)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

Since $f(x) = x$, $g(x) = -1 \cdot a \cdot f(x)$ where $a = \frac{1}{2}$.

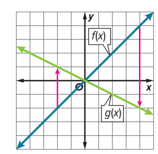
$g(x) = -\frac{1}{2}(x) \rightarrow g(x) = -\frac{1}{2}f(x)$

x	$f(x)$	$-\frac{1}{2}f(x)$	$(x, g(x))$
−2	−2	1	(−2, 1)
0	0	0	(0, 0)
4	4	−2	(4, −2)

The constant a is not grouped with x, and $|a|$ is less than 1, so the graph of $f(x) = x$ is vertically compressed.

The negative is not grouped with x, so the graph is also reflected across the x-axis.

The graph of $g(x) = -\frac{1}{2}(x)$ is the graph of the parent function vertically compressed and reflected across the x-axis.



Go Online You can complete an Extra Example online.

Go Online You can watch a video to see how to describe reflections of functions.

Watch Out!

Reflections of $f(x) = x$ When a reflection is the only transformation performed on the identity function, multiplying by -1 before or after evaluating the function appears to have the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Talk About It!

In the example, the slope of $g(x)$ is negative. Will this always be the case when multiplying a linear function by -1? Justify your argument.

No; sample answer: If you multiply a linear function that is already negative by -1, the resulting function will have a positive slope.

Lesson 4-4 • Transformations of Linear Functions 245

Interactive Presentation

Learn

TAP

Students tap on each flash card to learn more about reflections of functions.

DRAG & DROP

Students drag the correct term to identify if a reflection is over the x- or y-axis.

TYPE

Students write a function that is a dilation and reflection of a parent function.

Check

How can you tell whether multiplying −1 by the parent function will result in a reflection across the *x*-axis? **A**

A. If the constant is not grouped with *x*, the result will be a reflection across the *x*-axis.

B. If the constant is grouped with *x*, the result will be a reflection across the *x*-axis.

C. If the constant is greater than 0, the result will be a reflection across the *x*-axis.

D. If the constant is less than 0, the result will be a reflection across the *x*-axis.

Example 8 Reflections of Linear Functions Across the *y*-Axis

Describe the dilation in $g(x) = (-3x)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

Since $f(x) = x$, $g(x) = f(-1 \cdot a \cdot x)$ where $a = 3$.

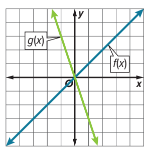
$g(x) = -3x \rightarrow g(x) = f(-3x)$

x	$-3x$	$f(-3x)$	$(x, g(x))$
−1	3	3	(−1, 3)
0	0	0	(0, 0)
1	−3	−3	(1, −3)

The constant a is grouped with x, and $|a|$ is greater than 1, so the graph of $f(x) = x$ is horizontally compressed.

The negative is grouped with x, so the graph is also reflected across the *y*-axis.

The graph of $g(x) = (-3x)$ is the graph of the parent function horizontally compressed and reflected across the *y*-axis.



Go Online You can watch a video to see how to graph transformations of a linear function using a graphing calculator.

Check

Describe the reflection in $g(x) = (-10x)$ as it relates to the graph of the parent function.

The graph of $g(x) = (-10x)$ is the graph of the parent function compressed horizontally and reflected across the ___?___. *y*-axis

Go Online You can complete an Extra Example online.

246 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation

Example 8

TAP

Students move through the steps to graph a reflection of a linear function across the *y*-axis.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 8 Reflections of Linear Functions Across the *y*-Axis

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the reflected function and the graph of the parent function used in this example.

Questions for Mathematical Discourse

AL How do you know the negative sign is grouped with the *x*? Sample answer: Because the negative sign is inside the parentheses with *x*.

OL How do you know if the parent function will be reflected over the *y*-axis? Sample answer: If the negative is inside the parentheses with *x*, the reflection will be over the *y*-axis.

BL How would the function have been written if the reflection was across the *x*-axis? $g(x) = -3f(x)$

Common Error

Students may have difficulty seeing how the graph of $g(x)$ is related to the graph of $f(x)$. For these students, you may want to show the transformation in two different steps, first dilating the graph by a factor of 3, and then reflecting the resulting graph across the *y*-axis.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–21
2	exercises that use a variety of skills from this lesson	22–29
2	exercises that extend concepts learned in this lesson to new contexts	30–33
3	exercises that emphasize higher-order and critical-thinking skills	34–36

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: BL

- Practice, Exercises 1–33 odd, 34–36
- Extension: Transformations of Other Families of Functions
- ALEKS® Equations of Lines

IF students score 66%–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–35 odd
- Remediation, Review Resources: Reflections
- Personal Tutors
- Extra Examples 1–8
- ALEKS® Reflections

IF students score 65% or less on the Checks, **THEN** assign: AL

- Practice, Exercises 1–21 odd
- Remediation, Review Resources: Reflections
- *Quick Review Math Handbook*: Transformations of Linear Functions
- Arrive**MATH** Take Another Look
- ALEKS® Reflections

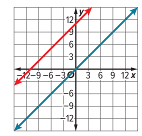
Practice

Go Online You can complete your homework online.

Examples 1 through 3

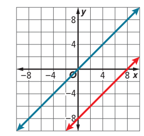
Describe the translation in each function as it relates to the graph of the parent function.

1. $g(x) = x + 11$



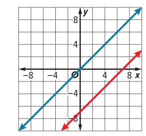
$g(x)$ is a translation of the parent function 11 units up

2. $g(x) = x - 8$



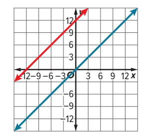
$g(x)$ is a translation of the parent function 8 units down

3. $g(x) = (x - 7)$



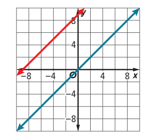
$g(x)$ is a translation of the parent function 7 units right

4. $g(x) = (x + 12)$



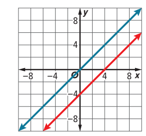
$g(x)$ is a translation of the parent function 12 units left

5. $g(x) = (x + 10) - 1$



$g(x)$ is a translation of the parent function 10 units left and 1 unit down

6. $g(x) = (x - 9) + 5$



$g(x)$ is a translation of the parent function 9 units right and 5 units up

Example 4

7. BOWLING The cost for Nobu to go bowling is \$4 per game plus an additional flat fee of \$3.50 for the rental of bowling shoes. The cost can be modeled by the function $f(x) = 4x + 3.5$, where x represents the number of games bowled. Describe the graph of $g(x)$ as it relates to $f(x)$ if Nobu does not rent bowling shoes. $g(x) = 4x$; $g(x)$ is the translation of $f(x)$ 3.5 units down.

8. SAVINGS Natalie has \$250 in her savings account, into which she deposits \$10 of her allowance each week. The balance of her savings account can be modeled by the function $f(w) = 250 + 10w$, where w represents the number of weeks. Write a function $g(w)$ to represent the balance of Natalie's savings account if she withdraws \$40 to purchase a new pair of shoes. Describe the translation of $f(w)$ that results in $g(w)$. $g(w) = 210 + 10w$; $g(w)$ is the translation of $f(w)$ 40 units down.

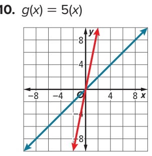
9. BOAT RENTAL The cost to rent a paddle boat at the county park is \$8 per hour plus a nonrefundable deposit of \$10. The cost can be modeled by the function $f(h) = 8h + 10$, where h represents the number of hours the boat is rented. Describe the graph of $g(h)$ as it relates to $f(h)$ if the nonrefundable deposit increases to \$15. $g(h) = 8h + 15$; $g(h)$ is the translation $f(h)$ of 5 units up.

Lesson 4-4 • Transformations of Linear Functions 247

Examples 5 and 6

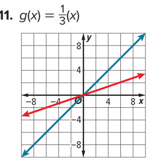
Describe the dilation in each function as it relates to the graph of the parent function.

10. $g(x) = 5(x)$



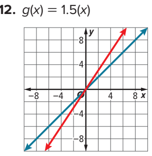
$g(x)$ is a vertical stretch of the parent function by a factor of 5

11. $g(x) = \frac{1}{3}(x)$



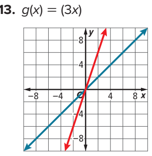
$g(x)$ is a vertical compression of the parent function by a factor of $\frac{1}{3}$

12. $g(x) = 1.5(x)$



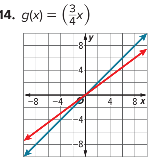
$g(x)$ is a vertical stretch of the parent function by a factor of 1.5

13. $g(x) = (3x)$



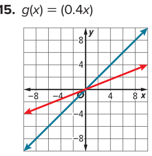
$g(x)$ is a horizontal compression of the parent function by a factor of $\frac{1}{3}$

14. $g(x) = \left(\frac{3}{4}x\right)$



$g(x)$ is a horizontal stretch of the parent function by a factor of $\frac{4}{3}$

15. $g(x) = (0.4x)$

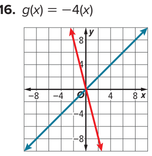


$g(x)$ is a horizontal stretch of the parent function by a factor of 2.5

Example 7

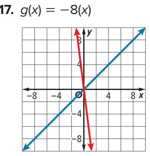
Describe the dilation in each function as it relates to the graph of the parent function.

16. $g(x) = -4(x)$



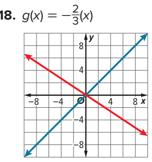
$g(x)$ is a vertical stretch of the parent function by a factor of 4 and a reflection across the x-axis

17. $g(x) = -8(x)$



$g(x)$ is a vertical stretch of the parent function by a factor of 8 and a reflection across the x-axis

18. $g(x) = -\frac{2}{3}(x)$



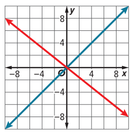
$g(x)$ is a vertical compression of the parent function by a factor of $\frac{2}{3}$ and a reflection across the x-axis

248 Module 4 • Linear and Nonlinear Functions

Example 8

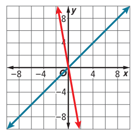
Describe the dilation in each function as it relates to the graph of the parent function.

19. $g(x) = \left(-\frac{4}{5}x\right)$



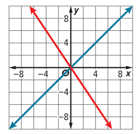
$g(x)$ is a horizontal stretch of the parent function by a factor of $\frac{5}{4}$ and a reflection across the y-axis

20. $g(x) = (-6x)$



$g(x)$ is a horizontal compression of the parent function by a factor of $\frac{1}{6}$ and a reflection across the y-axis

21. $g(x) = (-1.5x)$

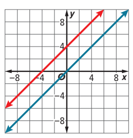


$g(x)$ is a horizontal compression of the parent function by a factor of $\frac{2}{3}$ and a reflection across the y-axis

Mixed Exercises

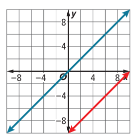
Describe the transformation in each function as it relates to the graph of the parent function.

22. $g(x) = x + 4$



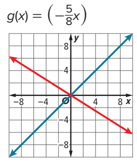
$g(x)$ is a translation of the parent function 4 units up

23. $g(x) = (x - 2) - 8$



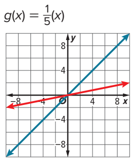
$g(x)$ is a translation of the parent function 2 units right and 8 units down

24. $g(x) = \left(-\frac{5}{8}x\right)$



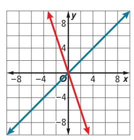
$g(x)$ is a horizontal stretch of the parent function by a factor of $\frac{8}{5}$ and a reflection across the y-axis

25. $g(x) = \frac{1}{5}(x)$



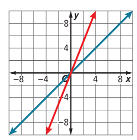
$g(x)$ is a vertical compression of the parent function by a factor of $\frac{1}{5}$

26. $g(x) = -3(x)$



$g(x)$ is a vertical stretch of the parent function by a factor of 3 and a reflection across the x-axis

27. $g(x) = (2.5x)$



$g(x)$ is a horizontal compression of the parent function by a factor of 0.4

Lesson 4-4 • Transformations of Linear Functions **249**

REASONING Write a function $g(x)$ to represent the translated graph.

28. $f(x) = 3x + 7$ translated 4 units up. $g(x) = 3x + 11$

29. $f(x) = x - 5$ translated 2 units down. $g(x) = x - 7$

30. PERIMETER The function $f(s) = 4s$ represents the perimeter of a square with side length s. Write a function $g(s)$ to represent the perimeter of a square with side lengths that are twice as great. Describe the graph of $g(s)$ compared to $f(s)$. $g(s) = 8s$; The graph of $g(s) = 8s$ is the graph of $f(s) = 4s$ stretched vertically by a factor of 2.

31. GAMES The function $f(x) = 0.50x$ gives the average cost in dollars for x cell phone game downloads that cost an average of \$0.50 each. Write a function $g(x)$ to represent the cost in dollars for x cell phone game downloads that cost \$1.50 each. Describe the graph of $g(x)$ compared to $f(x)$. $g(x) = 1.5x$; The graph of $g(x) = 1.5x$ is the graph of $f(x) = 0.50x$ stretched vertically by a factor of 3.

32. TRAINER The function $f(x) = 90x$ gives the cost of working out with a personal trainer, where \$90 is the trainer's hourly rate, and x represents the number of hours spent working out with the trainer. Describe the dilation, $g(x)$ of the function $f(x)$, if the trainer increases her hourly rate to \$100. $g(x) = 100x$; The graph of $g(x) = 100x$ is the graph of $f(x) = 90x$ stretched vertically by a factor of $\frac{10}{9}$.

33. DOWNLOADS Hannah wants to download songs. She researches the price to download songs from Site F. Hannah wrote the function $f(x) = x$, which represents the cost in dollars for x songs downloaded that cost \$1.00 each.

a. Hannah researches the price to download songs from Site G. Write a function $g(x)$ to represent the cost in dollars for x songs downloaded that cost \$1.29 each. $g(x) = 1.29x$

b. Describe the graph of $g(x)$ compared to the graph of $f(x)$. The graph of $g(x) = 1.29x$ is the graph of $f(x) = x$ stretched vertically by a factor of 1.29.

Higher-Order Thinking Skills

34. PERSEVERE For any linear function, replacing $f(x)$ with $f(x + k)$ results in the graph of $f(x)$ being shifted k units to the right for $k < 0$ and shifted k units to the left for $k > 0$. Does shifting the graph horizontally k units have the same effect as shifting the graph vertically $-k$ units? Justify your answer. Include graphs in your response. See margin.

35. CREATE Write an equation that is a vertical compression by a factor of a of the parent function $y = x$. What can you say about the horizontal dilation of the function? $y = \frac{1}{a}x$; The function is horizontally stretched by a factor of a.

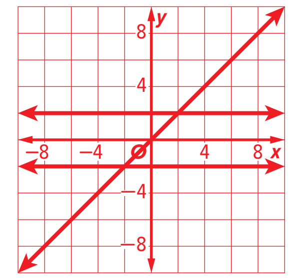
36. WHICH ONE DOESN'T BELONG Consider the four functions. Which one does not belong in this group? Justify your conclusion. $f(x) = -3x + 10$; This function is a reflection and translation of the parent graph. The other three functions are dilations and translations of the parent graph.

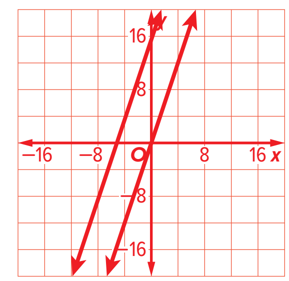
$f(x) = 2(x + 1) - 3$	$f(x) = \frac{1}{2}x - 4$	$f(x) = -3x + 10$	$f(x) = 5(x - 7) + 3$

250 **Module 4** • Linear and Nonlinear Functions

Answer

34. No; This is true only if the slope is 1. Consider any constant function. Shifting the graph of a constant function to the right or left doesn't result in any vertical shift of the same graph. If the slope m of the line described by $f(x)$ is something other than -1, 0, or 1, then a horizontal shift of k units is the same as a vertical shift of $-mk$ units. For example, if $f(x) = 3x$, then $f(x + 5) = 3(x + 5)$ or $3x + 15$. $f(x + 5)$ is shifted 5 units left of $f(x)$ or 15 units up from $f(x)$. Sample graphs shown.





Lesson 4-5

Arithmetic Sequences

F.BF.1a, F.BF.2, F.LE.2

LESSON GOAL

Students write and graph equations of arithmetic sequences.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Arithmetic Sequences

- Identify Arithmetic Sequences
- Find the Next Term

Explore: Common Differences

Develop:

Arithmetic Sequences as Linear Functions

- Find the *n*th Term
- Apply Arithmetic Sequences as Linear Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Add Integers	●	●		●
Extension: Arithmetic Series		●	●	●

Language Development Handbook

Assign page 24 of the *Language Development Handbook* to help your students build mathematical language related to arithmetic sequences.

ELL You can use the tips and suggestions on page T24 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Standards for Mathematical Practice:

3 Construct viable arguments and critique the reasoning of others.

8 Look for and express regularity in repeated reasoning.

Coherence

Vertical Alignment

Previous
Students identified the effects of transformations of the graphs of linear functions.
F.IF.7a, F.BF.3

Now
Students write and graph equations of arithmetic sequences.
F.BF.1a, F.BF.2, F.LE.2

Now
Students will graph piecewise-defined and step functions.
F.IF.4, F.IF.7b

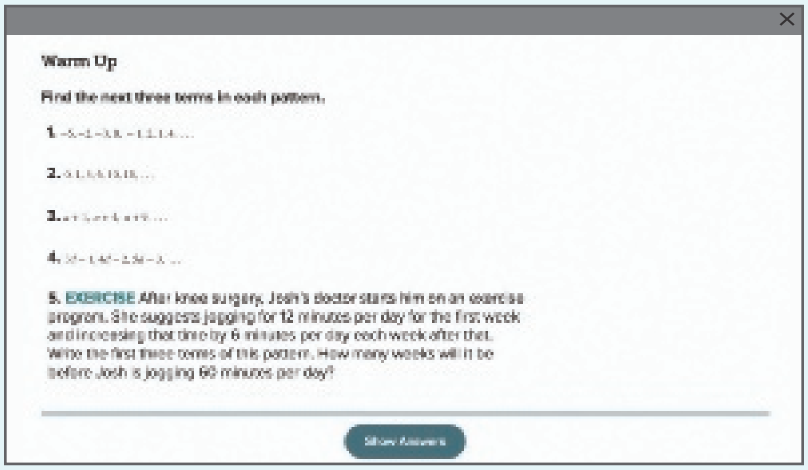
Rigor

The Three Pillars of Rigor

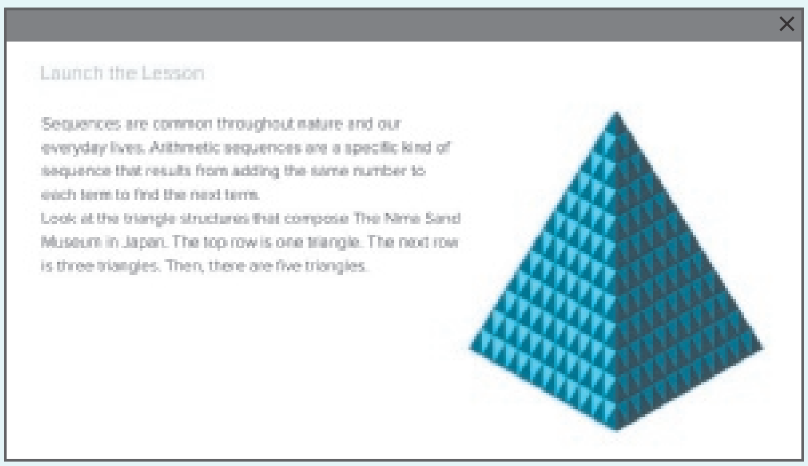
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students expand on their understanding of and build fluency with sequences (first studied in Grade 4) by writing formulas for arithmetic sequences and relating them to linear functions. They apply their understanding by solving real-world problems related to arithmetic sequences.

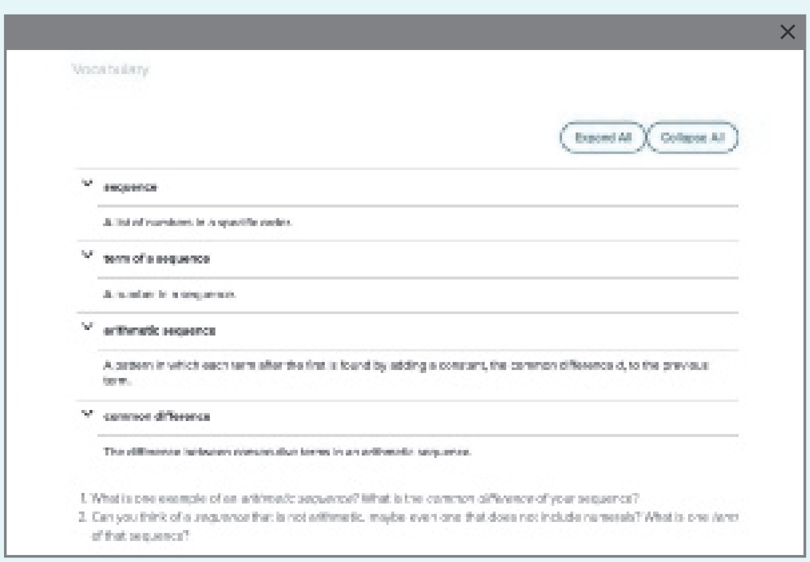
Interactive Presentation



Warm Up



Launch the Lesson



Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- finding the next terms in patterns

Answers:

1. 3, 6, 5
2. 21, 28, 36
3. a + 16, $a + 25$, $a + 36$
4. $6d - 4$, $7d - 5$, $8d - 6$
5. 12, 18, 24; 9 wk

Launch the Lesson

Teaching the Mathematical Practices

8 Look for a Pattern Help students to see the pattern in the triangle structures that compose The Nima Sand Museum and in the Pyramid of Oranges.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Mathematical Background

A sequence is a set of numbers in a specific order. The numbers in a sequence are called terms. If the terms of a sequence increase or decrease at a constant rate, the sequence is called an arithmetic sequence. The difference between successive terms of an arithmetic sequence is called the common difference. Any term of an arithmetic sequence can be found by adding the common difference to the preceding term. The formula for finding a specific term in an arithmetic sequence is $a_n = a_1 + (n - 1)d$, where a_n is the nth term, a_1 is the first term, and d is the common difference.

Explore Common Differences

Objective

Students use a sketch to explore the relationship between arithmetic sequences and linear functions.

Teaching the Mathematical Practices

4 Use Tools Point out that to solve the problem in this Explore, students will need to use the table and sketch.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

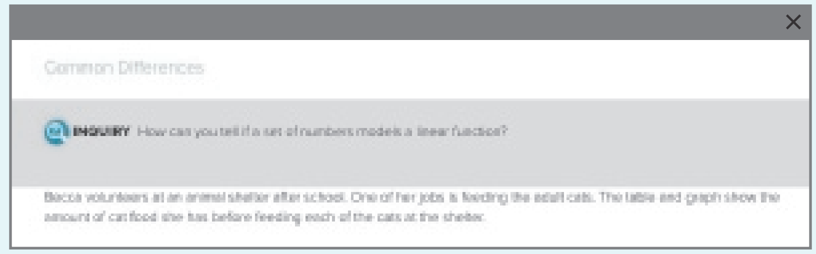
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

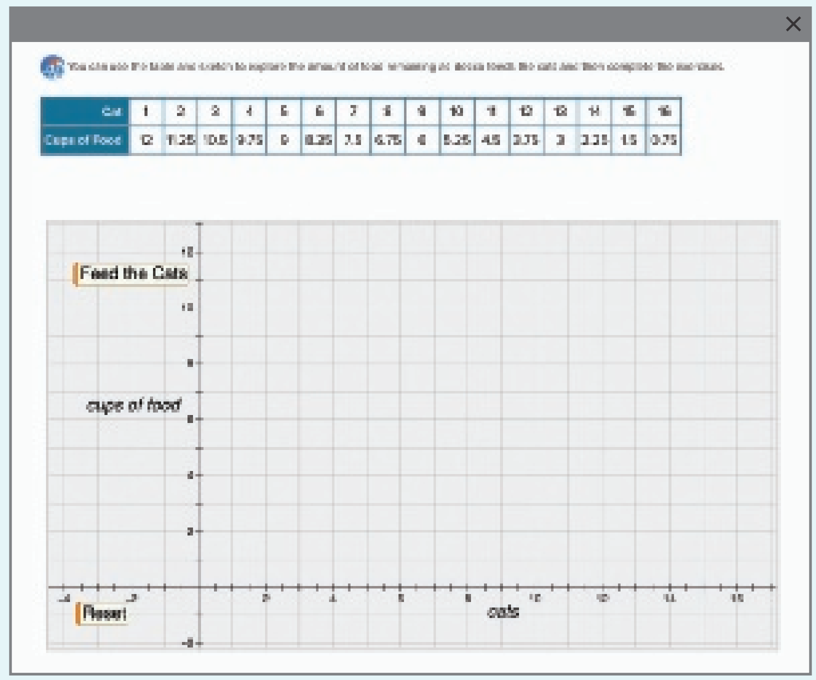
Students will complete guiding exercises throughout the Explore activity. Students will use the sketch to graph a linear function to solve a real-world problem. They will observe as the data points are plotted, and then answer questions related to the resulting graph. Then, students will answer the Inquiry Question.

(*continued on the next page*)

Interactive Presentation



Explore



Explore

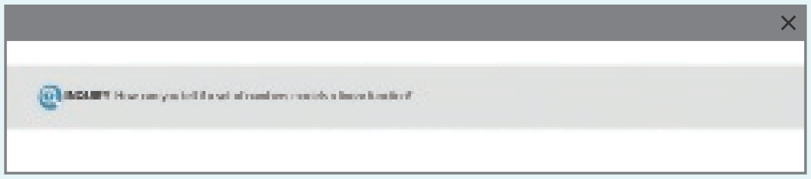
WEB SKETCHPAD

Students use a sketch to explore the amount of food remaining as cats are fed.

TYPE

Students answer questions about the pattern in the data.

Interactive Presentation



Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Common Differences (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- Why is the amount of food decreasing with each cat? Sample answer: Each cat is being fed a certain amount of food, so there will be less after each cat is fed.
- How does the amount of food each cat is fed relate to the slope of the linear function that models the situation? Sample answer: The amount of food each cat is fed represents the change in the amount of food, which is the slope of the function. As long as there is a constant change, or constant slope, then you have a linear function.

Inquiry

How can you tell if a set of numbers models a linear function? Sample answer: The points are on the same line and have a constant slope.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Arithmetic Sequences

Objective

Students construct arithmetic sequences by using the common difference.

Teaching the Mathematical Practices

8 Look for a Pattern Help students see the pattern in this Learn.

Common Misconception

Students may think that the terms of all arithmetic sequences must increase. They may believe this because the definition of an arithmetic sequence refers to the use of addition to find successive terms. Point out that when the number being added is negative, the terms will decrease.

Example 1 Identify Arithmetic Sequences

Teaching the Mathematical Practices

3 Reason Inductively In this example, students will use inductive reasoning to make plausible arguments.

Questions for Mathematical Discourse

AL How are the terms of an arithmetic sequence found? The same number is added to each term to find the next term.

OL What requirement must be met for the sequence to represent an arithmetic sequence? The difference between the terms must be constant.

BL Does the sequence follow a pattern? Explain. Yes; sample answer: The difference in the numbers repeats itself, so the next difference would be −3.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Lesson 4-5

Arithmetic Sequences

Learn Arithmetic Sequences

A **sequence** is a set of numbers that are ordered in a specific way. Each number within a sequence is called a **term of a sequence**.

In an **arithmetic sequence**, each term after the first is found by adding a constant, the **common difference** *d*, to the previous term.

Words	An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference.
Examples	22, 17, 12, 7, 2, ... (−5, −5, −5, −5) The common difference is −5. −13, −5, 3, 11, 19, ... (+8, +8, +8, +8) The common difference is 8.

Today's Goals
- Construct arithmetic sequences.
- Apply the arithmetic sequence formula.

Today's Vocabulary
sequence
term of sequence
arithmetic sequence
common difference
*n*th term of an arithmetic function

Example 1 Identify Arithmetic Sequences

Determine whether the sequence is an arithmetic sequence. Justify your reasoning.

17, 14, 10, 7, 3

Check the difference between terms.

17, 14, 10, 7, 3
−3, −4, −3, −4

This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

Think About It!
How are arithmetic sequences and number patterns alike and different?

Sample answer: An arithmetic sequence is a series of numbers that has a common difference between two consecutive terms, which is used to find other terms in the sequence. A number pattern is a series of numbers that follows a rule but might not have a common difference between terms.

Check

Determine whether the sequence is an arithmetic sequence. Justify your reasoning.

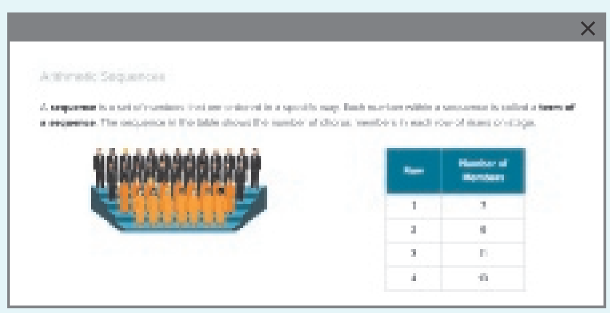
82, 73, 64, 55, . . .

yes, there is a common difference of −9

Go Online You can complete an Extra Example online.

Lesson 4-5 • Arithmetic Sequences 251

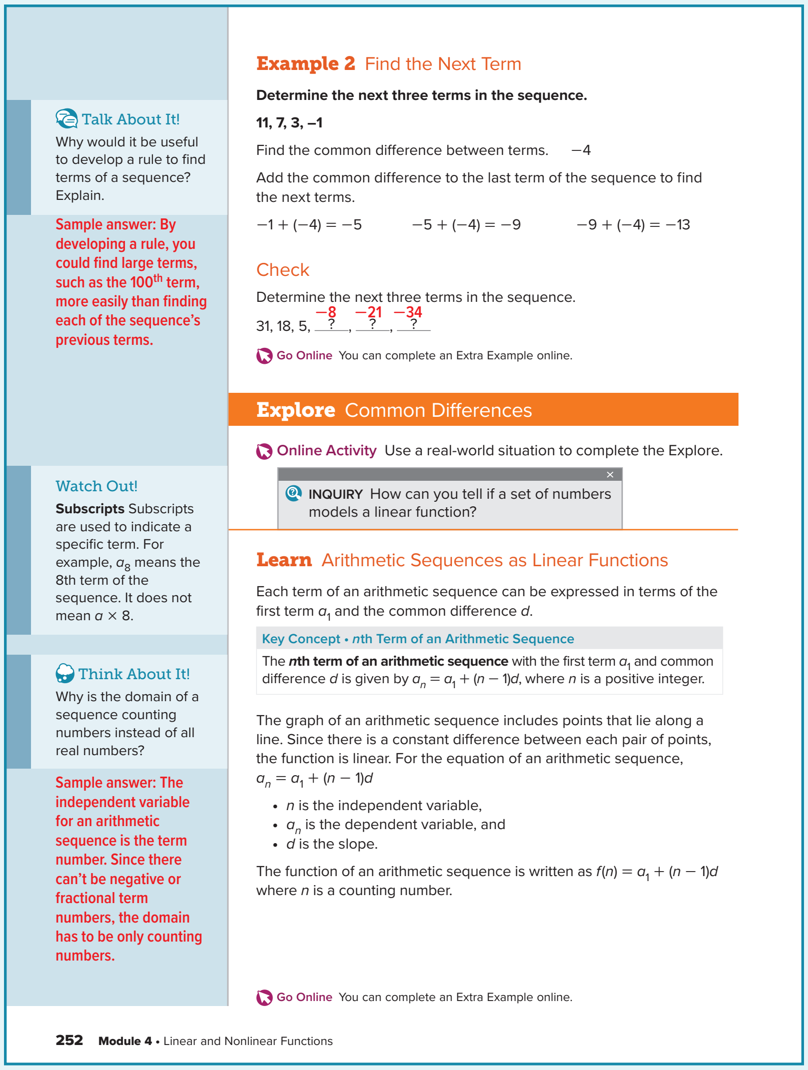
Interactive Presentation



Learn

TYPE

Students enter the number of chorus members in the 6th row.



Talk About It!
Why would it be useful to develop a rule to find terms of a sequence? Explain.

Sample answer: By developing a rule, you could find large terms, such as the 100th term, more easily than finding each of the sequence's previous terms.

Example 2 Find the Next Term

Determine the next three terms in the sequence.

11, 7, 3, −1

Find the common difference between terms. −4

Add the common difference to the last term of the sequence to find the next terms.

$-1 + (-4) = -5$ $-5 + (-4) = -9$ $-9 + (-4) = -13$

Check

Determine the next three terms in the sequence.

31, 18, 5, $\underline{-8}$, $\underline{-21}$, $\underline{-34}$

Go Online You can complete an Extra Example online.

Explore Common Differences

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you tell if a set of numbers models a linear function?

Watch Out!
Subscripts Subscripts are used to indicate a specific term. For example, a_8 means the 8th term of the sequence. It does not mean $a \times 8$.

Learn Arithmetic Sequences as Linear Functions

Each term of an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Key Concept • nth Term of an Arithmetic Sequence
The **nth term of an arithmetic sequence** with the first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

Think About It!
Why is the domain of a sequence counting numbers instead of all real numbers?

Sample answer: The independent variable for an arithmetic sequence is the term number. Since there can't be negative or fractional term numbers, the domain has to be only counting numbers.

The graph of an arithmetic sequence includes points that lie along a line. Since there is a constant difference between each pair of points, the function is linear. For the equation of an arithmetic sequence, $a_n = a_1 + (n - 1)d$

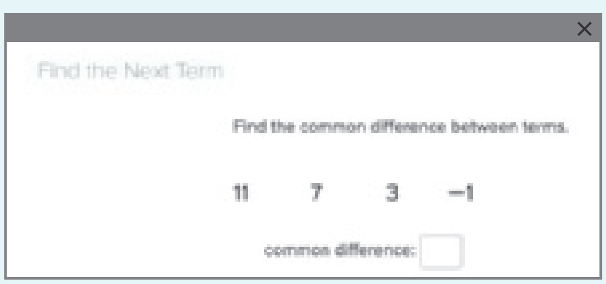
- n is the independent variable,
- a_n is the dependent variable, and
- d is the slope.

The function of an arithmetic sequence is written as $f(n) = a_1 + (n - 1)d$ where n is a counting number.

Go Online You can complete an Extra Example online.

252 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Example 2

TYPE

Students enter the consecutive members of the arithmetic sequences.

TYPE

Students explain how developing a rule or equation would be useful in finding terms of a sequence.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 2 Find the Next Term

Teaching the Mathematical Practices

1 Monitor and Evaluate Point out that in this example, students must stop and evaluate their progress when determining the next terms in the sequence.

Questions for Mathematical Discourse

AL What is the relationship between the terms? Each term is 4 less than the previous term.

OL If this sequence continues on, will all of the subsequent terms be negative, or will they go back to being positive? Explain. Sample answer: They will stay negative because each term is less than the one before.

BL What is the tenth term in the arithmetic sequence? −25

Learn Arithmetic Sequences as Linear Functions

Objective

Students apply the arithmetic sequence formula by examining the common differences in arithmetic sequences.

Teaching the Mathematical Practices

3 Construct Arguments In this Learn, students will use stated assumptions, definitions, and previously established results to construct an argument.

8 Look for a Pattern Help students to see the pattern in the formula for the nth term of an arithmetic sequence.

Important to Know

In the context of a function, the term numbers represent the input values, and the terms of the sequence represent the output values. The common difference is a constant that represents the slope. The function rule is linear, defining how each term is determined by its term number, n.

Common Misconception

Some students may think that the function rule contains more than one variable. Use several examples to show students that for any particular sequence, a_1 and d are known constants, and only n is variable.

Example 3 Find the *n*th Term

Teaching the Mathematical Practices

3 Reason Inductively In this example, students will use inductive reasoning to make plausible arguments.

Questions for Mathematical Discourse

AL What values do you need to know in order to write the equation? You need to know the first term, a_1, and the difference, d.

OL How are the values substituted to find the equation? Sample answer: $-4 + 3(n - 1) = -4 + 3n - 3 = 3n - 7$

BL Will n always be a positive number? Explain. Yes; sample answer: Since n refers to the number of the term, like the 1st term or the 15th term, it will always be a positive whole number.

DIFFERENTIATE

Reteaching Activity AL ELL

IF students have difficulty following the progression of steps that lead to the building of the equation,

THEN have them cycle through the steps again, using a simpler sequence, such as 1, 4, 7, 10, ...

DIFFERENTIATE

Enrichment Activity BL

Have students work with a partner. Tell them that you know of an arithmetic sequence in which the 4th term is 27 and the 8th term is 59. Ask them to find the first term and the common difference. Have pairs share how they solved the problem, and describe how they checked that their solution is correct. $a_1 = 3$, $d = 8$

Example 3 Find the *n*th Term

Use the arithmetic sequence −4, −1, 2, 5, . . . to complete the following.

Part A Write an equation.

$a_n = 3n - 7$

Part B Find the 16th term of the sequence.

Use the equation from Part A to find the 16th term in the arithmetic sequence.

$a_n = 3n - 7$	Equation from Part A
$a_{16} = 3(16) - 7$	Substitute 16 for n.
$a_{16} = 48 - 7$	Multiply.
$a_{16} = 41$	Simplify.

Check

RUNNING Randi has been training for a marathon, and it is important for her to keep a constant pace. She recorded her time each mile for the first several miles that she ran.

- At 1 mile, her time was 10 minutes and 30 seconds.
- At 2 miles, her time was 21 minutes.
- At 3 miles, her time was 31 minutes and 30 seconds.
- At 4 miles, her time was 42 minutes.

Part A Write a function to represent her sequence of data. Use n as the variable.

$f(n) = 10.5n$

Part B How long will it take her to run a whole marathon? Round your answer to the nearest thousandth if necessary. (Hint: a marathon is 26.2 miles.)

4.585 hours

Example 4 Apply Arithmetic Sequences as Linear Functions

MONEY **Laniqua opened a savings account to save for a trip to Spain. With the cost of plane tickets, food, hotel, and other expenses, she needs to save $1600. She opened the account with $525. Every month, she adds the same amount to her account using the money she earns at her after school job. From her bank statement, Laniqua can write a function that represents the balance of her savings account.**

(continued on the next page)

Go Online You can complete an Extra Example online.

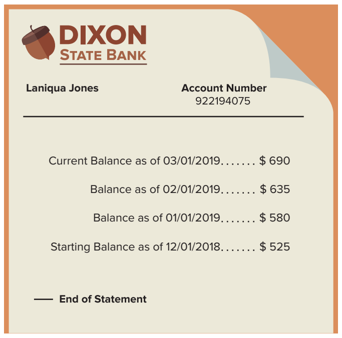
Lesson 4-5 • Arithmetic Sequences 253

Interactive Presentation

Example 3

EXPAND

Students can tap to see the steps to writing and using an equation for arithmetic sequences.


DIXON STATE BANK

Laniqua Jones

Account Number 922194075

Current Balance as of 03/01/2019....... $ 690

Balance as of 02/01/2019....... $ 635

Balance as of 01/01/2019....... $ 580

Starting Balance as of 12/01/2018....... $ 525

— End of Statement

Part A Create a function to represent the sequence.

First, find the common difference.

$$525 \underbrace{\quad}_{+55} 580 \underbrace{\quad}_{+55} 635 \underbrace{\quad}_{+55} 690$$

The common difference is 55.

The balance after 1 month is \$580, so let $a_1 = 580$. Notice that the starting balance is \$525. You can think of this starting point as $a_0 = 580$.

$f(n) = a_1 + (n - 1)d$ — Formula for the nth term.

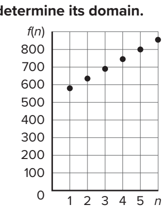
$= 580 + (n - 1)(55)$ — $a_1 = 580$ and $d = 55$

$= 580 + 55n - 55$ — Simplify.

$= 55n + 525$

Part B Graph the function and determine its domain.

n	$f(n)$
0	525
1	580
2	635
3	690
4	745
5	800
6	855



The domain is the number of months since Laniqua opened her savings account. The domain is {0, 1, 2, 3, 4, 5, ...}

Use a Source
Find the cost of a flight from the airport closest to you to Madrid, the capital of Spain. How many months would Laniqua need to save to afford the ticket?

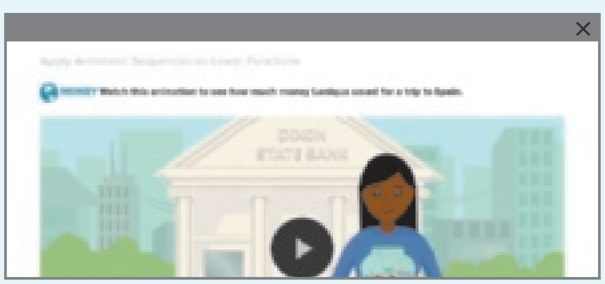
Sample answer: The cost to fly from the closest airport to Madrid is \$1416. To afford the ticket, Laniqua would have to save for 17 months.

Study Tip
Graphing You might not need to create a table of the sequence first. However, it might serve as a reminder that an arithmetic sequence is a series of points, not a line.

Go Online You can complete an Extra Example online.

254 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Example 4

WATCH

Students can watch an animation that describes the balance of an account over time.

TYPE

Students research costs of flights to Spain and determine the number of months they would need to save.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 4 Apply Arithmetic Sequences as Linear Functions

Teaching the Mathematical Practices

5 Use a Source Guide students to find external information to answer the questions posed in the Use a Source feature.

Questions for Mathematical Discourse

AL Why does the list of balances represent an arithmetic sequence? because there is a common difference between the balances

OL What does the common difference mean in the context of the problem? Laniqua is saving \$55 each month, so her account is increasing by \$55 per month.

BL Is the function discrete or continuous? Explain. Discrete; sample answer: The domain is the counting numbers, so the graph would consist of points, not a line.

DIFFERENTIATE

Enrichment Activity BL

Arithmetic sequences can be programmed into graphing calculators with results displayed in lists. Have advanced learners locate a set of directions for programming a sequence and develop a lesson for their classmates on analyzing sequences using the calculator.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–26
2	exercises that use a variety of skills from this lesson	27–34
2	exercises that extend concepts learned in this lesson to new contexts	35–37
3	exercises that emphasize higher-order and critical-thinking skills	38–46

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **THEN** assign: BL

- Practice, Exercises 1–37 odd, 38–46
- Extension: Arithmetic Series
- ALEKS® Arithmetic Sequences

IF students score 66%–89% on the Checks, **THEN** assign: OL

- Practice, Exercises 1–45 odd
- Remediation, Review Resources: Add Integers
- Personal Tutors
- Extra Examples 1–4
- ALEKS® Addition and Subtraction with Integers

IF students score 65% or less on the Checks, **THEN** assign: AL

- Practice, Exercises 1–25 odd
- Remediation, Review Resources: Add Integers
- *Quick Review Math Handbook*: Arithmetic Sequences as Linear Functions
- Arrive**MATH** Take Another Look
- ALEKS® Addition and Subtraction with Integers

Practice

Go Online You can complete your homework online.

Example 1

ARGUMENTS **Determine whether each sequence is an arithmetic sequence. Justify your reasoning.**

1. $-3, 1, 5, 9, \ldots$
This sequence has a common difference of 4 between its terms. This is an arithmetic sequence.

2. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

3. $-10, -7, -4, 1, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

4. $-12.3, -9.7, -7.1, -4.5, \ldots$
This sequence has a common difference of 2.6 between its terms. This is an arithmetic sequence.

5. $4, 7, 9, 12, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

6. $15, 13, 11, 9, \ldots$
This sequence has a common difference of –2 between its terms. This is an arithmetic sequence.

7. $7, 10, 13, 16, \ldots$
This sequence has a common difference of 3 between its terms. This is an arithmetic sequence.

8. $-6, -5, -3, -1, \ldots$
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

Example 2

Find the common difference of each arithmetic sequence. Then find the next three terms.

9. $0.02, 1.08, 2.14, 3.2, \ldots$
1.06; 4.26, 5.32, 6.38

10. $6, 12, 18, 24, \ldots$
6; 30, 36, 42

11. $21, 19, 17, 15, \ldots$
–2; 13, 11, 9

12. $-\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots$
$\frac{1}{2}; 1\frac{1}{2}, 2, 2\frac{1}{2}$

13. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \ldots$
$\frac{1}{3}; 3\frac{2}{3}, 4, 4\frac{1}{3}$

14. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, \ldots$
$\frac{3}{4}; 3\frac{7}{12}, 4\frac{1}{3}, 5\frac{1}{12}$

15. $3, 7, 11, 15, \ldots$
4; 19, 23, 27

16. $22, 19.5, 17, 14.5, \ldots$
–2.5; 12, 9.5, 7

17. $-13, -11, -9, -7, \ldots$
–2; –5, –3, –1

18. $-2, -5, -8, -11, \ldots$
–3; –14, –17, –20

Example 3

Use the given arithmetic sequence to write an equation and then find the 7th term of the sequence.

19. $-3, -8, -13, -18, \ldots$
$a_n = -5n + 2; -33$

20. $-2, 3, 8, 13, \ldots$
$a_n = 5n - 7; 28$

21. $-11, -15, -19, -23, \ldots$
$a_n = -4n - 7; -35$

22. $-0.75, -0.5, -0.25, 0, \ldots$
$a_n = 0.25n - 1; 0.75$

Lesson 4-5 • Arithmetic Sequences 255

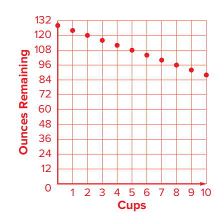
Example 4

23. SPORTS Wanda is the manager for the soccer team. One of her duties is to hand out cups of water at practice. Each cup of water is 4 ounces. She begins practice with a 128-ounce cooler of water.

a. Create a function to represent the arithmetic sequence. $f(n) = -4n + 128$

b. Graph the function.

c. How much water is remaining after Wanda hands out the 14th cup? 72 ounces

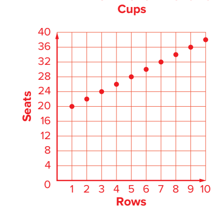


24. THEATER A theater has 20 seats in the first row, 22 in the second row, 24 in the third row, and so on for 25 rows.

a. Create a function to represent the arithmetic sequence. $f(n) = 2n + 18$

b. Graph the function.

c. How many seats are in the last row? 68 seats



25. POSTAGE The price to send a large envelope first class mail is 88 cents for the first ounce and 17 cents for each additional ounce. The table shows the cost for weights up to 5 ounces.

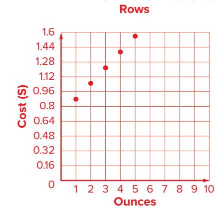
Weight (ounces)	1	2	3	4	5
Postage (dollars)	0.88	1.05	1.22	1.39	1.56

Source: United States Postal Service

a. Create a function to represent the arithmetic sequence. $f(n) = 0.17n + 0.71$

b. Graph the function.

c. How much did a large envelope weigh that cost $2.07 to send? 8 ounces

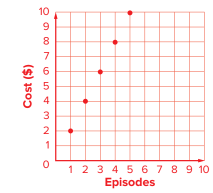


26. VIDEO DOWNLOADING Brian is downloading episodes of his favorite TV show to play on his personal media device. The cost to download 1 episode is $1.99. The cost to download 2 episodes is $3.98. The cost to download 3 episodes is $5.97.

a. Create a function to represent the arithmetic sequence. $f(n) = 1.99n$

b. Graph the function.

c. What is the cost to download 9 episodes? $17.91



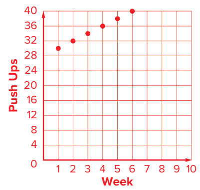
256 Module 4 • Linear and Nonlinear Functions

27. USE A MODEL Chapa is beginning an exercise program that calls for 30 push-ups each day for the first week. Each week thereafter, she has to increase her push-ups by 2.

a. Write a function to represent the arithmetic sequence. $f(n) = 2n + 28$

b. Graph the function.

c. Which week of her program will be the first one in which she will do 50 push-ups a day? 11th week



Mixed Exercises

CONSTRUCT ARGUMENTS **Determine whether each sequence is an arithmetic sequence. Justify your argument.**

28. −9, −12, −15, −18, ...
This sequence has a common difference of −3 between its terms. This is not an arithmetic sequence.

29. 10, 15, 25, 40, ...
This sequence does not have a common difference between its terms. This is not an arithmetic sequence.

30. −10, −5, 0, 5, ...
This sequence has a common difference of 5 between its terms. This is an arithmetic sequence.

31. −5, −3, −1, 1, ...
This sequence has a common difference of 2 between its terms. This is an arithmetic sequence.

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

32. 7, 13, 19, 25, ... See margin.

33. 30, 26, 22, 18, ... See margin.

34. −7, −4, −1, 2, ... See margin.

35. SAVINGS Fabiana decides to save the money she's earning from her after-school job for college. She makes an initial contribution of $3000 and each month deposits an additional $500. After one month she will have contributed $3500.

a. Write an equation for the *n*th term of the sequence. $a_n = 3000 + 500n$

b. How much money will Fabiana have contributed after 24 months? $15,000

36. NUMBER THEORY One of the most famous sequences in mathematics is the Fibonacci sequence. It is named after Leonardo de Pisa (1170–1250) or Filius Bonacci, alias Leonardo Fibonacci. The first several numbers in the Fibonacci sequence are shown.
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .
Does this represent an arithmetic sequence? Why or why not? No, because the difference between terms is not constant.

37. STRUCTURE Use the arithmetic sequence 2, 5, 8, 11, ...

a. Write an equation for the *n*th term of the sequence. $a_n = 3n - 1$

b. What is the 20th term in the sequence? 59

Higher-Order Thinking Skills

38. CREATE Write a sequence that is an arithmetic sequence. State the common difference, and find a_6. Sample answer: 2, 5, 8, 11, ...; The common difference is 3; $a_6 = 17$

39. CREATE Write a sequence that is not an arithmetic sequence. Determine whether the sequence has a pattern, and if so describe the pattern.
Sample answer: 5, 3, 8, 6, 11, 9, 14, ...; The pattern is to subtract 2 from the first term to find the second term, then add 5 to the second term to find the third term.

40. REASONING Determine if the sequence 1, 1, 1, 1, . . . is an arithmetic sequence. Explain your reasoning.
The sequence 1, 1, 1, 1, ... is a set of numbers whose difference between successive terms is the constant number 0. Thus, this sequence is an arithmetic sequence by the definition.

41. CREATE Create an arithmetic sequence with a common difference of −10.
Sample answer: 2, −8, −18, −28, ...

42. PERSEVERE Find the value of x that makes $x + 8$, $4x + 6$, and $3x$ the first three terms of an arithmetic sequence. −1

43. CREATE For each arithmetic sequence described, write a formula for the *n*th term of a sequence that satisfies the description.

a. first term is negative, common difference is negative
Sample answer: $a_n = -2 - 3n$

b. second term is −5, common difference is 7 $a_n = -19 + 7n$

c. $a_2 = 8$, $a_3 = 6$ $a_n = 12 - 2n$

Andre and Sam are both reading the same novel. Andre reads 30 pages each day. Sam created the table at the right. Refer to this information for Exercises 44–46.

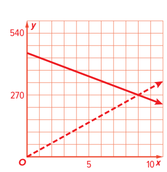
Sam's Reading Progress

Day	Pages Left to Read
1	430
2	410
3	390
4	370

44. ANALYZE Write arithmetic sequences to represent each boy's daily progress. Then write the function for the *n*th term of each sequence.
Andre: 30, 60, 90, 120, ...; $A(n) = 30n$; Sam: 430, 410, 390, 370, ...; $S(n) = 450 - 20n$

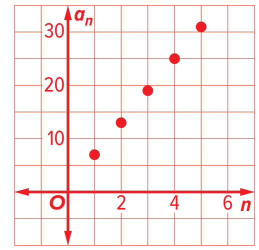
45. PERSEVERE Enter both functions from Exercise 44 into your calculator. Use the table to determine if there is a day when the number of pages Andre has read is equal to the number of pages Sam has left to read. If so, which day is it? Explain how you used the table feature to help you solve the problem.
On day 9, Andre has read 270 pages while Sam has 270 pages left to read. The table shows that both functions have a value of 270 when $x = 9$.

46. ANALYZE Graph both functions on your calculator, then sketch the graph. How can you use the graph to answer the equation from Exercise 45?
The intersection of the lines, which appears to be at (9, 270), represents the day (Day 9) when Andre has read the same number of pages that Sam has left to read (270 pages).

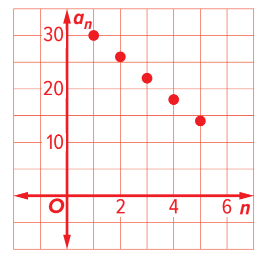


Answers

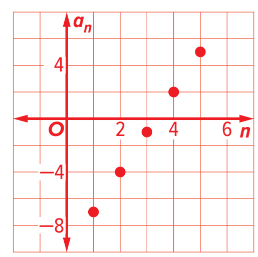
32. $a_n = 6n + 1$;



33. $a_n = -4n + 34$;



34. $a_n = 3n - 10$;



Piecewise and Step Functions

LESSON GOAL

Students graph piecewise-defined and step functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Develop:

Graphing Piecewise-Defined Functions
- Graph a Piecewise-Defined Function

Explore: Age as a Function

Develop:

Graphing Step Functions
- Graph a Greatest Integer Function
- Graph a Step Function

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Construct Linear Functions	●	●		●
Extension: Taxicab Graphs		●	●	●

Language Development Handbook

Assign page 25 of the *Language Development Handbook* to help your students build mathematical language related to piecewise-defined and step functions.

ELL You can use the tips and suggestions on page T25 of the handbook to support students who are building English proficiency.

Suggested Pacing

90 min	0.5 day
45 min	1 day

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

Standards for Mathematical Practice:

4 Model with mathematics.

6 Attend to precision.

Coherence

Vertical Alignment

Previous
Students wrote and graphed equations of arithmetic sequences.
F.BF.1a, F.BF.2, F.LE.2

Now
Students graph piecewise-defined and step functions.
F.IF.4, F.IF.7b

Next
Students will identify the effects of transformations of the graphs of absolute value functions.
F.IF.7b, F.BF.3

Rigor

The Three Pillars of Rigor

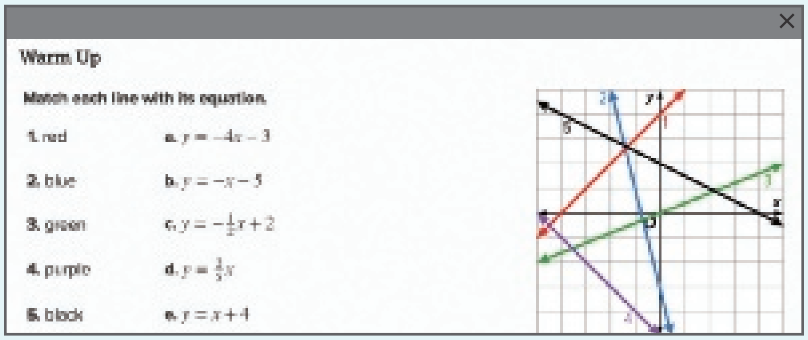
1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

Conceptual Bridge In this lesson, students extend their understanding of linear functions to piecewise-defined and step functions. They build fluency by graphing both types of functions, and they apply their understanding by solving real-world problems related to piecewise-defined and step functions.

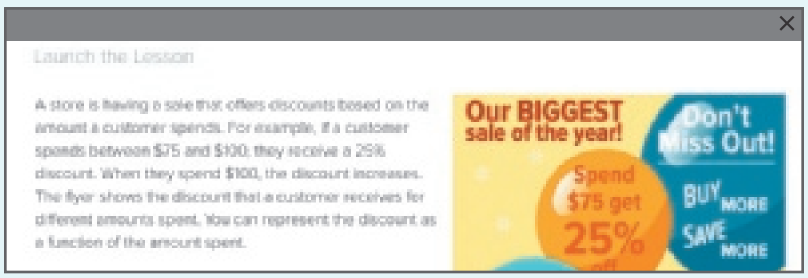
Mathematical Background

Piecewise-defined functions are functions that are defined by two or more functions, each with its own domain. The graph consists of the graph of each piece over its domain. A step function is a function whose graph consists of segments that look like a set of steps. The graph of the greatest integer function is an example of a step function.

Interactive Presentation



Warm Up



Launch the Lesson

Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- graphing linear functions

Answers:

1. e
2. a
3. d
4. b
5. c

Launch the Lesson

Teaching the Mathematical Practices

4 Apply Mathematics In this Launch, students learn how to apply what they have learned about special functions to a real-world situation about the discounts offered at a store.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?,* and connect these to the standards.

See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use these vocabulary terms in this lesson. You can expand each row if you wish to share the definitions. Then discuss the questions below with the class.

Explore Age as a Function

Objective

Students collect data to explore how real-world data can be represented by a step function.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

Students complete guiding exercises throughout the Explore activity. Students will explore how data in a real-world scenario involving age groups can be modeled by a step function. They will use their own age to create a table that shows the group in which they would be placed after various periods of time and answer questions regarding the data in their table. They will then explore how the graph of a step function represents this type of data. Then, students answer the Inquiry Question.

(continued on the next page)

Interactive Presentation

Explore

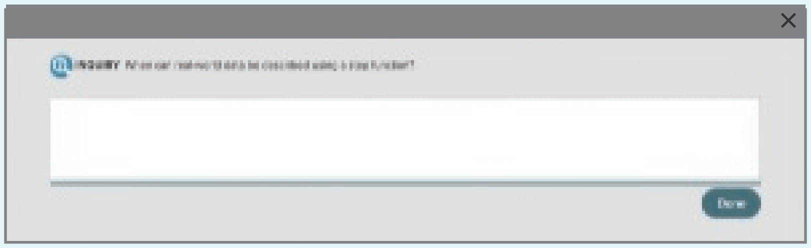
Elapsed Time	Exact Age (years and days)	Age Group
Today		
After 30 days		
After 200 days		
After 1 year		

Explore

TYPE

Students answer the questions and complete a table based on age.

Interactive Presentation



Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Age as a Function (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- In which age group would you place someone who will be 13 next week? Why? 11-12; Sample answer: According to the rules, the person would be in the 11-12 group because he or she is still 12.
- What other situations could be modeled by a step function? Sample answer: Movie ticket prices that depend on age could be modeled by a step function.

Inquiry

When can real-world data be described using a step function? Sample answer: When domain values in intervals have the same range value, real-world data can be described using a step function.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Graphing Piecewise-Defined Functions

Objective

Students graph piecewise-defined functions and identify their domain and range by determining the intervals where each part of the function should be graphed.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between equations and graphs of piecewise-defined and piecewise-linear functions.

Example 1 Graph a Piecewise-Defined Function

Teaching the Mathematical Practices

2 Different Properties Mathematically proficient students looks for different ways to solve problem. Encourage them to consider an alternate method in the Think About It! feature.

Questions for Mathematical Discourse

AL Why do you think this is called a piecewise-defined function? Sample answer: The function has different rules for different “pieces” of the graph.

OL Why is (1, 6) included in the graph, but (1, 2) is not? Sample answer: The first domain includes 1 because it states that $x \leq 1$ while the second domain does not include 1. So the y-value that corresponds with $x = 1$ is $2(1) + 4$, or 6.

BL Why is the range not the set of real numbers? There are no values of x that are paired with numbers greater than 6.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Lesson 4-6

Piecewise and Step Functions

Learn Graphing Piecewise-Defined Functions

Some functions cannot be described by a single expression because they are defined differently depending on the interval of x. These functions are **piecewise-defined functions.** A **piecewise-linear function** has a graph that is composed of some number of linear pieces.

Example 1 Graph a Piecewise-Defined Function

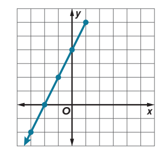
To graph a piecewise-defined function, graph each “piece” separately.

Graph $f(x) = \begin{cases} 2x + 4 \text{ if } x \leq 1 \\ -x + 3 \text{ if } x > 1 \end{cases}$. **State the domain and range.**

First, graph $f(x) = 2x + 4$ if $x \leq 1$.

- Create a table for $f(x) = 2x + 4$ using values of $x \leq 1$.
- Because x is *less than or equal to* 1, place a dot at (1, 6) to indicate that the endpoint is included in the graph.
- Then, plot the points and draw the graph beginning at (1, 6).

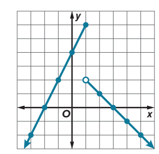
x	y
1	6
0	4
−1	2
−2	0
−3	−2



Next, graph $f(x) = -x + 3$ if $x > 1$.

- Create a table for $f(x) = -x + 3$ using values of $x > 1$.
- Because x is *greater than but not equal to* 1, place a circle at (1, 2) to indicate that the endpoint is not included in the graph.
- Then, plot the points and draw the graph beginning at (1, 2).

x	y
1	2
2	1
3	0
4	−1
5	−2



The domain is all real numbers. The range is $y \leq 6$.

Go Online You can complete an Extra Example online.

Today's Goals
- Identify and graph piecewise-defined functions.
- Identify and graph step functions.

Today's Vocabulary
piecewise-defined function
piecewise-linear function
step function
greatest integer function

Think About It!
What would be an advantage of graphing the entire expression and removing the portion that is not in the interval?

Sample answer: You can quickly graph the expressions using slope-intercept form or the intercepts and avoid having to make a table.

Go Online
An alternate method is available for this example.

Lesson 4-6 • Piecewise and Step Functions 259

Interactive Presentation

Example 1

TAP

Students select a method for graphing a piecewise-defined function.

TYPE

Students explain the advantage of graphing the entire expression and then removing the portion not in the interval.

Watch Out!

Circles and Dots Do not forget to examine the endpoint(s) of each piece to determine whether there should be a circle or a dot. > and < mean that a circle should be used, while ≥ and ≤ mean that a dot should be used.

Study Tip

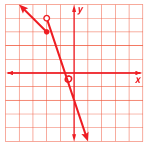
Piecewise-Defined Functions When graphing piecewise-defined functions, there should be a dot or line that contains each member of the domain.

Go Online

You can watch a video to see how to graph a piecewise-defined function on a graphing calculator

Check

Part A Graph $f(x) = \begin{cases} -x + 1 \text{ if } x \le -2 \\ -3x - 2 \text{ if } x > -2 \end{cases}$.



Part B Find the domain and range of the function.

D = $y \le -2$; R = $y > -2$

Explore Age as a Function

Online Activity Use a real-world situation to complete an Explore.

INQUIRY When can real-world data be described using a step function?

Learn Graphing Step Functions

A **step function** is a type of piecewise-linear function with a graph that is a series of horizontal line segments. One example of a step function is the **greatest integer function,** written as $f(x) = [\![x]\!]$ in which $f(x)$ is the greatest integer less than or equal to x.

Key Concept • Greatest Integer Function

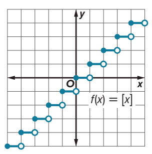
Type of graph: disjointed line segments

The graph of a step function is a series of disconnected horizontal line segments.

Domain: all real numbers; Because the dots and circles overlap, the domain is all real numbers.

Range: all integers; Because the function represents the greatest integer less than or equal to x, the range is all integers.

Parent function: $f(x) = [\![x]\!]$



Go Online You can complete an Extra Example online.

260 Module 4 • Linear and Nonlinear Functions

Interactive Presentation

Learn

TAP

Students tap on each marker to learn about graphing step functions.

TYPE

Students explain the relationship between the x- and y-values of each horizontal line segment.

CHECK

Students complete the Check online to determine whether they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | 2 FLUENCY | 3 APPLICATION

Learn Graphing Step Functions

Objective

Students graph step functions by making a table of values.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the x- and y-values of each horizontal line segment used in this Learn.

About the Key Concept

The graph of a greatest integer function always consists of infinitely many “steps,” each with one closed endpoint and one open endpoint. The parameters of the function determine the length of the steps. The greatest integer function is a type of piecewise-defined linear function, as the function is equal to a different constant for different intervals in the domain.

Common Misconception

Some students may think that the steps on the graph of a greatest integer function are always 1 unit long. Explain that while this is true of the graph of the parent greatest integer function, other greatest integer functions will contain parameters that may affect the length of each step.

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students have difficulty understanding the nature of the graph of the greatest integer function,

THEN have them create a table of values for the function. Instruct them to include decimals and fractions in their tables. Then have them describe how they determined the y-values for the x-values that they chose.

Example 2 Graph a Greatest Integer Function

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL Why do you think this is called a step function? The graph looks like steps on a staircase.

OL Why is (0, 1) included in the graph but (1, 1) is not? $[\![0 + 1]\!] = 1$ and $[\![1 + 1]\!] = 2$

BL The greatest integer function is sometimes called the floor function. Why do you think that is? Sample answer: The value truncates to the integer portion of the value, like standing on a chair on the 2nd floor still means you are on the 2nd floor.

Common Error

Some students may take the greatest integer of the x-value before adding 1. Explain that the greatest integer symbols act as grouping symbols, requiring that the operation inside the symbols be performed first, before finding the greatest integer of the resulting value.

DIFFERENTIATE

Enrichment Activity BL

Have students work with a partner. Ask them to create a story problem that can be modeled using the function $f(x) = 15[\![x]\!]$. Have students construct a graph for the model, and share their problems with the class.

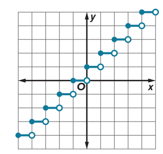
Example 2 Graph a Greatest Integer Function

Graph $f(x) = [\![x + 1]\!]$. State the domain and range.

First, make a table. Select a few values that are between integers.

x	$x + 1$	$[\![x + 1]\!]$	
−2	−1	−1	−1, −0.75, and 0.25 are greater than or equal to −1 but less than 0. So, −1 is the greatest integer that is not greater than −1, −0.75, or 0.25.
−1.75	−0.75	−1	
−1.25	−0.25	−1	
−1	0	0	0, 0.5, and 0.75 are greater than or equal to 0 but less than 1. So, 0 is the greatest integer that is not greater than 0, 0.5, or 0.75.
−0.5	0.5	0	
−0.25	0.75	0	
0	1	1	1, 1.25, and 1.5 are greater than or equal to 1 but less than 2. So, 1 is the greatest integer that is not greater than 1, 1.25, or 1.5.
0.25	1.25	1	
0.5	1.5	1	
1	2	2	2, 2.25, and 2.75 are greater than or equal to 2 but less than 3. So, 2 is the greatest integer that is not greater than 2, 2.25, or 2.75.
1.25	2.25	2	
1.75	2.75	2	

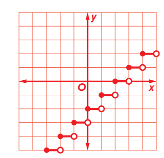
On the graph, dots represent included points. Circles represent points that are not included. The domain is all real numbers. The range is all integers. Note that this is the graph of $f(x) = [\![x]\!]$ shifted 1 unit to the left.



Check

Graph $f(x) = [\![x - 2]\!]$ by making a table. Copy and complete the table. Then graph the function.

x	$x - 2$	$[\![x - 2]\!]$
−1	−3	−3
−0.75	−2.75	−3
−0.25	−2.25	−3
0	−2	−2
0.25	−1.75	−2
0.5	−1.5	−2
1	−1	−1
1.25	−0.75	−1
1.5	−0.5	−1
2	0	0
2.25	0.25	0



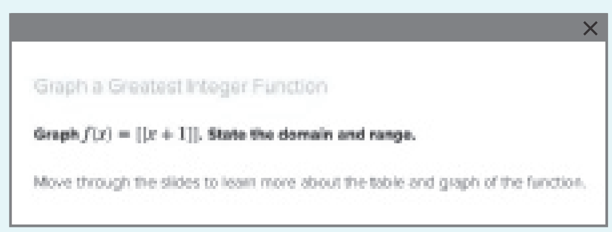
Go Online You can complete an Extra Example online.

Talk About It!
What do you notice about the symmetry, extrema, and end behavior of the function?

Sample answer: The function has no symmetry and no minimum or maximum values. As x increases, $f(x)$ increases, and as x decreases, $f(x)$ decreases.

Watch Out!
Greatest Integer Function When finding the value of a greatest integer function, do not round to the nearest integer. Instead, always round nonintegers down to the greatest integer that is not greater than the number.

Interactive Presentation



Example 2

TAP

Students move through the slides to learn more about the table and graph of a greatest integer function.

TYPE

Students describe what they noticed about the symmetry, extrema, and end behavior of the function.

Think About It!

How would the graph change if 1 certified lifeguard could watch up to 59 swimmers? For example, if there are greater than or equal to 60, but fewer than 120 swimmers, there must be 2 lifeguards on duty.

Sample answer: Every dot would become a circle, and every circle would become a dot.

Math History Minute

Oliver Heaviside (1850–1925) was a self-taught electrical engineer, mathematician, and physicist who laid much of the groundwork for telecommunications in the 21st century. Heaviside invented the Heaviside step function,

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } x = 0, \\ 1 & \text{if } x > 0 \end{cases}$$

which he used to model the current in an electric circuit.

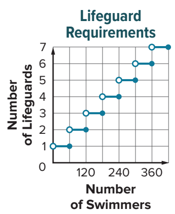
Example 3 Graph a Step Function

SAFETY **A state requires a ratio of 1 lifeguard to 60 swimmers in a swimming pool. This means that 1 lifeguard can watch up to and including 60 swimmers. Make a table and draw a graph that shows the number of lifeguards that must be on duty $f(x)$ based on the number of swimmers in the pool x.**

The number of lifeguards that must be on duty can be represented by a step function.

- If the number of swimmers is greater than 0 but fewer than or equal to 60, only 1 lifeguard must be on duty.
- If the number of swimmers is greater than 60 but fewer than or equal to 120, there must be 2 lifeguards on duty.
- If the number of swimmers is greater than 180 but fewer than or equal to 240, there must be 4 lifeguards on duty.

x	$f(x)$
$0 < x \leq 60$	1
$60 < x \leq 120$	2
$120 < x \leq 180$	3
$180 < x \leq 240$	4
$240 < x \leq 300$	5
$300 < x \leq 360$	6
$360 < x \leq 420$	7



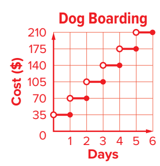
The circles mean that when there are more than a multiple of 60 swimmers, another lifeguard is required.

The dots represent the maximum number of swimmers that can be in the pool for that particular number of lifeguards on duty.

Check

PETS At Luciana's pet boarding facility, it costs $35 per day to board a dog. Every fraction of a day is rounded up to the next day. Copy and complete the table. Then graph the function.

Days	Cost ($)
$0 < x \leq 1$	35
$1 < x \leq 2$	70
$2 < x \leq 3$	105
$3 < x \leq 4$	140
$4 < x \leq 5$	175
$5 < x \leq 6$	210



Go Online You can complete an Extra Example online.

262 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation

Example 3

DRAG & DROP

Students drag the correct values to complete the table.

TAP

Students tap on each marker to see the difference between the circles and dots.

CHECK

Students complete the Check online to determine whether they are ready to move on.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | **3 APPLICATION**

Example 3 Graph a Step Function

Teaching the Mathematical Practices

4 Apply Mathematics In this example, students apply what they have learned about step functions to solving a real-world problem.

Questions for Mathematical Discourse

AL How many lifeguards are needed for 59 swimmers? 1 For 61 swimmers? 2

OL Why is this situation represented by a step function? Sample answer: Every x-value in each interval of 60 is paired with the same y-value, forming a graph that consists of steps.

BL How would the graph of the function change if the number of lifeguards required for the number of swimmers is cut in half? Sample answer: The graph would be stretched horizontally because more swimmers could be watched by each lifeguard.

Essential Question Follow-Up

Students have analyzed and graphed step functions.

Ask:

If you know that a function is a step function, what do you know about how the elements of the domain are paired with the elements of the range? Sample answer: The domain is grouped into intervals, and every number in the interval is paired with the same number in the range.

Exit Ticket

Recommended Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, go online to display the Exit Ticket prompt and ask students to respond verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–14
2	exercises that use a variety of skills from this lesson	15–20
2	exercises that extend concepts learned in this lesson to new contexts	21–22
3	exercises that emphasize higher-order and critical-thinking skills	23–31

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**

THEN assign:

- Practice, Exercises 1–21 odd, 23–31
- Extension: Taxicab Graphs

IF students score 66%–89% on the Checks, **OL**

THEN assign:

- Practice, Exercises 1–31 odd
- Remediation, Review Resources: Construct Linear Functions
- Personal Tutors
- Extra Examples 1–3
- **ALEKS**® Tables and Graphs of Lines

IF students score 65% or less on the Checks, **AL**

THEN assign:

- Practice, Exercises 1–13 odd
- Remediation, Review Resources: Construct Linear Functions
- *Quick Review Math Handbook*: Special Functions
- Arrive**MATH** Take Another Look
- **ALEKS**® Tables and Graphs of Lines

Answers

15. $f(x) = \begin{cases} 16.20 \text{ if } 0 < x \leq 1 \\ 19.30 \text{ if } 1 < x \leq 2 \\ 22.40 \text{ if } 2 < x \leq 3 \\ 25.50 \text{ if } 3 < x \leq 4 \\ 28.60 \text{ if } 4 < x \leq 5 \end{cases}$

16a.

x	0	4	8	12	16
$f(x)$	0	32	64	110	156

16b. $f(x) = \begin{cases} 8x & \text{if } x \leq 8 \\ 64 + 11.5(x - 8) & \text{if } x > 8 \end{cases}$

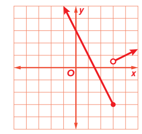
Practice

Go Online You can complete your homework online.

Example 1

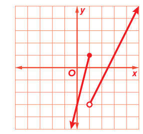
Graph each function. State the domain and range.

1. $f(x) = \begin{cases} \frac{1}{2}x - 1 \text{ if } x > 3 \\ -2x + 3 \text{ if } x \leq 3 \end{cases}$



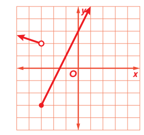
D = all real numbers, R = $f(x) \geq -3$

2. $f(x) = \begin{cases} 2x - 5 \text{ if } x > 1 \\ 4x - 3 \text{ if } x \leq 1 \end{cases}$



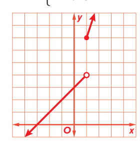
D = all real numbers, R = all real numbers

3. $f(x) = \begin{cases} 2x + 3 \text{ if } x \geq -3 \\ -\frac{1}{3}x + 1 \text{ if } x < -3 \end{cases}$



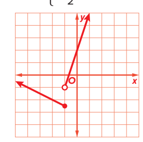
D = all real numbers, R = $f(x) \geq -3$

4. $f(x) = \begin{cases} 3x + 4 \text{ if } x \geq 1 \\ x + 3 \text{ if } x < 1 \end{cases}$



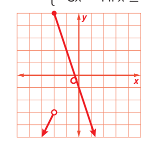
D = all real numbers, R = $f(x) < 4$ or $f(x) \geq 7$

5. $f(x) = \begin{cases} 3x + 2 \text{ if } x > -1 \\ -\frac{1}{2}x - 3 \text{ if } x \leq -1 \end{cases}$



D = all real numbers, R = $f(x) \geq -2.5$

6. $f(x) = \begin{cases} 2x + 1 \text{ if } x < -2 \\ -3x - 1 \text{ if } x \geq -2 \end{cases}$

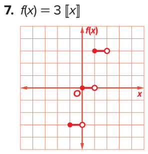


D = all real numbers, R = $f(x) \leq 5$

Example 2

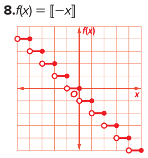
Graph each function. State the domain and range.

7. $f(x) = 3[\![x]\!]$



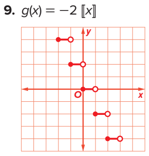
D = all real numbers, R = all integer multiples of 3

8. $f(x) = [\![-x]\!]$



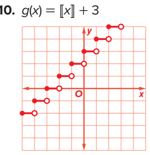
D = all real numbers, R = all integers

9. $g(x) = -2[\![x]\!]$



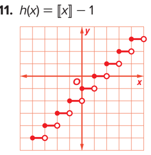
D = all real numbers, R = all even integers

10. $g(x) = [\![x]\!] + 3$



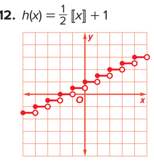
D = all real numbers, R = all integers

11. $h(x) = [\![x]\!] - 1$



D = all real numbers, R = all integers

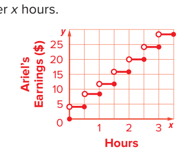
12. $h(x) = \frac{1}{2}[\![x]\!] + 1$



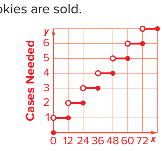
D = all real numbers, R = all multiples of $\frac{1}{2}$

Example 3

13. BABYSITTING Ariel charges $8 per hour as a babysitter. She rounds every fraction of an hour up to the next half-hour. Draw a graph to represent Ariel's total earnings y after x hours.



14 FUNDRAISING Students are selling boxes of cookies at a fundraiser. The boxes of cookies can only be ordered by the case, with 12 boxes per case. Draw a graph to represent the number of cases needed y when x boxes of cookies are sold.



Mixed Exercises

15. PRECISION A package delivery service determines rates for express shipping by the weight of a package, with every fraction of a pound rounded up to the next pound. The table shows the cost of express shipping packages that weigh no more than 5 pounds. Write a piecewise-linear function representing the cost to ship a package that weighs no more than 5 pounds. State the domain and range. See margin.

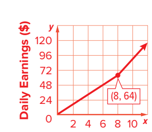
Weight (pounds)	Rate (dollars)
1	16.20
2	19.30
3	22.40
4	25.50
5	28.60

16. EARNINGS Kelly works in a hospital as a medical assistant. She earns $8 per hour the first 8 hours she works in a day and $11.50 per hour each hour thereafter. a–c. See margin.

a. Organize the information into a table. Include a row for hours worked x, and a row for daily earnings $f(x)$.

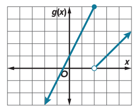
b. Write the piecewise equation describing Kelly's daily earnings $f(x)$ for x hours.

c. Draw a graph to represent Kelly's daily earnings.



17. REASONING Write a piecewise function that represents the graph.

$g(x) = \begin{cases} 2x + 1 \text{ if } x \le 2 \\ x - 2 \text{ if } x > 2 \end{cases}$



18. STRUCTURE Suppose $f(x) = 2[\![x - 1]\!]$.

a. Find $f(1.5)$. 0

b. Find $f(2.2)$. 2

c. Find $f(9.7)$. 16

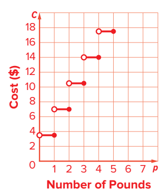
d. Find $f(-1.25)$. -6

19. RENTAL CARS Mr. Aronsohn wants to rent a car on vacation. The rate the car rental company charges is $19 per day. If any fraction of a day is counted as a whole day, how much would it cost for Mr. Aronsohn to rent a car for 6.4 days? $133.00

20. USE A MODEL A roadside fruit and vegetable stand determines rates for selling produce, with every fraction of a pound rounded up to the next pound. The table shows the cost of tomatoes by weight in pounds.

a. Write a piecewise-linear function representing the cost of purchasing tomatoes that weigh no more than 5 pounds, where C is the cost in dollars and p is the number of pounds. See margin.

Weight (pounds)	Rate (dollars)
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50



b. Graph the function.

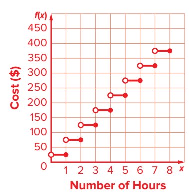
c. State the domain and range. $D = \{x \mid 0 < x \le 5\}$; $R = \{3.50, 7.00, 10.50, 14.00, 17.50\}$

d. What would be the cost of purchasing 8.3 pounds of tomatoes at the roadside stand? $31.50

21. ELECTRONIC REPAIRS Tech Repairs charges $25 for an electronic device repair that takes up to one hour. For each additional hour of labor, there is a charge of $50. The repair shop charges for the next full hour for any part of an hour.

a. Copy and complete the table to organize the information. Include a row for hours of repair x, and a row for total cost $f(x)$.

x	0	2	4	6	8
$f(x)$	0	75	175	275	375



b. Write a step function to represent the total cost for every hour h of repair. $f(x) = 25 + 50[\![x]\!]$

c. Graph the function.

d. Devesh was charged $125 to repair his tablet. How long did the repair take to complete? $2 < x \le 3$

22. INVENTORY Malik owns a bakery. Every week he orders chocolate chips from a supplier. The supplier's pricing is shown in the table.

Chocolate Chip Pricing	
$4 per pound	Up to 3 pounds
$1.50 per pound	For each pound over 3 pounds

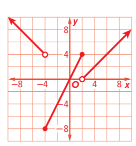
a. Write a function to represent the cost of chocolate chips. $f(x) = \begin{cases} 4x & \text{if } 0 \le x \le 3 \\ 12 + 1.5(x - 3) & \text{if } x > 3 \end{cases}$

b. Malik's budget for chocolate chips for the week is $25. How many whole pounds of chocolate chips can he order? 11 lbs

Higher-Order Thinking Skills

23. CREATE Write a piecewise-defined function with three linear pieces. Then graph the function.

Sample answer: $y = \begin{cases} -x & x < -4 \\ 2x & -4 \le x \le 2 \\ x - 2 & x > 2 \end{cases}$



24. FIND THE ERROR Amy graphed a function that gives the height of a car on a roller coaster as a function of time. She said her graph is the graph of a step function. Is this possible? Explain your reasoning.

No; the height of the car must change continuously, so it cannot "jump" from one constant height to another constant height as it would if the graph were a step function.

25. WRITE What is the difference between a step function and a piecewise-defined function?

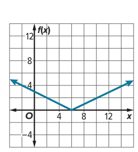
A step function has different constants over different intervals of its domain. A piecewise-defined function can have different algebraic rules over different intervals of its domain.

26. ANALYZE Does the piecewise relation $y = \begin{cases} -2x + 4 \text{ if } x \ge 2 \\ -\frac{1}{2}x - 1 \text{ if } x \le 4 \end{cases}$ represent a function? Justify your argument.

No; the pieces of the graph overlap vertically, so the graph fails the vertical line test.

ANALYZE **Refer to the graph for Exercises 27–31.**

27. Write a piecewise function to represent the graph. See margin.



28. What is the domain? D = all real numbers

29. What is the range? $R = f(x) \ge 0$

30. Find $f(8.5)$. 1.25

31. Find $f(1.2)$. 2.4

$$20a.\ C(p) = \begin{cases} 3.50 & \text{if } 0 < x \le 1 \\ 7.00 & \text{if } 1 < x \le 2 \\ 10.50 & \text{if } 2 < x \le 3 \\ 14.00 & \text{if } 3 < x \le 4 \\ 17.50 & \text{if } 4 < x \le 5 \end{cases}$$

$$27.\ f(x) = \begin{cases} \frac{1}{2}x - 3 & \text{if } x > 6 \\ -\frac{1}{2}x + 3 & \text{if } x \le 6 \end{cases}$$

Absolute Value Functions

LESSON GOAL

Students identify the effects of transformations of the graphs of absolute value functions.

1 LAUNCH

Launch the lesson with a **Warm Up** and an introduction.

2 EXPLORE AND DEVELOP

Explore: Parameters of an Absolute Value Function

Develop:

Graphing Absolute Value Functions; Translations of Absolute Value Functions

- Vertical Translations of Absolute Value Functions
- Horizontal Translations of Absolute Value Functions
- Multiple Translations of Absolute Value Functions
- Identify Absolute Value Functions from Graphs
- Identify Absolute Value Functions from Graphs (Multiple Translations)

Dilations of Absolute Value Functions

- Dilations of Form $a|x|$ When $x > 1$
- Dilations of the Form $|ax|$
- Dilations When $0 < a < 1$

Reflections of Absolute Value Functions

- Graphs of Reflections with Transformations
- Graphs of $y = -a|x|$
- Graphs of $y = |-ax|$

Transformations of Absolute Value Functions

- Graph an Absolute Value Function with Multiple Translations
- Graph an Absolute Value Function with Translations and Dilation
- Graph an Absolute Value Function with Translations and Reflection
- Apply Graphs of Absolute Value Functions

You may want your students to complete the **Checks** online.

3 REFLECT AND PRACTICE

Exit Ticket

Practice

Formative Assessment Math Probe

DIFFERENTIATE

View reports of student progress on the **Checks** after each example.

Resources	AL	OL	BL	ELL
Remediation: Integers: Opposites and Absolute Value	●	●		●
Extension: Parametric Equations		●	●	●

Suggested Pacing

90 min	1 day
45 min	2 days

Focus

Domain: Functions

Standards for Mathematical Content:

F.IF.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Standards for Mathematical Practice:

1 Make sense of problems and persevere in solving them.
5 Use appropriate tools strategically.
7 Look for and make use of structure.

Coherence

Previous
Students graphed piecewise-defined and step functions.
F.IF.4, F.IF.7b

Now
Students identify the effects of transformations of the graphs of absolute value functions.
F.IF.7b, F.BF.3

Next
Students will create linear equations in slope-intercept form.
A.CED.2, S.ID.7

Rigor

The Three Pillars of Rigor

1 CONCEPTUAL UNDERSTANDING	2 FLUENCY	3 APPLICATION

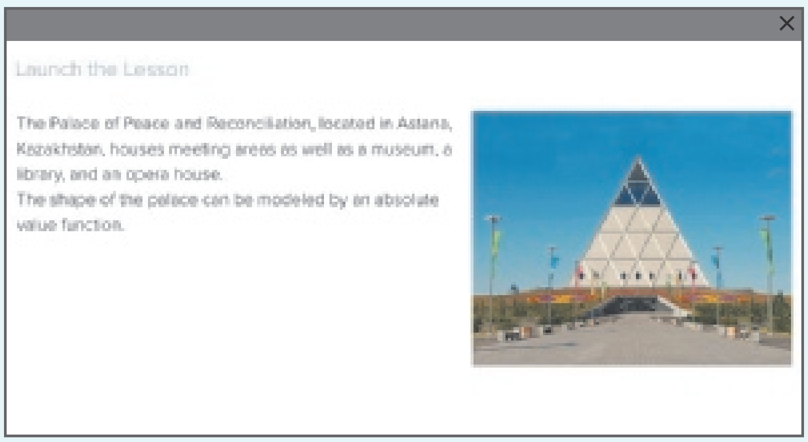
Conceptual Bridge In this lesson, students extend their understanding of absolute value to absolute value functions. They build fluency by graphing absolute value functions, and they apply their understanding by solving real-world problems related to absolute value functions.

Mathematical Background

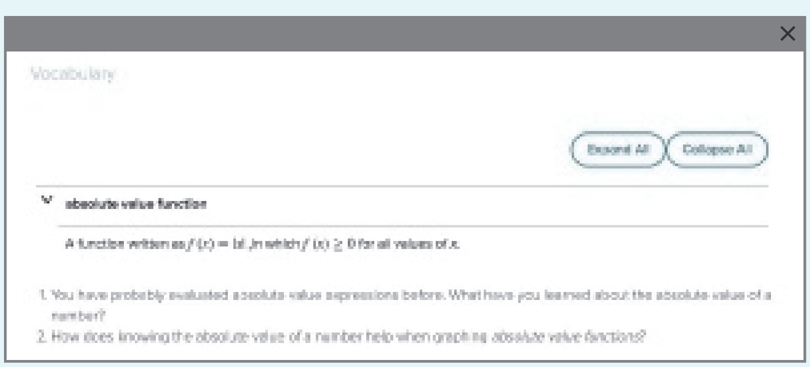
The graph of the absolute value parent function is V-shaped, with the vertex at the origin. The right side of the V is the graph of $y = x$; the left side is the graph of $y = -x$. Translations, dilations, and reflections of the graph of the absolute value parent function, $f(x) = |x|$, result in shifts, stretches or compressions, and flips (respectively), of the V-shaped graph.

Interactive Presentation

Warm Up



Launch the Lesson



Today's Vocabulary

Warm Up

Prerequisite Skills

The Warm Up exercises address the following prerequisite skill for this lesson:

- evaluating absolute value expressions

Answers:

1. $>$
2. $=$
3. $<$
4. $=$
5. $>$

Launch the Lesson

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationship between the shape of the Palace of Peace and Reconciliation and the graph of an absolute value function.

Go Online to find additional teaching notes and questions to promote classroom discourse.

Today's Standards

Tell students that they will address these content and practice standards in this lesson. You may wish to have a student volunteer read aloud *How can I meet these standards?* and *How can I use these practices?*, and connect these to the standards.

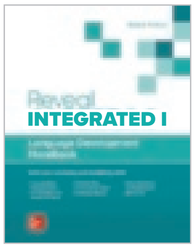
See the Interactive Presentation for I Can statements that align with the standards covered in this lesson.

Today's Vocabulary

Tell students that they will use this vocabulary term in this lesson. You can expand the row if you wish to share the definition. Then discuss the questions below with the class.

Language Development Handbook

Assign page 26 of the *Language Development Handbook* to help your students build mathematical language related to transformations of the graphs of absolute value functions.



ELL You can use the tips and suggestions on page T26 of the handbook to support students who are building English proficiency.

Explore Parameters of an Absolute Value Function

Objective

Students use a sketch to explore how changing the parameters changes the graphs of absolute value functions.

Teaching the Mathematical Practices

5 Use Mathematical Tools Point out that to complete this Explore activity, students will need to use the sketch. Work with students to explore and deepen their understanding of absolute value functions.

Ideas for Use

Recommended Use Present the Inquiry Question, or have a student volunteer read it aloud. Have students work in pairs to complete the Explore activity on their devices. Pairs should discuss each of the questions. Monitor student progress during the activity. Upon completion of the Explore activity, have student volunteers share their responses to the Inquiry Question.

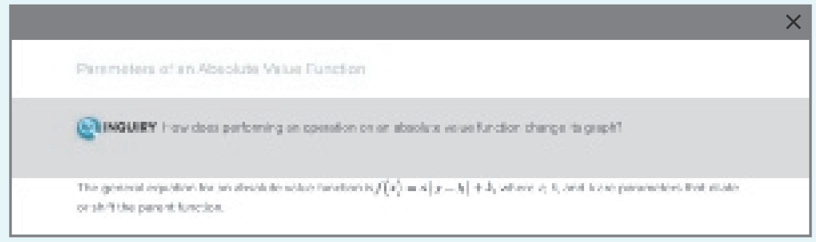
What if my students don't have devices? You may choose to project the activity on a whiteboard. A printable worksheet for each Explore is available online. You may choose to print the worksheet so that individuals or pairs of students can use it to record their observations.

Summary of the Activity

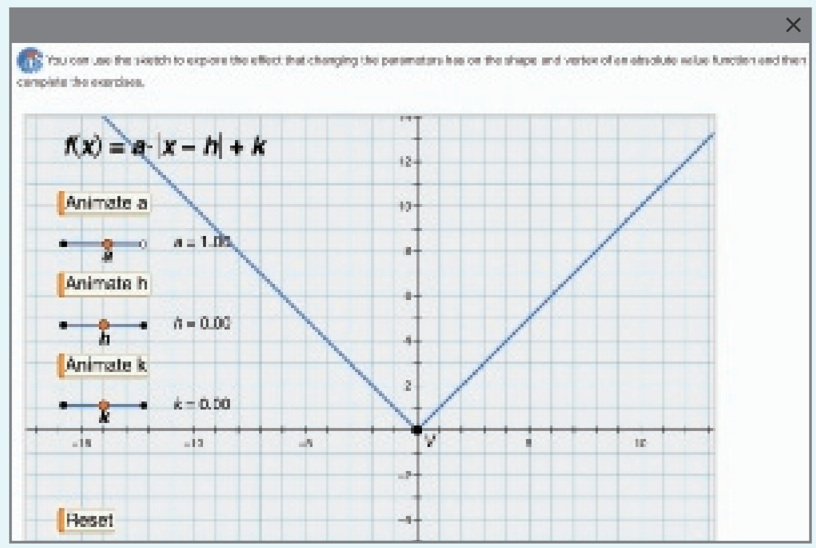
Students will complete guiding exercises throughout the Explore activity. Students will use a sketch to explore how changing the parameters of an absolute value function affects its graph. Students explore the graphs on their own and through an animation. They will answer questions and form generalizations based on their observations. Then, students will answer the Inquiry Question.

(continued on the next page)

Interactive Presentation



Explore



Explore

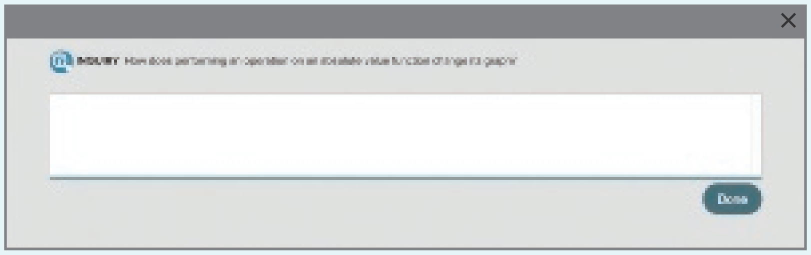
WEB SKETCHPAD

Students use a sketch to explore transformations of absolute value functions.

TYPE

Students answer questions about the transformations of absolute value functions.

Interactive Presentation



Explore

TYPE

Students respond to the Inquiry Question and can view a sample answer.

Explore Parameters of an Absolute Value Function (*continued*)

Questions

Have students complete the Explore activity.

Ask:

- How is changing the value of a for the absolute value graph similar to a linear function? Sample answer: The graphs get steeper as the value of a increases and less steep as a decreases.
- How can looking at point V help you determine the transformations in the function? Sample answer: Point V is moved up, down, left or right depending on how values were added or subtracted to the function.

Inquiry

How does performing an operation on an absolute value function change its graph? Sample answer: Adding a value to the function moves the graph up or down. Subtracting a value from x moves the graph left or right. Multiplying the function by a value makes the graph wider or narrower or flips it over the x-axis.

Go Online to find additional teaching notes and sample answers for the guiding exercises.

Learn Graphing Absolute Value Functions

Common Misconception

Some students may think that the graph of any absolute value function will lie completely above the *x*-axis. Explain that just as with other functions, transformations of the function will relocate the graph, and the resulting graph may, in fact, contain points that lie below the *x*-axis.

Learn Translations of Absolute Value Functions

Objective

Students identify the effect on the graph of an absolute value function by replacing $f(x)$ with $f(x) + k$ or $f(x - h)$ for positive and negative values.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 1 Vertical Translations of Absolute Value Functions

Teaching the Mathematical Practices

7 Use Structure Help students to use the structure of the transformed function to identify the translation in the function.

Questions for Mathematical Discourse

AL What type of transformation occurs in $g(x)$? a vertical translation How do you know? 3 is being subtracted from the parent function.

OL How is the *y*-value of each ordered pair in the parent function affected? Each *y*-value decreases by 3 units.

BL How would the graph of $f(x) = |x| + 3$ compare to this graph? Sample answer: It would be shifted up 3 instead of down 3.

Go Online

- Find additional teaching notes.
- View performance reports of the Checks.
- Assign or present an Extra Example.

Lesson 4-7

Absolute Value Functions

Explore Parameters of an Absolute Value Function

Online Activity Use graphing technology to complete the Explore.

INQUIRY How does performing an operation on an absolute value function change its graph?

Learn Graphing Absolute Value Functions

The **absolute value function** is a type of piecewise-linear function. An absolute value function is written as $f(x) = a|x - h| + k$, where a, h, and k are constants and $f(x) \geq 0$ for all values of x.

The **vertex** is either the lowest point or the highest point of a function. For the parent function, $y = |x|$, the vertex is at the origin.

Key Concept • Absolute Value Function	
Parent Function	$f(x) = |x|$, defined as $f(x) = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$
Type of Graph	V-Shaped
Domain:	all real numbers
Range:	all nonnegative real numbers

Learn Translations of Absolute Value Functions

Key Concept • Vertical Translations of Absolute Value Functions

If $k > 0$, the graph of $f(x) = |x|$ is translated k units up.
If $k < 0$, the graph of $f(x) = |x|$ is translated $|k|$ units down.

Key Concept • Horizontal Translations of Linear Functions

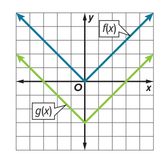
If $h > 0$, the graph of $f(x) = |x|$ is translated h units right.
If $h < 0$, the graph of $f(x) = |x|$ is translated h units left.

Example 1 Vertical Translations of Absolute Value Functions

Describe the translation in $g(x) = |x| - 3$ as it relates to the graph of the parent function.

Graph the parent function, $f(x) = |x|$, for absolute values.

The constant, k, is outside the absolute value signs, so k affects the *y*-values. The graph will be a vertical translation.



(continued on the next page)

Today's Goals
- Graph absolute value functions.
- Apply translations to absolute value functions.
- Apply dilations to absolute value functions.
- Apply reflections to absolute value functions.
- Interpret constants within equations of absolute value functions.

Today's Vocabulary
absolute value function
vertex

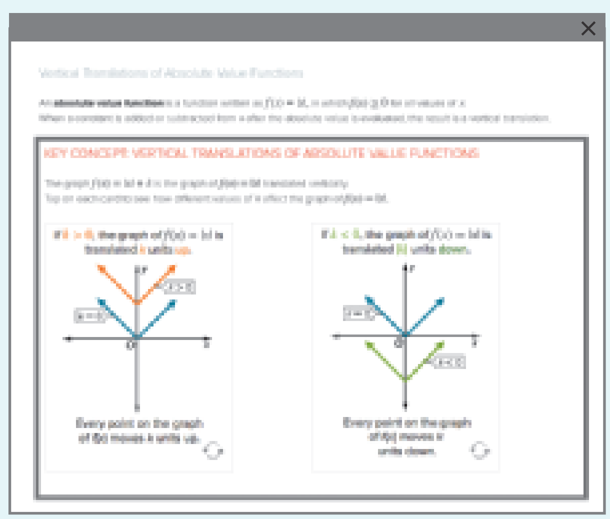
Think About It!
Why does adding a positive value of k shift the graph k units up?

Sample answer: This adds k to every value of the output of the absolute value. Since the output corresponds to the *y*-values, the graph moves up k units.

Go Online
You can watch a video to see how to describe translations of functions.

Study Tip
Horizontal Shifts Remember that the general form of an absolute value function is $y = a|x - h| + k$. So, $y = |x + 7|$ is actually $y = |x - (-7)|$ in the function's general form.

Interactive Presentation



Learn

TAP

Students tap on each card to see how vertical transformations affect the graph.

TYPE

Students explain why adding a positive value shifts the graph the same number of units up.

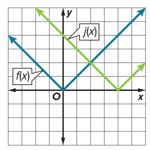
Since $f(x) = |x|$, $g(x) = f(x) + k$ where $k = -3$.
$g(x) = |x| - 3 \longrightarrow g(x) = f(x) + (-3)$

The value of k is less than 0, so the graph will be translated $|k|$ units down, or 3 units down.

$g(x) = |x| - 3$ is a translation of the graph of the parent function 3 units down.

Example 2 Horizontal Translations of Absolute Value Functions

Describe the translation in $j(x) = |x - 4|$ as it relates to the parent function.



Graph the parent function, $f(x) = |x|$, for absolute values.

The constant, h, is inside the absolute value signs, so h affects the input or, x-values. The graph will be a horizontal translation.

Since $f(x) = |x|$, $j(x) = f(x - h)$, where $h = 4$.
$j(x) = |x - 4| \longrightarrow j(x) = f(x - 4)$

The value of h is greater than 0, so the graph will be translated h units right, or 4 units right.

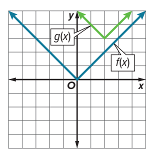
$j(x) = |x - 4|$ is the translation of the graph of the parent function 4 units right.

Think About It!
Since the vertex of the parent function is at the origin, what is a quick way to determine where the vertex is of $q(x) = |x - h| + k$?

Sample answer: The vertex will be at (h, k).

Example 3 Multiple Translations of Absolute Value Functions

Describe the translation in $g(x) = |x - 2| + 3$ as it relates to the graph of the parent function.



The equation has both h and k values. The input and output will be affected by the constants. The graph of $f(x) = |x|$ is vertically and horizontally translated.

Since $f(x) = |x|$, $g(x) = f(x - h) + k$ where $h = 2$ and $k = 3$.

Because $h = 2$ and $k = 3$, the graph is translated 2 units right and 3 units up.

$g(x) = |x - 2| + 3$ is the translation of the graph of the parent function 2 units right and 3 units up.

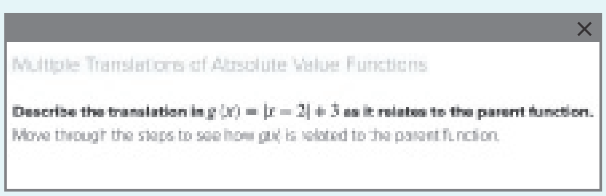
Emilio says that the graph of $g(x) = |x + 1| - 1$ is the same graph as $f(x) = |x|$. Is he correct? Why or why not?

No; sample answer: The graph of $g(x) = |x + 1| - 1$ is a graph that has been translated 1 unit left and 1 unit down from the parent function, $f(x) = |x|$.

Go Online You can complete an Extra Example online.

268 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Example 3

TAP

Students move through the steps to see how the graph of the function relates to the parent function.

TYPE

Students answer questions about the graphs of translated functions.

Example 2 Horizontal Translations of Absolute Value Functions

Teaching the Mathematical Practices

7 Use Structure Help students to determine the structure of the translated absolute value function in this example.

Questions for Mathematical Discourse

AL What type of transformation occurs in $j(x)$? a horizontal translation How do you know? The -4 is inside the absolute value symbols.

OL How would the graph of $f(x) = |x + 4|$ compare to this graph? The graph of the parent function would be shifted 4 units to the left instead of to the right.

BL How would the graph of $f(x) = |x| - 4$ compare to this graph? The graph of the parent function would be shifted 4 units down instead of to the right.

Example 3 Multiple Translations of Absolute Value Functions

Questions for Mathematical Discourse

AL Looking at only the equation, which value shifts the graph vertically? $+3$

OL Looking at only the equation, how do you know that the horizontal translation is to the right? Sample answer: If you use the form $f(x - h)$ for the translation, then $|x - 2|$ means that $h = 2$. This represents a translation to the right 2 units.

BL What would the function be if it was a horizontal translation of 2 units left and 3 units down? $g(x) = |x + 2| - 3$

Common Error

As the function becomes more complex, some students may have difficulty seeing the relationship to the parent function. Encourage them to rewrite functions like the one in this example using $f(x)$. For example, for the function in this problem, students would write $f(x - 2) + 3$. In this way, they can see that 2 is being subtracted from x, and 3 is being added to the function values (i.e., the y-values).

DIFFERENTIATE

Enrichment Activity AL BL ELL

IF students are having difficulty determining the direction of a translation, **THEN** have them create four examples of absolute value functions that represent each type of translation, and write each one on an index card. Have them sketch the transformation on a coordinate plane on the back of the card, and write the description. Then have them use the flash cards (in both directions) to practice what they have learned.

Example 4 Identify Absolute Value Functions from Graphs

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph and its equation used in this example.

Questions for Mathematical Discourse

AL What translation is shown on the graph? a horizontal shift of 1 to the right

OL Does this indicate that the value being added or subtracted should go inside or outside the absolute value symbols? inside

BL A classmate argues that the function should be $f(x) = |x + 1|$ since the shift is in the positive direction. Explain why this is incorrect. Sample answer: Translations are written in the form $f(x) = |x - h| + k$, so $f(x) = |x + 1|$ would be $f(x) = |x - (-1)|$, which would be a horizontal shift to the left.

Common Error

Some students may write the equation using a plus sign instead of a minus sign. Remind them that once they determine how many units and in what direction the graph is translated, they need to *subtract* that number from x.

Example 5 Identify Absolute Value Functions from Graphs (Multiple Translations)

Questions for Mathematical Discourse

AL How do you know that this graph represents a function with more than one transformation? Sample answer: The vertex is not on an axis.

OL How many transformations are there, and what type are they? 2; Sample answer: a horizontal translation of 2 units to the left and a vertical translation of 5 units down

BL What are the coordinates of the vertex? (–2, –5)
How does identifying the coordinates help you solve the problem? Sample answer: I can use the x-coordinate for h and the y-coordinate for k in the equation $g(x) = |x - h| + k$.

Learn Dilations of Absolute Value Functions

Objective

Students identify the effect on the graph of an absolute value function by replacing $f(x)$ with $af(x)$ or $f(ax)$.

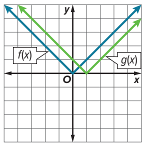
Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 4 Identify Absolute Value Functions from Graphs

Use the graph of the function to write its equation.

The graph is the translation of the parent graph 1 unit to the right.



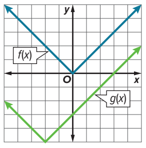
$g(x) = |x - h|$ General equation for a horizontal translation

$g(x) = |x - 1|$ The vertex is 1 unit to the right of the origin.

Example 5 Identify Absolute Value Functions from Graphs (Multiple Translations)

Use the graph of the function to write its equation.

The graph is a translation of the parent graph 2 units to the left and 5 units down.



$g(x) = |x - h| + k$ General equation for a translations

$g(x) = |x - (-2)| + k$ The vertex is 2 units left of the origin.

$g(x) = |x - (-2)| + (-5)$ The vertex is 5 units down from the origin.

$g(x) = |x + 2| - 5$ Simplify.

Learn Dilations of Absolute Value Functions

Multiplying by a constant a after evaluating an absolute value function creates a vertical change, either a stretch or compression.

Key Concept • Vertical Dilations of Absolute Value Functions

If $|a| > 1$, the graph of $f(x) = |x|$ is stretched vertically.
If $0 < |a| < 1$, the graph of $f(x) = |x|$ is compressed vertically.

When an input is multiplied by a constant a before for the absolute value is evaluated, a horizontal change occurs.

Key Concept • Horizontal Dilations of Absolute Value Functions

If $|a| > 1$, the graph of $f(x) = |x|$ is compressed horizontally.
If $0 < |a| < 1$, the graph of $f(x) = |x|$ is stretched horizontally.

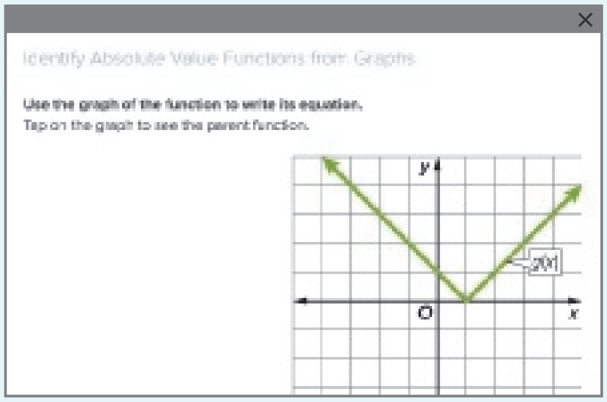
Talk About It!
How is the value of a in an absolute value function related to slope? Explain.

Sample answer: The value of a determines the slope of each part of the graph. The function $y = a|x|$ can also be written as $f(x) = \begin{cases} ax \text{ it } x \geq 0 \\ -ax \text{ if } x < 0 \end{cases}$ where a and $-a$ are the slopes of the rays.

Go Online You can complete an Extra Example online.

Lesson 4-7 • Absolute Value Functions 269

Interactive Presentation



Example 4

TAP

Students tap on the graph to see the parent function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Go Online You can watch a video to see how to describe dilations of functions.

Think About It! How are a|x| and |ax| evaluated differently?

Sample answer: In $a|x|$, the absolute value of the input is evaluated before multiplying by a. In $|ax|$, the input is first multiplied by a and then the absolute value is evaluated.

Watch Out!

Differences in Dilations Although $a|x|$ and $|ax|$ appear to have the same effect on a function, they are evaluated differently and that difference is more apparent when a function is dilated and translated horizontally. For a function with multiple transformations, it is best to first create a table.

Example 6 Dilations of the Form $a|x|$ When $a > 1$

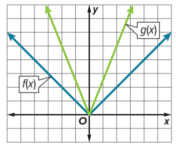
Describe the dilation in $g(x) = \frac{5}{2}|x|$ as it relates to the graph of the parent function.

Since $f(x) = |x|$, $g(x) = a \cdot f(x)$, where $a = \frac{5}{2}$.

$g(x) = \frac{5}{2}|x| \longrightarrow g(x) = \frac{5}{2} \cdot f(x)$

$g(x) = \frac{5}{2}|x|$ is a vertical stretch of the graph of the parent graph.

x	$\|x\|$	$\frac{5}{2}\|x\|$	$(x, g(x))$
−4	$\|-4\| = 4$	10	(−4, 10)
−2	$\|-2\| = 2$	5	(−2, 5)
0	$\|0\| = 0$	0	(0, 0)
2	$\|2\| = 2$	5	(2, 5)
4	$\|4\| = 4$	10	(4, 10)



Example 7 Dilations of the Form $|ax|$

Describe the dilation in $p(x) = |2x|$ as it relates to the graph of the parent function.

For $p(x) = |2x|$, $a = 2$. Since a is inside the absolute value symbols, the input is first multiplied by a. Then, the absolute value of ax is evaluated.

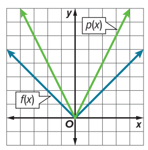
x	$\|2x\|$	$p(x)$	$(x, p(x))$
−4	$\|2(-4)\| = \|-8\|$	8	(−4, 8)
−2	$\|2(-2)\| = \|-4\|$	4	(−2, 4)
0	$\|2(0)\| = \|0\|$	0	(0, 0)
2	$\|2(2)\| = \|4\|$	4	(2, 4)
4	$\|2(4)\| = \|8\|$	8	(4, 8)

Plot the points from the table.

Since $f(x) = |x|$, $p(x) = f(ax)$ where $a = 2$.

$p(x) = |2x| \rightarrow p(x) = f(2x)$

$p(x) = |2x|$ is a horizontal compression of the graph of the parent graph.



Check

Tell whether each equation is an example of a *vertical stretch, vertical compression, horizontal stretch,* or *horizontal compression*.

A. $j(x) = \left|\frac{4}{3}x\right|$ horizontal compression

B. $q(x) = \left|\frac{1}{5}x\right|$ horizontal stretch

C. $p(x) = 6|x|$ vertical stretch

D. $g(x) = \frac{5}{7}|x|$ vertical compression

Go Online You can complete an Extra Example online.

270 **Module 4** • Linear and Nonlinear Functions

Example 6 Dilations of the Form $a|x|$ When $a > 1$

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graphs and equations of the dilated function and the parent function in this example.

Questions for Mathematical Discourse

AL Is $|a|$ greater than 1 or between 0 and 1? Why? Sample answer: a is greater than 1 because $a = \frac{5}{2}$, and $\frac{5}{2} > 1$.

OL What kind of dilation does this represent? Explain. It is a vertical stretch by a factor of $\frac{5}{2}$. Sample answer: The $\frac{5}{2}$ is outside of the absolute value symbols and it is greater than 1.

BL How would the function be different if it was a horizontal compression where $a = \frac{5}{2}$? Sample answer: The function would be $g(x) = \left|\frac{5}{2}x\right|$.

Example 7 Dilations of the Form $|ax|$

Teaching the Mathematical Practices

3 Construct Arguments In this example, students will use stated assumptions, definitions, and previously established results to construct an argument.

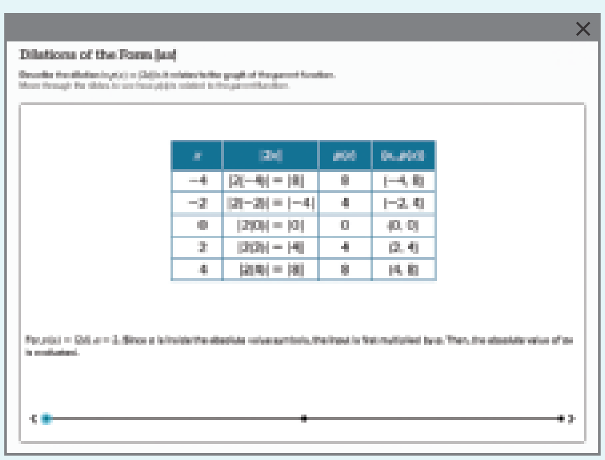
Questions for Mathematical Discourse

AL When the absolute value function is in the form $f(x) = |ax|$, what will be the effect of a? Sample answer: The graph will be horizontally stretched or compressed.

OL How would the transformation have changed if the function was $p(x) = \left|\frac{1}{2}x\right|$? Sample answer: It would be a horizontal stretch instead of a compression.

BL What would be an equivalent vertical dilation? Sample answer: a vertical stretch, $p(x) = 2|x|$

Interactive Presentation



Example 7

TAP

Students will move through the slides to see how to graph a dilation of an absolute value function.

Example 8 Dilations When $0 < a < 1$

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph and its equation used in this example.

Questions for Mathematical Discourse

AL Looking at only the equation, how do you know this is a vertical dilation and not a horizontal dilation? Sample answer: The $\frac{1}{3}$ is being multiplied on the outside of the function, not with x.

OL How would the dilation change if the function were $j(x) = 3|x|$? Sample answer: It would be a vertical stretch by a factor of 3.

BL How would this function change if it was a horizontal stretch where $a = \frac{1}{3}$? The function would be $j(x) = |\frac{1}{3}x|$.

Learn Reflections of Absolute Value Functions

Objective

Students identify the effect on the graph of an absolute value function by replacing $f(x)$ with $-af(x)$ or $f(-ax)$.

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Example 8 Dilations When $0 < a < 1$

Describe how the graph of $j(x) = \frac{1}{3}|x|$ as it relates to the graph of the parent function.

For $j(x) = \frac{1}{3}|x|$, $a = \frac{1}{3}$. Since a is outside the absolute value signs, the absolute value of the input is evaluated first. Then, the function is multiplied by a.

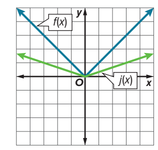
x	$\lvert x \rvert$	$\frac{1}{3}\lvert x \rvert$	$(x, j(x))$
-6	$\lvert -6 \rvert = 6$	2	$(-6, 2)$
-3	$\lvert -3 \rvert = 3$	1	$(-3, 1)$
0	$\lvert 0 \rvert = 0$	0	$(0, 0)$
3	$\lvert 3 \rvert = 3$	1	$(3, 1)$
6	$\lvert 6 \rvert = 6$	2	$(6, 2)$

Plot the points from the table.

Since $f(x) = |x|$, $j(x) = a \cdot f(x)$ where $a = \frac{1}{3}$.

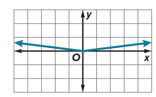
$j(x) = \frac{1}{3}|x| \rightarrow j(x) = \frac{1}{3} \cdot f(x)$

$j(x) = \frac{1}{3}|x|$ is a vertical compression of the graph of the parent graph.

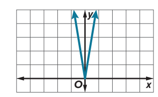


Check

Write an equation for each graph shown.



$j(x) = 0.125|x|$



$p(x) = |6x|$

Learn Reflections of Absolute Value Functions

The graph of $-a|x|$ appears to be flipped upside down compared to $a|x|$ and they are symmetric about the x-axis.

Key Concept • Reflections of Absolute Value Functions Across the x-axis

The graph of $-af(x)$ is the reflection of the graph of $af(x) = a|x|$ across the x-axis.

When the only transformation occurring is a reflection or a dilation and reflection, the graphs of $f(ax)$ and $f(-ax)$ appear the same.

Key Concept • Reflections of Absolute Value Functions Across the y-axis

The graph of $f(-ax)$ is the reflection of the graph of $f(ax) = |ax|$ across the y-axis.

Go Online You can watch a video to see how to describe reflections of functions.

Think About It! Why would $g(x) = |-2x|$ and $j(x) = |2x|$ appear to be the same graphs?

Sample answer: Since the absolute values of $-2x$ and $2x$ are the same nonnegative numbers, the graphs are the same.

Go Online You can complete an Extra Example online.

Lesson 4-7 • Absolute Value Functions 271

Interactive Presentation

Example 8

TAP

Students move through the steps to see how the given function relates to the parent function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

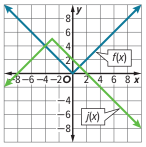
Example 9 Graphs of Reflections with Transformations

Describe the reflection in $j(x) = -|x + 3| + 5$ as it relates to the graph of the parent function.

x	$\lvert x + 3\rvert$	$-\lvert x + 3\rvert$	$-\lvert x + 3\rvert + 5$	$(x, j(x))$
-5	$\lvert -5 + 3\rvert = \lvert -2\rvert = 2$	-2	$-2 + 5 = 3$	$(-5, 3)$
-4	$\lvert -4 + 3\rvert = \lvert -1\rvert = 1$	-1	$-1 + 5 = 4$	$(-4, 4)$
-3	$\lvert -3 + 3\rvert = \lvert 0\rvert = 0$	0	$0 + 5 = 5$	$(-3, 5)$
-2	$\lvert -2 + 3\rvert = \lvert 1\rvert = 1$	-1	$-1 + 5 = 4$	$(-2, 4)$
-1	$\lvert -1 + 3\rvert = \lvert 2\rvert = 2$	-2	$-2 + 5 = 3$	$(-1, 3)$

First, the absolute value of $x + 3$ is evaluated. Then, the function is multiplied by $1 \cdot a$. Finally, 5 is added to the function.

Plot the points from the table.



Because $f(x) = |x|$, $j(x) = -1 \cdot a \cdot f(x)$ where $a = 1$, $h = -3$, and $k = 5$.

$j(x) = -|x + 3| + 5 \rightarrow j(x) = -1 \cdot f(x + 3) + 5$

$j(x) = -|x + 3| + 5$ is the graph of the parent function reflected across the x-axis, and translated 3 units left and 5 units up.

Example 10 Graphs of $y = -a|x|$

Describe the reflection in $q(x) = -\frac{3}{4}|x|$ as it relates to the graph of the parent function.

First, the absolute value of x is evaluated. Then, the function is multiplied by $1 \cdot a$.

Plot the points from the table.

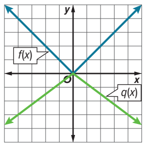
Because $f(x) = |x|$, $q(x) = -1 \cdot a \cdot f(x)$

x	$\lvert x\rvert$	$-\frac{3}{4}\lvert x\rvert$	$(x, q(x))$
-8	$\lvert -8\rvert = 8$	-6	$(-8, -6)$
-4	$\lvert -4\rvert = 4$	-3	$(-4, -3)$
0	$\lvert 0\rvert = 0$	0	$(0, 0)$
4	$\lvert 4\rvert = 4$	3	$(4, 3)$
8	$\lvert 8\rvert = 8$	6	$(8, 6)$

where $a = -\frac{3}{4}$.

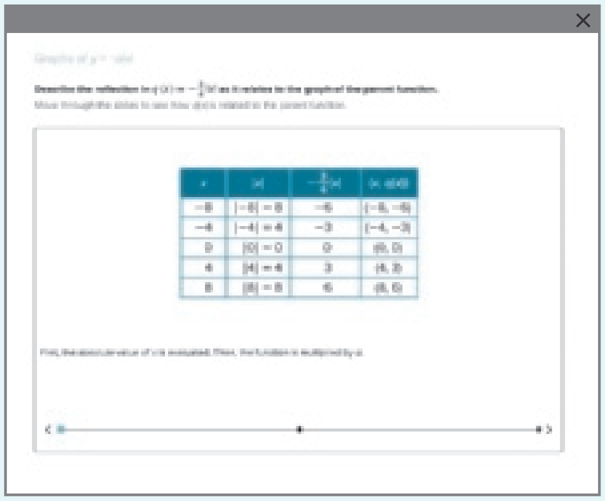
$q(x) = -\frac{3}{4}|x| \rightarrow q(x) = -\frac{3}{4}f(x)$

$q(x) = -\frac{3}{4}|x|$ is the graph of the parent function reflected across the x-axis and vertically compressed.



Go Online You can complete an Extra Example online.

Interactive Presentation



Example 10

TAP

Students move through the slides to see how a given function is related to the parent function.

1 CONCEPTUAL UNDERSTANDING | **2 FLUENCY** | 3 APPLICATION

Example 9 Graphs of Reflections with Transformations

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the reflected function and the graph of the parent function used in this example.

Questions for Mathematical Discourse

AL How do you know whether there is a horizontal translation? There is a 3 being added to x inside the absolute value symbols.

OL What is the effect of the negative in front of the absolute value symbols? It reflects the graph across the x-axis.

BL Why do you need to add 3 and take the absolute value before multiplying by -1? Sample answer: When evaluating to find the coordinates, you have to use the order of operations. In this case, you add 3 first because it is the operation inside the parentheses or grouping symbols.

Common Error

Remind students that the order in which they perform the operations when evaluating the function is important. Tell students that when creating the table, they must first add 3, then take the absolute value, then multiply by –1, then add 5.

Example 10 Graphs of $y = -a|x|$

Questions for Mathematical Discourse

AL How does the rule for $q(x)$ compare to the rule for the parent function? The rule for $q(x)$ is the rule for the parent function multiplied by $-\frac{3}{4}$.

OL How do you expect the vertex of $q(x)$ to compare to the vertex of the parent function? Explain. Sample answer: They will be the same because $q(x)$ has not been translated.

BL The point (12, 12) lies on the graph of the parent function. To what point does this map to on the graph of $q(x)$? $(12, -9)$

Common Error

Students may have difficulty seeing how the graph of $q(x)$ is related to the graph of the parent function. For these students, you may want to show the transformation in two different steps, first dilating the graph by a factor of $\frac{3}{4}$, and then reflecting the resulting graph across the x–axis.

DIFFERENTIATE

Enrichment Activity BL

Give students the function $f(x) = -|x - 4| - 2$. Have students create a step-by-step list of instructions for how to graph this function. Then have them graph the function.

Example 11 Graphs of $y = |-ax|$

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL What is the coefficient of x? -4

OL Looking at only the equation, what type of transformations does this function represent? a horizontal compression and a reflection across the y-axis

BL How would this function be different if it was a vertical stretch where $a = 4$ and a reflection across the x-axis? The function would be $f(x) = -4|x|$.

Common Error

Some students may think that this function is equivalent to $f(x) = -|4x|$. Have them create a table of values for both functions so that they can see that the two functions produce different sets of ordered pairs.

Learn Transformations of Absolute Value Functions

Objective

Students graph absolute value functions by interpreting constants within the equation or by making a table of values.

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the graph of the transformed functions and the graph of the parent function used in this Learn.

Common Misconception

Some students may think that translations should be applied before dilations and reflections. Use an example, such as $f(x) = -2|x - 3| + 4$, to show students that if they apply the vertical translation before the dilation and reflection, the resulting graph is not the same as when the transformations are applied in the correct order, with the vertical translation as the last transformation.

DIFFERENTIATE

Enrichment Activity BL

Have students work with a partner to create a poster showing examples of graphs that represent dilations of the graph of the parent function, including vertical and horizontal compressions and stretches, and have them use arrows to illustrate the stretch and compression. Have them also provide a description of each transformation.

Example 11 Graphs of $y = |-ax|$

Describe the reflection in $g(x) = |-4x|$ as it relates to the graph of the parent function.

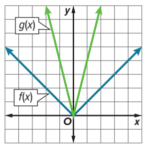
First, the input is multiplied by $1 \cdot a$. Then the absolute value of $-ax$ is evaluated.

Because $f(x) = |x|$, $g(x) = f(-1 \cdot a \cdot x)$

where $a = 4$.

$g(x) = |-4x| \rightarrow g(x) = f(-1 \cdot 4 \cdot x)$

$g(x) = |-4x|$ is the graph of the parent function reflected across the y-axis and horizontally compressed.



Learn Transformations of Absolute Value Functions

You can use the equation of a function to understand the behavior of the function. Because the constants a, h, and k affect the function in different ways, they can help develop an accurate graph of the function.

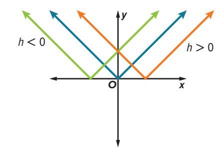
Concept Summary Transformations of Graphs of Absolute Value Functions

$g(x) = a|x - h| + k$

Horizontal Translation, h

If $h > 0$, the graph of $f(x) = |x|$ is translated h units right.

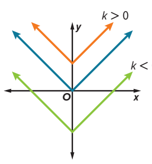
If $h < 0$, the graph of $f(x) = |x|$ is translated $|h|$ units left.



Vertical Translation, k

If $k > 0$, the graph of $f(x) = |x|$ is translated k units up.

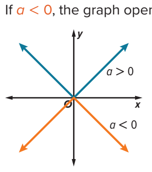
If $k < 0$, the graph of $f(x) = |x|$ is translated $|k|$ units down.



Reflection, a

If $a > 0$, the graph opens up.

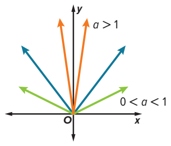
If $a < 0$, the graph opens down.



Dilation, a

If $|a| > 1$, the graph of $f(x) = |x|$ is stretched vertically.

If $0 < |a| < 1$, the graph is compressed vertically.



Think About It!

Describe how the graph of $y = |-ax|$ is related to the parent function.

Sample answer: Because x is multiplied by $-a$ before the absolute value is evaluated, the graph would be stretched or compressed, depending on the value of a, but not reflected across the x-axis.

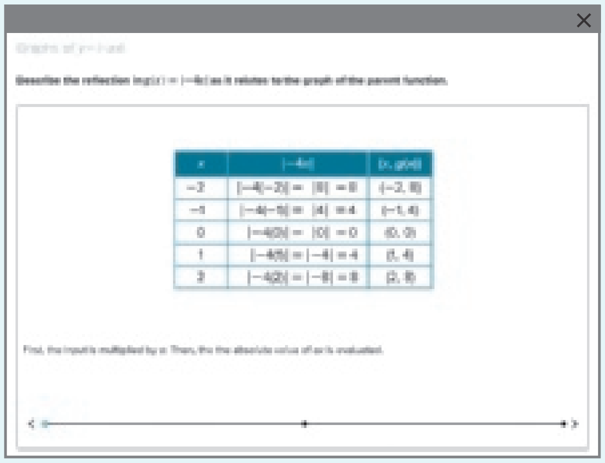
Why does there appear to be no reflection for the graph of $y = |-ax|$?

Sample answer: The graph is being reflected across the y-axis as a result of multiplying x by $-a$. There is a reflection occurring, but the final graph appears the same as $y = |ax|$.

Go Online You can complete an Extra Example online.

Lesson 4-7 • Absolute Value Functions 273

Interactive Presentation



Example 11

TAP

Students move through the slides to see how a given function is related to the parent function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Write the phrase that best describes how each parameter affects the graph of $g(x) = -5|x - 2| + 3$ in relation to the parent function.

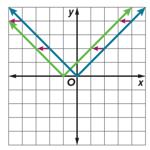
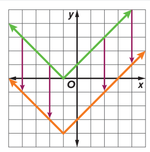
-5 Reflects and stretches vertically

2 Translates right

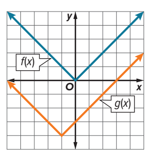
3 Translates up

Example 12 Graph an Absolute Value Function with Multiple Translations

Graph $g(x) = |x + 1| - 4$. State the domain and range.

$a = 1$	The graph is not reflected or dilated in relation to the parent function.	
$h = -1$	The graph is translated 1 unit left from the parent function.	
$k = -4$	The graph is translated 4 units down from the parent function.	

The graph of $g(x) = |x + 1| - 4$ is the graph of the parent function translated 1 unit left and 4 units down without dilation or reflection. The domain is all real numbers. The range is all real numbers greater than or equal to -4.



Watch Out!

Dilations and Translations

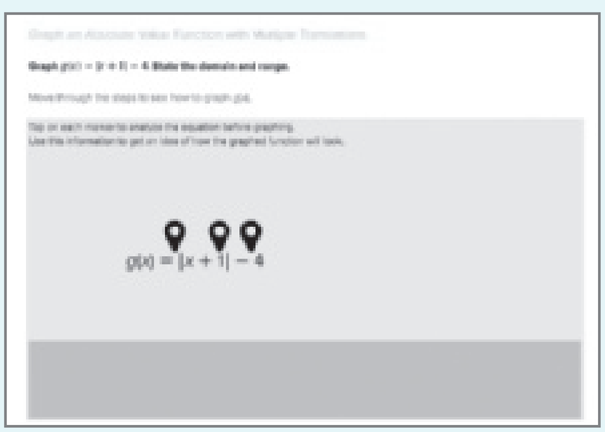
Don't assume that $j(x) = 2|x - 5| + 1$ and $p(x) = |2x - 5| + 1$ are the same graph. Functions are evaluated differently depending on whether a is inside or outside the absolute value symbols. It might be best to create a table to generate an accurate graph.

Go Online You can watch a video to see how to graph a transformed absolute value function.

Go Online You can complete an Extra Example online.

274 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Example 12

TAP

Students tap on each marker to analyze the parameters in the function.

Example 12 Graph an Absolute Value Function with Multiple Translations

Teaching the Mathematical Practices

7 Use Structure Helps students to use the structure of the transformed function to identify the transformations in $g(x)$ and graph $g(x)$.

Questions for Mathematical Discourse

AL Looking at only the equation, what transformations occur in $g(x)$? a horizontal translation 1 unit to the left and a vertical translation 4 units down

OL How do you know that there is no reflection in this transformation? Sample answer: There are no negative coefficients in the function.

BL How would the function be different if it also represented a reflection over the x-axis? Sample answer: The function would be $f(x) = -|x + 1| - 4$.

Example 13 Graph an Absolute Value Function with Translations and Dilation

Teaching the Mathematical Practices

1 Explain Correspondences Encourage students to explain the relationships between the equations and graphs of the transformed function and the parent function.

Questions for Mathematical Discourse

AL What types of transformations occur in $j(x)$? a horizontal compression and a horizontal shift

OL What is the vertex of the graph of $j(x)$? (2, 0)

BL How could the Distributive Property help explain the horizontal shift 2 units to the right? Sample answer: If we apply the Distributive Property to factor the expression inside the absolute value function, we get $|3(x - 2)|$. This shows that we would first perform a translation of 2 units to the right, then a horizontal compression of 3.

Example 14 Graph an Absolute Value Function with Translations and Reflection

Teaching the Mathematical Practices

6 Communicate Precisely Encourage students to routinely write or explain their solution methods. Point out that they should use clear definitions when they discuss their solutions with others.

Questions for Mathematical Discourse

AL Will the graph open up or down? How do you know? Down; sample answer: There is a negative sign in front of the absolute value symbols.

OL What types of transformations occur in $p(x)$? a horizontal translation 3 units to the right, a reflection across the x-axis, and a vertical translation 5 units up

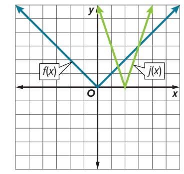
BL How would the function be different if the graph had been translated 3 units to the right and then reflected over the y-axis instead of over the x-axis? The function would be $f(x) = |-x - 3| + 5$.

Example 13 Graph an Absolute Value Function with Translations and Dilation

Graph $j(x) = |3x - 6|$. State the domain and range.

Because a is inside the absolute value symbols, the effect of h on the translation changes.

Evaluate the function for several values of x to find points on the graph.



x	$(x, j(x))$
0	(0, 6)
1	(1, 3)
2	(2, 0)
3	(3, 3)
4	(4, 6)

The graph of $j(x) = |3x - 6|$ is the graph of the parent function compressed horizontally and translated 2 units right.

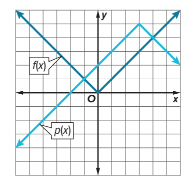
The domain is all real numbers. The range is all real numbers greater than or equal to 0.

Example 14 Graph an Absolute Value Function with Translations and Reflection

Graph $p(x) = -|x - 3| + 5$. State the domain and range.

In $p(x) = -|x - 3| + 5$, the parent function is reflected across the x-axis because the absolute value is being multiplied by -1.

The function is then translated 3 units right.



Finally, the function is translated 5 units up.

$p(x) = -|x - 3| + 5$ is the graph of the parent function translated 3 units right and 5 units up and reflected across the x-axis.

The domain is all real numbers. The range is all real numbers less than or equal to 5.

Think About It!

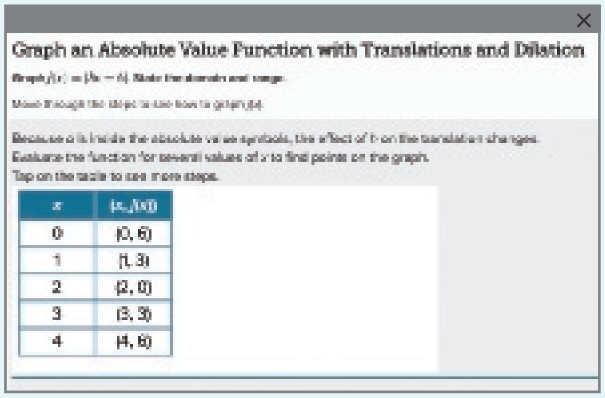
How is the vertical translation k of an absolute value function related to its range?

Sample answer: Since a vertical translation affects the location of the minimum or maximum point, the range will be greater than or equal to or less than or equal to the value of k, depending upon whether the function has been reflected.

Go Online You can complete an Extra Example online.

Lesson 4-7 • Absolute Value Functions 275

Interactive Presentation



Example 13

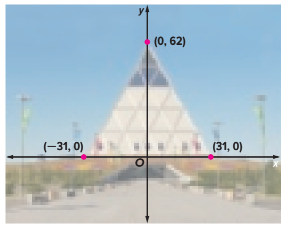
TAP

Students move through the steps to graph the function.

Example 15 Apply Graphs of Absolute Value Functions

BUILDINGS **Determine an absolute value function that models the shape of The Palace of Peace and Reconciliation.**

To write the equation for the absolute value function, we must determine the values of a, h, and k in $f(x) = a|x - h| + k$ from the graph.



If we consider the absolute value as a piecewise function, we can find the slope of one side of the graph to determine the value of a.

Because this function opens downward, the graph is a reflection of the parent graph across the x-axis. So we know that the a-value in the equation should be negative.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — The Slope Formula

$= \frac{0 - 62}{31 - 0}$ — $(0, 62) = (x_1, y_1)$ and $(31, 0) = (x_2, y_2)$

$= -\frac{62}{31}$ or -2

Next, notice that the vertex is not located at the origin. It has been translated. The absolute value function is not shifted left or right, but has been translated 62 units up from the origin.

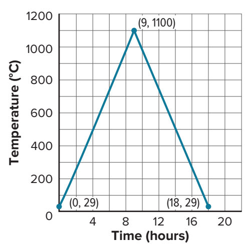
$y = -2|x - 0| + 62$ — $a = -2, h = 0, k = 2$

$y = -2|x| + 62$ — Simplify.

So, $y = -2|x| + 62$ models the shape of The Palace of Peace and Reconciliation.

Check

GLASS PRODUCTION Certain types of glass heat and cool at a nearly constant rate when they are melted to create new glass products. Use the graph to determine the equation that represents this process.



Dmitry Chulov/123RF

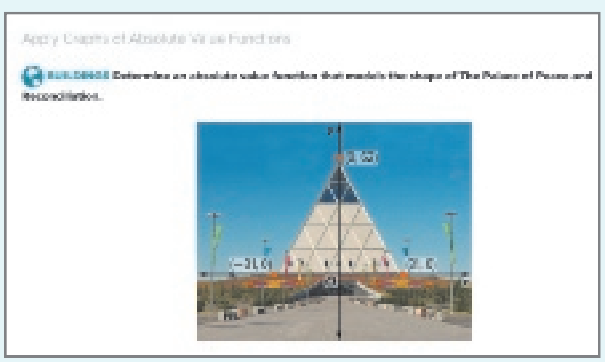
$y = \underline{\ ?\ }\,|x - \underline{\ ?\ }| + \underline{\ ?\ }$
−119 9 1100

Go Online to practice what you've learned about graphing special functions in the Put It All Together over Lessons 4–6 through 4–7.

Go Online You can complete an Extra Example online.

276 **Module 4** • Linear and Nonlinear Functions

Interactive Presentation



Example 15

TYPE

Students enter the values to find the slope of one side of the function.

CHECK

Students complete the Check online to determine whether they are ready to move on.

Example 15 Apply Graphs of Absolute Value Functions

Teaching the Mathematical Practices

4 Apply Mathematics Students will explore how to use an absolute value function to model the shape of a building. They will learn how to use the physical attributes of the building to calculate the parameters of the function.

Questions for Mathematical Discourse

AL How do you know that the value of a will be a negative number? Sample answer: The shape of the building is a V that opens down.

OL How do you know that the value of k will be 62? Sample answer: The vertex of the building is 62 units above the origin.

BL Why is it important to find the slope of the sides of the building? Sample answer: The slope tells you if there is a vertical or horizontal stretch or compression.

Common Error

After studying the photo, some students may try to incorporate a parameter representing a horizontal translation of 31 units. Help students to see that the diagram shows that the building is symmetric with respect to the y-axis, so there is no horizontal translation. Explain that the purpose of the marked points on the x-axis is for determining the dilation.

Exit Ticket

Recommended Use

At the end of class, have students respond to the Exit Ticket prompt using a separate piece of paper. Have students hand you their responses as they leave the room.

Alternate Use

At the end of class, have students respond to the Exit Ticket prompt verbally or by using a mini-whiteboard. Have students hold up their whiteboards so that you can see all student responses. Tap to reveal the answer when most or all students have completed the Exit Ticket.

Practice and Homework

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1, 2	exercises that mirror the examples	1–35
2	exercises that use a variety of skills from this lesson	26–42
2	exercises that extend concepts learned in this lesson to new contexts	43–48
3	exercises that emphasize higher-order and critical-thinking skills	49–52

ASSESS AND DIFFERENTIATE

Use the data from the **Checks** to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, **BL**
THEN assign:

- Practice, Exercises 1–47 odd, 49–52
- Extension: Parametric Equations
- ALEKS® Absolute Value Functions

IF students score 66%–89% on the Checks, **OL**
THEN assign:

- Practice, Exercises 1–51 odd
- Remediation, Review Resources: Absolute Value and Distance
- Personal Tutors
- Extra Examples 1–15
- ALEKS® Plotting and Comparing Signed Numbers

IF students score 65% or less on the Checks, **AL**
THEN assign:

- Practice, Exercises 1–35 odd
- Remediation, Review Resources: Absolute Value and Distance
- *Quick Review Math Handbook*: Special Functions
- Arrive**MATH** Take Another Look
- ALEKS® Plotting and Comparing Signed Numbers

Answers

19. The graph of $g(x)$ is a reflection of the parent function across the x-axis and a vertical stretch.
20. The graph of $g(x)$ is a reflection of the parent function across the x-axis and translated 2 units down.
21. The graph of $g(x)$ is a reflection of the parent function across the y-axis and a horizontal stretch.
22. The graph of $g(x)$ is a reflection of the parent function across the x-axis and translated 7 units right and 3 units up.
23. The graph of $g(x)$ is a reflection of the parent function across the y-axis and a horizontal compression.
24. The graph of $g(x)$ is a reflection of the parent function across the x-axis and a vertical compression.

Practice

Go Online You can complete your homework online.

Examples 1 through 3

Describe the translation in $g(x)$ as it relates to the graph of the parent function.

1. $g(x) = |x| - 5$
The graph of $g(x)$ is the parent function translated 5 units down.

2. $g(x) = |x + 6|$
The graph of $g(x)$ is the parent function translated 6 units left.

3. $g(x) = |x - 2| + 7$
The graph of $g(x)$ is the parent function translated 2 units right and 7 units up.

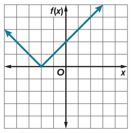
4. $g(x) = |x + 1| - 3$
The graph of $g(x)$ is the parent function translated 1 unit left and 3 units down.

5. $g(x) = |x| + 1$
The graph of $g(x)$ is the parent function translated 1 unit up.

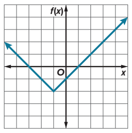
6. $g(x) = |x - 8|$
The graph of $g(x)$ is the parent function translated 8 units right.

Examples 4 and 5

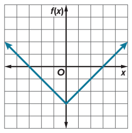
Use the graph of the function to write its equation.

7.


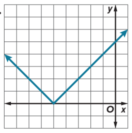
$f(x) = |x + 2|$

8.


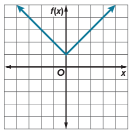
$f(x) = |x + 1| - 2$

9.


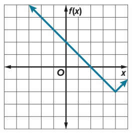
$f(x) = |x| - 3$

10.


$f(x) = |x + 5|$

11.


$f(x) = |x| + 1$

12.


$f(x) = |x - 4| - 2$

Examples 6 through 8

Describe the dilation in $g(x)$ as it relates to the graph of the parent function.

13. $g(x) = \frac{2}{5}|x|$
The graph of $g(x)$ is a vertical compression of the parent function.

14. $g(x) = |0.7x|$
The graph of $g(x)$ is a horizontal stretch of the parent function.

15. $g(x) = 1.3|x|$
The graph of $g(x)$ is a vertical stretch of the parent function.

16. $g(x) = |3x|$
The graph of $g(x)$ is a horizontal compression of the parent function.

17. $g(x) = \left|\frac{1}{6}x\right|$
The graph of $g(x)$ is a horizontal stretch of the parent function.

18. $g(x) = \frac{5}{4}|x|$
The graph of $g(x)$ is a vertical stretch of the parent function

Lesson 4-7 • Absolute Value Functions 277

Examples 9 through 11

Describe the reflection in $g(x)$ as it relates to the graph of the parent function. 19–24. See margin.

19. $g(x) = -3|x|$
20. $g(x) = -|x| - 2$
21. $g(x) = \left|-\frac{1}{4}x\right|$
22. $g(x) = -|x - 7| + 3$
23. $g(x) = |-2x|$
24. $g(x) = -\frac{2}{3}|x|$

Examples 12 through 14

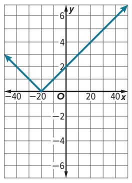
USE TOOLS **Graph each function. State the domain and range.** 25–33. See margin.

25. $g(x) = |x + 2| + 3$
26. $g(x) = |2x - 2| + 1$
27. $f(x) = \left|\frac{1}{2}x - 2\right|$
28. $f(x) = |2x - 1|$
29. $f(x) = \frac{1}{2}|x| + 2$
30. $h(x) = -2|x - 3| + 2$
31. $f(x) = -4|x + 2| - 3$
32. $g(x) = -\frac{2}{3}|x + 6| - 1$
33. $h(x) = -\frac{3}{4}|x - 8| + 1$

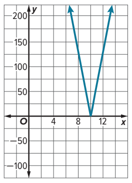
Example 15

Determine an absolute value function that models each situation.

34. ESCALATORS An escalator travels at a constant speed. The graph models the escalator's distance, in floors, from the second floor x seconds after leaving the ground floor. $y = 0.1|x + 20|$



35. TRAVEL The graph models the distance, in miles, a car traveling from Chicago, Illinois is from Annapolis, Maryland, where x is the number of hours since the car departed from Chicago, Illinois. $y = 65|10 - x|$



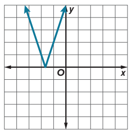
Mixed Exercises

MODELING **Graph each function. State the domain and range. Describe how each graph is related to its parent graph.** 36–38. See Mod 4 Answer Appendix.

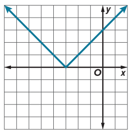
36. $f(x) = -4|x - 2| + 3$
37. $f(x) = |2x|$
38. $f(x) = |2x + 5|$

278 Module 4 • Linear and Nonlinear Functions

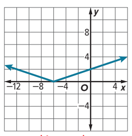
Use the graph of the function to write its equation.

39. 

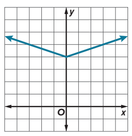
$f(x) = |-3x - 5|$

40. 

$f(x) = |-x - 3|$

41. 

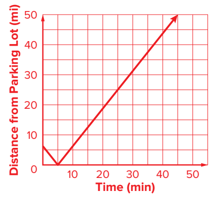
$f(x) = \left|\frac{1}{3}x + 2\right|$

42. 

$f(x) = \frac{1}{3}|x| + 4$

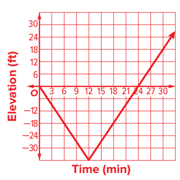
43. SUNFLOWER SEEDS A company produces and sells bags of sunflower seeds, s. A medium-sized bag of sunflower seeds must contain 16 ounces of seeds. If the amount of sunflower seeds in the medium-sized bag differs from the desired 16 ounces by more than x, the bag cannot be delivered to companies to be sold. Write an equation that can be used to find the highest and lowest amounts of sunflower seeds in a medium-sized bag. $x = |s - 16|$

44. REASONING The function $y = \frac{5}{4}|x - 5|$ models a car's distance in miles from a parking lot after x minutes. Graph the function. After how many minutes will the car reach the parking lot? 5 minutes



45. STATE YOUR ASSUMPTION A track coach set up an agility drill for members of the track team. According to the coach, 21.7 seconds is the target time to complete the agility drill. If the time differs from the desired 21.7 seconds by more than x, the track coach may require members of the track team to change their training. Write an equation that can be used to find the fastest and slowest times members of the track team can complete the agility drill so that their training does not have to change. If $x = 3.2$, what can you assume about the range of times the coach wants the members of the track team to complete the agility drill? Solve your equation for $x = 3.2$ and use the results to justify your assumption. $x = |t - 21.7|$; The range of times is twice the value of x, $3.2(2) = 6.4$ s; The solution to the equation is 24.9 and 18.5, which has a range of $24.9 - 18.5 = 6.4$ s.

46. SCUBA DIVING The function $y = 3|x - 12| - 36$ models a scuba diver's elevation in feet compared to sea level after x minutes. Graph the function. How far below sea level is the scuba diver at the deepest point in their dive? 36 feet below sea level



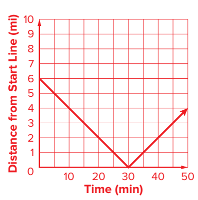
47. MANUFACTURING A manufacturing company produces boxes of cereal, b. A small box of cereal must have 12 ounces. If the amount of cereal in a small box differs from the desired 12 ounces by more than x, the box cannot be shipped for selling. Write an equation that can be used to find the highest and lowest amounts of cereal in a small box. $x = |b - 12|$

48. STRUCTURE Amelia is competing in a bicycle race. The race is along a circular path. She is 6 miles from the start line. She is approaching the start line at a speed of 0.2 mile per minute. After Amelia reaches the start line, she continues at the same speed, taking another lap around the track.

a. Organize the information into a table. Include a row for time in minutes x, and a row for distance from start line $f(x)$.

x	0	10	20	30	40	50
$f(x)$	6	4	2	0	2	4

b. Draw a graph to represent Amelia's distance from the start line.



Higher-Order Thinking Skills

49. WRITE Use transformations to describe how the graph of $h(x) = -|x + 2| - 3$ is related to the graph of the parent absolute value function.
To get the graph of $h(x)$, the parent absolute value function is reflected in the x-axis, then translated 2 units left and 3 units down.

50. ANALYZE On a straight highway, the town of Garvey is located at mile marker 200. A car is located at mile marker x and is traveling at an average speed of 50 miles per hour.

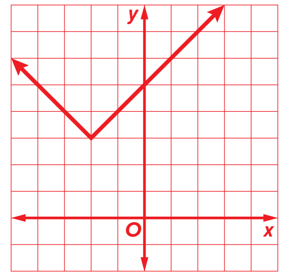
a. Write a function $T(x)$ that gives the time, in hours, it will take the car to reach Garvey. Then graph the function on the coordinate plane.
$T(x) = \frac{1}{50}|x - 200|$; See Mod 4. Answer Appendix for graph.

b. Does the graph have a maximum or minimum? If so, name it and describe what it represents in the context of the problem. Minimum at (200, 0); If the car is at mile marker 200, the time to reach Garvey is 0 hours.

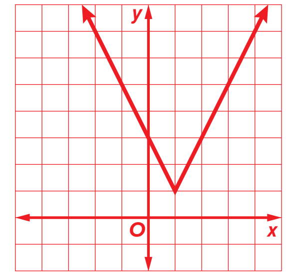
51. PERSEVERE Write the equation $y = |x - 3| + 2$ as a piecewise-defined function. Then graph the piecewise function.
$f(x) = \begin{cases} -x + 5 & \text{if } x < 3 \\ x - 1 & \text{if } x \geq 3 \end{cases}$; See Mod 4. Answer Appendix for graph.

52. CREATE Write an absolute value function, $f(x)$, that has a domain of all real numbers and a range that is greater than or equal to 4. Be sure your function also includes a dilation of the parent function. Describe how your function relates to the parent absolute value graph. Then graph your function. Sample answer: $f(x) = \frac{1}{3}|x| + 4$; The graph of $f(x)$ is the parent function translated 4 units up, and vertically compressed by a factor of 3. See Mod 4. Answer Appendix for graph.

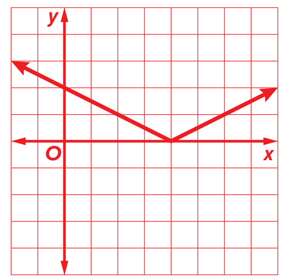
Answers

25. 

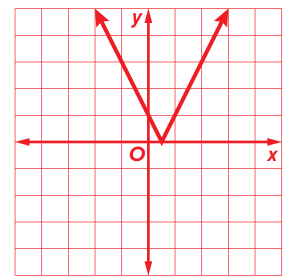
D = all real numbers,
R = $g(x) \geq 3$

26. 

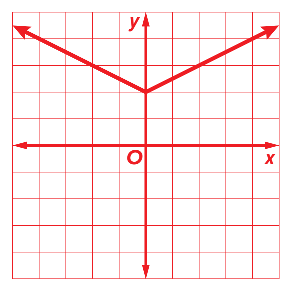
D = all real numbers,
R = $g(x) \geq 1$

27. 

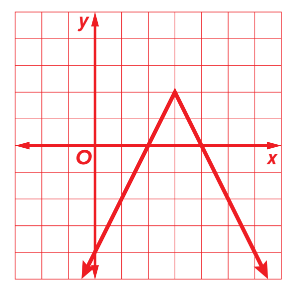
D = all real numbers,
R = $f(x) \geq 0$

28. 

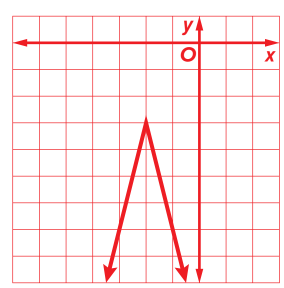
D = all real numbers,
R = $f(x) \geq 0$

29. 

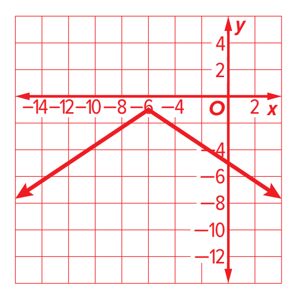
D = all real numbers,
R = $f(x) \geq 2$

30. 

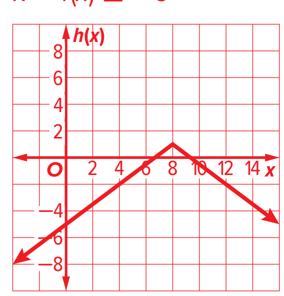
D = all real numbers,
R = $h(x) \leq 2$

31. 

D = all real numbers,
R = $f(x) \leq -3$

32. 

D = all real numbers,
R = $g(x) \leq -1$

33. 

Review

Rate Yourself!

Have students return to the Module Opener to rate their understanding of the concepts presented in this module. They should see that their knowledge and skills have increased. After completing the chart, have them respond to the prompts in their *Student Edition* and share their responses with a partner.

Answering the Essential Question

Before answering the Essential Question, have students review their answers to the Essential Question Follow-Up questions found throughout the module.

- Why is it helpful to have different ways to graph linear functions?
- What can you learn about the graph of a linear function by analyzing its equation?
- Why is it important to understand how the structure of a function models a situation?
- If you know that a function is a step function, what do you know about how the elements of the domain are paired with the elements of the range?

Then have them write their answer to the Essential Question.

DINAH ZIKE FOLDABLES

ELL A completed Foldable for this module should include the key concepts related to linear and nonlinear functions.

LS **LearnSmart** Use LearnSmart as part of your test preparation plan to measure student topic retention. You can create a student assignment in LearnSmart for additional practice on these topics for **Linear and Exponential Relationships, Descriptive Statistics**, and **Quadratic Functions and Modeling**.

- Interpret Expressions for Functions
- Build Linear and Exponential Function Models
- Interpret Linear Models
- Construct and Compare Linear, Quadratic, and Exponential Models and Solve Problems

Module 4 • Linear and Nonlinear Functions

Review

Essential Question

What can a function tell you about the relationship that it represents?
It can tell you about the rate of change, whether the relationship is positive or negative, the locations of the *x*- and *y*-intercepts, and what points fall on the graph.

Module Summary

Lessons 4-1 through 4-3

Graphing Linear Functions, Rate of Change, and Slope

- The graph of an equation represents all of its solutions.
- The x-value of the y-intercept is 0. The y-value of the x-intercept is 0.
- The rate of change is how a quantity is changing with respect to a change in another quantity. If x is the independent variable and y is the dependent variable, then rate of change $= \frac{\text{change in } y}{\text{change in } x}$.
- The slope m of a nonvertical line through any two points can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- A line with positive slope slopes upward from left to right. A line with negative slope slopes downward from left to right. A horizontal line has a slope of 0. The slope of a vertical line is undefined.

Lesson 4-4

Transformations of Linear Functions

- When a constant k is added to a linear function $f(x)$, the result is a vertical translation.
- When a linear function $f(x)$ is multiplied by a constant a, the result $a \cdot f(x)$ is a vertical dilation.
- When a linear function $f(x)$ is multiplied by -1 before or after the function has been evaluated, the result is a reflection across the x- or y-axis.

Lesson 4-5

Arithmetic Sequences

- An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference.
- The nth term of an arithmetic sequence with the first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

Lessons 4-6, 4-7

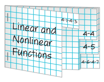
Special Functions

- A piecewise-linear function has a graph that is composed of a number of linear pieces.
- A step function is a type of piecewise-linear function with a graph that is a series of horizontal line segments.
- An absolute value function is V-shaped.

Study Organizer

Foldables

Use your Foldable to review this module. Working with a partner can be helpful. Ask for clarification of concepts as needed.



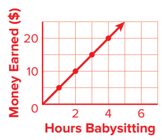
Module 4 Review • Linear and Nonlinear Functions 281

Test Practice

1. **GRAPH** Jalyn made a table of how much money she will earn from babysitting. (Lesson 4-1)

Hours Babysitting	Money Earned
1	5
2	10
3	15
4	20

Use the table to graph the function.



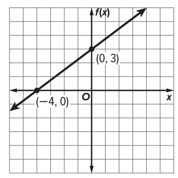
2. **OPEN RESPONSE** Copy and complete the table to find the missing values in the table that show the points on the graph of $f(x) = 2x - 4$. (Lesson 4-1)

x	−2	0	2	4	6
$f(x)$	−8	−4	0	4	8

3. **OPEN RESPONSE** Mr. Hernandez is draining his pool to have it cleaned. At 8:00 A.M., it had 2000 gallons of water and at 11:00 A.M. it had 500 gallons left to drain. What is the rate of change in the amount of water in the pool? (Lesson 4-2)

−500 gallons/hr

4. **MULTIPLE CHOICE** Find the slope of the graphed line. (Lesson 4-2)



A. $-\frac{4}{3}$

B. $-\frac{3}{4}$

C. $\frac{3}{4}$

D. $\frac{4}{3}$

5. **MULTIPLE CHOICE** Determine the slope of the line that passes through the points (4, 10) and (2, 10). (Lesson 4-2)

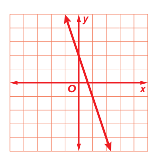
A. −1

B. 0

C. 1

D. undefined

6. **GRAPH** Graph the equation of a line with a slope of −3 and a y-intercept of 2. (Lesson 4-3)



7. **MULTIPLE CHOICE** What is the slope of the line that passes through (3, 4) and (−7, 4)? (Lesson 4-3)

A. 0

B. undefined

C. −2

D. −10

282 **Module 4 Review** • Linear and Nonlinear Functions

Review and Assessment Options

The following online review and assessment resources are available for you to assign to your students. These resources include technology-enhanced questions that are auto-scored, as well as essay questions.

Review Resources

Put It All Together: Lessons 4-1 through 4-7

Vocabulary Activity

Module Review

Assessment Resources

Vocabulary Test

AL Module Test Form B

OL Module Test Form A

BL Module Test Form C

Performance Task*

*The module-level performance task is available online as a printable document. A scoring rubric is included.

Practice

You can use these pages to help your students review module content and prepare for online assessments. Exercises 1–21 mirror the types of questions your students will see on online assessments.

Question Type	Description	Exercise(s)
Multiple Choice	Students select one correct answer.	4, 5, 7, 8, 11, 17
Multi-Select	Multiple answers may be correct. Students must select all correct answers.	18
Table Item	Students complete a table by entering in the correct values.	2, 15
Graph	Students create a graph on an online coordinate plane.	1, 6, 16
Open Response	Students construct their own response.	3, 9, 10, 12, 13, 14, 19, 20, 21

To ensure that students understand the standards, check students' success on individual exercises.

Standard	Lesson(s)	Exercise(s)
A.CED.2	4-3	8
A.REI.10	4-1	2
F.IF.2	4-6	15
F.IF.6	4-2	3, 4, 5, 7
F.IF.7	4-1, 4-3, 4-4, 4-6, 4-7	1, 6, 16, 21
F.BF.1a	4-5	12
F.BF.3	4-4, 4-7	9, 10, 17, 18, 19, 20
F.LE.2	4-5	11, 13, 14

8. **MULTIPLE CHOICE** A teacher buys 100 pencils to keep in her classroom at the beginning of the school year. She allows the students to borrow pencils, but they are not always returned. On average, she loses about 8 pencils a month. Write an equation in slope-intercept form that represents the number of pencils she has left, y, after a number of x months. (Lesson 4-3)

A. $y = -8x - 100$

B. $y = -8x + 100$

C. $y = 8x + 100$

D. $y = 8x - 100$

9. **OPEN RESPONSE** Name the transformation that changes the slope, or the steepness of, a graph. (Lesson 4-4)

dilation

10. **OPEN RESPONSE** Describe the dilation of $g(x) = \frac{1}{2}(x)$ as it relates to the graph of the parent function, $f(x) = x$. (Lesson 4-4)

$g(x)$ is a vertical compression of the parent function by a factor of $\frac{1}{2}$.

11. **MULTIPLE CHOICE** Arjun begins the calendar year with $40 in his bank account. Each week he receives an allowance of $20, half of which he deposits into his bank account. The situation describes an arithmetic sequence. Which function represents the amount in Arjun's account after n weeks? (Lesson 4-5)

A. $f(n) = 20n + 40$

B. $f(n) = 40n + 20$

C. $f(n) = 40 + 10n$

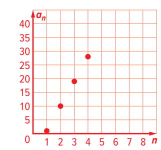
D. $f(n) = 10 + 40n$

12. **OPEN RESPONSE** What number can be used to complete the equation below that describes the nth term of the arithmetic sequence $-2, -1.5, -1, 0, 0.5, \ldots$? (Lesson 4-5)

$a_n = 0.5n - \underline{\ ?\ }$

2.5

13. **OPEN RESPONSE** Write and graph a function to represent the sequence 1, 10, 19, 28, … (Lesson 4-5)



$f(n) = 9n - 8$

14. **OPEN RESPONSE** Christa has a box of chocolate candies. The number of chocolates in each row forms an arithmetic sequence as shown in the table. (Lesson 4-5)

Row	1	2	3	4
Number of Chocolates	3	6	9	12

Write an arithmetic function that can be used to find the number of chocolates in each row.

$a_n = 3n$

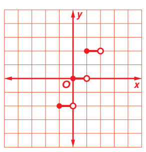
15. OPEN RESPONSE Daniel earns \$9 per hour at his job for the first 40 hours he works each week. However, his pay rate increases to \$13.50 per hour thereafter. This situation can be represented with the function

$$f(x) = \begin{cases} 9x, \text{ if } x \leq 40 \\ 360 + 13.5(x - 40), \text{ if } x > 40 \end{cases}$$

Use this function to copy and complete the table with the correct values. (Lesson 4-6)

Hours Worked, x	Money Earned, $f(x)$
30	270
35	315
40	360
45	427.5
50	495

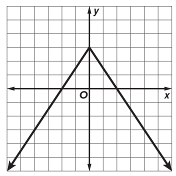
16. GRAPH Graph the function $f(x) = 2[[x]]$. (Lesson 4-6)



17. MULTIPLE CHOICE Which of the following describes the effect a dilation has upon the graph of the absolute value parent function? (Lesson 4-7)

A. Flipped across axis
B. Stretch or compression
C. Rotated about the origin
D. Shifted horizontally or vertically

18. MULTI-SELECT Describe the transformation(s) of the function graphed below in relation to the absolute value parent function. Select all that apply. (Lesson 4-7)



A. Reflected across x-axis
B. Vertical stretch
C. Vertical compression
D. Reflected across y-axis
E. Translated right 3
F. Translated up 3

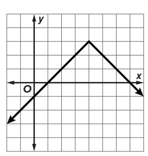
19. OPEN RESPONSE Describe the graph of $g(x) = |x| + 5$ in relation to the graph of the absolute value parent function. (Lesson 4-7)

Sample answer: It is translated 5 units up.

20. OPEN RESPONSE Across which axis is the graph of $h(x) = -5|x|$ reflected? (Lesson 4-7)

x-axis

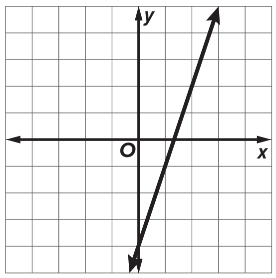
21. OPEN RESPONSE Use the graph of the function to write its equation. (Lesson 4-7)



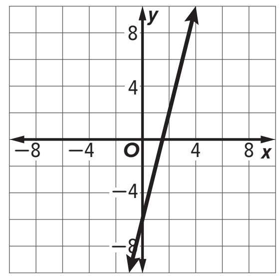
$f(x) = -|x - 4| + 3$

Lesson 4-3

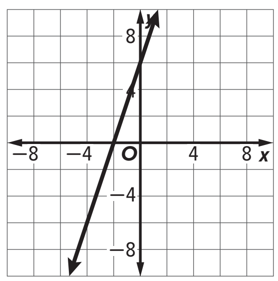
33.



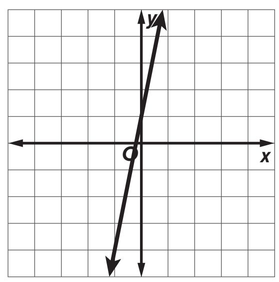
34.



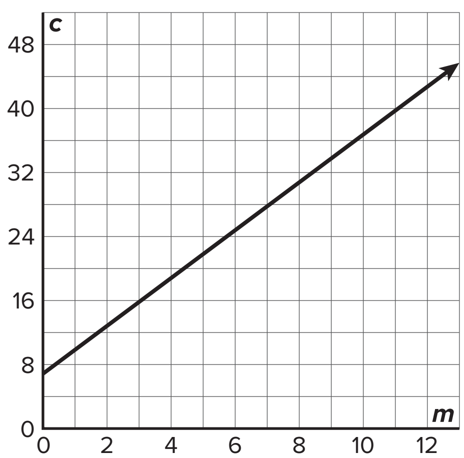
35.



36.



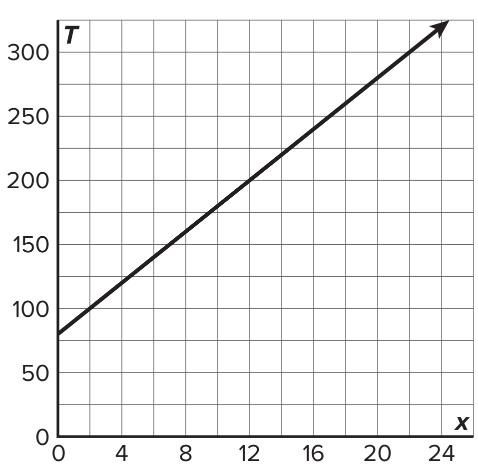
40b.



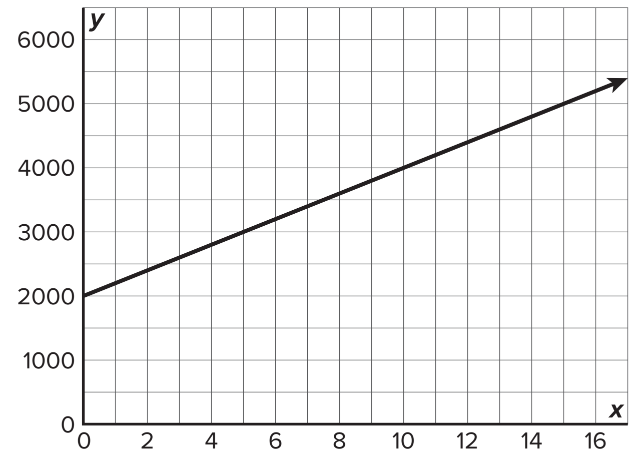
40c. Sample answer: Find 13 along the horizontal axis. Move up to the line. The corresponding value along the vertical axis is about 45. So, the cost of watching 13 movies from MovieMania is about $45.

40d. Sample answer: The cost of watching 13 movies from MovieMania is about $45, so divide $45 by 9 to get $5. So, the cost of watch a movie from SuperFlix is about $5.

41b.

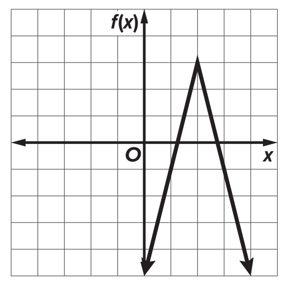


42b



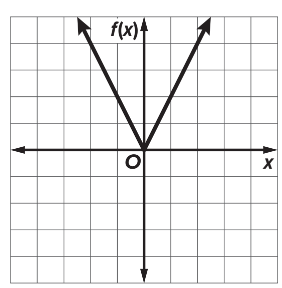
Lesson 4-7

36.



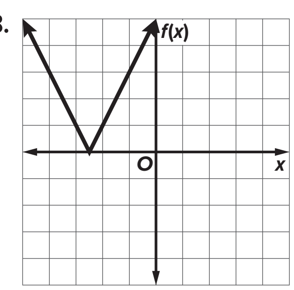
D = all real numbers,
R = $f(x) \leq 3$
The graph of $f(x)$ is a reflection of the parent function across the x-axis, vertically stretched by a factor of 4, and translated 2 units right and 3 units up.

37.



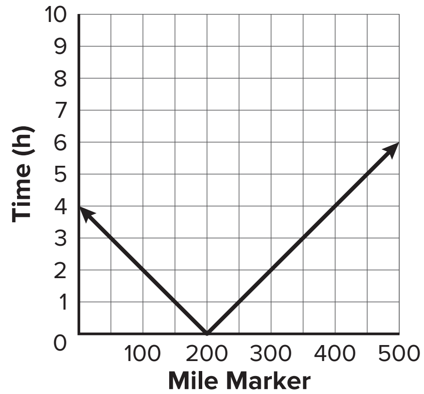
D = all real numbers,
R = $f(x) \geq 0$
The graph of $g(x)$ is the parent function horizontally compressed by a factor of $\frac{1}{2}$.

38.

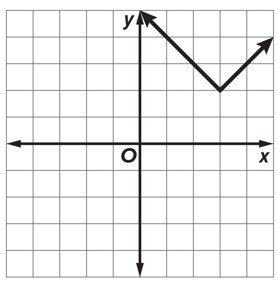


D = all real numbers,
R = $f(x) \geq 0$
The graph of $g(x)$ is the parent function horizontally compressed by a factor of $\frac{1}{2}$, and translated 2.5 units left.

50a.



51.



52.