



Promoting a Math-Positive Classroom: A Guide for K–12 Educators

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Dr. Raj Shah has always had an affinity for math. Powered by his love of math, he earned a Ph.D. in Physics in 1999, which led to a career in R&D at Intel. In 2008, he quit his job and founded Math Plus Academy, an after-school STEM enrichment program for kids from ages 5–14. His mission is to introduce kids and adults to the wonders of mathematics. Dr. Shah also contributes his time to Math Teacher Circles, the Julia Robinson Math Festival, and is a founding member of The Global Math Project. He believes that everyone can enjoy math, develop strong number sense, and become a perseverant problem solver.

Introduction

Children are born with a natural curiosity about the world they inhabit. Their curiosity includes fundamental mathematical notions such as quantity, shape, and patterns. Despite this natural human interest in mathematical ideas, math teachers have historically felt as if they are fighting a losing battle for the hearts and minds of students.

In a 2015 Ogilvy PR survey, 60% of Americans admitted to having difficulty dealing with numbers in simple, everyday situations. Nearly a third of those surveyed said they would rather clean the bathroom than try to solve a math problem. (Change the Equation, 2014).

This paper considers why this attitude towards math is so prevalent and what educators can do to address math anxiety or disinterest in K–12 classrooms.

Why Students Give Up on Math

Despite the best efforts of K–12 math educators, too many students lose interest in math or, worse, they just give up. The underlying issues can provide clues as to how to help students overcome negative emotions around math.

There are three main reasons why students give up on math:

1. A belief that mathematical talent is innate

The most pervasive and insidious myth about mathematics in our culture is that some people are born with an ability to do math and everyone else is not. The idea that a “math gene” might exist traps people into thinking they lack an innate ability to be successful in math. When a learner suffers from this false belief, any setback in his or her mathematical progress simply confirms to the learner that he/she does not possess the math gene. This thinking can lead to a vicious, negative feedback loop.

Sadly, a very large number of students (and adults) fall into this trap. In a poll of 1,000 American adults, over a third (36%) admit that they’ve found themselves saying, “I can’t do math.” For people ages 18–24, that number jumps to 53%! (Change the Equation, 2014).

It’s important to identify some of the key sources for this myth so that we can work to counter this false belief.

- In many instances, people like Einstein or Mozart are portrayed as geniuses who were simply born with special talents and abilities. Teachers and textbooks emphasize their achievements and leave out the stories of their challenges, failures, and setbacks. Stories of how these historical greats overcame challenges are conspicuously absent.
- We label students who show an early affinity for mathematics as “gifted.” This term reinforces the notion that some people have innate talent and others do not. We rarely identify students as gifted in art or music or even reading, but we regularly do so in math.
- Math is portrayed as difficult, meaningless, and uncool on television and in movies. The media is filled with images and scenes in which people disparage math, proudly admit their inability to do math, and/or portray people who enjoy math in a condescending manner. In the cases where movies show math in a positive light (e.g., *Good Will Hunting*, *A Beautiful Mind*), the focus is on mathematical genius—an innate quality that eludes most people as described above.

A study conducted at the Norwegian University of Science and Technology disproves the idea that math skills are innate by looking at how each individual math skill is attained. Researchers tested how well participants performed nine types of math tasks and found little to no correlation between being good at one skill versus another. For example, someone who can add fluently may not be good at word problems or geometry. Each math skill requires a different type of thinking, so one must practice each skill to be good at all of them. This understanding runs counter to the idea that math skills are innate, for if they were, those who are “good at math” would be naturally good at all math skills. The reality is that learners only master the math skills that they practice (Sigmundsson, 2014).

2. Failure in math is inevitable (eventually)

Unlike most school subjects, mathematical concepts get progressively more difficult from kindergarten through 12th grade and beyond. To make matters worse, math requires very diverse skills ranging from number sense, algebraic thinking, spatial representations, and much more. No one will excel in all these areas without experience and practice, and eventually, every math student will face what might feel like an insurmountable hurdle in mathematics. At these moments, learners must decide if they have “run out” of the natural ability to perform math or if they can overcome the challenge with more effort and practice.

3. School makes math boring

As mathematician, Paul Lockhart eloquently explains in his treatise, *A Mathematician’s Lament*, “If I had to design a mechanism for the express purpose of destroying a child’s natural curiosity and love of pattern-making, I couldn’t possibly do as good a job as is currently being done.” Lockhart goes on to explain how the current K–12 math education system presents math as a collection of unrelated facts, formulas, and procedures to memorize rather than a beautiful art, which is full of patterns, connections, and intriguing unsolved mysteries.

“By removing the creative process and leaving only the results of that process, you virtually guarantee that no one will have any real engagement with the subject, writes Lockhart” (Lockhart, 2009).

The only logical response to these factors is to find ways to re-engage students in the mystery and magic of mathematics. The following are suggestions for doing just that.

Three Key Steps for Supporting Math Enthusiasm and Achievement

The changes required to help students become more enthusiastic about math and to increase their achievement are more focused on mindset than skillset. When K–12 students believe that they can do math, and they are excited about accepting math challenges, their academic growth is assured.

Here are three ways K–12 math educators can encourage a more positive approach to math:

Step 1: Spark Curiosity (desire to learn)

Step 2: Develop Grit (ability to persist even when the path forward is unclear)

Step 3: Promote a Growth Mindset (how one copes with setbacks)

The good news is that teachers can make manageable changes in teaching methods and classroom culture to achieve all three goals.

STEP 1: SPARK CURIOSITY

“The greatest problem in education is the answering of questions that have not yet been asked.”

— Dr. Arthur Combs

The first step to better achievement is to begin every lesson with something that sparks student curiosity.

Curiosity Enhances Learning

A meta-analysis of research, which included over 200 studies and a total of about 50,000 students, found that curiosity does influence academic performance. In fact, it can have quite a large effect (about the same as conscientiousness). The study showed that, when put together, conscientiousness and curiosity can have as big an effect on performance as intelligence.

Co-author of the study, Sophie Von Stumm of the University of Edinburgh wasn't surprised that curiosity is so important to academic performance. “I'm a strong believer in the importance of a hungry mind for achievement, so I was just glad to finally have a good piece of evidence,” she says. “Teachers have a great opportunity to inspire curiosity in their students, to make them engaged and independent learners. That is very important” (Von Stumm, 2011).

Brain research has shown that people are better at learning information that they are curious about and that during states of high curiosity, memory is also enhanced (Gruber, 2014).

These findings highlight the importance of stimulating curiosity to create more effective learning experiences.

How We Betray Our Goals

- As educators, we know intuitively that curiosity and reasoning is important to learning. Yet, time and time again, we make choices that undermine what we know to be true.
- We say that reasoning is important, but our tests only reward correct answers.
- We say students should learn math deeply, but standardized tests force teachers to “cover” material superficially.
- We tell students that speed doesn't matter, but tests have time limits.
- We want students to achieve mastery, but after the unit test, teachers move on to the next topic.
- We tell students that it's OK to struggle, but we step in too quickly to help when they do.
- We want students to be curious, but we stress answer-getting over questioning.

How We Can Do Better: The Information Gap Theory

In his 1994 publication “The Psychology of Curiosity,” George Lowenstein posits his information-gap theory of curiosity. According to Lowenstein, curiosity arises when there is a gap between what one knows and what one wants to know. Such information gaps produce the feeling of deprivation labeled curiosity. The curious individual is motivated to obtain the missing information to reduce or eliminate the feeling of deprivation (Lowenstein, 1994).

Lowenstein enumerates several stimuli that can induce curiosity, including presenting a riddle or puzzle, exposure to a sequence of events with unknown resolution, and violation of expectations. These stimuli are enhanced when an individual generates a prediction of the outcome. The desire to know if the prediction is correct compounds the level of curiosity.

It is important to note that if the individual has too much background knowledge, they may perceive the gap to be too small and not be as curious. Conversely, if the individual has too little knowledge, they may perceive the gap to be too large to overcome and not be curious. This makes the teacher’s understanding of each child’s background-knowledge crucial in the generation of maximal curiosity.

Common Frameworks for Creating Information Gaps in the Mathematics Classroom

Framework 1: *Would You Rather*

This framework pits two similar scenarios against each other and asks students to make a prediction.

Example: In a store, you get 20% off but must pay 10% sales tax. Would you rather have the discount or the tax calculated first?

Example: Would you rather get a pound of nickels or a pound of dimes?

Example: Would you rather share 3 donuts with 4 friends or 7 donuts with 8 friends?

Resource: <http://www.wouldyourathermath.com>

Framework 2: *Partial Patterns*

This approach exposes students to a partial pattern to create an information gap that they will be compelled to try and close.

$$1=1$$

$$1+3=4$$

$$1+3+5=9$$

$$1+3+5+7=16$$

$$1+3+5+7+9=25$$

Example: Do you always get a square number when you add consecutive odd numbers? If so, why? If not, can you find a counter-example?

Framework 3: Start an Argument

Any time that educators can break expectations, curiosity will follow. In this case, the focus is on “controversial” questions. Choosing questions that expose common misconceptions are great fodder for arguments in math class.

Example: Is zero even or odd?

Example: Is 3.0 even because it ends with a zero?

Example: True or False: Multiplication always results in a larger number.

Teachers can compound the effect by forcing students to make an initial guess. Their desire to know if they are right will motivate their learning.

Framework 4: Pose Problems with an Open Middle

Posing problems that are easily stated and have a fixed goal can allow for many different solutions strategies. This is how the best video games are designed. Players know the starting condition, the rules, and the end goal. They are free to obtain that goal in any way that they can—an open middle. Also, giving students control of their choices increases their motivation to learn.

In many cases, simply taking an existing problem and switching the givens with the unknown can make an exercise feel like a riddle or a puzzle, which increases curiosity.

Example:

a) Traditional closed problem: Find the area of a rectangular garden that is 3 meters wide and 4 meters long.

The same problem with the givens and unknown swapped will result in an open question with many solutions.

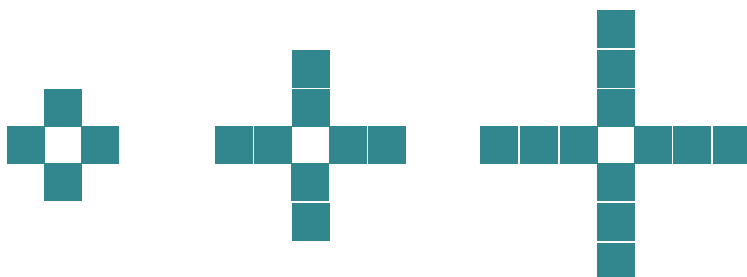
b) You would like to make a rectangular garden with an area of 12 square meters. What are the possible dimensions?

Resource: openmiddle.com (Robert Kaplinsky)

Framework 5: Let Students Pose Their Own Questions

Teachers are accustomed to posing all the questions in a mathematics classroom. One powerful way to spark curiosity is to present students with a scenario and simply ask them two questions: “What do you notice?” and “What do you wonder?”

Example: Show students this picture and ask both questions.



Resource: *Powerful Problem Solving* by Max Ray

Asking “what do you notice?” allows students to engage in the problem without feeling the pressure of having to produce an answer to the problem because no problem yet exists. Asking “what do you wonder?” will help the teacher discover the natural curiosity of their students. After they’ve voiced their questions, the teacher can choose one or two of the student-generated questions to explore.

Framework 6: Teach Students How to Generate Questions Using What-If-Not Framework

Teachers can show students how to create more questions by simply enumerating all the attributes of a given scenario, and then asking, “What if not?” for each attribute. This will help them create a plethora of new questions, some of which may be intriguing and challenging. Questions help highlight knowledge gaps, which in turn stimulates curiosity.

Example: The Fibonacci sequence (1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...) Some attributes of the sequence are: a. the starting two numbers are ones, and b. consecutive numbers are added to produce the next one.

Asking “what if not?” for each attribute could yield questions such as:

“What if the first two numbers were different?”

“What if consecutive numbers were multiplied instead of added?”

As you can imagine, this process can yield a wide variety of questions very quickly.

Resource: *The Art of Problem Posing* by Stephen Brown & Marion Walter

STEP 2: DEVELOP GRIT

“This quality of being able to sustain your passions, and also work really hard at them, over really disappointingly long periods of time, that’s grit.”

— Angela Duckworth, Psychology Professor at the University of Pennsylvania

Curiosity is the desire to learn, but it is transient. Curiosity fades when the challenge is very large or the time required is long. As the initial curiosity fades, some grit is required to sustain the effort. Curiosity sets the stage for hard work, but at some point, the learner must regulate his or her ability to persist in the face of an uncertain outcome.

Angela Duckworth calls the ability to persist over long-term goals “grit.” Duckworth’s research has shown that grit is a better predictor of GPA and graduation rates than IQ (Duckworth, 2007).

There is an active area of research around techniques that help develop grit, but there are few significant results at this point. That said, there are ways to challenge K–12 students and help them develop grit.

Get Comfortable with Being Uncomfortable

There is a sense of discomfort when one tries to learn something new. Math teachers must recognize and acknowledge this feeling in their students, remind them that it is a natural feeling, and encourage them to push through.

Allow Students Time to Struggle (don't be too helpful)

As discussed, students need opportunities to develop grit. When a student struggles to answer a question, teachers must resist the urge to swoop in and offer hints or solutions. Instead, they must allow students to try different solutions in the hope of finding a path forward. The idea is for kids to become comfortable with struggle, so they understand it is a normal part of learning.

Counterintuitively, the role of the teacher is not to provide answers; rather, it is to construct an environment in which students have the time and space to ask their own questions and develop a thirst for knowledge.

Recognize Improvements

Students easily lose sight of their progress in mathematics. Imagine a student who begins the year with a B grade in mathematics. At the end of the year, he/she remains a B student. The student may falsely believe he/she has made no learning gains because the grade has remained constant. What students often fail to recognize is that the learning goals increase throughout the year. In actuality, a student whose grade didn't change has still made a great deal of progress!

One powerful way to emphasize this fact is to provide standards-based grades to students. Standards-based grading measures a student's proficiency against each standard. This rating can change as the student learns. Reporting progress in this way allows both the teacher and the learner to see exactly which standards have been mastered and which ones have yet to be attained. By contrast, traditional letter grades make it difficult to decipher which standards each student has mastered.

An additional and important feature of standards-based grading is to allow students to re-take assessments and change the grade for a particular standard. This reinforces the notion that learning is a process where perseverance and grit are rewarded. With traditional letter grades, students generally take a test, get a grade and the class moves on to the next unit. The traditional model implicitly teaches students that grades (and ability) are fixed and that mastery is not the primary goal.

STEP 3: PROMOTE A GROWTH MINDSET

“It’s really hard to have high tolerance if you believe that your abilities or intelligence are fixed. Because if you believe ‘I can’t change my own abilities,’ then trying hard doesn’t make any sense.

It’s like pounding your head against the wall.”

— Eduardo Briceno, CEO of Mindset Works, a company he co-founded with Dr. Carol Dweck

Growth Mindset

Stanford psychologist, Dr. Carol Dweck has identified two learning mindsets, which she calls “fixed mindset” and “growth mindset.” Dweck’s research on this topic is summarized in her excellent book, *Mindset*.

A person with a fixed mindset believes that traits like intelligence are innate and cannot be changed. On the other hand, a person with a growth mindset believes that abilities can be improved with effort.

People with a fixed mindset would say Michael Jordan had an innate talent for basketball. People who believe in growth mindset might say that Michael Jordan practiced more often and more effectively than his peers and that is what led to his supreme ability and success.

If a student believes talent and ability are fixed, there’s a risk he or she won’t work as hard to improve.

Mindsets Predict Achievement

Dweck’s research has repeatedly shown that students with growth mindsets outperform those with fixed mindsets. Her team followed 373 students transitioning to 7th grade and monitored their math grades over the following two years. Their analysis showed significant improvement for students with growth mindset (Blackwell & Dweck, 2007).

The authors also note that “the impact of mindsets does not typically emerge until students face challenges or setbacks.” Growth mindset is a story we tell ourselves when we face setbacks. We are forced to ask ourselves, “Am I going to give up, because I’ve reached the limit of my talent, or will I push ahead, because I believe my efforts will lead to learning?”

How to Promote a Growth Mindset

Tell Students About the Two Mindsets. Research shows that just knowing two mindsets exist can help children move toward the growth mindset.

Praise Effort Over Outcomes. When a child goes home with an A on a math test, encourage parents to say, “I can see how your studying paid off. Great work,” rather than, “Wow! You’re so smart!” If a student got an A without much effort, try saying “Looks like that test was too easy for you. Maybe we can work on more challenging problems together.” This will teach them to value the process of learning more than the grades and that it’s OK to struggle.

Look for Signs that Reveal a Fixed Mindset. Students with a fixed mindset will try to hide their mistakes and make excuses to avoid trying because they are afraid to make mistakes. In their minds, mistakes are confirmation that they can’t do math instead of being just a natural part of the learning process.

Change Student’s Internal Dialogue. When students say, “I can’t” teachers should add the word, “yet.” This will reinforce the notion that learning is a process.

Give Descriptive Feedback. Descriptive feedback is constructive, helpful information that answers the question, “How can I do better?” Descriptive feedback needs to clearly indicate to the student which of his or her actions were correct and what has been done that is productive. It must enable the student to see what has and hasn’t been achieved and provide ideas on better ways to approach the problem.

Descriptive feedback answers the following questions:

- “Where am I now?”
- “What went well?”
- “Where am I going?”
- “How can I improve my work?”
- “What next?”

Teacher Mindsets Matter

Dweck has also conducted research on the impact a teacher’s mindset can have on students. Teachers were asked to provide feedback to 7th-grade students who had received a grade of 65% on an exam. Beforehand, half of the teachers had learned from a “scientific article” that math intelligence is fixed and half had learned that math intelligence is acquirable. Teachers who believed in a growth mindset were found to offer more encouragement and support, while also providing more strategies for improvement. Conversely, teachers with a fixed mindset were more likely to comfort the student by explaining that not everyone has math talent or that not everyone can excel in math.

Summary

Our culture promotes the notion that mathematical ability is innate. Students absorb this idea from the media they consume, and it is reinforced by an education system that focuses too heavily on grades and standardized test scores. Teachers are encouraged to “cover” math content in a way that emphasizes answer-getting over problem-solving, rote learning over understanding, and boredom over curiosity. Combined, these factors make it extremely difficult to engage K–12 students in mathematical discovery.

Education research gives us a strong set of actions to take to combat these factors, including promoting engagement, emphasizing the importance of grit, and redefining achievement.

To begin, teachers can leverage Lowenstein’s information gap theory for curiosity to help get more students become interested in mathematical questions. As discussed, common frameworks for creating information gaps can stimulate student interest. Such frameworks involve leading (and sometimes controversial) questions such as “Would you rather?” or “What if not?”

Once a high level of interest has been created, teachers can focus on helping students develop the grit they need to work towards long-term goals. Often, this means being more hands-off and allowing students to struggle. This also means reporting progress in a way that rewards grit and emphasizes mastery as the primary goal.

Finally, teachers can apply several strategies to help students develop a growth mindset so they can overcome the setbacks that they are certain to encounter in their mathematical learning journey. When students understand that there are two mindsets, they can choose to be more growth-oriented. Teachers can also help them change their inner dialogues and reinforce that learning is a process by giving helpful feedback and praising effort over outcomes.

Ultimately, learning is a process that teachers and students undertake together. A math-positive classroom creates an environment in which learning goals are based on the slow and steady acquisition of math skills and students feel both curious and successful as they attain new levels of mastery.

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