Module 4

Linear and Nonlinear Functions

Essential Question

What can a function tell you about the relationship that it represents?

What Will You Learn?

Place a check mark (✓) in each row that corresponds with how much you already know about each topic **before** starting this module.

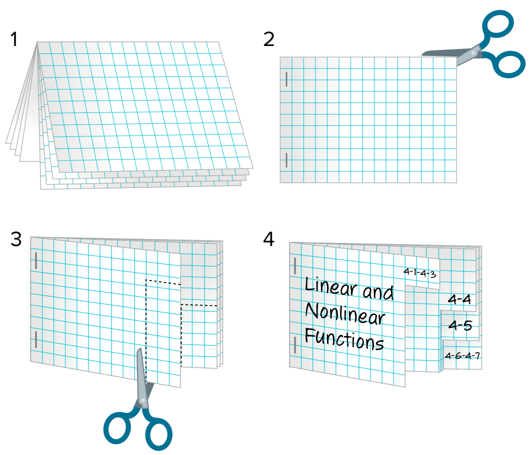
KEY

— I don't know. — I've heard of it. — I know it!

	Before			After		
graph linear equations by using a table						
graph linear equations by using intercepts						
find rates of change						
determine slopes of linear equations						
write linear equations in slope-intercept form						
graph linear functions in slope-intercept form						
translate, dilate, and reflect linear functions						
identify and find missing terms in arithmetic sequences						
write arithmetic sequences as linear functions						
model and use piecewise functions, step functions, and absolute value functions						
translate absolute value functions						

Foldables Make this Foldable to help you organize your notes about functions. Begin with five sheets of grid paper.

1. **Fold** five sheets of grid paper in half from top to bottom.
2. **Cut** along fold. Staple the eight half-sheets together to form a booklet.
3. **Cut** tabs into margin. The top tab is 4 lines wide, the next tab is 8 lines wide, and so on. When you reach the bottom of a sheet, start the next tab at the top of the page.
4. **Label** each tab with a lesson number. Use the extra pages for vocabulary.



What Vocabulary Will You Learn?

Check the box next to each vocabulary term that you may already know.

- ☐ absolute value function
- ☐ arithmetic sequence
- ☐ common difference
- ☐ constant function
- ☐ dilation
- ☐ family of graphs
- ☐ greatest integer function
- ☐ identity function
- ☐ interval
- ☐ *n*th term of an arithmetic sequence
- ☐ parameter
- ☐ parent function
- ☐ piecewise-defined function
- ☐ piecewise-linear function
- ☐ rate of change
- ☐ reflection
- ☐ sequence
- ☐ slope
- ☐ step function
- ☐ term of a sequence
- ☐ transformation
- ☐ translation
- ☐ vertex

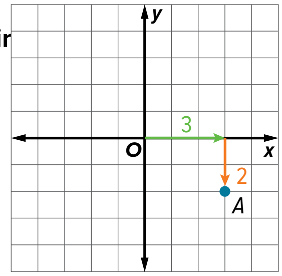
Are You Ready?

Complete the Quick Review to see if you are ready to start this module.
Then complete the Quick Check.

Quick Review

Example 1

Graph $A(3, -2)$ on a coordir



Start at the origin. Since the x-coordinate is positive, move 3 units to the right. Then move 2 units down since the y-coordinate is negative. Draw a dot and label it A.

Example 2

Solve $x - 2y = 8$ for y.

$x - 2y = 8$	Original expression
$x - x - 2y = 8 - x$	Subtract x from each side.
$-2y = 8 - x$	Simplify.
$\frac{-2y}{-2} = \frac{8 - x}{-2}$	Divide each side by -2.
$y = \frac{1}{2}x - 4$	Simplify.

Quick Check

Graph and label each point on the coordinate plane.

1. $B(-3, 3)$ **2.** $C(-2, 1)$ **3.** $D(3, 0)$

4. $E(-5, -4)$ **5.** $F(0, -3)$ **6.** $G(2, -1)$

Solve each equation for y.

7. $3x + y = 1$ **8.** $8 - y = x$

9. $5x - 2y = 12$ **10.** $3x + 4y = 10$

11. $3 - \frac{1}{2}y = 5x$ **12.** $\frac{y + 1}{3} = x + 2$

How did you do?

Which exercises did you answer correctly in the Quick Check? Shade those exercise numbers below.

1 2 3 4 5 6 7 8 9 10 11 12

Graphing Linear Functions

Explore Points on a Line

Online Activity Use an interactive tool to complete an Explore.

INQUIRY How is the graph of a linear equation related to its solutions?

Learn Graphing Linear Functions by Using Tables

A table of values can be used to graph a linear function. Every ordered pair that makes the equation true represents a point on its graph. So, a graph represents all the solutions of an equation.

Linear functions can be represented by equations in two variables.

Example 1 Graph by Making a Table

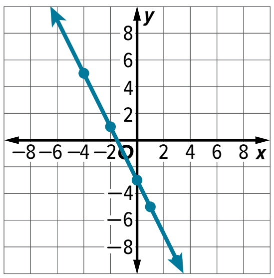
Graph $-2x - 3 = y$ by making a table.

Step 1 Choose any values of x from the domain and make a table.

Step 2 Substitute each x-value into the equation to find the corresponding y-value. Then, write the x- and y-values as an ordered pair.

x	$-2x - 3$	y	(x, y)
-4			
-2			
0			
1			
3			

Step 3 Graph the ordered pairs in the table and connect them with a line.



Go Online You can complete an Extra Example online.

Today's Goals
- Graph linear functions by making tables of values.
- Graph linear functions by using the x- and y-intercepts.

Talk About It!
What values of x might be easiest to use when graphing a linear equation when the x-coefficient is a whole number? Justify your argument.

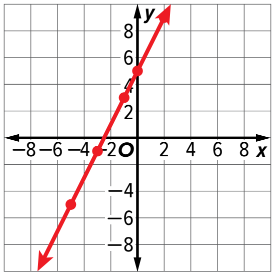
Study Tip
Exactness Although only two points are needed to graph a linear function, choosing three to five x-values that are spaced out can verify that your graph is correct.

Your Notes

Check

Graph $y = 2x + 5$ by using a table.

x	y
−5	
−3	
−1	
0	
2	



Example 2 Choose Appropriate Domain Values

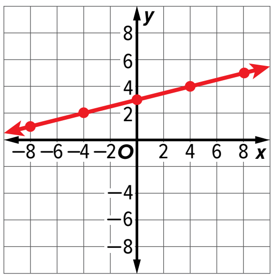
Graph $y = \frac{1}{4}x + 3$ by making a table.

Step 1 Make a table.

Step 2 Find the y-values.

Step 3 Graph the ordered pairs in the table and connect them with a line.

x	$\frac{1}{4}x + 3$	y	(x, y)
−8			
−4			
0			
4			
8			



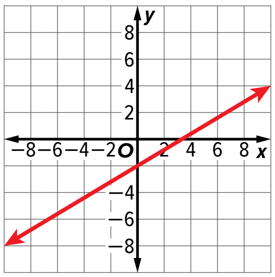
Watch Out!

Equivalent Equations
Sometimes, the variables are on the same side of the equal sign. Rewrite these equations by solving for y to make it easier to find values for y.

Check

Graph $y = \frac{3}{5}x - 2$ by making a table.

x	y
−10	
−5	
0	
5	
10	



Go Online You can complete an Extra Example online.

Example 3 Graph $y = a$

Graph $y = 5$ by making a table.

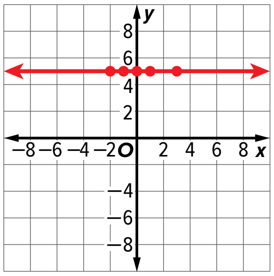
Step 1 Rewrite the equation.
$y = 0x + 5$

Step 2 Make a table.

x	$0x + 5$	y	(x, y)
-2			
-1			
0			
1			
2			

Step 3 Graph the line.

The graph of $y = 5$ is a horizontal line through $(x, 5)$ for all values of x in the domain.



Think About It!

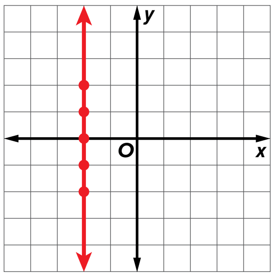
In general, what does the graph of an equation of the form $y = a$, where a is any real number, look like?

Example 4 Graph $x = a$

Graph $x = -2$.

You learned in the previous example that equations of the form $y = a$ have graphs that are horizontal lines. Equations of the form $x = a$ have graphs that are vertical lines.

The graph of $x = -2$ is a vertical line through $(-2, y)$ for all real values of y. Graph ordered pairs that have x-coordinates of -2 and connect them with a vertical line.

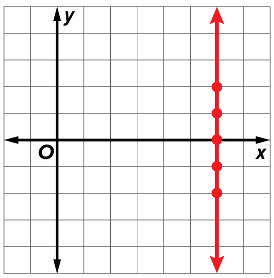


Think About It!

Is $x = a$ a function? Why or why not?

Check

Graph $x = 6$.



Go Online You can complete an Extra Example online.

Go Online
You can watch a video to see how to graph linear functions.

Think About It!
Why are the x- and y-intercepts easy to find?

Think About It!
What does a line that only has an x-intercept look like? a line that only has a y-intercept?

Study Tip
Tools When drawing lines by hand, it is helpful to use a straightedge or a ruler.

Explore Lines Through Two Points

Online Activity Use graphing technology to complete an Explore.

INQUIRY How many lines can be formed with two given points?

Learn Graphing Linear Functions by Using the Intercepts

You can graph a linear function given only two points on the line. Using the x- and y-intercepts is common because they are easy to find. The intercepts provide the ordered pairs of two points through which the graph of the linear function passes.

Example 5 Graph by Using Intercepts

Graph $-x + 2y = 8$ by using the x- and y-intercepts.

To find the x-intercept, let ________.

________	Original equation
________	Replace y with 0.
________	Simplify.
________	Divide.

This means that the graph intersects the x-axis at ________.

To find the y-intercept, let ________.

________	Original equation
________	Replace x with 0.
________	Simplify.
________	Divide.

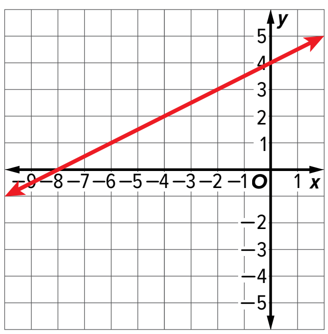
This means that the graph intersects the y-axis at ________.

Graph the equation.

Step 1 Graph the x-intercept.

Step 2 Graph the y-intercept.

Step 3 Draw a line through the points.



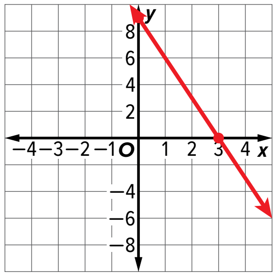
Go Online You can complete an Extra Example online.

Check

Graph $4y = -12x + 36$ by using the x- and y- intercepts.

x-intercept: ______

y-Intercept: ______



Example 6 Use Intercepts

PETS Angelina bought a 15-pound bag of food for her dog. The bag contains about 60 cups of food, and she feeds her dog $2\frac{1}{2}$ or $\frac{5}{2}$ cups of food per day. The function $y + \frac{5}{2}x = 60$ represents the amount of food left in the bag y after x days. Graph the amount of dog food left in the bag as a function of time.

Part A

Find the *x*- and *y*-intercepts and interpret their meaning in the context of the situation.

To find the x-intercept, let ______.

______ Original equation

______ Replace y with 0.

______ Simplify.

______ Multiply each side by $\frac{2}{5}$.

The x-intercept is 24. This means that the graph intersects the x-axis at ______. So, after 24 days, there is no dog food left in the bag.

To find the y-intercept, let ______.

______ Original equation

______ Replace x with 0.

______ Simplify.

The y-intercept is 60. This means that the graph intersects the y-axis at ______. So, after 0 days, there are 60 cups of food in the bag.

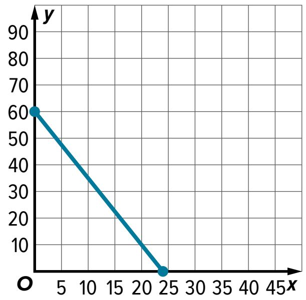
(continued on the next page)

Go Online You can watch a video to see how to use a graphing calculator with this example.

Think About It! Find another point on the graph. What does it mean in the context of the problem?

Part B

Graph the equation by using the intercepts.



Check

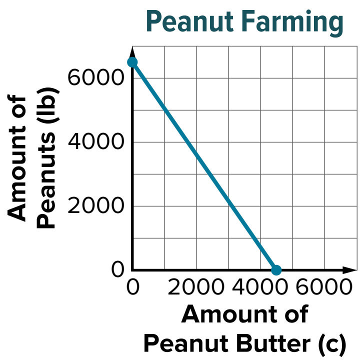
PEANUTS A farm produces about 4362 pounds of peanuts per acre. One cup of peanut butter requires about $\frac{2}{3}$ pound of peanuts. If one acre of peanuts is harvested to make peanut butter, the function $y = -\frac{2}{3}x + 4362$ represents the pounds of peanuts remaining y after x cups of peanut butter are made.

x-intercept: ______

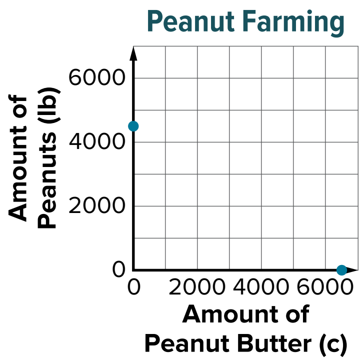
y-intercept: ______

Which graph uses the x- and y-intercepts to correctly graph the equation? ____

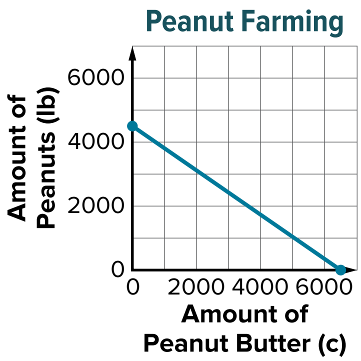
A.



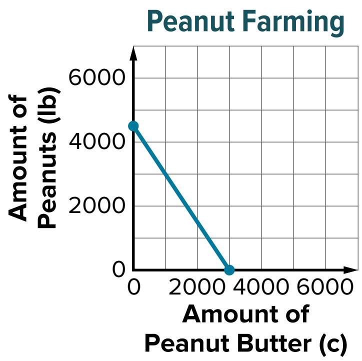
B.



C.



D.



Think About It!

What assumptions did you make about the amount of food Angelina feeds her dog each day?

Go Online

You can watch a video to see how to graph a linear function using a graphing calculator.

Go Online You can complete an Extra Example online.

Name ______________________ Period ____________ Date ____________

Practice

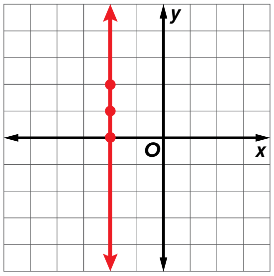
Go Online You can complete your homework online.

Examples 1 through 4

Graph each equation by making a table.

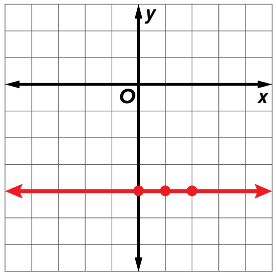
1. $x = -2$

x	y



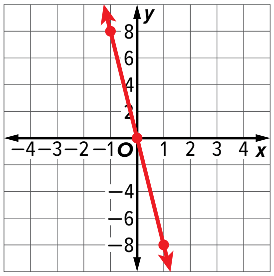
2. $y = -4$

x	y



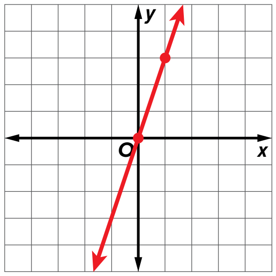
3. $y = -8x$

x	y



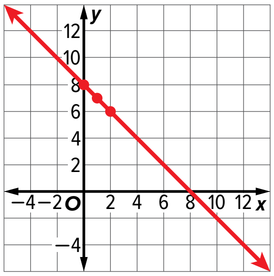
4. $3x = y$

x	y



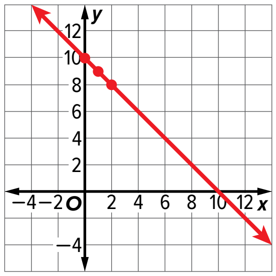
5. $y - 8 = -x$

x	y



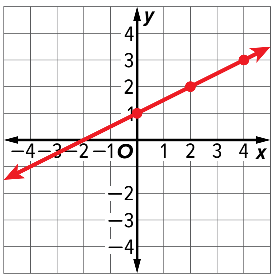
6. $x = 10 - y$

x	y



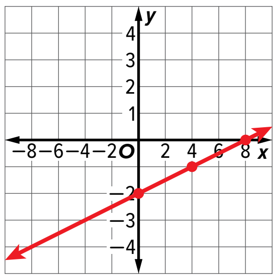
7. $y = \frac{1}{2}x + 1$

x	y



8. $y + 2 = \frac{1}{4}x$

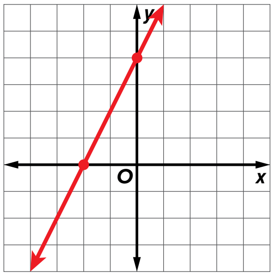
x	y



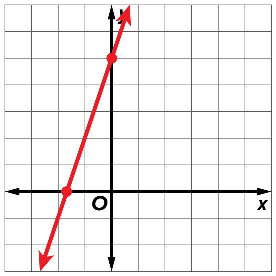
Example 5

Graph each equation by using the *x*-and *y*-intercepts.

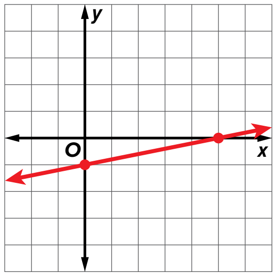
9. $y = 4 + 2x$



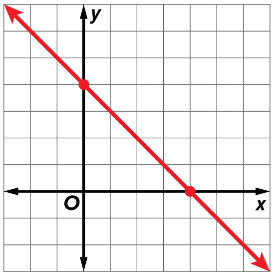
10. $5 - y = -3x$



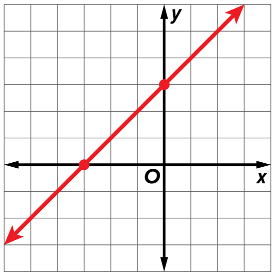
11. $x = 5y + 5$



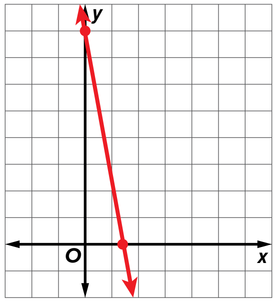
12. $x + y = 4$



13. $x - y = -3$



14. $y = 8 - 6x$



Example 6

15. SCHOOL LUNCH Amanda has \$210 in her school lunch account. She spends \$35 each week on school lunches. The equation $y = 210 - 35x$ represents the total amount in Amanda's school lunch account y for x weeks of purchasing lunches.

a. Find the x- and y-intercepts and interpret their meaning in the context of the situation.

b. Graph the equation by using the intercepts.

16. SHIPPING The *OOCL Shenzhen,* one of the world's largest container ships, carries 8063 TEUs (1280-cubic-feet containers). Workers can unload a ship at a rate of 1 TEU every minute. The equation $y = 8063 - 60x$ represents the number of TEUs on the ship y after x hours of the workers unloading the containers from the *Shenzhen*.

a. Find the x- and y-intercepts and interpret their meaning in the context of the situation.

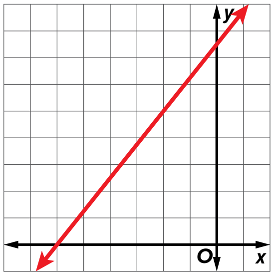
b. Graph the equation by using the intercepts.

Name ______________________ Period ____________ Date ____________

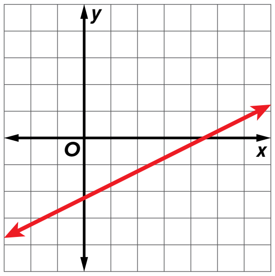
Mixed Exercises

Graph each equation.

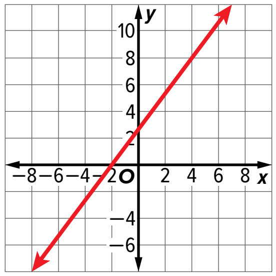
17. $1.25x + 7.5 = y$



18. $2x - 3 = 4y + 6$



19. $3y - 7 = 4x + 1$



Find the *x*-intercept and *y*-intercept of the graph of each equation.

20. $5x + 3y = 15$

21. $2x - 7y = 14$

22. $2x - 3y = 5$

23. $6x + 2y = 8$

24. $y = \frac{1}{4}x - 3$

25. $y = \frac{2}{3}x + 1$

26. HEIGHT The height of a woman can be predicted by the equation $h = 81.2 + 3.34r$, where h is her height in centimeters and r is the length of her radius bone in centimeters.

a. Is this a linear function? Explain.

b. What are the r- and h-intercepts of the equation? Do they make sense in the situation? Explain.

c. Graph the equation by using the intercepts.

d. Use the graph to find the approximate height of a woman whose radius bone is 25 centimeters long.

27. TOWING Pick-M-Up Towing Company charges \$40 to hook a car and \$1.70 for each mile that it is towed. Write an equation that represents the total cost y for x miles towed. Graph the equation. Find the y-intercept, and interpret its meaning in the context of the situation.

28. USE A MODEL Elias has \$18 to spend on peanuts and pretzels for a party. Peanuts cost \$3 per pound and pretzels cost \$2 per pound. Write an equation that relates the number of pounds of pretzels y and the number of pounds of peanuts x. Graph the equation. Find the x- and y-intercepts. What does each intercept represent in terms of context?

29. REASONING One football season, a football team won 4 more games than they lost. The function $y = x + 4$ represents the number of games won y and the number of games lost x. Find the x- and y-intercepts. Are the x- and y-intercepts reasonable in this situation? Explain.

30. WRITE Consider real-world situations that can be modeled by linear functions.

a. Write a real-world situation that can be modeled by a linear function.

b. Write an equation to model your real-world situation. Be sure to define variables. Then find the x- and y-intercepts. What does each intercept represent in your context?

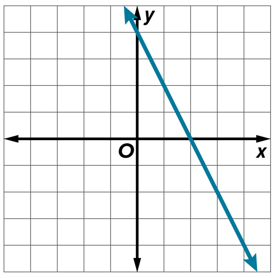
c. Graph your equation by making a table. Include a title for the graph as well as labels and titles for each axis. Explain how you labeled the x- and y-axes. State a reasonable domain for this situation. What does the domain represent?

31. FIND THE ERROR Geroy claims that every line has both an x- and a y-intercept. Is he correct? Explain your reasoning.

32. WHICH ONE DOESN'T BELONG? Which equation does not belong with the other equations? Justify your conclusion.

$y = 2 - 3x$	$5x = y - 4$	$y = 2x + 5$	$y - 4 = 0$

33. ANALYZE Robert sketched a graph of a linear equation $2x + y = 4$. What are the x- and y-intercepts of the graph? Explain how Robert could have graphed this equation using the x- and y-intercepts.



34. ANALYZE Compare and contrast the graph of $y = 2x + 1$ with the domain $\{1, 2, 3, 4\}$ and $y = 2x + 1$ with the domain all real numbers.

CREATE **Give an example of a linear equation in the form $Ax + By = C$ for each condition. Then describe the graph of the equation.**

35. $A = 0$ **36.** $B = 0$ **37.** $C = 0$

Rate of Change and Slope

Learn Rate of Change of a Linear Function

The **rate of change** is how a quantity is changing with respect to a change in another quantity.

If x is the independent variable and y is the dependent variable, then rate of change $= \frac{\text{change in } y}{\text{change in } x}$.

Example 1 Find the Rate of Change

COOKING **Find the rate of change of the function by using two points from the table.**

Amount of Flour x (cups)	Pancakes y
2	12
4	24
6	36

$$\text{rate of change} = \frac{\text{change in } y}{\text{change in } x}$$
$$= \frac{\text{change in pancakes}}{\text{change in flour}}$$
$$= \frac{24 - 12}{4 - 2}$$
$$= \_\_\_\_ \text{ or } \_\_\_\_$$

The rate is ____ or ____. This means that you could make ____ pancakes for each cup of flour.

Check

Find the rate of change.

$\_\_\_\_ \ \frac{\text{dollars}}{\text{gallons}}$

Amount of Gasoline Purchased (Gallons)	Cost (Dollars)
4.75	15.77
6	19.92
7.25	24.07
8.5	28.22

Go Online You can complete an Extra Example online.

Today's Goals
- Calculate and interpret rate of change.
- Calculate and interpret slope.

Today's Vocabulary
rate of change

slope

Think About It!
Suppose you found a new recipe that makes 6 pancakes when using 2 cups of flour, 12 pancakes when using 4 cups of flour, and 18 pancakes when using 6 cups of flour. How does this change the rate you found for the original recipe?

Study Tip
Placement Be sure that the dependent variable is in the numerator and the independent variable is in the denominator. In this example, the number of pancakes you can make *depends* on the amount of flour you can use.

Your Notes

Think About It!

How is a greater increase or decrease of funds represented graphically?

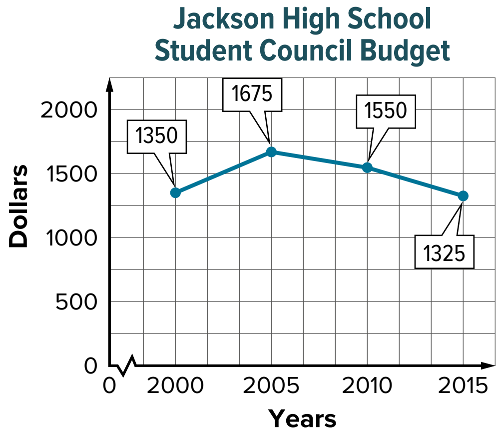
Study Tip

Assumptions In this example, we assumed that the rate of change for the budget was constant between each 5-year period. Although the budget might have varied from year to year, analyzing in larger periods of time allows us to see trends within data.

Example 2 Compare Rates of Change

STUDENT COUNCIL **The Jackson High School Student Council budget varies based on the fundraising of the previous year.**

Part A Find the rate of change for 2000–2005 and describe its meaning in the context of the situation.



$\frac{\text{change in budget}}{\text{change in time}} =$

$\frac{1675 - 1350}{2005 - 2000} =$ ______, or ______

This means that the student council's budget increased by $______ over the ______ -year period, with a rate of change of $______ per year.

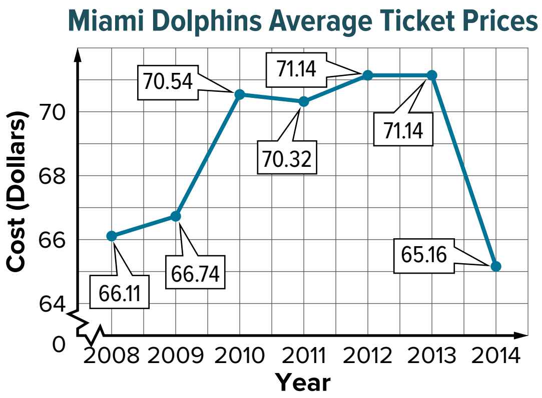
Part B Find the rate of change for 2010–2015 and describe its meaning in the context of the situation.

$\frac{\text{change in budget}}{\text{change in time}} = \frac{1325 - 1550}{2015 - 2010} =$ ______, or ______

This means that the student council's budget was reduced by $______ over the ______-year period, with a rate of change of −$______ per year.

Check

TICKETS The graph shows the average ticket prices for the Miami Dolphins football team.



Part A Find the rate of change in ticket prices between 2009–2010.

______ $\frac{\text{dollars}}{\text{year}}$

Part B The ticket prices have the greatest rate of change between ______.

Part C Between ______ and ______, the rate of change is negative.

Go Online You can complete an Extra Example online.

Example 3 Constant Rate of Change

Determine whether the function is linear. If it is, state the rate of change.

x	y
11	−5
8	−3
5	−1
2	1
−1	3

Find the changes in the x-values and the changes in the y-values.

Notice that the rate of change for each pair of points shown is ____

The rates of change are ________, so the function is ______. The rate of change is ____.

Example 4 Rate of Change

Determine whether the function is linear. If it is, state the rate of change.

x	y
22	−4
29	−1
36	1
43	4
50	6

Find the changes in the x-values and the changes in the y-values.

The rates of change are ____________. Between some pairs of points the rate of change is ____, and between the other pairs it is ____. Therefore, this is

____________________.

Study Tip

Linear Versus Not Linear Remember that the word *linear* means that the graph of the function is a straight line. For the graph of a function to be a line, it has to be increasing or decreasing at a constant rate.

Check

Complete the table so that the function is linear.

x	y
	−2.25
	1
11	
10.5	7.5
10	10.75
9.5	

Go Online You can complete an Extra Example online.

Go Online You can watch a video to see how to find the slope of a nonvertical line.

Think About It! If the point (1, 3) is on a line, what other point could be on the line to make the slope positive? negative? zero? undefined?

Think About It! Can a line that passes through two specific points, such as the origin and (2, 4), have more than one slope? Explain your reasoning.

Think About It! How would lines with slopes of $m = \frac{1}{8}$ and $m = 80$ compare on the same coordinate plane?

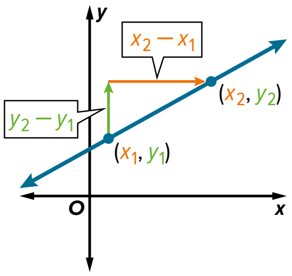
Explore Investigating Slope

Online Activity Use graphing technology to complete an Explore.

INQUIRY How does slope help to describe a line?

Learn Slope of a Line

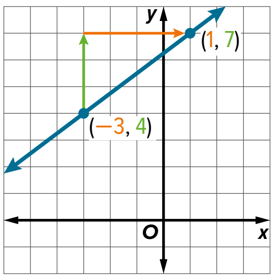
The **slope** of a line is the rate of change in the *y*-coordinates (rise) for the corresponding change in the *x*-coordinates (run) for points on the line.

Key Concept • Slope	
Words	The slope of a nonvertical line is the ratio of the rise to the run.
Symbols	The slope m of a nonvertical line through any two points (x_1, y_1) and (x_2, y_2) can be found as follows. $m = \frac{y_2 - y_1}{x_2 - x_1}$
Example	

The slope of a line can show how a quantity changes over time. When finding the slope of a line that represents a real-world situation, it is often referred to as the *rate of change*.

Example 5 Positive Slope

Find the slope of a line that passes through (−3, 4) and (1, 7).



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - 4}{1 - (-3)}$$

$$= \underline{\hspace{2em}}$$

Check

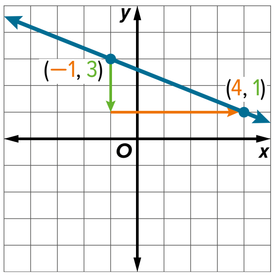
Determine the slope of a line passing through the given points. If the slope is undefined, write *undefined*. Enter your answer as a decimal if necessary.

(−1, 8) and (7, 10) ______

Go Online You can complete an Extra Example online.

Example 6 Negative Slope

Find the slope of a line that passes through (−1, 3) and (4, 1).



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - 3}{4 - (-1)}$$

$$= \_\_\_\_$$

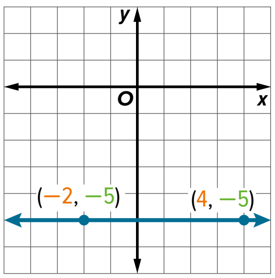
Check

Determine the slope of a line passing through the given points. If the slope is undefined, write *undefined*. Enter your answer as a decimal if necessary.

a. (5, −4) and (0, 1) ____

Example 7 Slopes of Horizontal Lines

Find the slope of a line that passes through (−2, −5) and (4, −5).



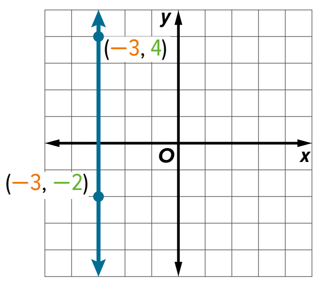
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-5 - (-5)}{4 - (-2)}$$

$= \_\_\_\_$ or $\_\_\_\_$

Example 8 Slopes of Vertical Lines

Find the slope of a line that passes through (−3, 4) and (−3, −2).



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-2 - 4}{-3 - (-3)}$$

$= \_\_\_\_$ or $\_\_\_\_\_\_\_\_$

Go Online You can complete an Extra Example online.

Study Tip

Positive and Negative Slope To know whether a line has a positive or negative slope, read the graph of the line just like you would read a sentence, from left to right. If the line "goes uphill," then the slope is positive. If the line "goes downhill," then the slope is negative.

Talk About It!

Why is the slope for vertical lines always undefined? Justify your argument.

Study Tip

Converting Slope When solving for an unknown coordinate, like the previous example, converting a slope from a decimal or mixed number to an improper fraction might make the problem easier to solve. For example, a slope of $1.\overline{333}$ can be rewritten as $\frac{4}{3}$.

Think About It!

If a crab is walking along the ocean floor 112 meters away from the shoreline to 114 meters away from the shoreline, how far does it descend?

Example 9 Find Coordinates Given the Slope

Find the value of *r* so that the line passing through (−4, 5) and (4, *r*) has a slope of $\frac{3}{4}$.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ — Use the Slope Formula.

$\frac{3}{4} = \frac{r - 5}{4 - (-4)}$ — $(-4, 5) = (x_1, y_1)$ and $(4, r) = (x_2, y_2)$

$\frac{3}{4} = \frac{r - 5}{8}$ — Subtract.

______ = _________ — Multiply each side by 8.

______ = _________ — Simplify.

______ = _________ — Add 5 to each side.

_____ = _____ — Simplify.

Check

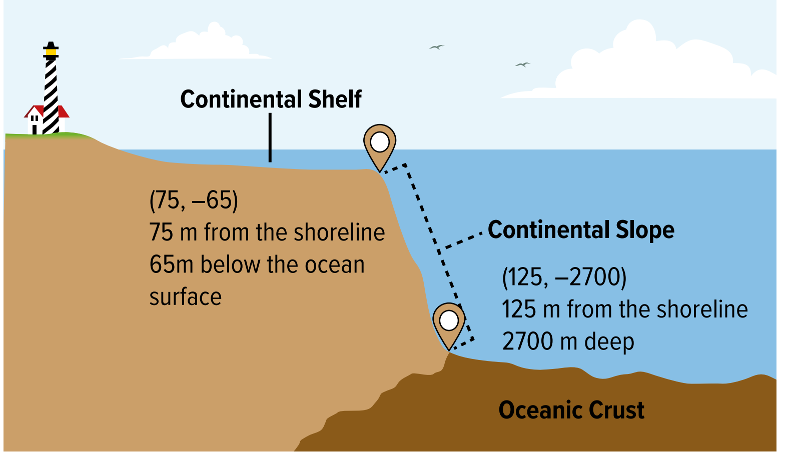
Find the value of *r* so that the line passing through (−3, *r*) and (7, −6) has a slope of $2\frac{2}{5}$.

$r =$ _______

Example 10 Use Slope

OCEANS

What is the slope of the continental slope at Cape Hatteras?



$m = \frac{y_2 - y_1}{x_2 - x_1}$ — Use the Slope Formula

$= \frac{-2700 - (-65)}{125 - 75}$ — $(75, -65) = (x_1, y_1)$ and $(125, -2700) = (x_2, y_2)$

= ________ or ______ — Simplify.

The continental slope at Cape Hatteras has a slope of _______.

Go Online You can complete an Extra Example online.

Name ________________ Period ________ Date ________

Practice

Go Online You can complete your homework online.

Example 1

Find the rate of change of the function by using two points from the table.

1.

x	y
5	2
10	3
15	4
20	5

2.

x	y
1	15
2	9
3	3
4	−3

3. POPULATION DENSITY The table shows the population density for the state of Texas in various years. Find the average annual rate of change in the population density from 2000 to 2009.

Population Density	
Year	**People Per Square Mile**
1930	22.1
1960	36.4
1980	54.3
2000	79.6
2009	96.7

Source: Bureau of the Census, U.S. Dept. of Commerce

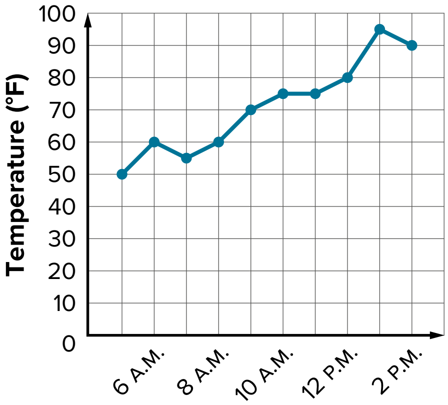
4. BAND In 2012, there were approximately 275 students in the Delaware High School band. In 2018, that number increased to 305. Find the annual rate of change in the number of students in the band.

Example 2

5. TEMPERATURE The graph shows the temperature in a city during different hours of one day.

a. Find the rate of change in temperature between 6 A.M. and 7 A.M. and describe its meaning in the context of the situation.

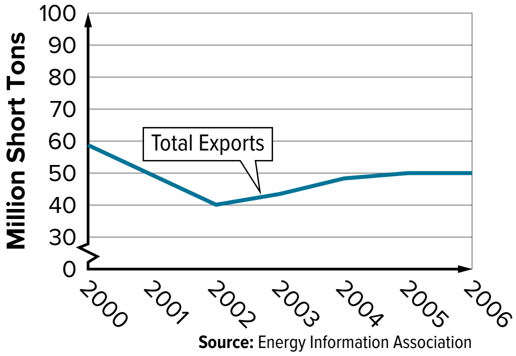
b. Find the rate of change in temperature from 1 P.M. and 2 P.M. and describe its meaning in the context of the situation.



6. COAL EXPORTS The graph shows the annual coal exports from U.S. mines in millions of short tons.

a. Find the rate of change in coal exports between 2000 and 2002 and describe its meaning in the context of the situation.

b. Find the rate of change in coal exports between 2005 and 2006 and describe its meaning in the context of the situation.



Examples 3 and 4

Determine whether the function is linear. If it is, state the rate of change.

7.

x	4	2	0	−2	−4
y	−1	1	3	5	7

8.

x	−7	−5	−3	−1	0
y	11	14	17	20	23

9.

x	−0.2	0	0.2	0.4	0.6
y	0.7	0.4	0.1	0.3	0.6

10.

x	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$
y	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$

Examples 5 through 8

Find the slope of the line that passes through each pair of points.

11. (4, 3), (−1, 6)

12. (8, −2), (1, 1)

13. (2, 2), (−2, −2)

14. (6, −10), (6, 14)

15. (5, −4), (9, −4)

16. (11, 7), (−6, 2)

17. (−3, 5), (3, 6)

18. (−3, 2), (7, 2)

19. (8, 10), (−4, −6)

20. (−12, 15), (18, −13)

21. (−8, 6), (−8, 4)

22. (−8, −15), (−2, 5)

23. (2, 5), (3, 6)

24. (6, 1), (−6, 1)

25. (4, 6), (4, 8)

26. (−5, −8), (−8, 1)

27. (2, 5), (−3, −5)

28. (9, 8), (7, −8)

29. (5, 2), (5, −2)

30. (10, 0), (−2, 4)

31. (17, 18), (18, 17)

32. (−6, −4), (4, 1)

33. (−3, 10), (−3, 7)

34. (2, −1), (−8, −2)

35. (5, −9), (3, −2)

36. (12, 6), (3, −5)

37. (−4, 5), (−8, −5)

Example 9

Find the value of *r* so the line that passes through each pair of points has the given slope.

38. $(12, 10), (-2, r), m = -4$

39. $(r, -5), (3, 13), m = 8$

40. $(3, 5), (-3, r), m = \frac{3}{4}$

41. $(-2, 8), (r, 4), m = -\frac{1}{2}$

42. $(r, 3), (5, 9), m = 2$

43. $(5, 9), (r, -3), m = -4$

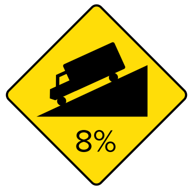
44. $(r, 2), (6, 3), m = \frac{1}{2}$

45. $(r, 4), (7, 1), m = \frac{3}{4}$

Name ________ Period ________ Date ________

Example 10

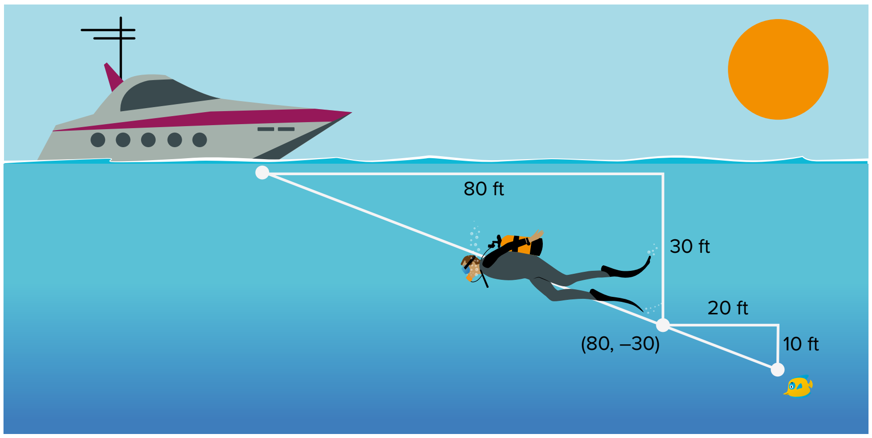
46. ROAD SIGNS Roadway signs such as the one shown are used to warn drivers of an upcoming steep down grade. What is the grade, or slope, of the hill described on the sign?



47. HOME MAINTENANCE Grading the soil around the foundation of a house can reduce interior home damage from water runoff. For every 6 inches in height, the soil should extend 10 feet from the foundation. What is the slope of the soil grade?

48. USE A SOURCE Research the Americans with Disabilities Act (ADA) regulation for the slope of a wheelchair ramp. What is the maximum slope of an ADA regulation ramp? Use the slope to determine the length and height of an ADA regulation ramp.

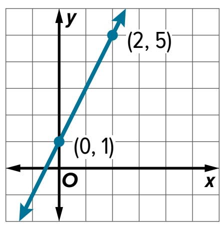
49. DIVERS A boat is located at sea level. A scuba diver is 80 feet along the surface of the water from the boat and 30 feet below the water surface. A fish is 20 feet along the horizontal plane from the scuba diver and 10 feet below the scuba diver. What is the slope between the scuba diver and fish?



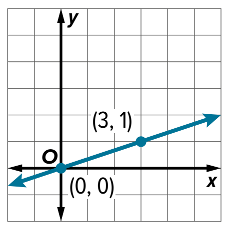
Mixed Exercises

STRUCTURE **Find the slope of the line that passes through each pair of points.**

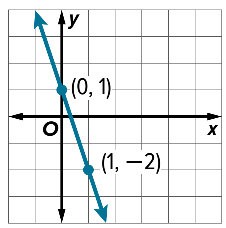
50.



51.



52.



53. (6, −7), (4, −8)

54. (0, 5), (5, 5)

55. (−2, 6), (−5, 9)

56. (5, 8), (−4, 6)

57. (9, 4), (5, −3)

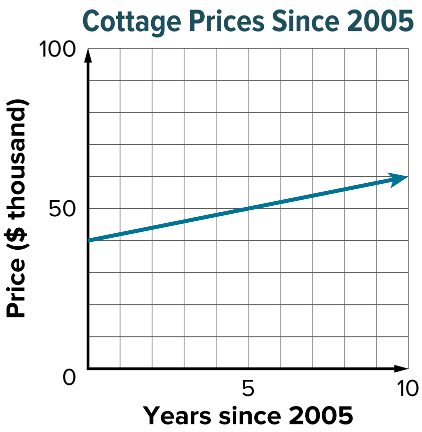
58. (1, 4), (3, −1)

59. **REASONING** Find the value of r that gives the line passing through (3, 2) and (r, −4) a slope that is undefined.

60. **REASONING** Find the value of r that gives the line passing through (−5, 2) and (3, r) a slope of 0.

61. **CREATE** Draw a line on a coordinate plane so that you can determine at least two points on the graph. Describe how you would determine the slope of the graph and justify the slope you found.

62. **ARGUMENTS** The graph shows median prices for small cottages on a lake since 2005. A real estate agent says that since 2005, the rate of change for house prices is $10,000 each year. Do you agree? Use the graph to justify your answer.

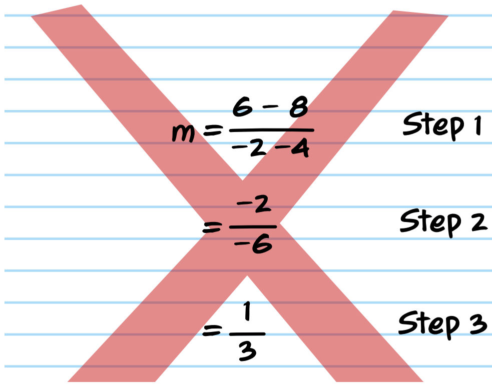


63. **CREATE** Use what you know about rate of change to describe the function represented by the table.

Time (wk)	Height of Plant (in.)
4	9.0
6	13.5
8	18.0

64. **WRITE** Explain how the rate of change and slope are related and how to find the slope of a line.

65. **FIND THE ERROR** Fern is finding the slope of the line that passes through (−2, 8) and (4, 6). Determine in which step she made an error. Explain your reasoning.



66. **PERSEVERE** Find the value of d so that the line that passes through (a, b) and (c, d) has a slope of $\frac{1}{2}$.

67. **ANALYZE** Why is the slope undefined for vertical lines? Explain.

68. **WRITE** Tarak wants to find the value of a so that the line that passes through (10, a) and (−2, 8) has a slope of $\frac{1}{4}$. Explain how Tarak can find the value of a.

Slope-Intercept Form

Learn Writing Linear Equations in Slope-Intercept Form

An equation of the form $y = mx + b$, where m is the slope and b is the y-intercept, is written in slope-intercept form. When an equation is not in slope-intercept form, it might be easier to rewrite it before graphing. An equation can be rewritten in slope-intercept form by using the properties of equality.

Key Concept • Slope-Intercept Form	
Words	The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept.
Example	$y = mx + b$ $y = -2x + 7$

Today's Goals

- Rewrite linear equations in slope-intercept form.
- Graph and interpret linear functions.

Today's Vocabulary

parameter

constant function

Think About It!

Explain why the y-intercept of a linear equation can be written as $(0, b)$, where b is the y-intercept.

Example 1 Write Linear Equations in Slope-Intercept Form

Write an equation in slope-intercept form for the line with a slope of $\frac{4}{7}$ and a y-intercept of 5.

Write the equation in slope-intercept form.

$y = mx + b$	Slope-intercept form.
$y = (\_\_\_\_)x + 5$	$m = \frac{4}{7}$, $b = 5$
$y = \_\_\_\_\_\_\_\_$	Simplify.

Check

Write an equation for the line with a slope of −5 and a y-intercept of 12. __________

Go Online You can complete an Extra Example online.

Your Notes

Think About It!

Can $x = 5$ be rewritten in slope-intercept form? Justify your argument.

Example 2 Rewrite Linear Equations in Slope-Intercept Form

Write $-22x + 8y = 4$ in slope-intercept form.

$-22x + 8y = 4$	Original equation
______________	Add $22x$ to each side.
______________	Simplify.
______________	Divide each side by 8.
______________	Simplify.

Check

What is the slope intercept form of $-16x - 4y = -56$? ______

Think About It!

When $x = 2$, describe the meaning of the equation in the context of the situation.

Example 3 Write Linear Equations

JOBS **The number of job openings in the United States during a recent year increased by an average of 0.06 million per month since May. In May, there were about 4.61 million job openings in the United States. Write an equation in slope-intercept form to represent the number of job openings in the United States in the months since May.**

Use the given information to write an equation in slope-intercept form.

- You are given that there were ______ million job openings in May.
- Let ______ and ______ ______.
- Because the number of job openings is 4.61 million when ______, ______, and because the number of job openings has increased by ______ million each month, ______.
- So, the equation ______ represents the number of job openings in the United States since May.

Check

SOCIAL MEDIA In the first quarter of 2012, there were 183 million users of a popular social media site in North America. The number of users increased by an average of 9 million per year since 2012. Write an equation that represents the number of users in millions of the social media site in North America after 2012.

______

Go Online You can complete an Extra Example online.

Explore Graphing Linear Functions by Using the Slope-Intercept Form

Online Activity Use graphing technology to complete an Explore.

INQUIRY How do the quantities m and b affect the graph of a linear function in slope-intercept form?

Learn Graphing Linear Functions in Slope-Intercept Form

The slope-intercept form of a linear equation is $y = mx + b$, where m is the slope and b is the y-intercept. The variables m and b are called **parameters** of the equation because changing either value changes the graph of the function.

A **constant function** is a linear function of the form $y = b$. Constant functions where $b \neq 0$ do not cross the x-axis. The graphs of constant functions have a slope of 0. The domain of a constant function is all real numbers, and the range is b.

Example 4 Graph Linear Functions in Slope-Intercept Form

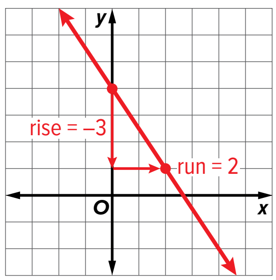
Graph a linear function with a slope of $-\frac{3}{2}$ and a y-intercept of 4.

Write the equation in slope-intercept form and graph the function.

$y = mx + b$

$y = (\_\_\_\_)\,x\ \_\_\_\_$

$y = \_\_\_\_\_\_\_\_$



Study Tip

Negative Slope When counting rise and run, a negative sign may be associated with the value in the numerator or denominator. In this case, we associated the negative sign with the numerator. If we had associated it with the denominator, we would have moved up 3 and left 2 to the point (−2, 7). Notice that this point is also on the line. The resulting line will be the same whether the negative sign is associated with the numerator or denominator.

Think About It!

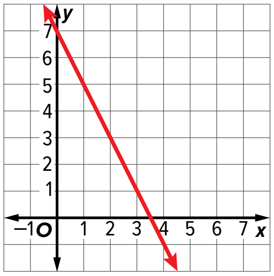
Use the slope to find another point on the graph. Explain how you found the point.

Talk About It!

Why is it useful to write an equation in slope-intercept form before graphing the function?

Check

Graph a linear function with a slope of -2 and a y-intercept of 7.



Example 5 Graph Linear Functions

Graph $12x - 3y = 18$.

Rewrite the equation in slope-intercept form.

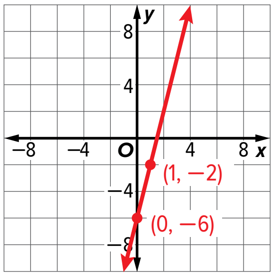
$12x - 3y = 18$	Original equation
__________	Subtract $12x$ from each side.
__________	Simplify.
__________	Divide each side by -3.
__________	Simplify.

Graph the function.

Plot the y-intercept $(0, -6)$.

The slope is $\frac{rise}{run} = 4$. From $(0, -6)$, move up 4 units and right 1 unit. Plot the point $(1, -2)$.

Draw a line through the points $(0, -6)$ and $(1, -2)$.



Go Online You can complete an Extra Example online.

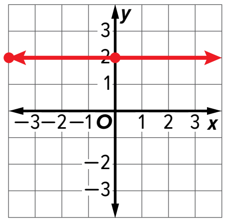
Example 6 Graph Constant Functions

Graph $y = 2$.

Step 1 Plot (0, 2).

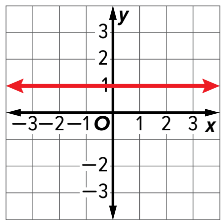
Step 2 The slope of $y = 2$ is 0.

Step 3 Draw a line through all the points that have a y-coordinate of 2.



Check

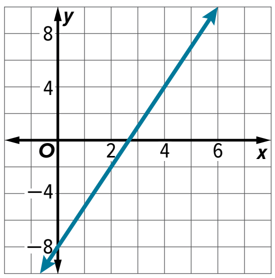
Graph $y = 1$.



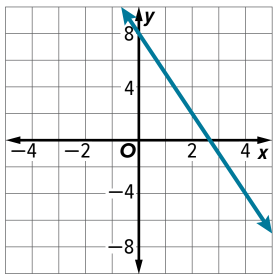
Match each graph with its function.

____ $y = 8$	____ $3x + 7y = -28$	____ $y = \frac{3}{7}x - 4$
____ $y = -4$	____ $y = -3x + 8$	____ $3x - y = 8$

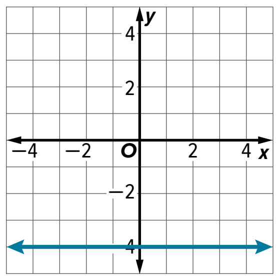
A.



B.



C.



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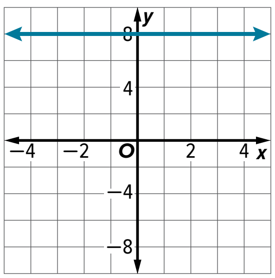
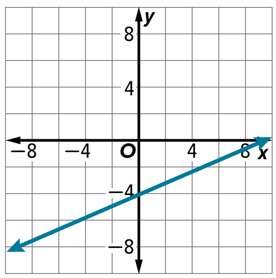
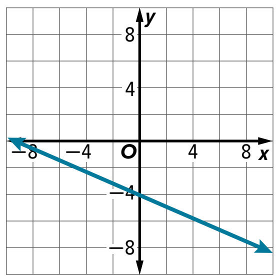


E.



F.



Go Online You can complete an Extra Example online.

Think About It!

How do you know that the graph of $y = 2$ has a slope of 0?

Watch Out!

Slope A line with zero slope is not the same as a line with no slope. A line with zero slope is horizontal, and a line with no slope is vertical.

Apply Example 7 Use Graphs of Linear Functions

SHOPPING **The number of online shoppers in the United States can be modeled by the equation $-5.88x + y = 172.3$, where y represents the number of millions of online shoppers in the United States x years after 2010. Estimate the number of people shopping online in 2020.**

1. What is the task?

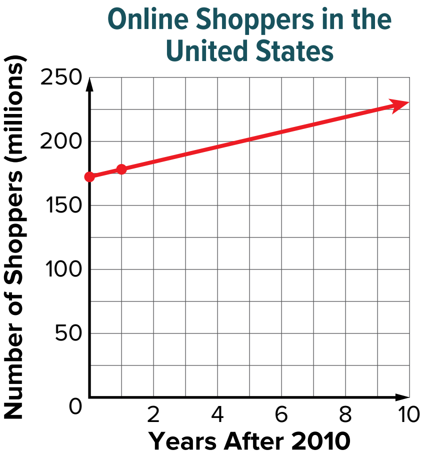
Describe the task in your own words. Then list any questions that you may have. How can you find answers to your questions?

2. How will you approach the task? What have you learned that you can use to help you complete the task?

3. What is your solution?

Use your strategy to solve the problem. Graph the function.

In 2020, there will be approximately ____________ online shoppers in the United States.



4. How can you know that your solution is reasonable?

Write About It! Write an argument that can be used to defend your solution.

Think About It!
Estimate the year when the number of online shoppers in the United States will reach 271 million.

Go Online
to learn about intervals in linear growth patterns in Expand 4-3.

Go Online You can complete an Extra Example online.

Name ______________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

Example 1

Write an equation of a line in slope-intercept form with the given slope and *y*-intercept.

1. slope: 5, y-intercept: -3

2. slope: -2, y-intercept: 7

3. slope: -6, y-intercept: -2

4. slope: 7, y-intercept: 1

5. slope: 3, y-intercept: 2

6. slope: -4, y-intercept: -9

7. slope: 1, y-intercept: -12

8. slope: 0, y-intercept: 8

Example 2

Write each equation in slope-intercept form.

9. $-10x + 2y = 12$

10. $4y + 12x = 16$

11. $-5x + 15y = -30$

12. $6x - 3y = -18$

13. $-2x - 8y = 24$

14. $-4x - 10y = -7$

Example 3

15. SAVINGS Wade's grandmother gave him \$100 for his birthday. Wade wants to save his money to buy a portable game console. Each month, he adds \$25 to his savings. Write an equation in slope-intercept form to represent Wade's savings y after x months.

16. FITNESS CLASSES Toshelle wants to take strength training classes at the community center. She has to pay a one-time enrollment fee of \$25 to join the community center, and then \$45 for each class she wants to take. Write an equation in slope-intercept form for the cost of taking x classes.

17. EARNINGS Macario works part time at a clothing store in the mall. He is paid \$9 per hour plus 12% commission on the items he sells in the store. Write an equation in slope-intercept form to represent Macario's hourly wage y.

18. ENERGY From 2002 to 2005, U.S. consumption of renewable energy increased an average of 0.17 quadrillion BTUs per year. About 6.07 quadrillion BTUs of renewable power were produced in the year 2002. Write an equation in slope-intercept form to find the amount of renewable power P in quadrillion BTUs produced in year y between 2002 and 2005.

Example 4

Graph a linear function with the given slope and *y*-intercept.

19. slope: 5, y-intercept: 8

20. slope: 3, y-intercept: 10

21. slope: -4, y-intercept: 6

22. slope: -2, y-intercept: 8

Examples 5 and 6

Graph each function.

23. $5x + 2y = 8$

24. $4x + 9y = 27$

25. $y = 7$

26. $y = -\frac{2}{3}$

27. $21 = 7y$

28. $3y - 6 = 2x$

Example 7

29. **STREAMING** An online company charges \$13 per month for the basic plan. They offer premium channels for an additional \$8 per month.

a. Write an equation in slope-intercept form for the total cost c of the basic plan with p premium channels in one month.

b. Graph the function.

c. What would the monthly cost be for a basic plan plus 3 premium channels?

30. **CAR CARE** Suppose regular gasoline costs \$2.76 per gallon. You can purchase a car wash at the gas station for \$3.

a. Write an equation in slope-intercept form for the total cost y of purchasing a car wash and x gallons of gasoline.

b. Graph the function.

c. Find the cost of purchasing a car wash and 8 gallons of gasoline.

Mixed Exercises

Write an equation of a line in slope-intercept form with the given slope and *y*-intercept.

31. slope: $\frac{1}{2}$, y-intercept: -3

32. slope: $\frac{2}{3}$, y-intercept: -5

Graph a function with the given slope and *y*-intercept.

33. slope: 3, y-intercept: -4

34. slope: 4, y-intercept: -6

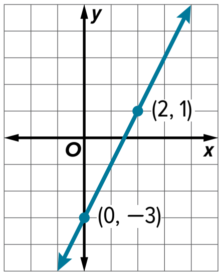
Graph each function.

35. $-3x + y = 6$

36. $-5x + y = 1$

Write an equation in slope-intercept form for each graph shown.

37.



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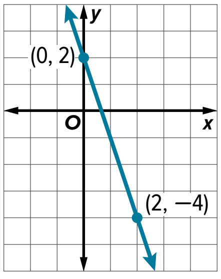

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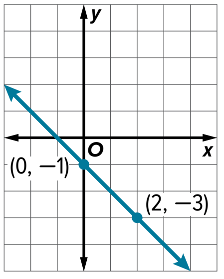


40. **MOVIES** MovieMania, an online movie rental Web site charges a one-time fee of \$6.85 and \$2.99 per movie rental. Let *m* represent the number of movies you watch and let *C* represent the total cost to watch the movies.

a. Write an equation that relates the total cost to the number of movies you watch from MovieMania.

b. Graph the function.

c. Explain how to use the graph to estimate the cost of watching 13 movies at MovieMania.

d. SuperFlix has no sign-up fee, just a flat rate per movie. If renting 13 movies at MovieMania costs the same as renting 9 movies at SuperFlix, what does SuperFlix charge per movie? Explain your reasoning.

e. Write an equation that relates the total cost to the number of movies you watch from SuperFlix. Round to the nearest whole number.

41. **FACTORY** A factory uses a heater in part of its manufacturing process. The product cannot be heated too quickly, nor can it be cooled too quickly after the heating portion of the process is complete.

a. The heater is digitally controlled to raise the temperature inside the chamber by 10°F each minute until it reaches the set temperature. Write an equation to represent the temperature, *T*, inside the chamber after *x* minutes if the starting temperature is 80°F.

b. Graph the function.

c. The heating process takes 22 minutes. Use your graph to find the temperature in the chamber at this point.

d. After the heater reaches the temperature determined in **part c**, the temperature is kept constant for 20 minutes before cooling begins. Fans within the heater control the cooling so that the temperature inside the chamber decreases by 5°F each minute. Write an equation to represent the temperature, T, inside the chamber x minutes after the cooling begins.

42. **SAVINGS** When Santo was born, his uncle started saving money to help pay for a car when Santo became a teenager. Santo's uncle initially saved \$2000. Each year, his uncle saved an additional \$200.

a. Write an equation that represents the amount, in dollars, Santo's uncle saved *y* after *x* years.

b. Graph the function.

c. Santo starts shopping for a car when he turns 16. The car he wants to buy costs \$6000. Does he have enough money in the account to buy the car? Explain.

43. **STRUCTURE** Jazmin is participating in a 25.5-kilometer charity walk. She walks at a rate of 4.25 km per hour. Jazmin walks at the same pace for the entire event.

a. Write an equation in slope-intercept form for the remaining distance y in kilometers of walking for x hours.

b. Graph the function.

c. What do the x- and y-intercepts represent in this situation?

d. After Jazmin has walked 17 kilometers, how much longer will it take her to complete the walk? Explain how you can use your graph to answer the question.

For Exercises 44 and 45, refer to the equation $y = -\frac{4}{5}x + \frac{2}{5}$ where $-2 \leq x \leq 5$.

44. **ANALYZE** Complete the table to help you graph the function $y = -\frac{4}{5}x + \frac{2}{5}$ over the interval. Identify any values of x where maximum or minimum values of y occur.

x	$-\frac{4}{5}x + \frac{2}{5}$	y	(x, y)
-2			
0			
5			

45. **WRITE** A student says you can find the solution to $-\frac{4}{5}x + \frac{2}{5} = 0$ using the graph. Do you agree? Explain your reasoning. Include the solution to the equation in your response.

46. **PERSEVERE** Consider three points that lie on the same line, (3, 7), (−6, 1), and (9, p). Find the value of p and explain your reasoning.

47. **CREATE** Linear equations are useful in predicting future events. Create a linear equation that models a real-world situation. Make a prediction from your equation.

Transformations of Linear Functions

Explore Transforming Linear Functions

Online Activity Use graphing technology to complete an Explore.

INQUIRY How does performing an operation on a linear function change its graph?

Learn Translations of Linear Functions

A **family of graphs** includes graphs and equations of graphs that have at least one characteristic in common. The **parent function** is the simplest of the functions in a family.

The family of linear functions includes all lines, with the parent function $f(x) = x$, also called the **identity function**. A **transformation** moves the graph on the coordinate plane, which can create new linear functions.

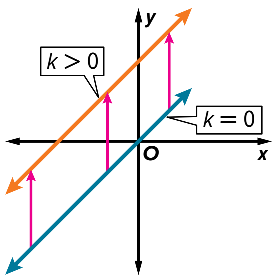
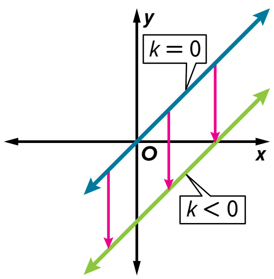
One type of transformation is a translation. A **translation** is a transformation in which a figure is slid from one position to another without being turned. A linear function can be slid up, down, left, right, or in two directions.

Vertical Translations

When a constant k is added to a linear function $f(x)$, the result is a vertical translation. The y-intercept of $f(x)$ is translated up or down.

Key Concept • Vertical Translations of Linear Functions

The graph of $g(x) = x + k$ is the graph of $f(x) = x$ translated vertically.

If $k > 0$, the graph of $f(x)$ is translated k units up.	If $k < 0$, the graph of $f(x)$ is translated $\lvert k \rvert$ units down.
	
Every point on the graph of $f(x)$ moves k units up.	Every point on the graph of $f(x)$ moves $\lvert k \rvert$ units down.

Today's Goals
- Apply translations to linear functions.
- Apply dilations to linear functions.
- Apply reflections to linear functions.

Today's Vocabulary

family of graphs
parent function
identity function
transformation
translation
dilation
reflection

Study Tip

Slope When translating a linear function, the graph of the function moves from one location to another, but the slope remains the same.

Watch Out!

Translations of $f(x)$ When a translation is the only transformation performed on the identity function, adding a constant before or after evaluating the function has the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Your Notes

Think About It!

What do you notice about the y-intercepts of vertically translated functions compared to the y-intercept of the parent function?

Example 1 Vertical Translations of Linear Functions

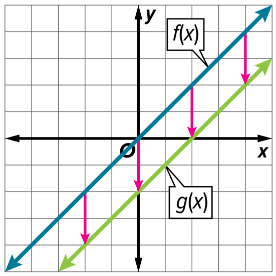
Describe the translation in $g(x) = x - 2$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

x	$f(x)$	$f(x) - 2$	$(x, g(x))$
-2	-2	-4	$(-2, -4)$
0	0	-2	$(0, -2)$
1	1	-1	$(1, -1)$

Because $f(x) = x$, $g(x) = f(x) + k$ where ________.

$g(x) = x - 2 \rightarrow$ ____________



The constant k is not grouped with x, so k affects the ________, or ________.
The value of k is less than 0, so the graph of $f(x) = x$ is translated ____ units down, or 2 units down.

$g(x) = x - 2$ is the translation of the graph of the parent function 2 units down.

Check

Describe the translation in $g(x) = x - 1$ as it relates to the graph of the parent function.

The graph of $g(x) = x - 1$ is a translation of the graph of the parent function 1 unit ________.

Go Online You can watch a video to see how to describe translations of functions.

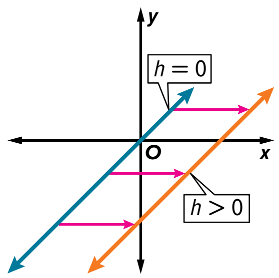
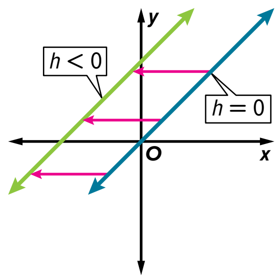
Go Online You may want to complete the Concept Check to check your understanding.

Horizontal Translations

When a constant h is subtracted from the x-value before the function $f(x)$ is performed, the result is a horizontal translation. The x-intercept of $f(x)$ is translated right or left.

Key Concept • Horizontal Translations of Linear Functions

The graph of $g(x) = (x - h)$ is the graph of $f(x) = x$ translated horizontally.

If $h > 0$, the graph of $f(x)$ is translated h units right.	If $h < 0$, the graph of $f(x)$ is translated $\lvert h \rvert$ units left.
	
Every point on the graph of $f(x)$ moves h units right.	Every point on the graph of $f(x)$ moves $\lvert h \rvert$ units left.

Go Online You can complete an Extra Example online.

Example 2 Horizontal Translations of Linear Functions

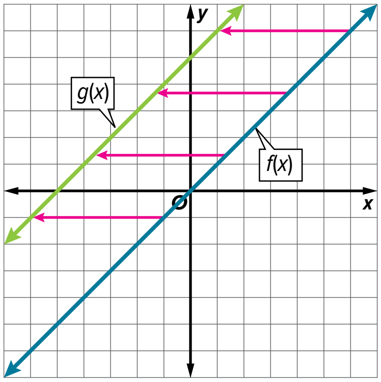
Describe the translation in $g(x) = (x + 5)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

Because $f(x) = x$, ______________ where $h = -5$.

$g(x) = (x + 5) \rightarrow$ ______________

x	$x + 5$	$f(x + 5)$	$(x, g(x))$
−2	3	3	(−2, 3)
0	5	5	(0, 5)
1	6	6	(1, 6)



The constant h is grouped with x, so k affects the __________, or __________. The value of h is less than 0, so the graph of $f(x) = x$ is translated _____ units left, or 5 units left.

$g(x) = (x + 5)$ is the translation of the graph of the parent function 5 units left.

Check

Describe the translation in $g(x) = (x + 12)$ as it relates to the graph of the parent function.

The graph of $g(x) = (x + 12)$ is a translation of the graph of the parent function 12 units __________.

Example 3 Multiple Translations of Linear Functions

Describe the translation in $g(x) = (x - 6) + 3$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

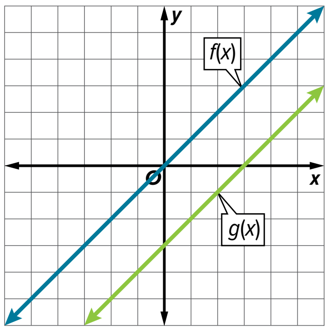
Because $f(x) = x$,

____

where ________ and ________.

$g(x) = (x - 6) + 3 \rightarrow$ ______________

x	$x - 6$	$f(x - 6)$	$f(x - 6) + 3$	$(x, g(x))$
−2	−8	−8	−5	(−2, −5)
0	−6	−6	−3	(0, −3)
1	−5	−5	−2	(1, −2)



The value of h is grouped with x and is greater than 0, so the graph of $f(x) = x$ is translated ______________.

The value of k is not grouped with x and is greater than 0, so the graph of $f(x) = x$ is translated ______________.

$g(x) = (x - 6) + 3$ is the translation of the graph of the parent function 6 units right and 3 units up.

Go Online You can complete an Extra Example online.

Think About It!

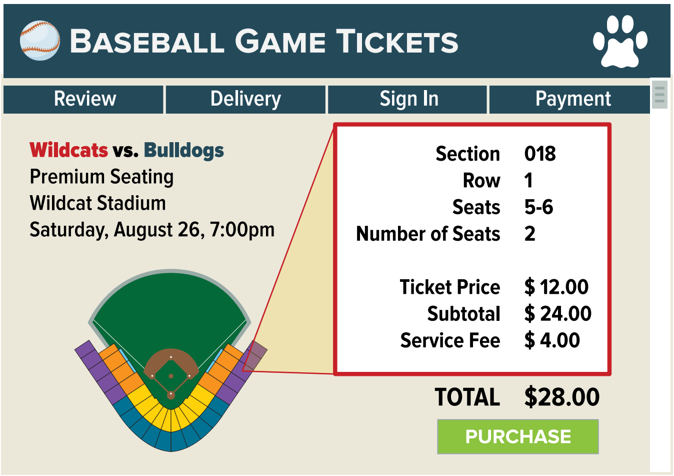
What do you notice about the x-intercepts of horizontally translated functions compared to the x-intercept of the parent function?

Think About It!

Eleni described the graph of $g(x) = (x - 6) + 3$ as the graph of the parent function translated down 3 units. Is she correct? Explain your reasoning.

Example 4 Translations of Linear Functions

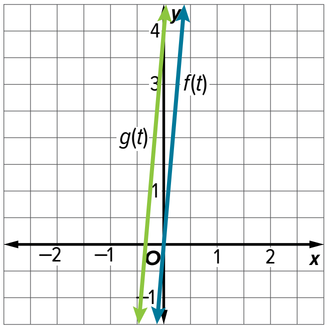
TICKETS A Web site sells tickets to concerts and sporting events. The total price of the tickets to a certain game can be modeled by $f(t) = 12t$, where t represents the number of tickets purchased. The Web site then charges a standard service fee of $4 per order. The total price of an order can be modeled by $g(t) = 12t + 4$. Describe the translation of $g(t)$ as it relates to $f(t)$.



Complete the steps to describe the translation of $g(t)$ as it relates to $f(t)$.
Because $f(t) = 12t$, $g(t) = f(t) + k$, where $k = 4$. $g(t) = 12t + 4 \rightarrow f(t) +$ ____

The constant k is added to $f(t)$ after the total price of the tickets has been evaluated and is greater than 0, so the graph will be shifted 4 units up. $g(t) = 12t + 4$ is the translation of the graph of $f(t)$ ____ units ____.

Graph the parent function and the translated function.



Check

RETAIL Jerome is buying paint for a mural. The total cost of the paint can be modeled by the function $f(p) = 6.99p$. He has a coupon for $5.95 off his purchase at the art supply store, so the final cost of his purchase can be modeled by $g(p) = 6.99p - 5.95$. Describe the translation in $g(p)$ as it relates to $f(p)$.

____

Go Online You can complete an Extra Example online.

Learn Dilations of Linear Functions

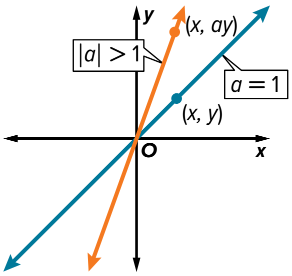
A **dilation** stretches or compresses the graph of a function.

When a linear function $f(x)$ is multiplied by a positive constant a, the result $a \cdot f(x)$ is a vertical dilation.

Key Concept • Vertical Dilations of Linear Functions

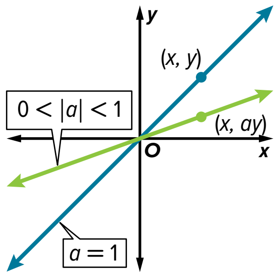
The graph of $g(x) = ax$ is the graph of $f(x) = x$ stretched or compressed vertically.

If $|a| > 1$, the graph of $f(x)$ is stretched vertically away from the x-axis.



The slope of the graph of $a \cdot f(x)$ is steeper than that of the graph of $f(x)$.

If $0 < |a| < 1$, the graph of $f(x)$ is compressed vertically toward the x-axis.



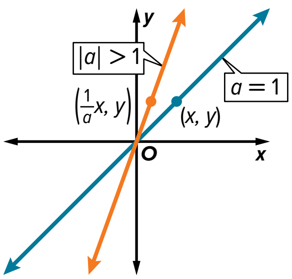
The slope of the graph of $a \cdot f(x)$ is less steep than that of the graph of $f(x)$.

When x is multiplied by a positive constant a before a linear function $f(a \cdot x)$ is evaluated, the result is a horizontal dilation.

Key Concept • Horizontal Dilations of Linear Functions

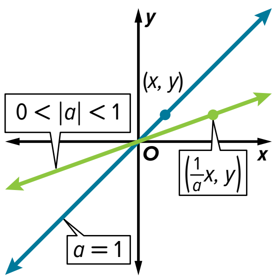
The graph of $g(x) = (a \cdot x)$ is the graph of $f(x) = x$ stretched or compressed horizontally.

If $|a| > 1$, the graph of $f(x)$ is compressed horizontally toward the y-axis.



The slope of the graph of $f(a \cdot x)$ is steeper than that of the graph of $f(x)$.

If $0 < |a| < 1$, the graph of $f(x)$ is stretched horizontally away from the y-axis.



The slope of the graph of $f(a \cdot x)$ is less steep than that of the graph of $f(x)$.

Go Online You can complete an Extra Example online.

Watch Out!

Dilations of $f(x) = x$
When a dilation is the only transformation performed on the identity function, multiplying by a constant before or after evaluating the function has the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Go Online
You can watch a video to see how to describe dilations of functions.

Think About It!
What do you notice about the slope of the vertical dilation $g(x)$ compared to the slope of $f(x)$?

How does this relate to the constant a in the vertical dilation?

Example 5 Vertical Dilations of Linear Functions

Describe the dilation in $g(x) = 2(x)$ as it relates to the graph of the parent function.

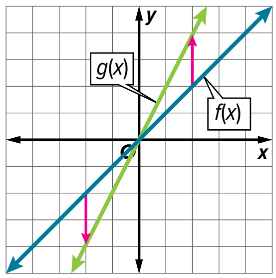
Graph the parent graph for linear functions.

x	$f(x)$	$2f(x)$	$(x, g(x))$
−2	−2	−4	(−2, −4)
0	0	0	(0, 0)
1	1	2	(1, 2)

Since $f(x) = x$, ________________
where ________.

$g(x) = 2(x) \rightarrow$ ________________

The positive constant a is not grouped with x, and $|a|$ is greater than 1, so the graph of $f(x) = x$ is ____________________ by a factor of a, or _____.



$g(x) = 2(x)$ is a vertical stretch of the graph of the parent function. The slope of the graph of $g(x)$ is steeper than that of $f(x)$.

Check

Describe the transformation in $g(x) = 6(x)$ as it relates to the graph of the parent function.

The graph of $g(x) = 6(x)$ is a ____________________ of the graph of the parent function.

The slope of the graph $g(x)$ is __________ than that of the parent function.

Think About It!
What do you notice about the slope of the horizontal dilation $g(x)$ compared to the slope of $f(x)$?

How does this relate to the constant a in the horizontal dilation?

Example 6 Horizontal Dilations of Linear Functions

Describe the dilation in $g(x) = \left(\frac{1}{4}x\right)$ as it relates to the graph of the parent function.

Graph the parent graph for linear functions.

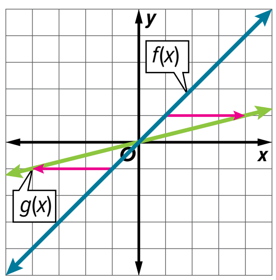
x	$\frac{1}{4}x$	$f\left(\frac{1}{4}x\right)$	$(x, g(x))$
−4	−1	−1	(−4, −1)
0	0	0	(0, 0)
4	1	1	(4, 1)

Since $f(x) = x$, ____________________

where ________.

$g(x) = \left(\frac{1}{4}x\right) \rightarrow$ ____________

The positive constant a is grouped with x, and $|a|$ is between 0 and 1, so the graph of $f(x) = x$ is ______________________ by a factor of $\frac{1}{a}$, or 4.



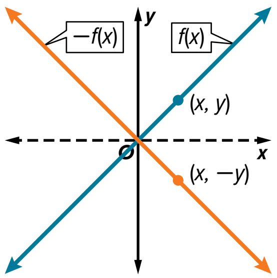
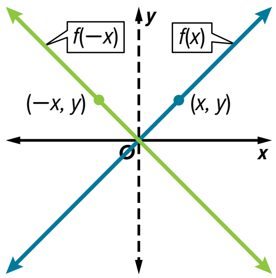
$g(x) = \left(\frac{1}{4}x\right)$ is a horizontal stretch of the graph of the parent function. The slope of the graph of $g(x)$ is less steep than that of $f(x)$.

Go Online You can complete an Extra Example online.

Learn Reflections of Linear Functions

A **reflection** is a transformation in which a figure, line, or curve, is flipped across a line. When a linear function $f(x)$ is multiplied by -1 before or after the function has been evaluated, the result is a reflection across the x- or y-axis. Every x- or y-coordinate of $f(x)$ is multiplied by -1.

Key Concept • Reflections of Linear Functions

The graph of $-f(x)$ is the reflection of the graph of $f(x) = x$ across the x-axis.	The graph of $f(-x)$ is the reflection of the graph of $f(x) = x$ across the y-axis.
	
Every y-coordinate of $-f(x)$ is the corresponding y-coordinate of $f(x)$ multiplied by -1.	Every x-coordinate of $f(-x)$ is the corresponding x-coordinate of $f(x)$ multiplied by -1.

Example 7 Reflections of Linear Functions Across the x-Axis

Describe how the graph of $g(x) = -\frac{1}{2}(x)$ is related to the graph of the parent function.

Graph the parent graph for linear functions.

x	$f(x)$	$-\frac{1}{2}f(x)$	$(x, g(x))$
-2	-2	1	$(-2, 1)$
0	0	0	$(0, 0)$
4	4	-2	$(4, -2)$

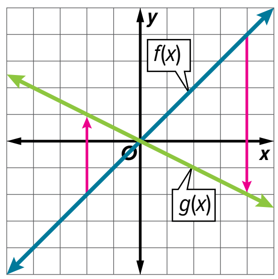
Since $f(x) = x$, ____________

where ________.

$g(x) = -\frac{1}{2}(x) \rightarrow$ ____________

The constant a is not grouped with x, and $|a|$ is less than 1, so the graph of $f(x) = x$ is

____________.



The negative is not grouped with x, so the graph is also reflected across the ________.

The graph of $g(x) = -\frac{1}{2}(x)$ is the graph of the parent function vertically compressed and reflected across the x-axis.

Go Online You can complete an Extra Example online.

Go Online
You can watch a video to see how to describe reflections of functions.

Watch Out!

Reflections of $f(x) = x$
When a reflection is the only transformation performed on the identity function, multiplying by -1 before or after evaluating the function appears to have the same effect on the graph. However, when more than one type of transformation is applied, this will not be the case.

Talk About It!
In the example, the slope of $g(x)$ is negative. Will this always be the case when multiplying a linear function by -1? Justify your argument.

Check

How can you tell whether multiplying −1 by the parent function will result in a reflection across the *x*-axis? ____

A. If the constant is not grouped with *x*, the result will be a reflection across the *x*-axis.

B. If the constant is grouped with *x*, the result will be a reflection across the *x*-axis.

C. If the constant is greater than 0, the result will be a reflection across the *x*-axis.

D. If the constant is less than 0, the result will be a reflection across the *x*-axis.

Example 8 Reflections of Linear Functions Across the *y*-Axis

Describe how the graph of $g(x) = (-3x)$ is related to the graph of the parent function.

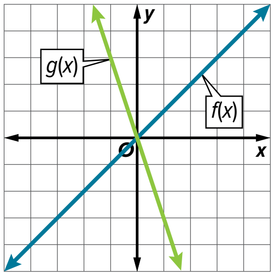
Graph the parent graph for linear functions.

Since $f(x) = x$, ____________ where ______.

$g(x) = -3x \rightarrow$ ____________

x	$-3x$	$f(-3x)$	$(x, g(x))$
−1	3	3	(−1, 3)
0	0	0	(0, 0)
1	−3	−3	(1, −3)

The constant *a* is grouped with *x*, and $|a|$ is greater than 1, so the graph of $f(x) = x$ is ____________.



The negative is grouped with *x*, so the graph is also reflected across the ________.

The graph of $g(x) = (-3x)$ is the graph of the parent function horizontally compressed and reflected across the *y*-axis.

Check

Describe how the graph of $g(x) = (-10x)$ is related to the graph of the parent function.

The graph of $g(x) = (-10x)$ is the graph of the parent function compressed horizontally and reflected across the ________.

Go Online You can watch a video to see how to graph transformations of a linear function using a graphing calculator.

Go Online You can complete an Extra Example online.

Name ____________________ Period ________ Date ________

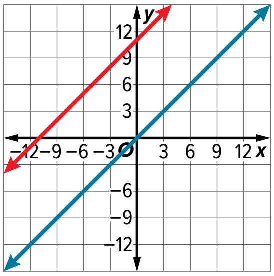
Practice

Go Online You can complete your homework online.

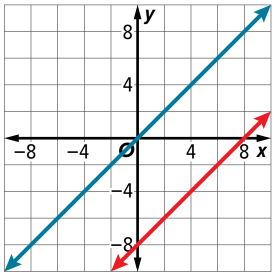
Examples 1 through 3

Describe the translation in each function as it relates to the graph of the parent function.

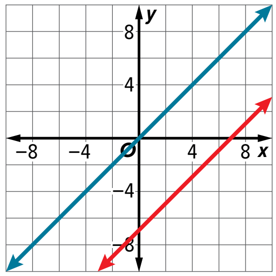
1. $g(x) = x + 11$



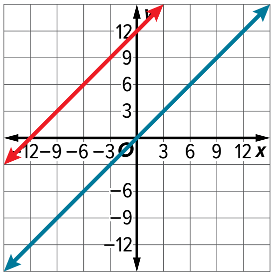
2. $g(x) = x - 8$



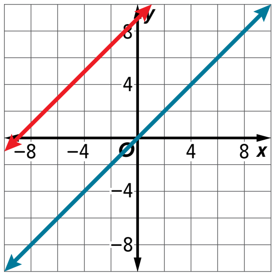
3. $g(x) = (x - 7)$



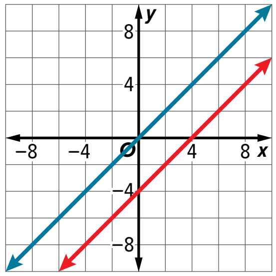
4. $g(x) = (x + 12)$



5. $g(x) = (x + 10) - 1$



6. $g(x) = (x - 9) + 5$



Example 4

7. BOWLING The cost for Nobu to go bowling is \$4 per game plus an additional flat fee of \$3.50 for the rental of bowling shoes. The cost can be modeled by the function $f(x) = 4x + 3.5$, where x represents the number of games bowled. Describe the graph of $g(x)$ as it relates to $f(x)$ if Nobu does not rent bowling shoes.

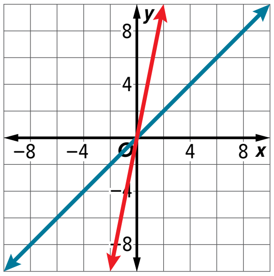
8. SAVINGS Natalie has \$250 in her savings account, into which she deposits \$10 of her allowance each week. The balance of her savings account can be modeled by the function $f(w) = 250 + 10w$, where w represents the number of weeks. Write a function $g(w)$ to represent the balance of Natalie's savings account if she withdraws \$40 to purchase a new pair of shoes. Describe the translation of $f(w)$ that results in $g(w)$.

9. BOAT RENTAL The cost to rent a paddle boat at the county park is \$8 per hour plus a nonrefundable deposit of \$10. The cost can be modeled by the function $f(h) = 8h + 10$, where h represents the number of hours the boat is rented. Describe the graph of $g(h)$ as it relates to $f(h)$ if the nonrefundable deposit increases to \$15.

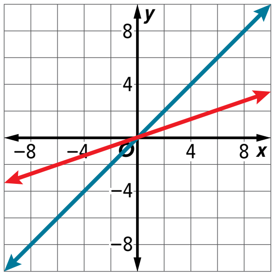
Examples 5 and 6

Describe the dilation in each function as it relates to the graph of the parent function.

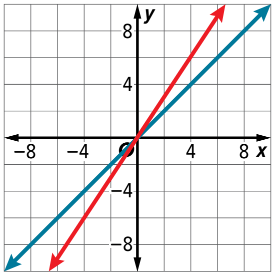
10. $g(x) = 5(x)$



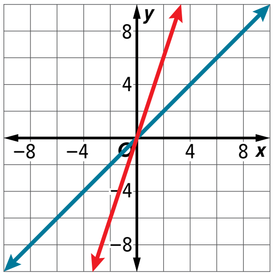
11. $g(x) = \frac{1}{3}(x)$



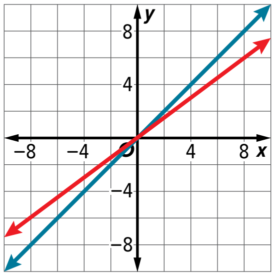
12. $g(x) = 1.5(x)$



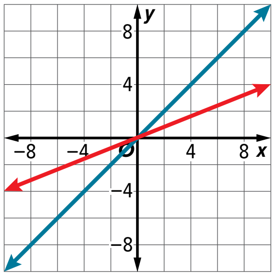
13. $g(x) = (3x)$



14. $g(x) = \left(\frac{3}{4}x\right)$



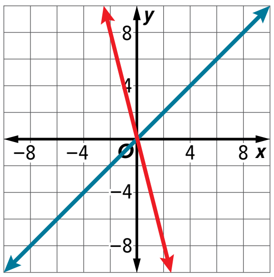
15. $g(x) = (0.4x)$



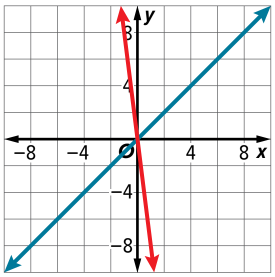
Example 7

Describe how the graph of each function is related to the graph of the parent function.

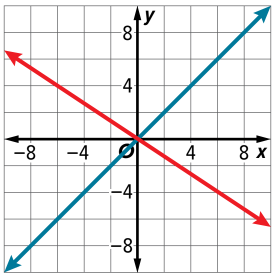
16. $g(x) = -4(x)$



17. $g(x) = -8(x)$



18. $g(x) = -\frac{2}{3}(x)$

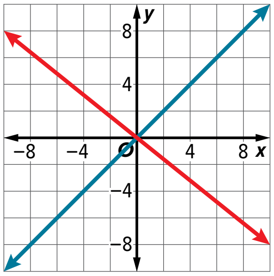


Name ____________________ Period __________ Date __________

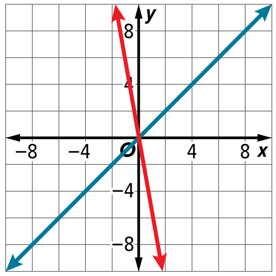
Example 8

Describe how the graph of each function is related to the graph of the parent function.

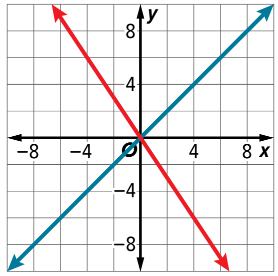
19. $g(x) = \left(-\frac{4}{5}x\right)$



20. $g(x) = (-6x)$



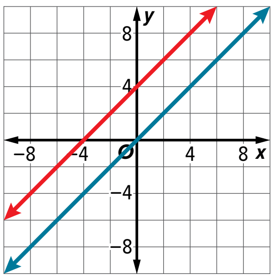
21. $g(x) = (-1.5x)$



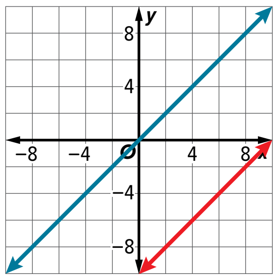
Mixed Exercises

Describe the transformation in each function as it relates to the graph of the parent function.

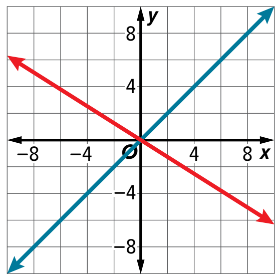
22. $g(x) = x + 4$



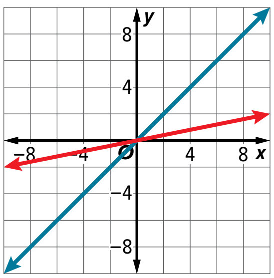
23. $g(x) = (x - 2) - 8$



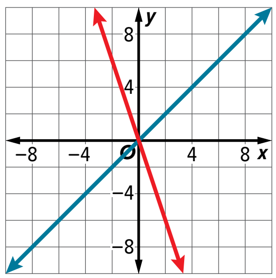
24. $g(x) = \left(-\frac{5}{8}x\right)$



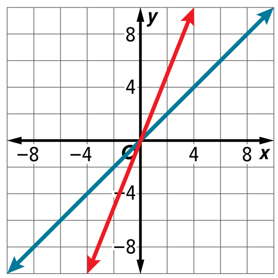
25. $g(x) = \frac{1}{5}(x)$



26. $g(x) = -3(x)$



27. $g(x) = (2.5x)$



REASONING Write a function $g(x)$ to represent the translated graph.

28. $f(x) = 3x + 7$ translated 4 units up.

29. $f(x) = x - 5$ translated 2 units down.

30. **PERIMETER** The function $f(s) = 4s$ represents the perimeter of a square with side length s. Write a function $g(s)$ to represent the perimeter of a square with side lengths that are twice as great. Describe the graph of $g(s)$ compared to $f(s)$.

31. **GAMES** The function $f(x) = 0.50x$ gives the average cost in dollars for x cell phone game downloads that cost an average of \$0.50 each. Write a function $g(x)$ to represent the cost in dollars for x cell phone game downloads that cost \$1.50 each. Describe the graph of $g(x)$ compared to $f(x)$.

32. **TRAINER** The function $f(x) = 90x$ gives the cost of working out with a personal trainer, where \$90 is the trainer's hourly rate, and x represents the number of hours spent working out with the trainer. Describe the dilation, $g(x)$ of the function $f(x)$, if the trainer increases her hourly rate to \$100.

33. **DOWNLOADS** Hannah wants to download songs. She researches the price to download songs from Site F. Hannah wrote the function $f(x) = x$, which represents the cost in dollars for x songs downloaded that cost \$1.00 each.

a. Hannah researches the price to download songs from Site G. Write a function $g(x)$ to represent the cost in dollars for x songs downloaded that cost \$1.29 each.

b. Describe the graph of $g(x)$ compared to the graph of $f(x)$.

34. **PERSEVERE** For any linear function, replacing $f(x)$ with $f(x + k)$ results in the graph of $f(x)$ being shifted k units to the right for $k < 0$ and shifted k units to the left for $k > 0$. Does shifting the graph horizontally k units have the same effect as shifting the graph vertically $-k$ units? Justify your answer. Include graphs in your response.

35. **CREATE** Write an equation that is a vertical compression by a factor of a of the parent function $y = x$. What can you say about the horizontal dilation of the function?

36. **WHICH ONE DOESN'T BELONG** Consider the four functions. Which one does not belong in this group? Justify your conclusion.

$f(x) = 2(x + 1) - 3$	$f(x) = \frac{1}{2}x - 4$	$f(x) = -3x + 10$	$f(x) = 5(x - 7) + 3$

Arithmetic Sequences

Learn Arithmetic Sequences

A **sequence** is a set of numbers that are ordered in a specific way. Each number within a sequence is called a **term of a sequence**.

In an **arithmetic sequence**, each term after the first is found by adding a constant, the **common difference** *d*, to the previous term.

Words	An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference.
Examples	22, 17, 12, 7, 2, … −5 −5 −5 −5 The common difference is −5. −13, −5, 3, 11, 19, … +8 +8 +8 +8 The common difference is 8.

Today's Goals

- Construct arithmetic sequences.
- Apply the arithmetic sequence formula.

Today's Vocabulary

sequence
term of sequence
arithmetic sequence
common difference
*n*th term of an arithmetic function

Example 1 Identify Arithmetic Sequences

Determine whether the sequence is an arithmetic sequence. Justify your reasoning.

17, 14, 10, 7, 3

Check the difference between terms.

17 14 10 7 3
−3 −4 −3 −4

This sequence ________ a common difference between its terms. This ______ an arithmetic sequence.

Think About It!

How are arithmetic sequences and number patterns alike and different?

Check

Determine whether the sequence is an arithmetic sequence. Justify your reasoning.

82, 73, 64, 55, . . .

____

Go Online You can complete an Extra Example online.

Your Notes

Talk About It!

Why would it be useful to develop a rule to find terms of a sequence? Explain.

Watch Out!

Subscripts Subscripts are used to indicate a specific term. For example, a_8 means the 8th term of the sequence. It does not mean $a \times 8$.

Think About It!

Why is the domain of a sequence counting numbers instead of all real numbers?

Example 2 Find the Next Term

Determine the next three terms in the sequence.

11, 7, 3, –1

Find the common difference between terms. ____

Add the common difference to the last term of the sequence to find the next terms.

$-1 + (\_\_\_) = \_\_\_$ $\qquad$ $\_\_\_ + (\_\_\_) = \_\_\_$ $\qquad$ $\_\_\_ + (\_\_\_) = \_\_\_$

Check

Determine the next three terms in the sequence.

31, 18, 5, ____, ____, ____

Go Online You can complete an Extra Example online.

Explore Common Differences

Online Activity Use a real-world situation to complete the Explore.

INQUIRY How can you tell if a set of numbers models a linear function?

Learn Arithmetic Sequences as Linear Functions

Each term of an arithmetic sequence can be expressed in terms of the first term a_1 and the common difference d.

Key Concept • *n*th Term of an Arithmetic Sequence

The ***n*th term of an arithmetic sequence** with the first term a_1 and common difference d is given by $a_n = a_1 + (n - 1)d$, where n is a positive integer.

The graph of an arithmetic sequence includes points that lie along a line. Because there is a constant difference between each pair of points, the function is linear. For the equation of an arithmetic sequence, $a_n = a_1 + (n - 1)d$

- n is the independent variable,
- a_n is the dependent variable, and
- d is the slope.

The function of an arithmetic sequence is written as $f(n) = a_1 + (n - 1)d$, where n is a counting number.

Go Online You can complete an Extra Example online.

Example 3 Find the *n*th Term

Use the arithmetic sequence −4, −1, 2, 5, . . . to complete the following.

Part A Write an equation.

______

Part B Find the 16th term of the sequence.

Use the equation from Part A to find the 16th term in the arithmetic sequence.

______ Equation from Part *A*

______ Substitute 16 for *n*.

______ Multiply.

______ Simplify.

Check

RUNNING Randi has been training for a marathon, and it is important for her to keep a constant pace. She recorded her time each mile for the first several miles that she ran.

- At 1 mile, her time was 10 minutes and 30 seconds.
- At 2 miles, her time was 21 minutes.
- At 3 miles, her time was 31 minutes and 30 seconds.
- At 4 miles, her time was 42 minutes.

Part A Write a function to represent her sequence of data. Use *n* as the variable.

______

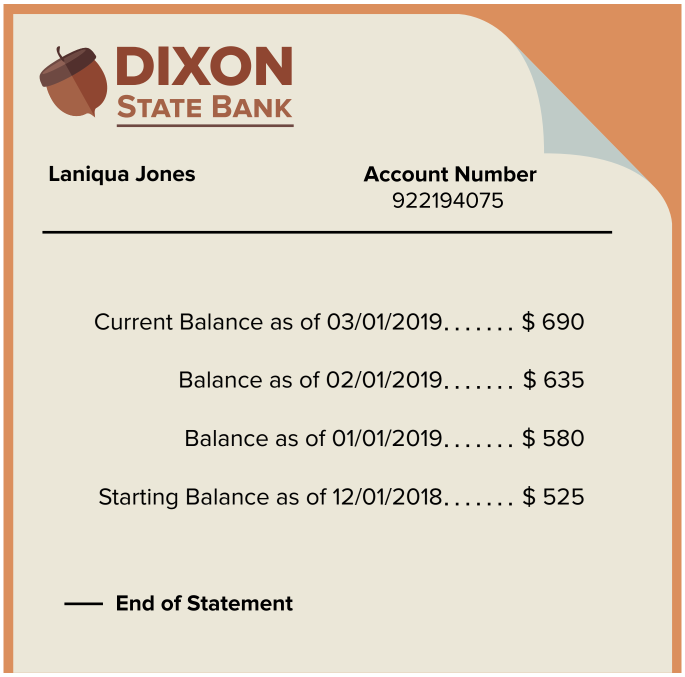
Part B How long will it take her to run a whole marathon? Round your answer to the nearest thousandth if necessary. (Hint: a marathon is 26.2 miles.)

______

Example 4 Apply Arithmetic Sequences as Linear Functions

MONEY Laniqua opened a savings account to save for a trip to Spain. With the cost of plane tickets, food, hotel, and other expenses, she needs to save $1600. She opened the account with $525. Every month, she adds the same amount to her account using the money she earns at her after school job. From her bank statement, Laniqua can write a function that represents the balance of her savings account.

Go Online You can complete an Extra Example online.


DIXON
STATE BANK

Laniqua Jones

Account Number
922194075

Current Balance as of 03/01/2019....... $ 690

Balance as of 02/01/2019....... $ 635

Balance as of 01/01/2019....... $ 580

Starting Balance as of 12/01/2018....... $ 525

—— End of Statement

Part A Create a function to represent the sequence.

First, find the ________________.

525 $\xrightarrow{+55}$ 580 $\xrightarrow{+55}$ 635 $\xrightarrow{+55}$ 690

The common difference is ____.

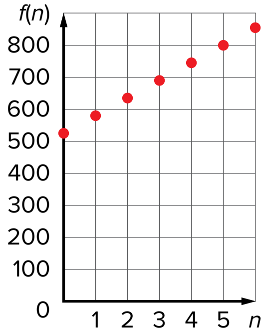
The balance after 1 month is \$580, so let $a_1 = 580$. Notice that the starting balance is \$525. You can think of this starting point as $a_0 = 580$.

________________	Formula for the *n*th term
________________	$a_1 = 580$ and $d = 55$
________________	Simplify.
____________	

> ### Use a Source
> Find the cost of a flight from the airport closest to you to Madrid, the capital of Spain. How many months would Laniqua need to save to afford the ticket?

Part B Graph the function and determine its domain.

n	*f*(*n*)
0	____
1	____
2	____
3	____
4	____
5	____
6	____



> ### Study Tip
> **Graphing** You might not need to create a table of the sequence first. However, it might serve as a reminder that an arithmetic sequence is a series of points, not a line.

The domain is the number of months since Laniqua opened her savings account. The domain is {0, 1, 2, 3, 4, 5, ...}.

Go Online You can complete an Extra Example online.

Name ______________________ Period __________ Date __________

Practice

Go Online You can complete your homework online.

Example 1

ARGUMENTS **Determine whether each sequence is an arithmetic sequence. Justify your reasoning.**

1. $-3, 1, 5, 9, \ldots$

2. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{7}{16}, \ldots$

3. $-10, -7, -4, 1, \ldots$

4. $-12.3, -9.7, -7.1, -4.5, \ldots$

5. $4, 7, 9, 12, \ldots$

6. $15, 13, 11, 9, \ldots$

7. $7, 10, 13, 16, \ldots$

8. $-6, -5, -3, -1, \ldots$

Example 2

Find the common difference of each arithmetic sequence. Then find the next three terms.

9. $0.02, 1.08, 2.14, 3.2, \ldots$

10. $6, 12, 18, 24, \ldots$

11. $21, 19, 17, 15, \ldots$

12. $-\frac{1}{2}, 0, \frac{1}{2}, 1, \ldots$

13. $2\frac{1}{3}, 2\frac{2}{3}, 3, 3\frac{1}{3}, \ldots$

14. $\frac{7}{12}, 1\frac{1}{3}, 2\frac{1}{12}, 2\frac{5}{6}, \ldots$

15. $3, 7, 11, 15, \ldots$

16. $22, 19.5, 17, 14.5, \ldots$

17. $-13, -11, -9, -7, \ldots$

18. $-2, -5, -8, -11, \ldots$

Example 3

Use the given arithmetic sequence to write an equation and then find the 7th term of the sequence.

19. $-3, -8, -13, -18, \ldots$

20. $-2, 3, 8, 13, \ldots$

21. $-11, -15, -19, -23, \ldots$

22. $-0.75, -0.5, -0.25, 0, \ldots$

Example 4

23. **SPORTS** Wanda is the manager for the soccer team. One of her duties is to hand out cups of water at practice. Each cup of water is 4 ounces. She begins practice with a 128-ounce cooler of water.

a. Create a function to represent the arithmetic sequence.

b. Graph the function.

c. How much water is remaining after Wanda hands out the 14th cup?

24. **THEATER** A theater has 20 seats in the first row, 22 in the second row, 24 in the third row, and so on for 25 rows.

a. Create a function to represent the arithmetic sequence.

b. Graph the function.

c. How many seats are in the last row?

25. **POSTAGE** The price to send a large envelope first class mail is 88 cents for the first ounce and 17 cents for each additional ounce. The table shows the cost for weights up to 5 ounces.

Weight (ounces)	1	2	3	4	5
Postage (dollars)	0.88	1.05	1.22	1.39	1.56

Source: United States Postal Service

a. Create a function to represent the arithmetic sequence.

b. Graph the function.

c. How much did a large envelope weigh that cost $2.07 to send?

26. **VIDEO DOWNLOADING** Brian is downloading episodes of his favorite TV show to play on his personal media device. The cost to download 1 episode is $1.99. The cost to download 2 episodes is $3.98. The cost to download 3 episodes is $5.97.

a. Create a function to represent the arithmetic sequence.

b. Graph the function.

c. What is the cost to download 9 episodes?

27. USE A MODEL Chapa is beginning an exercise program that calls for 30 push-ups each day for the first week. Each week thereafter, she has to increase her push-ups by 2.

a. Write a function to represent the arithmetic sequence.

b. Graph the function.

c. Which week of her program will be the first one in which she will do at least 50 push-ups a day?

Mixed Exercises

CONSTRUCT ARGUMENTS **Determine whether each sequence is an arithmetic sequence. Justify your argument.**

28. −9, −12, −15, −18, ...

29. 10, 15, 25, 40, ...

30. −10, −5, 0, 5, ...

31. −5, −3, −1, 1, ...

Write an equation for the *n*th term of each arithmetic sequence. Then graph the first five terms of the sequence.

32. 7, 13, 19, 25, ...

33. 30, 26, 22, 18, ...

34. −7, −4, −1, 2, ...

35. SAVINGS Fabiana decides to save the money she's earning from her after-school job for college. She makes an initial contribution of $3000 and each month deposits an additional $500. After one month, she will have contributed $3500.

a. Write an equation for the *n*th term of the sequence.

b. How much money will Fabiana have contributed after 24 months?

36. NUMBER THEORY One of the most famous sequences in mathematics is the Fibonacci sequence. It is named after Leonardo de Pisa (1170–1250) or Filius Bonacci, alias Leonardo Fibonacci. The first several numbers in the Fibonacci sequence are shown.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . .

Does this represent an arithmetic sequence? Why or why not?

37. STRUCTURE Use the arithmetic sequence 2, 5, 8, 11, ...

a. Write an equation for the *n*th term of the sequence.

b. What is the 20th term in the sequence?

38. **CREATE** Write a sequence that is an arithmetic sequence. State the common difference, and find a_6.

39. **CREATE** Write a sequence that is not an arithmetic sequence. Determine whether the sequence has a pattern, and if so describe the pattern.

40. **REASONING** Determine if the sequence 1, 1, 1, 1, . . . is an arithmetic sequence. Explain your reasoning.

41. **CREATE** Create an arithmetic sequence with a common difference of −10.

42. **PERSEVERE** Find the value of x that makes $x + 8$, $4x + 6$, and $3x$ the first three terms of an arithmetic sequence.

43. **CREATE** For each arithmetic sequence described, write a formula for the nth term of a sequence that satisfies the description.

a. first term is negative, common difference is negative

b. second term is –5, common difference is 7

c. $a_2 = 8$, $a_3 = 6$

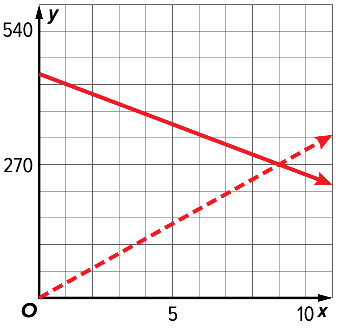
Andre and Sam are both reading the same novel. Andre reads 30 pages each day. Sam created the table at the right. Refer to this information for Exercises 44–46.

Sam's Reading Progress	
Day	Pages Left to Read
1	430
2	410
3	390
4	370

44. **ANALYZE** Write arithmetic sequences to represent each boy's daily progress. Then write the function for the nth term of each sequence.

45. **PERSEVERE** Enter both functions from Exercise 44 into your calculator. Use the table to determine if there is a day when the number of pages Andre has read is equal to the number of pages Sam has left to read. If so, which day is it? Explain how you used the table feature to help you solve the problem.

46. **ANALYZE** Graph both functions on your calculator, then sketch the graph in the coordinate plane at the right. How can you use the graph to answer the equation from Exercise 45?



Piecewise and Step Functions

Learn Graphing Piecewise-Defined Functions

Some functions cannot be described by a single expression because they are defined differently depending on the interval of *x*. These functions are **piecewise-defined functions.** A **piecewise-linear function** has a graph that is composed of some number of linear pieces.

Example 1 Graph a Piecewise-Defined Function

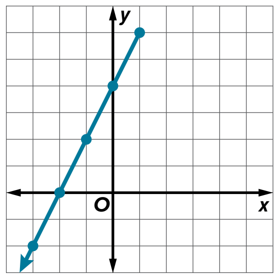
To graph a piecewise-defined function, graph each "piece" separately.

Graph $f(x) = \begin{cases} 2x + 4 \text{ if } x \leq 1 \\ -x + 3 \text{ if } x > 1 \end{cases}$. **State the domain and range.**

First, graph $f(x) = 2x + 4$ if $x \leq 1$.

- Create a table for $f(x) = 2x + 4$ using values of $x > 1$.
- Because x is *less than or equal to* 1, place a _____ at _____ to indicate that the endpoint is included in the graph.
- Then, plot the points and draw the graph beginning at (1, 6).

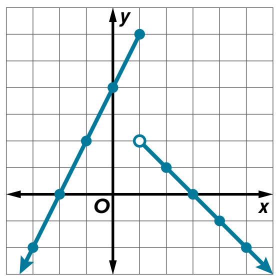
x	y
1	
0	
−1	
−2	
−3	



Next, graph $f(x) = -x + 3$ if $x > 1$.

- Create a table for $f(x) = -x + 3$ using values of $x > 1$.
- Because x is *greater than but not equal to* 1, place a _____ at _____ to indicate that the endpoint is not included in the graph.
- Then, plot the points and draw the graph beginning at (1, 2).

x	y
1	
2	
3	
4	
5	



The domain is all real numbers. The range is $y \leq$ _____.

Go Online You can complete an Extra Example online.

Today's Goals
- Identify and graph piecewise-defined functions.
- Identify and graph step functions.

Today's Vocabulary
piecewise-defined function
piecewise-linear function
step function
greatest integer function

Think About It!
What would be an advantage of graphing the entire expression and removing the portion that is not in the interval?

Go Online
An alternate method is available for this example.

Your Notes

Watch Out!

Circles and Dots Do not forget to examine the endpoint(s) of each piece to determine whether there should be a circle or a dot. > and < mean that a circle should be used, while ≥ and ≤ mean that a dot should be used.

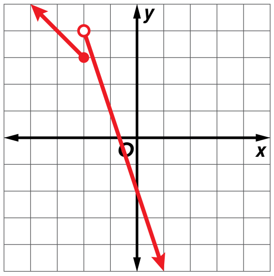
Study Tip

Piecewise-Defined Functions When graphing piecewise-defined functions, there should be a dot or line that contains each member of the domain.

Go Online You can watch a video to see how to graph a piecewise-defined function on a graphing calculator

Check

Part A Graph $f(x) = \begin{cases} -x + 1 \text{ if } x \leq -2 \\ -3x - 2 \text{ if } x > -2 \end{cases}$.



Part B Find the domain and range of the function.

____

Explore Age as a Function

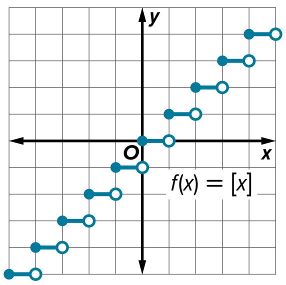
Online Activity Use a real-world situation to complete an Explore.

INQUIRY When can real-world data be described using a step function?

Learn Graphing Step Functions

A **step function** is a type of piecewise-linear function with a graph that is a series of horizontal line segments. One example of a step function is the **greatest integer function,** written as $f(x) = [\![x]\!]$ in which $f(x)$ is the greatest integer less than or equal to x.

Key Concept • Greatest Integer Function

Type of graph: disjointed line segments The graph of a step function is a series of disconnected horizontal line segments. Domain: all real numbers; Because the dots and circles overlap, the domain is all real numbers. Range: all integers; Because the function represents the greatest integer less than or equal to x, the range is all integers.	Parent function: $f(x) = [\![x]\!]$ 

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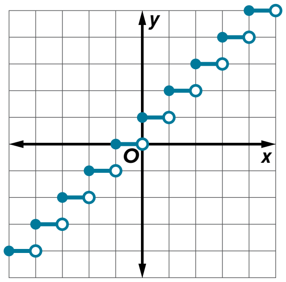
Example 2 Graph a Greatest Integer Function

Graph $f(x) = [\![x + 1]\!]$. State the domain and range.

First, make a table. Select a few values that are between integers.

x	$x + 1$	$[\![x + 1]\!]$	
−2	−1	−1	−1, −0.75, and −0.25 are greater than or equal to −1 but less than 0. So, −1 is the greatest integer that is not greater than −1, −0.75, or −0.25.
−1.75	−0.75	−1	
−1.25	−0.25	−1	
−1	0	0	0, 0.5, and 0.75 are greater than or equal to 0 but less than 1. So, 0 is the greatest integer that is not greater than 0, 0.5, or 0.75.
−0.5	0.5	0	
−0.25	0.75	0	
0	1	1	1, 1.25, and 1.5 are greater than or equal to 1 but less than 2. So, 1 is the greatest integer that is not greater than 1, 1.25, or 1.5.
0.25	1.25	1	
0.5	1.5	1	
1	2	2	2, 2.25, and 2.75 are greater than or equal to 2 but less than 3. So, 2 is the greatest integer that is not greater than 2, 2.25, or 2.75.
1.25	2.25	2	
1.75	2.75	2	

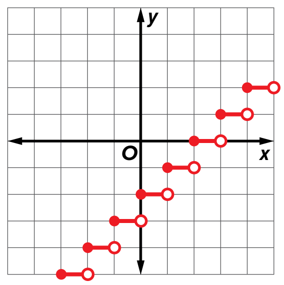
On the graph, dots represent included points. Circles represent points that are not included. The domain is all real numbers. The range is all integers. Note that this is the graph of $f(x) = [\![x]\!]$ shifted 1 unit to the left.



Check

Graph $f(x) = [\![x - 2]\!]$ by making a table.

x	$x - 2$	$[\![x - 2]\!]$
−1	−3	
−0.75	−2.75	
−0.25	−2.25	−3
0	−2	−2
0.25	−1.75	−2
0.5	−1.5	
1	−1	
1.25	−0.75	−1
1.5	−0.5	
2	0	
2.25	0.25	



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Talk About It!

What do you notice about the symmetry, extrema, and end behavior of the function?

Watch Out!

Greatest Integer Function When finding the value of a greatest integer function, do not round to the nearest integer. Instead, always round nonintegers down to the greatest integer that is not greater than the number.

Think About It!

How would the graph change if 1 certified lifeguard could watch up to 59 swimmers? For example, if there are greater than or equal to 60, but fewer than 120 swimmers, there must be 2 lifeguards on duty.

Math History Minute

Oliver Heaviside (1850–1925) was a self-taught electrical engineer, mathematician, and physicist who laid much of the groundwork for telecommunications in the 21st century. Heaviside invented the Heaviside step function,

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } x = 0, \\ 1 & \text{if } x > 0 \end{cases}$$

which he used to model the current in an electric circuit.

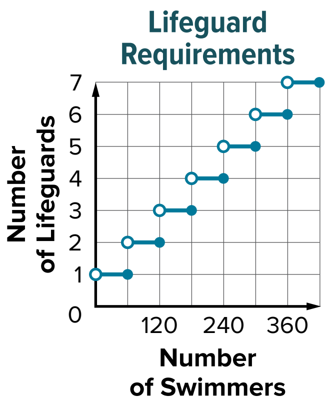
Example 3 Graph a Step Function

SAFETY A state requires a ratio of 1 lifeguard to 60 swimmers in a swimming pool. This means that 1 lifeguard can watch up to and including 60 swimmers. Make a table and draw a graph that shows the number of lifeguards that must be on duty *f*(*x*) based on the number of swimmers in the pool *x*.

The number of lifeguards that must be on duty can be represented by a step function.

- If the number of swimmers is greater than 0 but fewer than or equal to 60, only 1 lifeguard must be on duty.
- If the number of swimmers is greater than 60 but fewer than or equal to 120, there must be 2 lifeguards on duty.
- If the number of swimmers is greater than 180 but fewer than or equal to 240, there must be 4 lifeguards on duty.

x	$f(x)$
$0 < x \le 60$	1
$60 < x \le 120$	2
$120 < x \le 180$	
$180 < x \le 240$	4
$240 < x \le 300$	
$300 < x \le 360$	
$360 < x \le 420$	



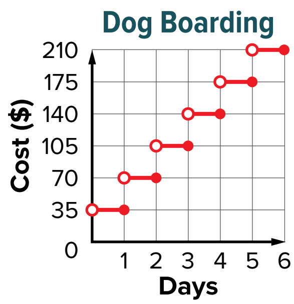
The circles mean that when there are more than a multiple of 60 swimmers, ______________________.

The dots represent the __________ number of swimmers that can be in the pool for that particular number of __________ on duty.

Check

PETS At Luciana's pet boarding facility, it costs \$35 per day to board a dog. Every fraction of a day is rounded up to the next day. Graph the function representing this situation by making a table.

Days	Cost (\$)
$0 < x \le 1$	
$1 < x \le 2$	
$2 < x \le 3$	
$3 < x \le 4$	
$4 < x \le 5$	
$5 < x \le 6$	



Go Online You can complete an Extra Example online.

Name ______________________ Period ____________ Date ____________

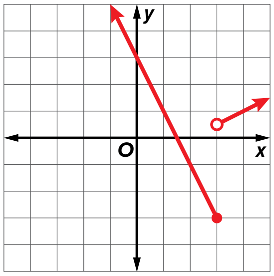
Practice

Go Online You can complete your homework online.

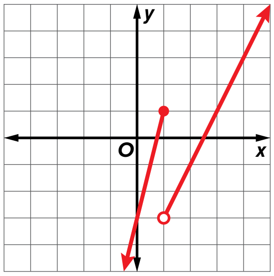
Example 1

Graph each function. State the domain and range.

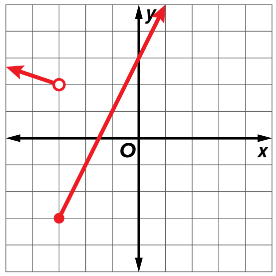
1. $f(x) = \begin{cases} \frac{1}{2}x - 1 \text{ if } x > 3 \\ -2x + 3 \text{ if } x \leq 3 \end{cases}$



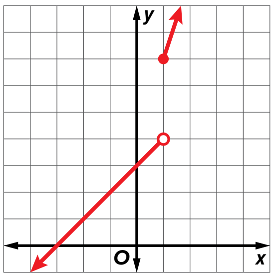
2. $f(x) = \begin{cases} 2x - 5 \text{ if } x > 1 \\ 4x - 3 \text{ if } x \leq 1 \end{cases}$



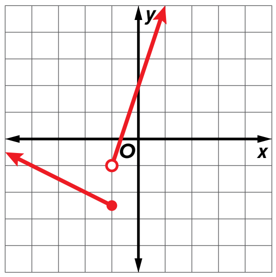
3. $f(x) = \begin{cases} 2x + 3 \text{ if } x \geq -3 \\ -\frac{1}{3}x + 1 \text{ if } x < -3 \end{cases}$



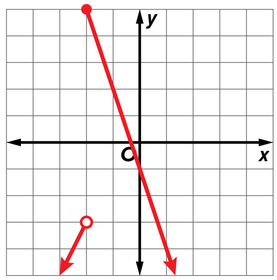
4. $f(x) = \begin{cases} 3x + 4 \text{ if } x \geq 1 \\ x + 3 \text{ if } x < 1 \end{cases}$



5. $f(x) = \begin{cases} 3x + 2 \text{ if } x > -1 \\ -\frac{1}{2}x - 3 \text{ if } x \leq -1 \end{cases}$



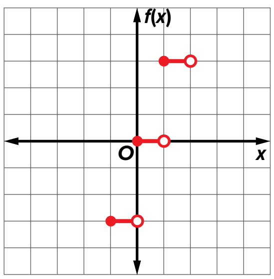
6. $f(x) = \begin{cases} 2x + 1 \text{ if } x < -2 \\ -3x - 1 \text{ if } x \geq -2 \end{cases}$



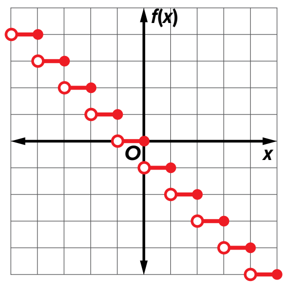
Example 2

Graph each function. State the domain and range.

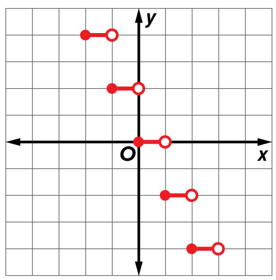
7. $f(x) = 3\,[\![x]\!]$



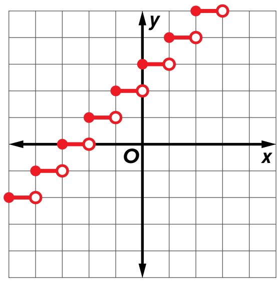
8. $f(x) = [\![-x]\!]$



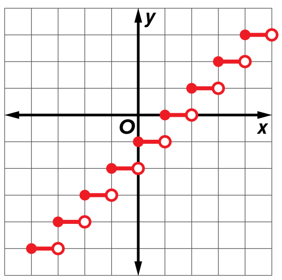
9. $g(x) = -2\,[\![x]\!]$



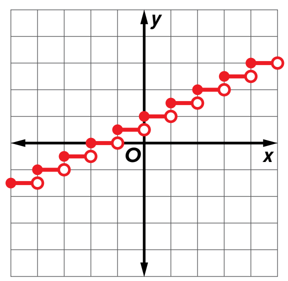
10. $g(x) = [\![x]\!] + 3$



11. $h(x) = [\![x]\!] - 1$

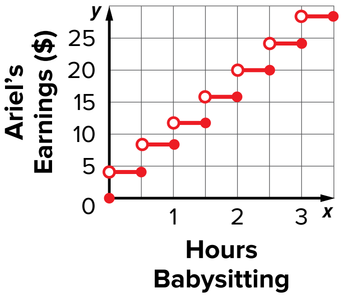


12. $h(x) = \frac{1}{2}[\![x]\!] + 1$

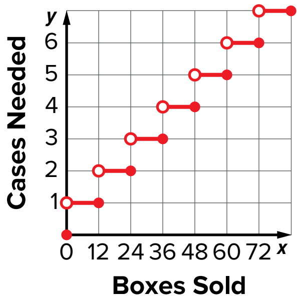


Example 3

13. **BABYSITTING** Ariel charges \$8 per hour as a babysitter. She rounds every fraction of an hour up to the next half-hour. Draw a graph to represent Ariel's total earnings *y* after *x* hours.



14 **FUNDRAISING** Students are selling boxes of cookies at a fundraiser. The boxes of cookies can only be ordered by the case, with 12 boxes per case. Draw a graph to represent the number of cases needed *y* when *x* boxes of cookies are sold.



Mixed Exercises

15. **PRECISION** A package delivery service determines rates for express shipping by the weight of a package, with every fraction of a pound rounded up to the next pound. The table shows the cost of express shipping packages that weigh no more than 5 pounds. Write a piecewise-linear function representing the cost to ship a package that weighs no more than 5 pounds. State the domain and range.

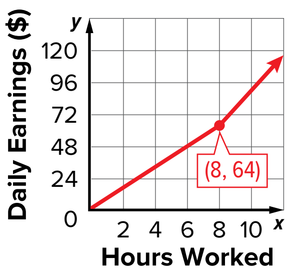
Weight (pounds)	Rate (dollars)
1	16.20
2	19.30
3	22.40
4	25.50
5	28.60

16. **EARNINGS** Kelly works in a hospital as a medical assistant. She earns \$8 per hour the first 8 hours she works in a day and \$11.50 per hour each hour thereafter.

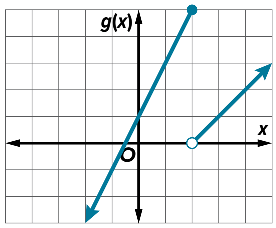
a. Organize the information into a table. Include a row for hours worked *x*, and a row for daily earnings *f*(*x*).

b. Write the piecewise equation describing Kelly's daily earnings *f*(*x*) for *x* hours.

c. Draw a graph to represent Kelly's daily earnings.



17. **REASONING** Write a piecewise function that represents the graph.



18. **STRUCTURE** Suppose $f(x) = 2[\![x - 1]\!]$.

a. Find $f(1.5)$. **b.** Find $f(2.2)$.

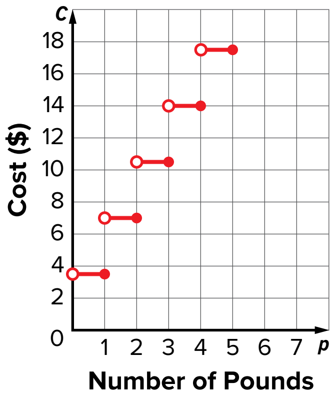
c. Find $f(9.7)$. **d.** Find $f(-1.25)$.

19. **RENTAL CARS** Mr. Aronsohn wants to rent a car on vacation. The rate the car rental company charges is \$19 per day. If any fraction of a day is counted as a whole day, how much would it cost for Mr. Aronsohn to rent a car for 6.4 days?

20. **USE A MODEL** A roadside fruit and vegetable stand determines rates for selling produce, with every fraction of a pound rounded up to the next pound. The table shows the cost of tomatoes by weight in pounds.

a. Write a piecewise-linear function representing the cost of purchasing 0 to 5 pounds of tomatoes, where C is the cost in dollars and p is the number of pounds.

Weight (pounds)	Rate (dollars)
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50



b. Graph the function.

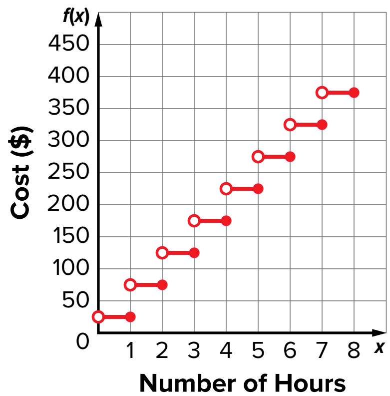
c. State the domain and range.

d. What would be the cost of purchasing 4.3 pounds of tomatoes at the roadside stand?

21. **ELECTRONIC REPAIRS** Tech Repairs charges \$25 for an electronic device repair that takes up to one hour. For each additional hour of labor, there is a charge of \$50. The repair shop charges for the next full hour for any part of an hour.

a. Complete the table to organize the information. Include a row for hours of repair x, and a row for total cost $f(x)$.

x	0	2	4	6	8
$f(x)$					



b. Write a step function to represent the total cost for every hour x of repair.

c. Graph the function.

d. Devesh was charged \$125 to repair his tablet. How long did the repair take to complete?

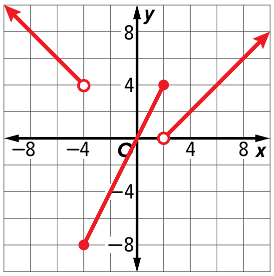
22. INVENTORY Malik owns a bakery. Every week he orders chocolate chips from a supplier. The supplier's pricing is shown in the table.

Chocolate Chip Pricing	
\$4 per pound	Up to 3 pounds
\$1.50 per pound	For each pound over 3 pounds

a. Write a function to represent the cost of chocolate chips.

b. Malik's budget for chocolate chips for the week is \$25. How many whole pounds of chocolate chips can he order?

23. CREATE Write a piecewise-defined function with three linear pieces. Then graph the function.

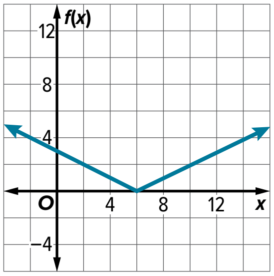


24. FIND THE ERROR Amy graphed a function that gives the height of a car on a roller coaster as a function of time. She said her graph is the graph of a step function. Is this possible? Explain your reasoning.

25. WRITE What is the difference between a step function and a piecewise-defined function?

26. ANALYZE Does the piecewise relation $y = \begin{cases} -2x + 4 \text{ if } x \geq 2 \\ -\frac{1}{2}x - 1 \text{ if } x \leq 4 \end{cases}$ represent a function? Justify your argument.

ANALYZE **Refer to the graph for Exercises 27–31.**



27. Write a piecewise function to represent the graph.

28. What is the domain?

29. What is the range?

30. Find $f(8.5)$.

31. Find $f(1.2)$.

Absolute Value Functions

Explore Parameters of an Absolute Value Function

Online Activity Use graphing technology to complete the Explore.

INQUIRY How does performing an operation on an absolute value function change its graph?

Learn Graphing Absolute Value Functions

The **absolute value function** is a type of piecewise-linear function. An absolute value function is written as $f(x) = a|x - h| + k$, where a, h, and k are constants and $f(x) \geq 0$ for all values of x.

The **vertex** is either the lowest point or the highest point of a function. For the parent function, $y = |x|$, the vertex is at the origin.

Key Concept • Absolute Value Function

Parent Function	$f(x) = |x|$, defined as $f(x) = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$
Type of Graph	V-Shaped
Domain:	all real numbers
Range:	all nonnegative real numbers

Learn Translations of Absolute Value Functions

Key Concept • Vertical Translations of Absolute Value Functions

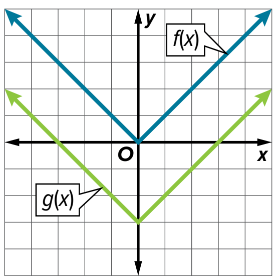
If $k > 0$, the graph of $f(x) = |x|$ is translated k units up.
If $k < 0$, the graph of $f(x) = |x|$ is translated $|k|$ units down.

Key Concept • Horizontal Translations of Linear Functions

If $h > 0$, the graph of $f(x) = |x|$ is translated h units right.
If $h < 0$, the graph of $f(x) = |x|$ is translated h units left.

Example 1 Vertical Translations of Absolute Value Functions

Describe the translation in $g(x) = |x| - 3$ as it relates to the graph of the parent function.



Graph the parent function, $f(x) = |x|$, for absolute values.

The constant, k, is outside the absolute value signs, so k affects the y-values. The graph will be a vertical translation.

(continued on the next page)

Today's Goals
- Graph absolute value functions.
- Apply translations to absolute value functions.
- Apply dilations to absolute value functions.
- Apply reflections to absolute value functions.
- Interpret constants within equations of absolute value functions.

Today's Vocabulary
absolute value function

vertex

Think About It!
Why does adding a positive value of k shift the graph k units up?

Go Online
You can watch a video to see how to describe translations of functions.

Study Tip
Horizontal Shifts Remember that the general form of an absolute value function is $y = a|x - h| + k$. So, $y = |x + 7|$ is actually $y = |x - (-7)|$ in the function's general form.

Your Notes

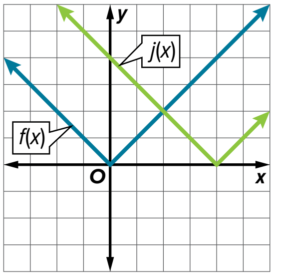
Since $f(x) = |x|$, $g(x) = f(x) + k$ where $k = -3$.
$g(x) = |x| - 3 \longrightarrow g(x) = f(x) + (-3)$

The value of k is less than 0, so the graph will be translated $|k|$ units down, or 3 units down.

$g(x) = |x| - 3$ is a translation of the graph of the parent function 3 units down.

Example 2 Horizontal Translations of Absolute Value Functions

Describe the translation in $j(x) = |x - 4|$ as it relates to the parent function.



Graph the parent function, $f(x) = |x|$, for absolute values.

The constant, h, is inside the absolute value signs, so h affects the input or, x-values. The graph will be a horizontal translation.

Since $f(x) = |x|$, $j(x) = f(x - h)$, where $h = 4$.
$j(x) = |x - 4| \longrightarrow j(x) = f(x - 4)$

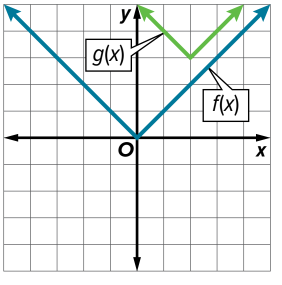
The value of h is greater than 0, so the graph will be translated h units right, or 4 units right.

$j(x) = |x - 4|$ is the translation of the graph of the parent function 4 units right.

Think About It!
Since the vertex of the parent function is at the origin, what is a quick way to determine where the vertex is of $q(x) = |x - h| + k$?

Example 3 Multiple Translations of Absolute Value Functions

Describe the translation in $g(x) = |x - 2| + 3$ as it relates to the graph of the parent function.



The equation has both h and k values. The input and output will be affected by the constants. The graph of $f(x) = |x|$ is vertically and horizontally translated.

Since $f(x) = |x|$, $g(x) = f(x - h) + k$ where $h = 2$ and $k = 3$.

Because ________ and ________, the graph is translated 2 units ________ and 3 units ________.

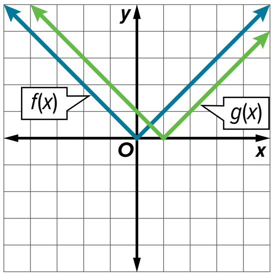
$g(x) = |x - 2| + 3$ is the translation of the graph of the parent function 2 units right and 3 units________.

Emilio says that the graph of $g(x) = |x + 1| - 1$ is the same graph as $f(x) = |x|$. Is he correct? Why or why not?

Go Online You can complete an Extra Example online.

Example 4 Identify Absolute Value Functions from Graphs

Use the graph of the function to write its equation.



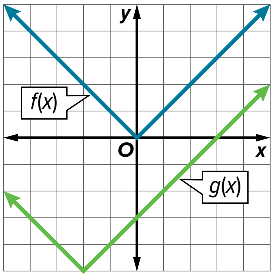
The graph is the translation of the parent graph 1 unit to the right.

$g(x) = |x - h|$ General equation for a horizontal translation

$g(x) = |x - 1|$ The vertex is 1 unit to the right of the origin.

Example 5 Identify Absolute Value Functions from Graphs (Multiple Translations)

Use the graph of the function to write its equation.



The graph is a translation of the parent graph 2 units to the left and 5 units down.

$g(x) = |x - h| + k$ General equation for translations

$g(x) = |x - (-2)| + k$ The vertex is 2 units left of the origin.

$g(x) = |x - (-2)| + (-5)$ The vertex is 5 units down from the origin.

$g(x) = |x + 2| - 5$ Simplify.

Learn Dilations of Absolute Value Functions

Multiplying by a constant a after evaluating an absolute value function creates a vertical change, either a stretch or compression.

Key Concept • Vertical Dilations of Absolute Value Functions

If $|a| > 1$, the graph of $f(x) = |x|$ is stretched vertically.
If $0 < |a| < 1$, the graph of $f(x) = |x|$ is compressed vertically.

When an input is multiplied by a constant a before for the absolute value is evaluated, a horizontal change occurs.

Key Concept • Horizontal Dilations of Absolute Value Functions

If $|a| > 1$, the graph of $f(x) = |x|$ is compressed horizontally.
If $0 < |a| < 1$, the graph of $f(x) = |x|$ is stretched horizontally.

Go Online You can complete an Extra Example online.

Talk About It!
How is the value of a in an absolute value function related to slope? Explain.

Example 6 Dilations of the Form $a|x|$ When $a > 1$

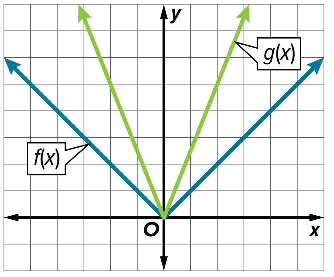
Describe the dilation in $g(x) = \frac{5}{2}|x|$ as it relates to the graph of the parent function.

Since $f(x) = |x|$, $g(x) = a \cdot f(x)$, where $a = \frac{5}{2}$.

$g(x) = \frac{5}{2}|x| \longrightarrow g(x) = \frac{5}{2} \cdot f(x)$

$g(x) = \frac{5}{2}|x|$ is a vertical stretch of the graph of the parent graph.

x	$\|x\|$	$\frac{5}{2}\|x\|$	$(x, g(x))$
−4	$\|-4\| = 4$	10	(−4, 10)
−2	$\|-2\| = 2$	5	(−2, 5)
0	$\|0\| = 0$	0	(0, 0)
2	$\|2\| = 2$	5	(2, 5)
4	$\|4\| = 4$	10	(4, 10)



Example 7 Dilations of the Form $|ax|$

Describe the dilation in $p(x) = |2x|$ as it relates to the graph of the parent function.

For $p(x) = |2x|$, ________.
Since a is inside the absolute value symbols, the input is first multiplied by a. Then, the absolute value of ax is evaluated.

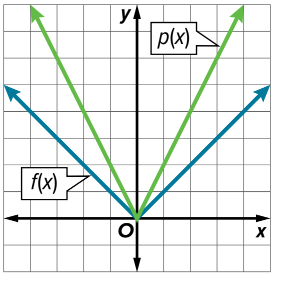
x	$\|2x\|$	$p(x)$	$(x, p(x))$
−4	$\|2(-4)\| = \|-8\|$	8	(−4, 8)
−2	$\|2(-2)\| = \|-4\|$	4	(−2, 4)
0	$\|2(0)\| = \|0\|$	0	(0, 0)
2	$\|2(2)\| = \|4\|$	4	(2, 4)
4	$\|2(4)\| = \|8\|$	8	(4, 8)

Plot the points from the table.

Since $f(x) = |x|$, ____________ where ________.

$p(x) = |2x| \rightarrow$ ____________

$p(x) = |2x|$ is a ________________________ of the graph of the parent graph.



Check

Match each description of the dilation with its equation.

stretched vertically	$j(x) = \left\|\frac{4}{3}x\right\|$
compressed vertically	$q(x) = \left\|\frac{1}{5}x\right\|$
stretched horizontally	$p(x) = 6\|x\|$
compressed horizontally	$g(x) = \frac{5}{7}\|x\|$

Go Online You can complete an Extra Example online.

Go Online
You can watch a video to see how to describe dilations of functions.

Think About It!
How are $a|x|$ and $|ax|$ evaluated differently?

Watch Out!

Differences in Dilations Although $a|x|$ and $|ax|$ appear to have the same effect on a function, they are evaluated differently and that difference is more apparent when a function is dilated and translated horizontally. For a function with multiple transformations, it is best to first create a table.

Example 8 Dilations When $0 < a < 1$

Describe the dilation in $j(x) = \frac{1}{3}|x|$ as it relates to the graph of the parent function.

For $j(x) = \frac{1}{3}|x|$, ________.
Because a is outside the absolute value signs, the absolute value of the input is evaluated first. Then, the function is multiplied by a.

x	$\|x\|$	$\frac{1}{3}\|x\|$	$(x, j(x))$
-6	$\|-6\| = 6$	2	$(-6, 2)$
-3	$\|-3\| = 3$	1	$(-3, 1)$
0	$\|0\| = 0$	0	$(0, 0)$
3	$\|3\| = 3$	1	$(3, 1)$
6	$\|6\| = 6$	2	$(6, 2)$

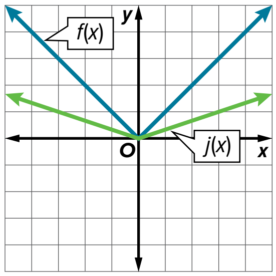
Plot the points from the table.

Because $f(x) = |x|$, ______________

where ________.

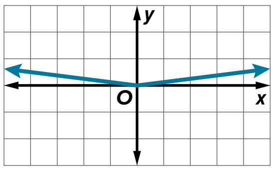
$j(x) = \frac{1}{3}|x| \rightarrow$ ____________

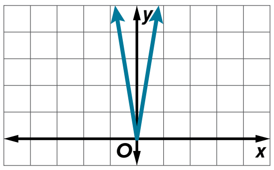
$j(x) = \frac{1}{3}|x|$ is a ____________________ of the graph of the parent function.



Check

Write an equation for each graph shown.





________________ ________________

Learn Reflections of Absolute Value Functions

The graph of $-a|x|$ appears to be flipped upside down compared to $a|x|$, and they are symmetric about the x-axis.

Key Concept • Reflections of Absolute Value Functions Across the x-axis

The graph of $-af(x)$ is the reflection of the graph of $af(x) = a|x|$ across the x-axis.

When the only transformation occurring is a reflection or a dilation and reflection, the graphs of $f(ax)$ and $f(-ax)$ appear the same.

Key Concept • Reflections of Absolute Value Functions Across the y-axis

The graph of $f(-ax)$ is the reflection of the graph of $f(ax) = |ax|$ across the y-axis.

Go Online You can complete an Extra Example online.

Go Online
You can watch a video to see how to describe reflections of functions.

Think About It!
Why would $g(x) = |-2x|$ and $j(x) = |2x|$ appear to be the same graphs?

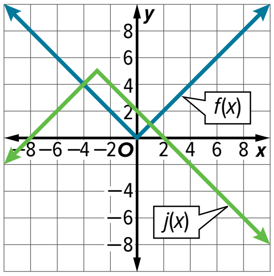
Example 9 Graphs of Reflections with Transformations

Describe how the graph of $j(x) = -|x + 3| + 5$ is related to the graph of the parent function.

x	$\lvert x + 3\rvert$	$-\lvert x + 3\rvert$	$-\lvert x + 3\rvert + 5$	$(x, j(x))$
-5	$\lvert -5 + 3\rvert = \lvert -2\rvert = 2$	-2	$-2 + 5 = 3$	$(-5, 3)$
-4	$\lvert -4 + 3\rvert = \lvert -1\rvert = 1$	-1	$-1 + 5 = 4$	$(-4, 4)$
-3	$\lvert -3 + 3\rvert = \lvert 0\rvert = 0$	0	$0 + 5 = 5$	$(-3, 5)$
-2	$\lvert -2 + 3\rvert = \lvert 1\rvert = 1$	-1	$-1 + 5 = 4$	$(-2, 4)$
-1	$\lvert -1 + 3\rvert = \lvert 2\rvert = 2$	-2	$-2 + 5 = 3$	$(-1, 3)$

First, the absolute value of $x + 3$ is evaluated. Then, the function is multiplied by $-1 \cdot a$. Finally, 5 is added to the function.

Plot the points from the table.



Because $f(x) = |x|$, ______________
where ______________.

$j(x) = -|x + 3| + 5 \rightarrow$ ______________

$j(x) = -|x + 3| + 5$ is the graph of the parent function reflected across the ________ and translated 3 units ________ and 5 units ________.

Example 10 Graphs of $y = -a|x|$

Describe how the graph of $q(x) = -\frac{3}{4}|x|$ is related to the graph of the parent function.

First, the absolute value of x is evaluated. Then, the function is multiplied by $-1 \cdot a$.

Plot the points from the table.

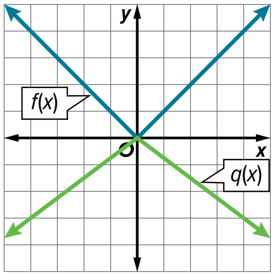
x	$\lvert x\rvert$	$-\frac{3}{4}\lvert x\rvert$	$(x, q(x))$
-8	$\lvert -8\rvert = 8$	-6	$(-8, -6)$
-4	$\lvert -4\rvert = 4$	-3	$(-4, -3)$
0	$\lvert 0\rvert = 0$	0	$(0, 0)$
4	$\lvert 4\rvert = 4$	3	$(4, 3)$
8	$\lvert 8\rvert = 8$	6	$(8, 6)$

Because $f(x) = |x|$, ______________

where ________.

$q(x) = -\frac{3}{4}|x| \rightarrow$ ______________

$q(x) = -\frac{3}{4}|x|$ is the graph of the parent function reflected across the ________ and ______________.



Go Online You can complete an Extra Example online.

Example 11 Graphs of $y = |-ax|$

Describe how the graph of $g(x) = |-4x|$ is related to the graph of the parent function.

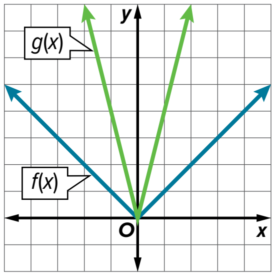
First, the input is multiplied by $-1 \cdot a$. Then the absolute value of $-ax$ is evaluated.

Because $f(x) = |x|$, ____________________________

where ____________.

$g(x) = |-4x| \rightarrow$ ____________________________

$g(x) = |-4x|$ is the graph of the parent function reflected across the __________ and ____________________________.



Learn Transformations of Absolute Value Functions

You can use the equation of a function to understand the behavior of the function. Because the constants a, h, and k affect the function in different ways, they can help develop an accurate graph of the function.

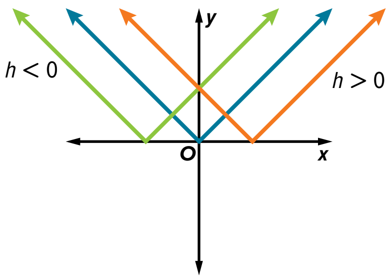
Concept Summary Transformations of Graphs of Absolute Value Functions

$g(x) = a|x - h| + k$

Horizontal Translation, *h*

If __________, the graph of $f(x) = |x|$ is translated ________ units __________.

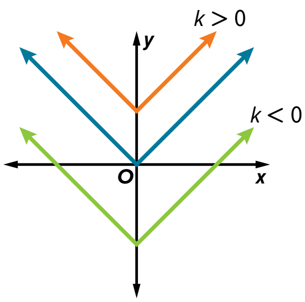
If $h < 0$, the graph of $f(x) = |x|$ is translated $|h|$ units left.



Vertical Translation, *k*

If __________, the graph of $f(x) = |x|$ is translated ________ units __________.

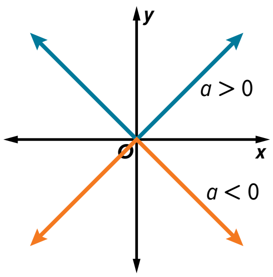
If $k < 0$, the graph of $f(x) = |x|$ is translated $|k|$ units down.



Reflection, *a*

If $a > 0$, the graph opens up.

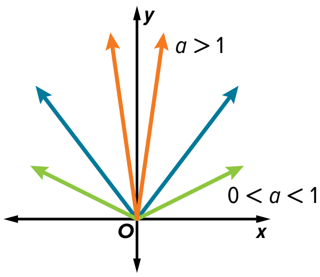
If __________, the graph opens down.



Dilation, *a*

If __________, the graph of $f(x) = |x|$ is stretched vertically.

If $0 < |a| < 1$, the graph is compressed vertically.



Think About It!

Describe how the graph of $y = |-ax|$ is related to the parent function.

Why does there appear to be no reflection for the graph of $y = |-ax|$?

Go Online You can complete an Extra Example online.

Choose the phrase that best describes how each parameter affects the graph of $g(x) = -5|x - 2| + 3$ in relation to the parent function.

-5 ______________________

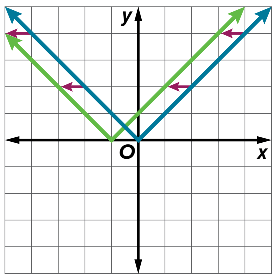
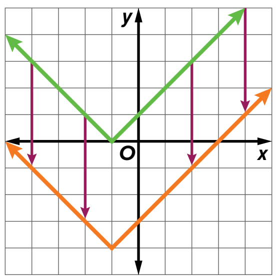
2 ______________________

3 ______________________

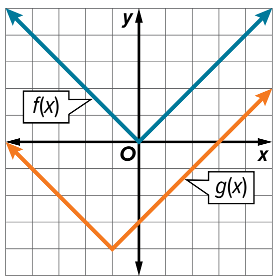
Translates right	Translates left
Translates up	Translates down
Stretches vertically only	Reflects and compresses vertically
Compresses vertically only	Reflects and stretches vertically

Example 12 Graph an Absolute Value Function with Multiple Translations

Graph $g(x) = |x + 1| - 4$. State the domain and range.

$a = 1$	The graph is not reflected or dilated in relation to the parent function.	
$h = -1$	The graph is __________ 1 unit left from the parent function.	
$k = -4$	The graph is translated _____ units down from the parent function.	

The graph of $g(x) = |x + 1| - 4$ is the graph of the parent function translated 1 unit left and 4 units down without dilation or reflection. The domain is ________________. The range is ____.



Watch Out!

Dilations and Translations

Don't assume that $j(x) = 2|x - 5| + 1$ and $p(x) = |2x - 5| + 1$ are the same graph. Functions are evaluated differently depending on whether a is inside or outside the absolute value symbols. It might be best to create a table to generate an accurate graph.

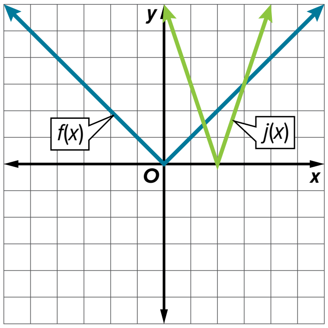
Go Online

You can watch a video to see how to graph a transformed absolute value function.

Go Online You can complete an Extra Example online.

Example 13 Graph an Absolute Value Function with Translations and Dilation

Graph $j(x) = |3x - 6|$. State the domain and range.



Because a is inside the absolute value symbols, the effect of h on the translation changes.

Evaluate the function for several values of x to find points on the graph.

x	$(x, j(x))$
0	(0, 6)
1	(1, 3)
2	(2, 0)
3	(3, 3)
4	(4, 6)

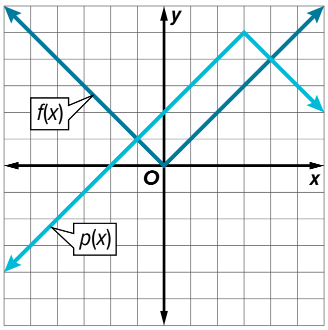
The graph of $j(x) = |3x - 6|$ is the graph of the parent function compressed horizontally and translated 2 units right.

The domain is ______________. The range is ____________________

______________.

Example 14 Graph an Absolute Value Function with Translations and Reflection

Graph $p(x) = -|x - 3| + 5$. State the domain and range.



In $p(x) = -|x - 3| + 5$, the parent function is reflected across the x-axis because the absolute value is being multiplied by -1.

The function is then translated 3 units right.

Finally, the function is translated 5 units up.

$p(x) = -|x - 3| + 5$ is the graph of the parent function translated 3 units ____ and 5 units ____ and reflected across the ____.

The domain is ______________. The range is ________________

______________.

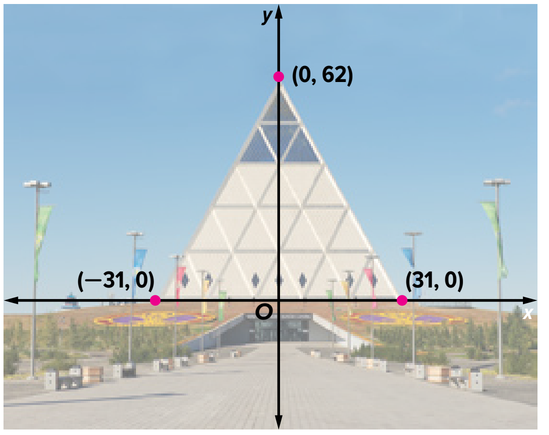
Think About It!

How is the vertical translation k of an absolute value function related to its range?

Go Online You can complete an Extra Example online.

Example 15 Apply Graphs of Absolute Value Functions

BUILDINGS **Determine an absolute value function that models the shape of The Palace of Peace and Reconciliation.**



To write the equation for the absolute value function, we must determine the values of a, h, and k in $f(x) = a|x - h| + k$ from the graph.

If we consider the absolute value as a piecewise function, we can find the slope of one side of the graph to determine the value of a.

Because this function opens downward, the graph is a reflection of the parent graph across the x-axis. So we know that the a-value in the equation should be negative.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$	The Slope Formula
$= \underline{\qquad}$	$(0, 62) = (x_1, y_1)$ and $(31, 0) = (x_2, y_2)$
$= \underline{\qquad}$ or $\underline{\qquad}$	

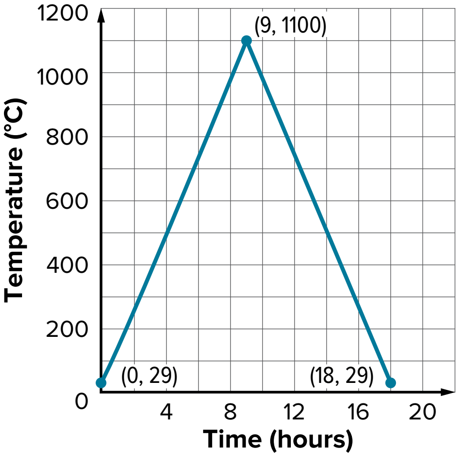
Next, notice that the vertex is not located at the origin. It has been translated. The absolute value function is not shifted left or right, but has been translated 62 units up from the origin.

$y = \underline{\qquad\qquad}$	$a = -2, h = 0, k = 2$
$y = \underline{\qquad\qquad}$	Simplify.

So, $y = \underline{\qquad\qquad}$ models the shape of The Palace of Peace and Reconciliation.

Check

GLASS PRODUCTION Certain types of glass heat and cool at a nearly constant rate when they are melted to create new glass products. Use the graph to determine the equation that represents this process.



$y = \underline{\qquad} |x - \underline{\quad}| + \underline{\qquad}$

Go Online to practice what you've learned about graphing special functions in the Put It All Together over Lessons 4–6 through 4–7.

Go Online You can complete an Extra Example online.

Name ________________________ Period ____________ Date ____________

Practice

Go Online You can complete your homework online.

Examples 1 through 3

Describe the translation in $g(x)$ as it relates to the graph of the parent function.

1. $g(x) = |x| - 5$

2. $g(x) = |x + 6|$

3. $g(x) = |x - 2| + 7$

4. $g(x) = |x + 1| - 3$

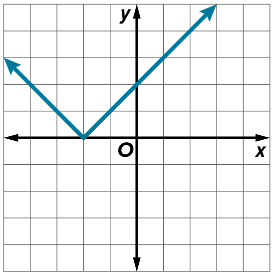
5. $g(x) = |x| + 1$

6. $g(x) = |x - 8|$

Examples 4 and 5

Use the graph of the function to write its equation.

7.



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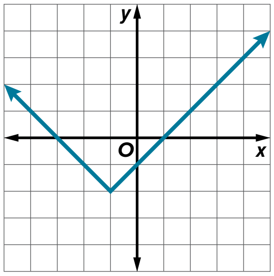

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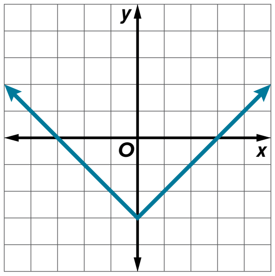

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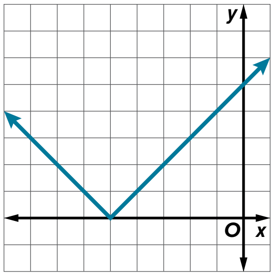

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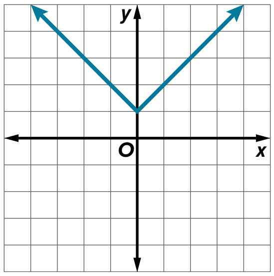

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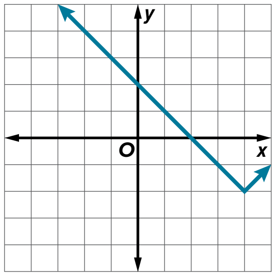


Examples 6 through 8

Describe the dilation in $g(x)$ as it relates to the graph of the parent function.

13. $g(x) = \frac{2}{5}|x|$

14. $g(x) = |0.7x|$

15. $g(x) = 1.3|x|$

16. $g(x) = |3x|$

17. $g(x) = \left|\frac{1}{6}x\right|$

18. $g(x) = \frac{5}{4}|x|$

Examples 9 through 11

Describe how the graph of $g(x)$ is related to the graph of the parent function.

19. $g(x) = -3|x|$

20. $g(x) = -|x| - 2$

21. $g(x) = \left|-\frac{1}{4}x\right|$

22. $g(x) = -|x - 7| + 3$

23. $g(x) = |-2x|$

24. $g(x) = -\frac{2}{3}|x|$

Examples 12 through 14

USE TOOLS **Graph each function. State the domain and range.**

25. $g(x) = |x + 2| + 3$

26. $g(x) = |2x - 2| + 1$

27. $f(x) = \left|\frac{1}{2}x - 2\right|$

28. $f(x) = |2x - 1|$

29. $f(x) = \frac{1}{2}|x| + 2$

30. $h(x) = -2|x - 3| + 2$

31. $f(x) = -4|x + 2| - 3$

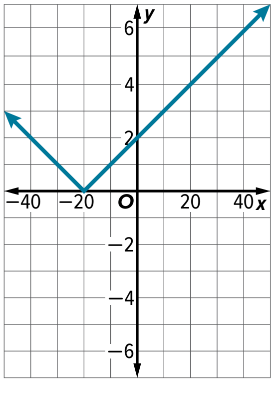
32. $g(x) = -\frac{2}{3}|x + 6| - 1$

33. $h(x) = -\frac{3}{4}|x - 8| + 1$

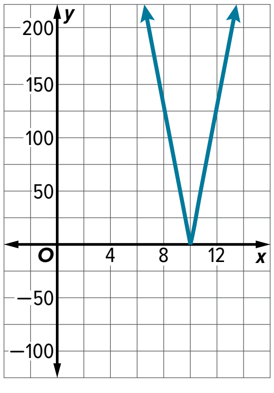
Example 15

Determine an absolute value function that models each situation.

34. ESCALATORS An escalator travels at a constant speed. The graph models the escalator's distance, in floors, from the second floor x seconds after leaving the ground floor.



35. TRAVEL The graph models the distance, in miles, a car traveling from Chicago, Illinois is from Annapolis, Maryland, where x is the number of hours since the car departed from Chicago, Illinois.



Mixed Exercises

MODELING **Graph each function. State the domain and range. Describe how each graph is related to its parent graph.**

36. $f(x) = -4|x - 2| + 3$

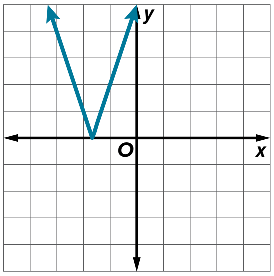
37. $f(x) = |2x|$

38. $f(x) = |2x + 5|$

Name ______________________ Period ____________ Date ____________

Use the graph of the function to write its equation.

39.



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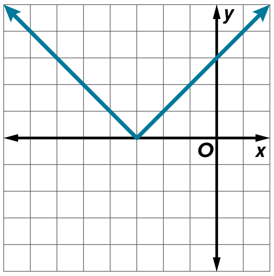

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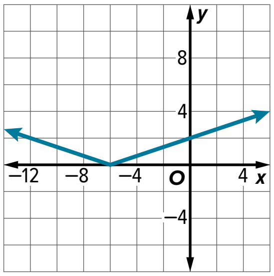

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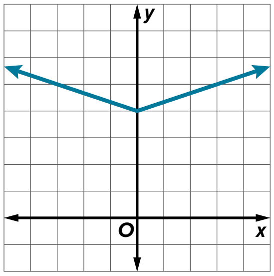


43. SUNFLOWER SEEDS A company produces and sells bags of sunflower seeds. A medium-sized bag of sunflower seeds must contain 16 ounces of seeds. If the amount of sunflower seeds s in the medium-sized bag differs from the desired 16 ounces by more than x, the bag cannot be delivered to companies to be sold. Write an equation that can be used to find the highest and lowest amounts of sunflower seeds in a medium-sized bag.

44. REASONING The function $y = \frac{5}{4}|x - 5|$ models a car's distance in miles from a parking lot after x minutes. Graph the function. After how many minutes will the car reach the parking lot?

45. STATE YOUR ASSUMPTION A track coach set up an agility drill for members of the track team. According to the coach, 21.7 seconds is the target time to complete the agility drill. If the time differs from the desired 21.7 seconds by more than x, the track coach may require members of the track team to change their training. Write an equation that can be used to find the fastest and slowest times members of the track team can complete the agility drill so that their training does not have to change. If $x = 3.2$, what can you assume about the range of times the coach wants the members of the track team to complete the agility drill? Solve your equation for $x = 3.2$ and use the results to justify your assumption.

46. SCUBA DIVING The function $y = 3|x - 12| - 36$ models a scuba diver's elevation in feet compared to sea level after x minutes. Graph the function. How far below sea level is the scuba diver at the deepest point in their dive?

47. **MANUFACTURING** A manufacturing company produces boxes of cereal. A small box of cereal must have 12 ounces. If the amount of cereal b in a small box differs from the desired 12 ounces by more than x, the box cannot be shipped for selling. Write an equation that can be used to find the highest and lowest amounts of cereal in a small box.

48. **STRUCTURE** Amelia is competing in a bicycle race. The race is along a circular path. She is 6 miles from the start line. She is approaching the start line at a speed of 0.2 mile per minute. After Amelia reaches the start line, she continues at the same speed, taking another lap around the track.

    a. Organize the information into a table. Include a row for time in minutes x, and a row for distance from start line $f(x)$.

    b. Draw a graph to represent Amelia's distance from the start line.

49. **WRITE** Use transformations to describe how the graph of $h(x) = -|x + 2| - 3$ is related to the graph of the parent absolute value function.

50. **ANALYZE** On a straight highway, the town of Garvey is located at mile marker 200. A car is located at mile marker x and is traveling at an average speed of 50 miles per hour.

    a. Write a function $T(x)$ that gives the time, in hours, it will take the car to reach Garvey. Then graph the function on the coordinate plane.

    b. Does the graph have a maximum or minimum? If so, name it and describe what it represents in the context of the problem.

51. **PERSEVERE** Write the equation $y = |x - 3| + 2$ as a piecewise-defined function. Then graph the piecewise function.

52. **CREATE** Write an absolute value function, $f(x)$, that has a domain of all real numbers and a range that is greater than or equal to 4. Be sure your function also includes a dilation of the parent function. Describe how your function relates to the parent absolute value graph. Then graph your function.

Review

Essential Question

What can a function tell you about the relationship that it represents?

Module Summary

Lessons 4-1 through 4-3

Graphing Linear Functions, Rate of Change, and Slope

- The graph of an equation represents all of its solutions.
- The *x*-value of the *y*-intercept is 0. The *y*-value of the *x*-intercept is 0.
- The rate of change is how a quantity is changing with respect to a change in another quantity. If *x* is the independent variable and *y* is the dependent variable, then rate of change $= \frac{\text{change in } y}{\text{change in } x}$.
- The slope *m* of a nonvertical line through any two points can be found using $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- A line with positive slope slopes upward from left to right. A line with negative slope slopes downward from left to right. A horizontal line has a slope of 0. The slope of a vertical line is undefined.

Lesson 4-4

Transformations of Linear Functions

- When a constant *k* is added to a linear function *f*(*x*), the result is a vertical translation.
- When a linear function *f*(*x*) is multiplied by a constant *a*, the result $a \cdot f(x)$ is a vertical dilation.
- When a linear function *f*(*x*) is multiplied by −1 before or after the function has been evaluated, the result is a reflection across the *x*- or *y*-axis.

Lesson 4-5

Arithmetic Sequences

- An arithmetic sequence is a numerical pattern that increases or decreases at a constant rate called the common difference.
- The *n*th term of an arithmetic sequence with the first term a_1 and common difference *d* is given by $a_n = a_1 + (n - 1)d$, where *n* is a positive integer.

Lessons 4-6, 4-7

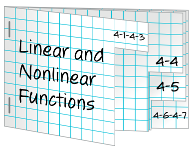
Special Functions

- A piecewise-linear function has a graph that is composed of a number of linear pieces.
- A step function is a type of piecewise-linear function with a graph that is a series of horizontal line segments.
- An absolute value function is V-shaped.

Study Organizer

Foldables

Use your Foldable to review this module. Working with a partner can be helpful. Ask for clarification of concepts as needed.



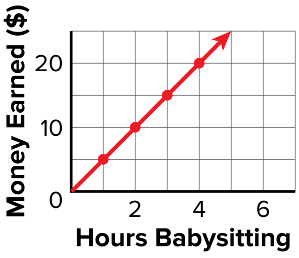
Name ______________________ Period __________ Date __________

Test Practice

1. GRAPH Jalyn made a table of how much money she will earn from babysitting. (Lesson 4-1)

Hours Babysitting	Money Earned
1	5
2	10
3	15
4	20

Use the table to graph the function in the coordinate grid.

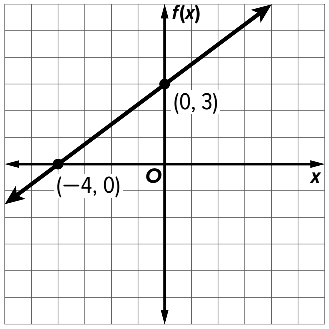


2. TABLE ITEM What are the missing values in the table that show the points on the graph of $f(x) = 2x - 4$? (Lesson 4-1)

x	-2	0	2	4	6
$f(x)$	-8	-4			

3. OPEN RESPONSE Mr. Hernandez is draining his pool to have it cleaned. At 8:00 A.M., it had 2000 gallons of water and at 11:00 A.M. it had 500 gallons left to drain. What is the rate of change in the amount of water in the pool? (Lesson 4-2)

4. MULTIPLE CHOICE Find the slope of the graphed line. (Lesson 4-2)



(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

5. MULTIPLE CHOICE Determine the slope of the line that passes through the points (4, 10) and (2, 10). (Lesson 4-2)

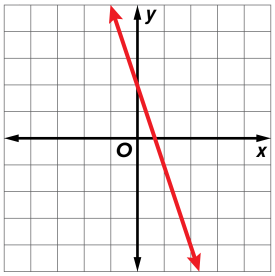
(A) -1

(B) 0

(C) 1

(D) undefined

6. GRAPH Graph the equation of a line with a slope of -3 and a y-intercept of 2. (Lesson 4-3)



7. MULTIPLE CHOICE What is the slope of the line that passes through (3, 4) and (−7, 4)? (Lesson 4-3)

(A) 0

(B) undefined

(C) -2

(D) -10

Name ______________________ Period ____________ Date ____________

8. **MULTIPLE CHOICE** A teacher buys 100 pencils to keep in her classroom at the beginning of the school year. She allows the students to borrow pencils, but they are not always returned. On average, she loses about 8 pencils a month. Write an equation in slope-intercept form that represents the number of pencils she has left y after a number of x months. (Lesson 4-3)

   (A) $y = -8x - 100$
   (B) $y = -8x + 100$
   (C) $y = 8x + 100$
   (D) $y = 8x - 100$

9. **OPEN RESPONSE** Name the transformation that changes the slope, or the steepness of a graph. (Lesson 4-4)

10. **OPEN RESPONSE** Describe the dilation of $g(x) = \frac{1}{2}(x)$ as it relates to the graph of the parent function, $f(x) = x$. (Lesson 4-4)

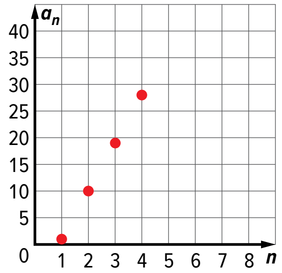
11. **MULTIPLE CHOICE** Arjun begins the calendar year with $40 in his bank account. Each week he receives an allowance of $20, half of which he deposits into his bank account. The situation describes an arithmetic sequence. Which function represents the amount in Arjun's account after n weeks? (Lesson 4-5)

   (A) $f(n) = 20n + 40$
   (B) $f(n) = 40n + 20$
   (C) $f(n) = 40 + 10n$
   (D) $f(n) = 10 + 40n$

12. **OPEN RESPONSE** What number can be used to complete the equation below that describes the nth term of the arithmetic sequence $-2, -1.5, -1, 0, 0.5, \ldots$? (Lesson 4-5)

   $a_n = 0.5n -$ ____

13. **OPEN RESPONSE** Write and graph a function to represent the sequence 1, 10, 19, 28, … (Lesson 4-5)



14. **OPEN RESPONSE** Christa has a box of chocolate candies. The number of chocolates in each row forms an arithmetic sequence, as shown in the table. (Lesson 4-5)

Row	1	2	3	4
Number of Chocolates	3	6	9	12

Write an arithmetic function that can be used to find the number of chocolates in each row.

Name ______________________ Period __________ Date __________

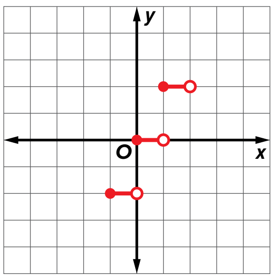
15. TABLE ITEM Daniel earns \$9 per hour at his job for the first 40 hours he works each week. However, his pay rate increases to \$13.50 per hour thereafter. This situation can be represented with the function

$$f(x) = \begin{cases} 9x, \text{ if } x \leq 40 \\ 360 + 13.5(x - 40), \text{ if } x > 40 \end{cases}$$

Use this function to complete the table with the correct values. (Lesson 4-6)

Hours Worked, x	Money Earned, $f(x)$
30	
35	315
40	
45	427.5
50	

16. GRAPH Graph the function $f(x) = 2[[x]]$. (Lesson 4-6)



17. MULTIPLE CHOICE Which of the following describes the effect a dilation has upon the graph of the absolute value parent function? (Lesson 4-7)

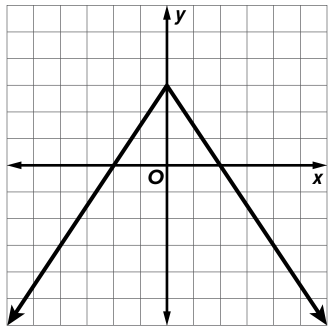
Ⓐ Flipped across axis

Ⓑ Stretch or compression

Ⓒ Rotated about the origin

Ⓓ Shifted horizontally or vertically

18. MULTI-SELECT Describe the transformation(s) of the function graphed below in relation to the absolute value parent function. Select all that apply. (Lesson 4-7)



Ⓐ Reflected across x-axis

Ⓑ Vertical stretch

Ⓒ Vertical compression

Ⓓ Reflected across y-axis

Ⓔ Translated right 3

Ⓕ Translated up 3

19. OPEN RESPONSE Describe the graph of $g(x) = |x| + 5$ in relation to the graph of the absolute value parent function. (Lesson 4-7)

20. OPEN RESPONSE Across which axis is the graph of $h(x) = -5|x|$ reflected? (Lesson 4-7)

21. OPEN RESPONSE Use the graph of the function to write its equation. (Lesson 4-7)