



Grades 6–12

Reveal MATH™

Instructional Model
Research Base





Introduction

At the core of the middle school and high school *Reveal Math™* program are eight key areas of research that have emerged from a number of learning science domains. These research foundations have informed the way we crafted the program, ranging from the instructional model and scope and sequence, right down to the design of individual tasks. These areas are the rich tasks, productive struggle, mathematical language, collaborative learning, mathematical discourse, valuing common student errors/misconceptions as learning opportunities, learning targets, and formative assessment. This brochure highlights the recommendations from research in these areas and reviews how each area informed the design of the *Reveal Math* program.

In addition to the academic research outlined here, several other critical inputs contributed to the development of the program, including extensive field testing, focus groups, user testing, and direct feedback from hundreds of educators in the field.

Lesson Model

Key Research Areas

- Rich Tasks
- Productive Struggle
- Mathematical Language
- Collaborative Learning
- Mathematical Discourse
- Error Perception/Misconceptions
- Learning Targets
- Formative Assessment

Module Components

- Essential Question (Student Edition)
- What Will You Learn? (Student Edition)
- What Vocabulary Will You Learn? (Student Edition)
- Are You Ready? (Student Edition)
- Foldables® (Student Edition)
- Ignite! Activity (online)
- Launch the Module Videos (online)
- Cheryl Tobey Formative Assessment Math Probes (Teacher Edition)
- Mindset Matters (Teacher Edition)

Launch

WARM UP

Daily warm-up covering prerequisite skills needed for lesson. Introduction of standards (both content and practices) and lesson vocabulary.

LAUNCH THE LESSON

Hook to engage students; real-world when appropriate; includes 1-2 questions students should be able to answer by the end of the lesson.

Explore and Develop

EXPLORE

Students explore new concepts through rich tasks that promote productive struggle and mathematical discourse.

LEARN

Students' emerging understanding of lesson concepts is formalized through guided instruction.

EXAMPLES & CHECK

Students work through one or more Examples tied to the key concepts, followed by a quick Check (formative assessment) to see if students “get it.”

(Note: 6–8 Math has a Check after each Example; 9–12 Math has a Check after one or more Examples.)

Reflect and Practice

EXIT TICKET

Students complete a quick formative assessment that encourages them to reflect on their learning during the lesson.

PRACTICE

Students complete practice problems that help them to apply what they've learned and build procedural fluency.

I Can... understand the meaning of a percent as a rate per 100, and model percents using 10×10 grids and bar diagrams.

The screenshot shows a lesson page for 'Understand Percents' (Lesson 2-1). It features an 'I Can...' statement, a 'Learn' section with a definition of percent and an example of a 10x10 grid, and a 'Talk About It!' section with a question about a shaded grid. A blue box at the bottom of the screenshot reads 'I Can statement, Course 1 Student Edition'.

Launch

What It's About

Each lesson opens with a warm-up routine as well as an introduction to the objectives of the lesson ahead. This Launch design serves to both activate students' prior knowledge and to orient them to the concepts and targeted skills that they will be developing.

Key Design Attributes

- The **Warm-Up** helps students connect lesson content with prerequisite skills and prior knowledge in order to prepare for new concepts in the lesson.
- **"I Can"** statements (also known as learning targets or learning intentions) are introduced to students in age-appropriate and mathematically-accurate language.
- The **Launch the Lesson** resource introduces students to the concepts in the lesson in a real-world context.

Why We Engineered It This Way

"Introducing students to the learning target and revisiting this target throughout a core set of learning activities (a lesson) is foundational to the formative assessment process."

(Tobey 2017, p. 1)

The first part of the Launch is a warm-up question that helps students reflect on what they already know about a concept relevant to the new material that will be introduced. Research has shown that activating prior knowledge helps learners to more accurately and easily acquire new concepts and ideas. At the same time, it gives students a chance to practice familiar routines and build fluency with those skills (Kelemanik, Lucenta, & Creighton, 2016; Newton, 2014).

As Hattie and colleagues (2017, p. 44) note, “Once prior knowledge is activated, students can make connections between their knowledge and the lesson’s learning intentions,” which in *Reveal Math* are the next elements of the Launch phase. Learning targets for each lesson are crafted so that they employ language that is simultaneously student-friendly and mathematically accurate. These “I Can” statements help to frame the activities that follow. They also serve as meaningful targets that students return to at the end of the lesson to self-assess what they have learned and reflect on how they got there.

Explore

What It’s About

- The whole class observes a mathematical situation or problem and makes observations about the question or problem to be explored.
- Students then engage in an exploration of the content, freely generating their own models and possible strategies in small groups.
- Students are asked to share and discuss their strategies and engage in a broader discussion with the class about how and why those strategies worked (or didn’t work).

Key Design Attributes

- The Explore phase is anchored in a rich task, designed to drive inquiry, provide a high-cognitive demand, allow for multiple entry points, and that often employs multiple strategies.
- Each student records their thinking, models, and approach (and may return to this page later on in the lesson after further development of key concepts and strategies).
- It is not necessary for each student to arrive at a solution or full understanding of the problem raised in the Explore task; rather, the intention is to give every student an opportunity to grapple with the concepts and potential strategies.
- Teachers are provided with guidance around potential strategies, misconceptions, questions for generating discourse, and for sharing and encouraging students to visually show and explain their reasoning to others in the group.
- For students who have difficulty getting started, the teacher notes include sample questions for them to use in supporting the activity without intervening too explicitly (in a way that might inadvertently undermine the pedagogical purpose of the task).



Explore activity, Course 3
Student Digital Center

Why We Engineered It This Way

"Mathematics educators and researchers suggest that struggling to make sense of mathematics is a necessary component of learning mathematics with understanding...Start by giving students an engaging problem (either contextual or purely mathematical), and have them explore it, either alone or, preferably, with a partner or within a small group... When students have some time to explore and even struggle with a problem, our role as teacher becomes one of facilitating and stimulating conversation among students to ensure that they uncover and discuss the important mathematical ideas that lie within the problem. We want to structure ways for students to talk about different approaches they might have tried or solutions they've found. We try to ensure that students justify their thinking by responding to our probing questions and eventually learn to ask their own clarifying questions of each other."

(Seeley, 2016, p. 33)

At the heart of recent research on productive struggle is a certain tension between what we know with increasing confidence is best for learners and what for many well-meaning educators is a natural inclination to reduce discomfort and difficulty for the learner (Pasquale, 2015, p. 1; Seeley, 2016, p. 22). Research has shown that teachers often perceive student struggle or delay at arriving at correct answers as a reason to "rescue" students and show them an effective procedure too soon, ultimately to the detriment of students' learning and conceptual understanding of new mathematical ideas (NCTM 2014, p. 48). A number of experts have concluded that students' struggles with learning mathematics are often viewed as a problem and cast in a negative light in mathematics classrooms (Hiebert & Wearne, 1993; Borasi, 1996), which harms both engagement and learning outcomes.

Ideally, learning environments ought to instead frame struggle and occasional mistakes as a natural and positive part of progressing toward understanding.

In an Edutopia article, Ellie Cowen (2016) highlights five important benefits of a classroom environment that support productive struggle:

1. Prioritizing student-centered lesson activities.
2. Building student engagement.
3. Emphasizing that math makes sense to the student.
4. Creating ample opportunity for assessment, intervention, and feedback.
5. Building perseverance.

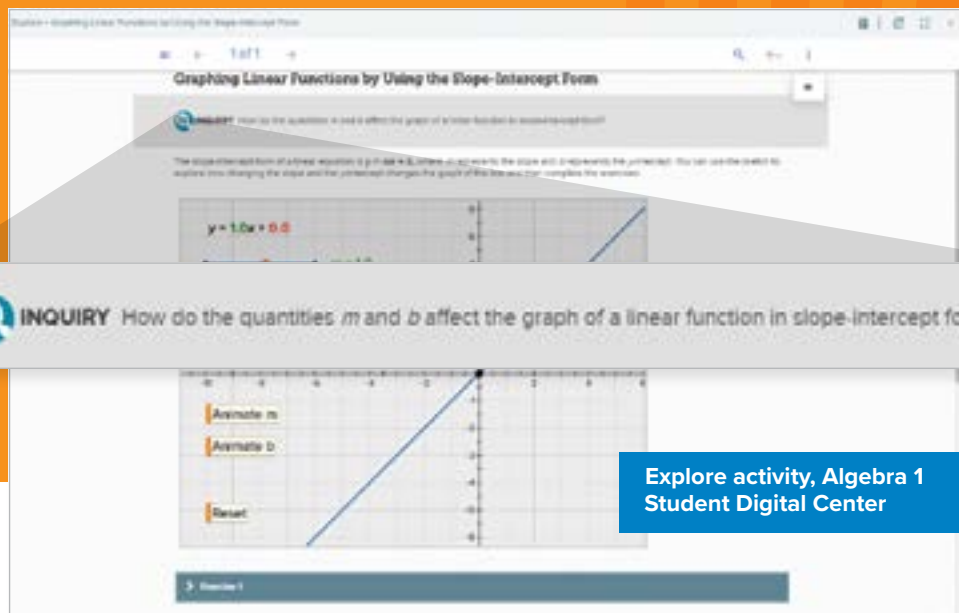


Among the hallmarks of productive struggle in practice is the notion that for student-centered activities to have their intended impact, they must appear in the lesson (and be allotted necessary time) in such a way that allows for them to take center stage, rather than appearing as optional or time-permitting only (Cowen, 2016). The instructional model of *Reveal Math* aligns with this idea by typically featuring the Explore activity at the outset of the lesson, essentially posing the problem to students before teaching particular methods, as advocated by leading researchers in mathematical instruction (Boaler & Dweck, 2016, p. 81).

During these activities early in the lesson, students are introduced to a new concept by starting with a rich task that has multiple points of entry (Hattie, 2017, pp.72–73 and p. 181)—or, put differently, a task that has a “low floor and high ceiling” (see Boaler & Dweck, 2016, pp. 84–85). Individual learners are encouraged to suggest strategies for working through the problem before procedures, formulas, and new concepts are formally introduced. While teacher-led instruction also plays an important role in each lesson, it is intentionally sequenced after this exploratory activity.

Many leading researchers identify productive struggle as an essential component of effective mathematics classrooms (Boaler & Dweck, 2016; Kapur, 2014; Sternberg & Preiss, 2010; see also Hattie, 2014, p. 85 on how productive struggle relates to perseverance and rich discourse) and urge teachers to include opportunities for students to attempt solving problems that target concepts that are new to them, rather than limiting those opportunities to tasks with familiar/ known skills (Hattie, 2017, p. 117).

Critically, during the Explore activity, it is not necessarily important that each student arrives at the correct solution to the problem/ task at this phase of the lesson—rather, the aim is to elicit students’ intuitions and existing knowledge, and to give each student an opportunity to engage in reasoning about the nature of the problem (e.g., What would you like to know? How is this problem like problems you have seen before?). This task is designed to maximize engagement and set the stage for the introduction of new concepts, vocabulary, and procedures later on in the lesson.



According to leading math education expert Linda Gojak (2017), in “A Key to Deep Understanding: The Importance of Rich Tasks in K–12 Mathematics,” rich tasks are distinguished by seven key characteristics, namely they:

1. Are engaging.
2. Require substantive mathematics to reach a solution.
3. Are accessible to all students in the class.
4. Have multiple solution paths.
5. Have multiple solutions.
6. Encourage student discourse and questions.
7. Treat the solution process and the solution as being of equal importance.

While not all rich tasks exhibit every one of these characteristics, the list to the left calls attention to features that distinguish rich tasks from other sorts of mathematical exercises and problems (e.g., which may be appropriate in certain circumstances, for students to practice with skills and strategies they’ve learned). In an often-cited rubric for a rich mathematical task, a problem is qualitatively evaluated as rich to the extent that it builds conceptual understanding, requires students to engage in explanation and justification, and involves using mathematical language in an age-appropriate and mathematically-accurate way (see Hattie, 2017, p. 65, for more detail on this rubric-based definition).

It is also important to note that rich tasks can take the form of a contextualized word problem or a problem presented strictly with numbers; what is essential for a rich task is not the appearance of complexity, but rather the underlying cognitive demand and deeper conceptual thinking that the task promotes for the learner.

Develop

What It's About

- The teacher formalizes students' understanding through guided instruction via **Learn** resources, **Examples**, and discussion.
- The students discuss and relate multiple representations, strategies, and procedures. They also have to explain why they work and record their own thinking while doing so.
- Students work through one or more examples tied to the key concepts, followed by a quick formative assessment, called **Check**, to ensure their understanding.
- The **Check** is an embedded formative assessment opportunity designed to inform the teacher as to an individual student's current levels of concept mastery, fluency and confidence, and to help identify skill gaps.

Key Design Attributes

- Teacher-guided examples are connected to strategies used and uncovered during the Explore activity (and students' own discussion of their strategies and thinking).
- Teachers build a “conceptual bridge” to help students make connections between conceptual understanding, problem contexts, and the use of strategies/procedures.
- Students are encouraged to explain and justify their thinking to their peers, supported by appropriate teacher prompts and questioning.
- Teachers are equipped with advance knowledge of common misconceptions related to the lesson content, so they can anticipate the emergence of those errors in students' discussion and provide scaffolding accordingly.
- Students spend the majority of their time during this phase connecting and applying skills and strategies (as opposed to learning skills and procedures in isolation).
- Ample resources are provided and meaningfully integrated into the lesson for “**Talk About It**” and “**Think About It**” activities, which support student reflection and provide opportunities for students to engage in meaningful discourse as they learn new procedures and vocabulary.
- The **Check** question is designed to be completed by students in a short amount of time in order to provide the teacher with information about how each student should proceed in the lesson.
- Opportunities for differentiation are clearly called out in teacher notes, facilitating an efficient transition to additional small group work, targeted remediation, and extension activities.

Questions for Mathematical Discourse
SLIDE 2

AL What does the graph describe? **the total cost based on the number of muffins purchased.**

OL What is the change in y -values between the two points? the change in x -values? **The y -value increases by 2 and the x -value increases by 1.**

OL Study the graph. Is this relationship proportional? Explain. If so, which point on the graph illustrates the slope? **Yes, the relationship is proportional because the graph is a straight line that passes through the origin. The slope is illustrated by the point (1, 2), which is also the unit rate.**

BL Find two other points that lie on this line. **Sample answer: (4, 8) and (5, 10)**

BL Describe the relationship between each x -value and its corresponding y -value using words and an equation. **Sample answer: Each y -value is equal to twice its corresponding x -value; $y = 2x$**

Find additional teaching notes and the talk about it question to promote mathematical discourse.

View performance reports of the Checks.

Assign or present an Extra Example.

152 Module 4 • Linear Relationships and Slope

Why We Engineered It This Way

“Questions about the task that get students to think about errors or confusions are particularly helpful in encouraging students’ self-reflections. And they will often result in the students’ understanding the mathematics for the first time. Such questions also give the teacher really important information that can guide their teaching.”

(Boaler & Dweck, 2016, p. 47)

“Research indicates that making formative assessment processes an integral component of instruction is associated with improved student learning.”

(NCTM 2014, p. 94)

When deployed at the appropriate stage of the lesson, teacher-guided instruction helps to reinforce student-driven learning and discovery, building upon the exploration and discussion activities. There is evidence that helping students to build computational and procedural skills can be supported through direct instruction (Flores & Kaylor, 2007), particularly when it follows students’ own exploration of a concept—rather than being used as the first introduction of a new mathematical concept. In order to help bridge conceptual understanding and procedural fluency, NCTM recommends that the teacher play an active role in “connecting student-generated strategies and methods to more efficient procedures as appropriate” (NCTM, 2014, p. 47). Including some teacher-led instruction in the correct context and stage is known to be helpful to math learners generally, but it is also worth noting that researchers have found it to be particularly crucial when supporting struggling learners (e.g., Morgan, Farkas, & Maczuga, 2015).

When learners are asked to explain their thinking—either to a peer/teacher, or in a self-explanation exercise—the act of justifying their reasoning has been shown to both enhance students’ conceptual understanding and to boost engagement (National Council of Teachers of Mathematics (NCTM), 2013a; Whitenack & Yackel, 2002). In fact, many researchers have found that when discussion allows for disagreements to arise between students’ justifications, there are opportunities for uncovering misconceptions that might have otherwise gone undetected; these also provide learners with a chance to perceive their own thinking more clearly, in the course of making it transparent to others (Horn, 2008; Rittle-Johnson, 2006).



To support effective self-questioning or collaboration, teachers can best support students by utilizing prompts that encourage reflection and ask students to justify their reasoning or choice of strategy—or at a minimum, to elaborate on their thought process (Booth et al., 2017; Hattie, 2017, p. 152). *Reveal Math* provides these prompts in both student- and teacher-facing materials.

This approach allows learners improved metacognitive insights into their own thinking—connecting intuition, model, and conceptual representation—and is at the very heart of the mathematical practices that foster deep learning (Hattie, 2017, p. 136). Further, it aligns with a movement away from the teacher as the sole authority in the classroom, and towards a more effective and engaging mode of student-driven learning. An added benefit to this approach is that when teachers use strong focusing questions, they are also modeling how to ask clarifying questions in a way that will serve students better in later phases of learning—when they ask themselves those clarifying questions.

Small group discussion involving the justification of an individual student’s reasoning also provides an opportunity to reinforce the notion of valuing mistakes and framing them in a safe, positive way for learners (Tobey, 2017; Steuer et al., 2013), which is linked to both stronger outcomes and levels of engagement.

Think About It!
What are the locations of the angles with respect to the parallel lines?

Example 2 • Classify Angle Pairs
Classify the relationship between $\angle 2$ and $\angle 6$ in the figure as alternate interior, alternate exterior, or corresponding.

Think About It!
What are the locations of the angles with respect to the parallel lines?

Think About It!
Name another pair of corresponding angles. How many pairs of corresponding angles are there when two parallel lines are cut by a transversal? Will this happen when any two parallel lines are cut by a transversal? Explain.

Pause and Reflect
How did what you know about alternate interior, alternate exterior, and corresponding angles help you solve the problem?

Think About It and Talk About It, Course 3 Student Edition

Practice & Reflect

What It's About

- Students complete a quick formative assessment, the **Exit Ticket**, so teachers can determine whether students have a conceptual understanding of the lesson.
- The students work either independently or in small groups to collaboratively work through **Practice** problems, applying strategies and skills learned/discussed during the **Explore and Develop** phase.

Key Design Attributes

- Students are encouraged to reflect on their learning targets and think, “How am I making progress? How can I show that I am making progress?”
- Many problems allow for multiple approaches and/or models and strategies to be applied, such that the conceptual connections between strategies emerges through the collaborative small group work.

DOK	Topic	Exercises
1,2	exercises that mirror the examples	1–7
2	exercises that use a variety of skills from this lesson	8–10
2	exercises that extend concepts learned in this lesson to new contexts	11–16
3	exercises that emphasize higher-order and critical thinking skills	17–23

3 REFLECT AND PRACTICE

Practice and Homework

The Practice pages are meant to be used as a homework assignment. Students can complete the practice exercises in their Interactive Student Edition.

Suggested Assignments

Use the table below to select appropriate exercises.

DOK	Topic	Exercises
1,2	exercises that mirror the examples	1–7
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3	exercises that emphasize higher-order and critical thinking skills	17–23

ASSESS AND DIFFERENTIATE

Use the data from the Checks to determine whether to provide resources for extension, remediation, or intervention.

IF students score 90% or more on the Checks, THEN assign:

- Practice, Exercises 1–15 odd, 17–23
- Extension: Axioms and Propositions
- ALEKS: Proofs Involving Segments and Angles

IF students score 66–89% or more on the Checks, THEN assign:

- Practice, Exercises 1–23 odd
- Remediation: Deductive Reasoning
- Watch the Personal Tutors again.
- Extra Examples 1–3
- ALEKS: making a valid argument about algebra

IF students score less than 65% on the Checks, THEN assign:

- Practice, Exercises 1–7 odd
- ALEKS: making a valid argument about algebra

Assess and Differentiate, Geometry Teacher Edition

Lesson 3.5: Proving Segment Relationships 195-196

Why We Engineered It This Way

“The role of the learner is not to passively receive information, but to actively participate in the construction of new meaning.”

(Shapiro, 1994, p.8)

“Teachers need better ways of determining where their students are in their thinking and understanding prior to and throughout the instructional process. Students need to be actively involved in the assessment process so that they are learning through assessment as well as providing useful feedback to the teacher.”

(Keeley & Tobey, 2011, p. 10)

At this phase of the instructional model, each lesson concludes with an Exit Ticket—an embedded formative assessment component—which provides the teacher with feedback about an individual student’s current level of conceptual understanding. Inclusion and utilization of these short-cycle assessments have been shown across a number of studies to be effective at raising student achievement, with large effect sizes and relative ease of implementation with appropriate teacher supports (National Council of Teachers of Mathematics (NCTM), 2013b; Wiliam, 2010).

Formative assessment items in *Reveal Math* were carefully crafted to align with best practices emerging from research for math learners (e.g., Creighton et al., 2015).

In order for these formative assessments to be actionable and maximally impactful, it is important for teachers to be provided with information about how to proceed with each student based on his/her Check response and depth of understanding of the lesson content. The Assess and Differentiate chart in the Teacher Edition suggests appropriate resources for students based on their needs, while the DOK chart suggests specific Practice exercises for each level.

Further Reading

Several of the key research areas discussed in this paper are explored in a series of McGraw-Hill white papers authored by various K–12 education experts and *Reveal Math* advisors with whom we’ve consulted throughout the program’s development. We encourage you to review these white papers for a more extensive discussion of these key research topics. Ask your McGraw-Hill sales representative for a copy of any of the white papers mentioned below.



Cathy Seeley

Mathematical Discourse

Talking About Math: How K–12 Classroom Discourse Can Develop Mathematical Thinkers and Problem-Solvers



Cheryl Rose Tobey

Learning Targets

Developing Learning Targets

Math Probes

Identifying Student Misconceptions with Formative Assessment Math Probes



Linda Gojak

Rich Math Tasks

A Key to Deep Understanding: The Importance of Rich Tasks in K-12 Mathematics



Raj Shah

Fostering Interest in Math

Promoting a Math-Positive Classroom: A Guide for K–12 Educators



Ryan Baker

Digital Engagement

Data-Driven Instructional Practices: Why They Work

Family Involvement & Growth Mindset

A New Perspective on Math at Home



Dinah Zike

Foldables as Effective Graphic Organizers

Using Foldables® in the Classroom

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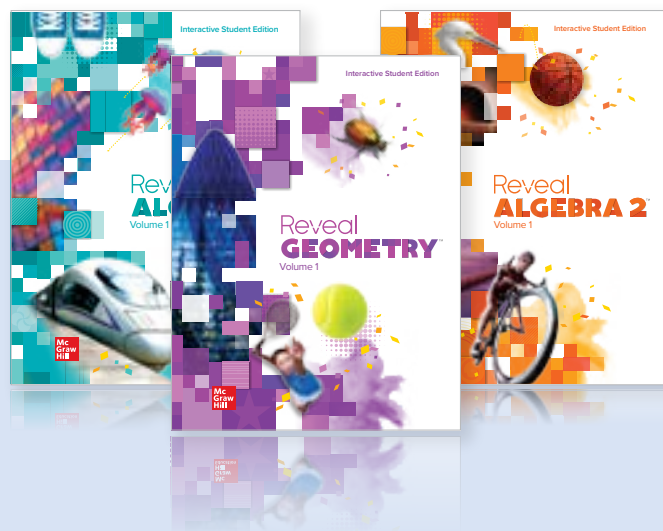
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Grades 6–12

6–8



9–12



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