Math Workstations: Purposeful Practice

By Nicki Newton, Ed. D.

Introduction

Today's 21st-century classroom is filled with so many types of different learners and more possibilities than ever to reach and teach all those learners. We have students who are working below grade level, on grade level, and above grade level. Tomlinson asked the question over 20 years ago, "How do I divide time, resources, and myself so that I am an effective catalyst for maximizing talent in all my students?" (2014, p. 2) Math workstation is one of the ways to address the varied needs of students and curricular demands of classrooms. They take up the task of being able to engage students with the curriculum through "different approaches to learning, by appealing to a range of interests and by using varied rates of instruction along with varied degrees of complexity and differing support systems" (Tomlinson, 2014, p. 3).

Burns (2016) argues that giving students the opportunities to practice the math through a variety of choice activities is important and productive. National Council of Teachers of Mathematics (NCTM) (2004) pointed out that "Small groups provide a forum in which students can ask questions, discuss ideas, make mistakes, learn to listen to others' ideas, and offer constructive criticism." Protheroe (2007) notes that students who work together in pairs and groups on math activities showed increased achievement (p. 56). There is a long ideological history of students working on tasks, by themselves, in partners, and in small groups (Bruner, 1961, 2009; Dewey & Dewey, 1915; Fosnot & Dolk, 2001a; Piaget, 2013; Vygotsky, 1978; L. Williams, personal

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communication). “They can extend learning ...” In writing about workstations, Cosgrove (1992) argued that:

> “They can extend learning by being interesting, meaningful, relevant, and social to the learners because they will give students opportunities to participate in thought-provoking activities and stimulate curiosity to learn within a cooperative setting. Learning centers capitalize on the inherent social nature of the classroom by encouraging students to communicate, share projects and jointly solve real problems while meeting individual needs, styles, interests and curriculum demands.”

Math workstations are powerful opportunities for students to work in their *zone of proximal development* (Vygotsky, 1978). They provide students the opportunity to practice and build a deeper understanding of math concepts. They help to build mathematical proficiency, defined by the NAP (The National Academies Press) study as conceptual understanding, procedural fluency, adaptive reasoning, strategic competence, and productive mathematical disposition (Kilpatrick, Swafford, & Finell, 2001). In math workstations, these strands are woven together to help students understand and be able to do the math.

One of the major parts of a math workstation is students working on the conceptual understanding of the concepts through discovery. Bruner (1962/1979) notes that:

> “the virtues of encouraging discovery [are]of two kinds. In the first place, the child will make what he learns his own, will fit his discovery into the interior world of culture that he creates for himself. Equally important, discovery and the sense of confidence it provides is the proper reward for learning.”

Students get time to think about concepts and work with them at the concrete, pictorial, and abstract levels. In workstations, students get to build addition facts, draw out subtraction facts, and build and split numbers so they understand the distributive property. They get to work on building procedural fluency at many levels as well. Procedural fluency is defined as “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately” (Kilpatrick et al., p. 116). Throughout math workstations, students work on unpacking and understanding different math skills. For example, students might build out the multiplication array by using base ten blocks or practice multiplying decimals on decimal grids. Workstations allow students to understand the procedures they are doing, so that they do them with understanding. When they do them with understanding, they can then reason about them and determine whether their answer is correct or not. Part of the purpose of math workstations is that students get ample opportunity to work on comprehending math “concepts, operations and relations” (Kilpatrick et al., 2001, p. 116).

Reasoning is a big part of math workstation work in general. Adaptive reasoning is defined as “the capacity for logical thought, reflection, explanation and justification” (Kilpatrick et al., 2001, p. 116). The idea is to get students to think about numbers by contextualizing and decontextualizing them. We want students to be able to logically consider problems and their own thinking as well as the thinking of others. In workstations, students get a chance to use manipulatives and templates to work out problems, see whether they make sense and if they don’t, then to persevere in doing them until they do make sense. Math workstations are set up in a way that allows students not only to engage in self-reflection but also in explaining, defending, and justifying their thinking to and with others. Workstations should be organized in a way that students are required to listen to the thinking of others, follow the logic of others, and be able to decide if it makes sense or not. Workstations are about students making sense of the math, for themselves and with others.

Math workstations allow students to build strategic competence by considering problems in many ways. Strategic competence is “the ability to formulate, represent and solve” math problems (Kilpatrick et al., 2001). In math workstations, students work on problem-solving throughout the various activities. There is an emphasis on solving one way and checking another. They solve using different tools and templates. They think about their solutions and decide which one was most efficient.

Furthermore, in workstations, students are asked to consider their mathematical journey as they do the work. A productive disposition is “a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (Kilpatrick et al., 2001, p. 116). There are plenty of opportunities for students to monitor their own learning and reflect upon what they know and what they still are learning. Good math workstations have structures embedded in them to help students to make sense of math and to persevere while doing it.

What Is a Workstation?

A workstation is a structure for students to practice the current unit of study ideas and engage in ongoing review. It isn’t a specific place; it is a structure for practicing the standards. They can be located anywhere. They can be a game, an activity, or a project. They can be done with paper and pencil or digital tools. Some workstations stay up all year while others rotate throughout the year. Fluency, word problem, and place value workstations should stay up all year long. Stations for the current unit of study, such as measurement or geometry, are worked on during that unit of study.

Students practice in a variety of ways including by themselves, with a partner, and sometimes in a small group. Students usually spend 10–15 min at each workstation with no more than three to five students at that station. In math workstations, “Students are involved not only in discovery and invention but in a social discourse involving explanation, negotiation, sharing and evaluation” (Kamii & Lewis, 2009). It is through this social interaction and public thinking that students make meaning of the math they are doing. Tomlinson (2014) reminds us that when differentiating the focus is to “ensure that students compete against themselves as they grow and develop more than they compete against one another, always moving toward—and often beyond—designated content goals.”

Everything a student does in a workstation should be meaningful, engaging, standards-based, and rigorous. Cross notes that “when students are actively involved in . . . learning . . ., they learn more than when they are passive recipients of instruction” (1987, p. 4, cited in Bonwell & Eison, 1991). Workstations should never be boring or frustrating. Students should be engaging in “just right activities” that scaffold their learning in alignment with the grade-level standards. Students should be familiar with the ideas that they are working on in the workstation. The activities are meant to solidify learning the knowledge and skills through purposeful practice.

Why Do Workstations?

Workstations allow students to practice the concepts they are learning. Mastery is achieved over time by distributing the practice across the year. Fosnot notes,

> "Teaching mathematics is about facilitating mathematical development. This means that you cannot get all learners to the same landmarks at the same time, in the same way, any more than you can get all toddlers to walk at the same time, in the same way! All you can do is provide a rich environment, turn your classroom into a mathematical community, and support the development of each child in the journey toward the horizon."

Workstations are often leveled. This allows students to work in their zone of proximal development, gaining mastery throughout the year. Researchers warn us that because of "the enormous variability in young children's development," we mustn't set up arbitrary "fixed timeline[s] for children to reach each specific learning objective" but rather give students the time to learn the concepts well and develop deep understandings of the math" (National Association for the Education of Young Children and NCTM, 2002). Workstations offer structures for students to "learn as deeply as possible and as quickly as possible, without assuming one student's road map for learning is identical to anyone else's" (Tomlinson, 2014, p. 11). Hilberg, Chang, and Epaloose (2003) found that the goal of workstations "is to allow the teacher to provide the highest quality instruction to a small group of students, while other students work productively, independently, and cooperatively in a variety of interconnected tasks at other activity centers" (p. 14). Workstations allow practice to be distributed across time, at varied levels, so that students can actually gain mastery of the concepts.

Framework for Workstation Activities

There are many different types of activities that can happen in a workstation. Pellegrino (2007) found that for workstations to "provide authentic learning experiences, they must have a direct objective, be related to the curriculum, meet the needs of all learners, and encourage higher order thinking in students" (p. 47). Throughout workstations, activities should build on the concrete, pictorial, and abstract approaches to learning (Hoong, Kin, & Pien, 2015). Students need many opportunities to model ideas with tools. Moreover, the intentional and well-planned use of manipulatives and other tools is also positively related to student achievement and attitudes about math (Liggett, 2017; National Council of Supervisors of Mathematics, 2013; Swan & Marshall, 2010). Taking time to build this concrete understanding is very important. Students should unpack key ideas and understandings through building arrays with 1-inch tiles, using meter sticks to discover measurement around the room and building fraction sets with circles and bars.

Pictorial representations are also very important (Zorfass, Han, & PowerUp WHAT WORKS, 2014). Marzano (2010) notes that there are many different forms of nonlinguistic representations including graphic organizers, sketches, pictographs (sticks figures and symbols), concept maps, dramatizations, and more. Boaler (2016) notes that visual math is an important part of learning and that new brain research shows how it is connected with students understanding the numbers. Yin (2010) has identified at least seven different roles that visualization plays in students making sense of math problems: (1) understanding the problem,

(2) simplifying the problem, (3) seeing connections to a related problem, (4) connecting to learning styles, (5) in place of computation, (6) tools to check the work, and (7) mathematize the problem. Throughout math workstations, there is an emphasis on not skipping this middle step between concrete approaches and abstract approaches. Students have plenty of opportunities to draw out their thinking, use graphic organizers, and look at pictorial representations of problems to make sense of the math.

Finally, students must have opportunities to practice skills at the abstract level through a variety of games. These include board games, card games, and dice games, where students are working mainly with numbers. They could be exploring concepts through traditional games such as *tic-tac-toe* and *bingo* or newer games such as *bump* and *four-in-a-row*. Marzano reminds us that practice is one of the nine best instructional strategies. Citing Rosenshine (2002), he notes that *guided practice* is essential.

> "guided practice is the place where students—working alone, with other students, or with the teacher—engage in the cognitive processing activities of organizing, reviewing, rehearsing, summarizing, comparing, and contrasting. However, it is important that all students engage in these activities. (p. 7)"

In the abstract-level activities, students *are reviewing and rehearsing* the knowledge and skills that they have learned in the unit and throughout the year.

Workstation Grouping Formats

In workstations, students can work alone, with partners, or in small groups. Students can work alone by building out word problems. They might work with partners by playing a dice or card game. They might play a board game in small groups. The game structures stay the same throughout the year, but the content changes. Once students are familiar with the format of the activity, they can get right to the math! Researchers have found that workstations also contribute to the "development of the values necessary for a successful classroom community—fairness, harmony, inclusion and academic excellence" (Hilberg, Chang, & Epaloose, 2003, p. 13).

Engaging Activities

Students develop their understanding of the math by engaging in hands-on, minds-on activities.

> "Knowledge is actively created or invented by the child, not passively received from the environment. This idea can be illustrated by the Piagetian position that mathematical ideas are made by children, not found like a pebble or accepted from others like a gift."
>
> (Sinclair, in Steffe & Cobb (1988), cited in Clements & Battista, 1990)

In math workstations, students actively engage with mathematical ideas by experiencing the math. They aren't going on "what their teacher told them" but "what they know because they did the math!" They experience the math. They learn that learning "involves risk, error and personal triumph" (Tomlinson, 2014, p. 11). Astin stated that "students learn by becoming involved . . . student involvement refers to the amount of physical and psychological energy that the student devotes to the academic experience (1985, pp. 133–134, cited in Bonwell & Eison, 1991). Task card activities are often hands-on, with students either building or drawing to unpack a concept. The card games, dice games, and board games often provide the abstract practice that students need to solidify their understanding of a concept or skill. The project pages are usually end of the unit tasks that give students a platform to showcase their full understanding of the big ideas, concepts, and skills in a unit of study. There are a variety of different ways that students can practice throughout the year because deliberate practice is how we master concepts (Gladwell, 2008).

Standards-Based

Workstations should be standards-based. Standards-based does not only mean that the standards are written on the board in an *I can statement* or some type of learning objective, it means that students understand what they are learning about (Cosgrove, 1992; Ford & Opitz, 2002). McDonald and Boud (2013) state that self-assessment is "the involvement of students in identifying standards and/or criteria to apply to their work and making judgments about the extent to which they met these criteria and standards" (p. 209). As Hattie et al. (2016) have noted, it is really important that students know what they are supposed to be learning, what it looks like, and what the criteria for success is.

In math workstations, students know what the math is that they are working on during the activity. They can explain not only how to play the game but what they are practicing. For example, if you ask a first grader, what they are working on, they would be able to explain "counting on as an addition strategy." They would not only be able to tell you what it is, but why it is important and how to do it. If you go up to a third grader playing a partial sums addition game, they can explain that they are working on breaking numbers apart and adding more efficiently. This sense of knowing what the math is is crucial to a successful workstation. Teachers should write the *I can statement* on the workstation as a visual reminder of what the math ideas and concepts are in that station.

Mathematical Practices/Processes

Math workstations work not only on developing conceptual understanding but also on developing problem-solving, reasoning, modeling, communicating, using tools, using precise language and calculations, and understanding structure and looking for patterns (NCTM, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Throughout the workstation activities, there is a balanced emphasis on not just getting the answer but understanding that answer and being able to communicate that understanding with others (Daro, n.d.). Math workstations go "Beyond answer getting!" (Daro, n.d.).

Academically Rigorous

It is important to think about the levels of rigor that students are engaging in throughout the workstation activity. The workstations span a variety of DOK (Depth of Knowledge) levels (Webb, 2002). The emphasis is on getting students to practice the concepts and then apply that knowledge as part of the practice. There is an emphasis on students working together to explain their thinking, justify their explanations, and prove that they are correct. Oftentimes, it is small tweak that makes a big difference in the rigor level in a station. We also are always thinking about the intersection between Bloom's (1956) and Webb's (2002). Hess (2004) gives a framework to look at how these intersect. Pellegrino (2007) notes that math workstations should be "challenging critical thinking activities and important reinforcement for classroom instruction." Good workstations don't just happen, they are planned for and then evaluated and adjusted. A good workstation is a working curriculum space subject to change whenever needed.

Student Accountability

One of the big questions about math workstations is "How do we know that the students are doing what they are supposed to do?" Math workstations have recording sheets, so that students are accountable for the activities and games that they are doing. There are different types of recording sheets. Some sheets only require students to write down a few of the problems that they solved while playing a board game. Other sheets require that students record all of the work that they did in those stations. For example, if students are playing a comparison game, they write down each of their turns and record the comparison with a symbol. Students can also record their work in a math journal or even take pictures of their work with their electronic devices.

Researchers have found that it is very effective to have students monitor, record, and reflect on their work. All students can do it, although it will look different depending on their grade and the individual student. In math workstations, students have the opportunity to set goals, think about their strategies, and monitor how it is going, which is what the research tells us helps improve students learning. Reflection can be in many forms including verbal, drawings, and written. The prompts are very important for the scaffolding of good reflections (Anghileri, 2006; Attard, 2017).

> "Modifying tasks to include a *self-correcting* element can provide further feedback that supports pupils' autonomous learning, not only in finding a solution, but also in reflecting on the processes involved in such a solution." (Anghileri, 2006)

Every math workstation should have some way for students to self-correct. Moreover, teachers must provide opportunities for students to reflect on the math that they are practicing, sometime during the week on entrance and/or exit slips. Teachers must also keep track of the work and reflect on it. (Hilberg et al., 2003) note that the teacher sets up meaningful, standards-based activities, monitors and "assesses student's levels of understanding by observing, listening and questioning and then provides responsive assistance to students' developmental levels and advances their understanding" (p. 16). Teachers must reflect on and monitor the stations so that they can make sure that everyone is practicing with purpose. Here are a few guiding questions for reflecting on the stations:

- How do I know whether my students understand the concept?
- What is the evidence that they are fluent with the skill?
- How do I measure that growth overtime?
- What pieces of evidence will I collect that prove the student is making progress? (Newton, in press)

Accountable Language

Math is a language. If students don't know the words and aren't required to speak it, they never learn it. So, we have to hold students accountable to using the math vocabulary during their work in stations (Coleman, n.d.; Jordan, 2013). One way to do this is to have language frames that scaffold the language and the phraseology for the students at the workstations. For example, at a workstation where students are comparing numbers, there might be a language strip that says:

I can use my math words!
_________ is greater than ____________.
_________ is less than ______________.
_________is equal to ________________.

(Newton, in press).

Anecdotals

Math workstation work is another form of ongoing assessment. Teachers should keep anecdotals of what they see students doing in the workstation. This is done as the teacher walks around and sometimes even joins in to observe what is happening in the workstation.

- Who is working well together?
- Are they able to express their mathematical thinking?
- Do they understand the concept?
- Who is having difficulty?
- What is the nature of the difficulty?
- What are the error patterns or misunderstandings?
  (Newton, in press)

Teachers should use these observations as they confer with students about their math achievement. Notes should be taken on individual students, partner work, small group work, and overall class reflections.

Notes on the Individual Student
Juan Carlos has learned his doubles. I watched him in the workstation today and he is doing it fluently.

Notes on Partner Board Game
Kelly struggled with elapsed time. Maria was a great partner and helped her use the elapsed time ruler.

Scaffolded to Unscaffolded Activities

Scaffolding workstation activities is important. Scaffolding in math help students to access the big ideas, enduring understanding and specific skills that they are working on in a unit of study (Anghileri, 2006). In workstations, students use a variety of visuals, graphic organizers, templates, tools, and intentional grouping structures (alone, partner, and small group) to scaffold the math. Many of the workstations have the tools and templates built into the actual activity. For example, there are number paths, decimal grids, and fraction circles on the actual gameboards, so that students can reference them when needed. Also, students are encouraged to use various types of manipulatives such as counters, place value blocks, and geoboards to make sense of the math they are doing.

Scaffolding is temporary but often necessary. As students learn the concepts and understand the math they are doing and become proficient with the skills, the scaffolding is phased out. Not all students need the same level of scaffolding. For example, during a board game on elapsed time, one student might need an elapsed time ruler and the other student might be able just to draw a number line diagram. Another example is the use of *hint cards*. In some games, there are two levels of cards. The first level has cards with hints, and the second level has the cards without hints.

In this game, students pick a card and move around the gameboard trying to be the first to reach finish.	
Scaffolded	**Unscaffolded**
Estimate 19 x 18. *Hint: Round 19 and 18 to 20, then multiply.* **If correct, move forward 2 spaces.**	Estimate 19 x 18. **If correct, move forward 2 spaces.**

Getting Started—The First 20 Days

The first 20 days are essential. If you don't take time to teach *the how* of workstations during the first 20 days, you will end up teaching it all year long. During the first 20 days, students learn all the routines, rituals, and protocols of math workshop. They learn how to move to and from the workstations, how to get out the workstations, how to start games, how to stop games, and how to rotate around the room. They also learn to win with grace and lose with dignity. They learn how to play well together, discuss their thinking, listen to the thinking of others, and communicate in respectful ways. They also learn to collaborate and compete in friendly ways. The first 20 days of workshop rollout is the bedrock for the entire year, never skip them. See here for more information http://www.drnickinewton.com/downloads/

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