McGraw Hill

Grades K–8

Research Foundations

A Research Summary of Program-Focused Outcomes

Reveal MATH®

As a learning science company, McGraw Hill designs all our products and services to unlock the potential of each learner. Not only do we strive to create products that improve and accelerate the learning process, but we also look to reflect a diverse range of perspectives and approaches that cater to the whole child and each student's individual learning journey. Moreover, we support teachers as they work to create inclusive classrooms that embrace the needs of all learners.

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Introduction

Reveal Math is built on principles that honor the full potential in each student mathematician by setting high expectations for all and providing support, extensions, and delivery options to incorporate the core values of curiosity, connections, communication, collaboration, and confidence. The overall instructional goal is to empower every teacher to orchestrate rich mathematics learning in order to reveal the full potential in every student. To do this, we grounded the development of *Reveal Math* in salient research and evidence-based best practices.

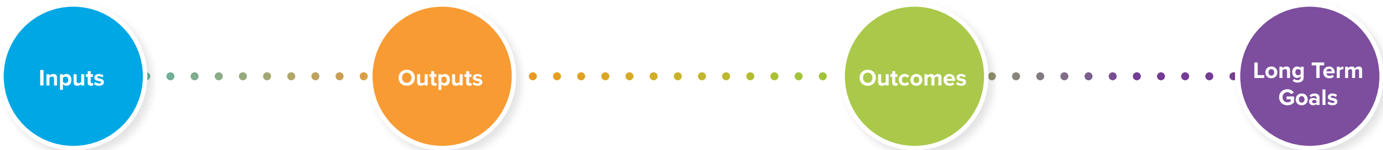
At the core of *Reveal Math* are specific areas of focus that have emerged from numerous learning science domains essential to strengthening the teaching and learning of elementary mathematics (NCTM, 2017). The foci chosen for *Reveal Math* offer a balanced approach to mathematics instruction that encompasses both student-centered and teacher-facilitated instructional activities. They inform how the program was crafted, starting from the development of the overarching program goals to the construction of the *Reveal Math* learning interactions and instructional model.

This research brochure provides theoretical and empirical foundational analyses that support the instructional underpinnings of *Reveal Math*, including discourse on the following topics:

- **Equitable Classrooms:** Teachers employ a variety of learner-focused practices to develop an equitable classroom designed for all students.
- **Classroom Discourse and Language:** Students engage in class discourse, use appropriate math vocabulary, and learn to critique the math thinking of others.
- **Sense-Making:** Students make sense of problem situations and use sense-making to develop problem solving skills.
- **Fluency:** Students develop fluency by using flexible strategies to practice math content.
- **Instructional Routines:** Teachers use instructional routines to provide structure and set expectations that create productive classroom interactions with students.
- **Student Agency:** Students draw on their agency through the expectation of ownership and accountability in their learning. These areas support student agency:
    - **Metacognition**—Students use metacognition to reflect on their learning throughout the lesson.
    - **Productive Struggle**—Students engage in productive struggle as they grapple with mathematical ideas and relationships.
    - **Social and Emotional Learning**—Students use social and emotional learning competencies to become academically and socially engaged classroom members.

The *Reveal Math* Logic Model

A program logic model, which delineates the path through which the program can meet the anticipated goals, was developed to build the program with the end results in mind and provides a big picture overview of the main features of *Reveal Math*. The logic model is also an important component of the program research plan because it guided the development of the program research foundation, program research questions, and effectiveness and efficacy studies.



Inputs

Launch—Prepare to Learn

- Math Is...
- Ignite!
- Be Curious
- Focus, Coherence & Rigor Supports

Explore & Develop—Teach and Learn Together

- Activity-Based Exploration
- Guided Exploration
- Instructional Routines
- Effective Teaching Practices

Practice & Reflect—Practice, Apply and Extend

- Math Replay
- STEM Adventures
- Interactive Digital Practices
- Student Practice Book

Assess—Evaluate and Apply Evidence

- Exit Tickets
- Math Probes
- Assessments: Course, Unit, and Lesson

Differentiate—Learning Supports

- Workstations: Games, Application, and Small Group (K–5)
- Digital and Print Independent Activities (6–8)
- Take Another Look
- eToolkit

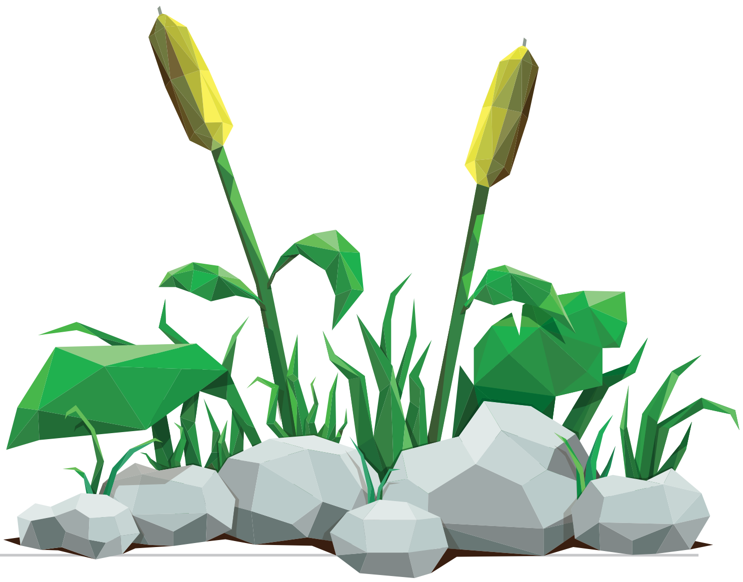
Outputs

Students:

- Think about and share what they know about math.
- Talk about and help create classroom norms, interactions, and expectations for learning math.
- Engage in self-reflection and discussion with others about classroom norms and expectations.
- Ask questions and talk with others about math problems, representations, strategies, or ideas.
- Learn about specific job & real-world related math skills.
- Apply math practices to solve problems.
- Regularly practice math skills and concepts.
- Self-reflect on how they learn math tasks/concepts and understand their areas of strength and growth.
- Engage in independent practice in areas of growth to enhance their own math learning.

Teachers:

- Model and foster meaningful math discussion practices.
- Use instructional routines to model specific mathematical practices.
- Monitor student progress and use data to make instructional decisions for student growth.
- Use standards, effective teaching practices, student data, and interests to select appropriate lesson models and tasks.



Outcomes

Students:

- **Problem-solving**—Make sense of problem situations and use sense-making to develop problem-solving skills.
- **Discourse**—Engage in class discourse, use appropriate math vocabulary, and learn to critique the math thinking of others.
- **SEL**—Use social and emotional learning competencies to become academically and socially engaged classroom members.
- **Fluency**—Develop fluency by using flexible strategies to practice math content.
- **Agency**—Draw on their agency through the expectation of ownership and accountability in their learning. Three areas support student agency:
  - *Growth Mindset*—Resilience in problem-solving and the learning process.
  - *Productive Struggle*—Deep engagement with mathematical ideas and relationships.
  - *Metacognition*—Promotion of student reflection on their learning through contemplative, communicative practice.
- **Reasoning**—Construct objective, logical arguments and share discipline-specific thought processes.
- **Sense-making**—Engage in the dynamic process of building or revising an explanation in order to "figure something out."
- **Perseverance**—Improve their ability to continue working on learning tasks even when difficult and/or tedious.
- **Mastery**—Successfully demonstrate learning (measured against a "mastery" benchmark) of a given skill, concept, or disciplinary disposition, typically achieved through individually paced learning experiences.

Teachers:

- Use instructional routines to provide structure and set expectations that create productive classroom interactions with students.
- Employ a variety of learner-focused practices to develop an equitable classroom designed for all students.

Long Term Goals

Students:

- Consider multiple strategies, play with math, and practice perseverance.
- Understand that math is not just a series of operations, but a rich language that calls for specific ways of thinking and habits of mind.
- Engage in mathematical discourse as they listen actively, formulate thoughtful responses, and translate big ideas through their discussions.
- Respect and reflect different points of view, and support and inspire each other.
- Think about their own learning processes and develop agency as active and supported learners.

Teachers:

- Create a mathematically sound and equitable classroom through understanding students academically, socially, and personally.

01 | Equity

Equity in the Classroom

Educational equity is essential to the success of our nation's schools and classrooms. Equity can be considered "the driving force behind ensuring that all students, everywhere, receive rigorous, rich educational experiences that are designed to meet their specific learning needs" (Snyder, Trowery, & McGrath, 2019, p. 3).

The National School Board Association highlights the role of school district practices and resources, defining educational equity as "the intentional allocation of resources, instruction, and opportunities according to need, requiring that discriminatory practices, prejudices, and beliefs be identified and eradicated" (NSBA, 2020). Geneva Gay (1988), in her work on designing relevant curricula for diverse learners, posits that a focus on the equitable outputs should lead the development and selection of the inputs, or materials and practices, used in classrooms: "...the real focus of equity is not sameness of content for all students, but equivalency of effect potential, quality status, and significance of learning opportunities" (p. 329).

From a school mathematics perspective, the National Council of Teachers of Mathematics states, "Acknowledging and addressing factors that contribute to differential outcomes among groups of students is critical to ensuring that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful" (NCTM, 2020). Thus, an equitable classroom is one where all students are supported as they learn rigorous academics and where teachers leverage the materials and practices needed to support positive academic outcomes for all students.

Effective Practices in Equitable Math Classrooms

Research findings on equitable and culturally relevant mathematics teaching demonstrate how teachers can make effective connections to students' lives and communities with real-world applications of mathematics (Ensign, 2003; Enyedy & Mukhopadhyay, 2007; Gutstein, Lipman, Hernandez, de los Reyes, 1997; Rosa & Orey, 2010; Tate, 1995). Gutierrez (2009), for example, suggests that to move toward equitable mathematics teaching, teachers must know their students through a variety of lenses—academically, socially, and personally—without being reductive. Matthews (2003), in his work with four elementary mathematics teachers enacting culturally relevant teaching, suggests teachers should work to form an open relationship with their students so that informal/cultural knowledge and critical thinking in the classroom community can be used to build bridges to mathematics knowledge and the culture of school.

Research demonstrates that classroom culture influences how students learn math and retain mathematical knowledge. Therefore, recognizing this impact classroom culture plays is a vital is in creating equitable classroom environments where all students have opportunities to learn math at high levels (Waddell, 2014).

"Acknowledging and addressing factors that contribute to differential outcomes among groups of students is critical to ensuring that all students routinely have opportunities to experience high-quality mathematics instruction, learn challenging mathematics content, and receive the support necessary to be successful."

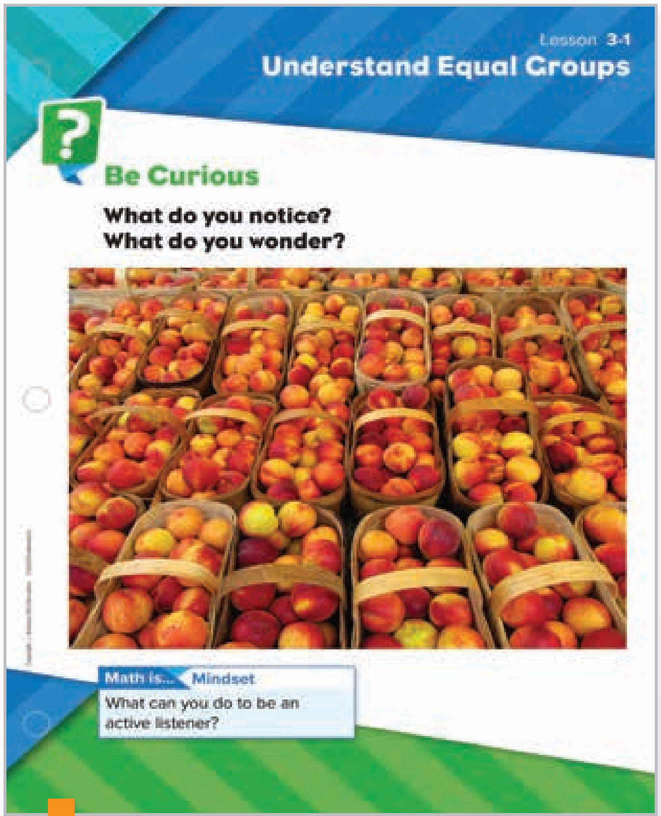
-NCTM, 2020

Equity in *Reveal Math*

Reveal Math supports the development of equitable math classrooms through a variety of resources and practices embedded in the program. *Reveal Math* places an emphasis on creating a positive and productive classroom culture where all students have common access to rigorous instruction while fostering the development of growth mindset and a positive math identity.

For the lesson's main instruction, the teacher can choose between two equivalent methods: an Activity-Based Exploration or a Guided Exploration. These two options provide access to the same rigorous content while allowing for a variety of modalities to experience the math. Both methods offer students the opportunity to develop deep understanding of the material through meaningful discourse. Embedded teacher support helps to ensure that students have the appropriate scaffolds to process and understand the lesson content. These supports include English Learner scaffolds, math language routines, and questioning grounded in effective teaching practices.

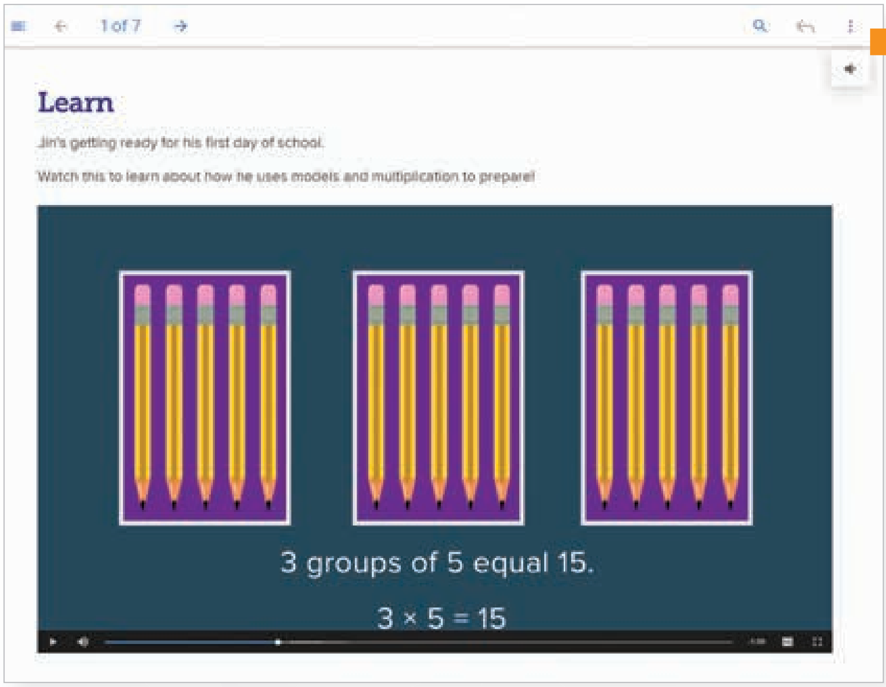
Reveal Math also provides rich differentiation resources, including intervention resources, to support all students in the learning process. Each option includes multiple modalities, offering rich differentiation that not only supports students' understanding but also challenges them through games, STEM simulations, and application cards*. Targeted intervention aligns to item analysis of assessments to help support and target specific misunderstandings and gaps in learning. These features provide the foundation for all students to receive high-quality, rigorous mathematics instruction and reach their academic goals.



The **Be Curious** activity found in each lesson uses sense-making routines to engage students in a low floor, high ceiling discussion, creating an equitable classroom culture where all ideas are welcome and respected.

**Application cards for K–5 only.*

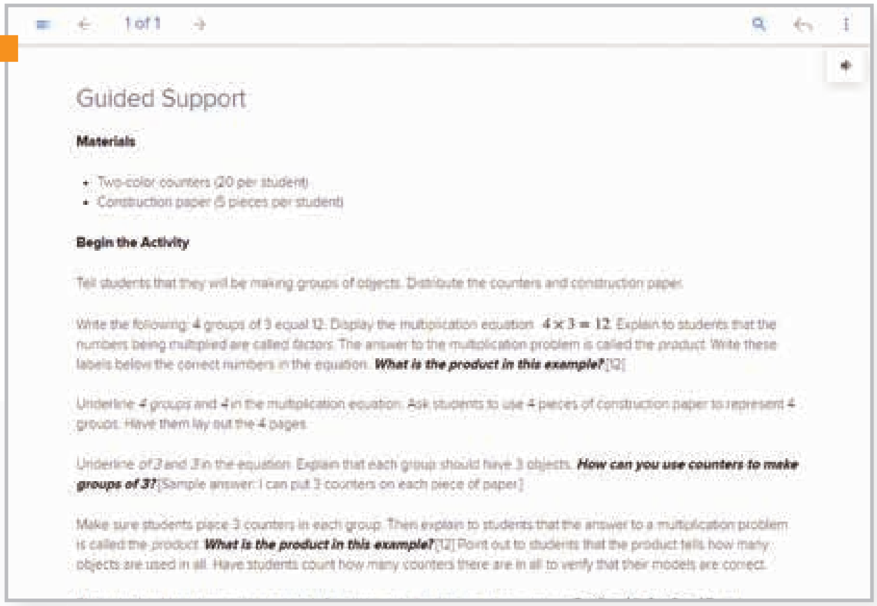
Targeted Intervention Resources



Take Another Look digital mini-lessons provide quick, actionable data to help inform instruction while supporting each student with a three-part, gradual-release activity:

- Modeling
- Interactive Practice
- Lesson Check

Guided Support provides a teacher-facilitated small group mini-lesson that uses concrete modeling and discussion to build conceptual understanding.



02 | Agency

Student Agency

Promoting agency, or the capacity for individuals to make choices, is an important aspect of supporting all students in their efforts to learn. Agentic decisions are thought to be influenced by people's habits and beliefs, external structures and events, and goals (Adie, Willis, & Van der Kleij, 2018; Giddens, 1984 in Deed et al., 2014; Klemencic, 2015; Vaughn, 2019;). Additionally, agency is time-bound: individuals draw on their present patterns, habits, and identity to set goals or outcomes, create plans or actions toward reaching those goals, and evaluate how well the plan and actions are helping meet the goals in the current context (Adie, Willis, & Van der Kleij, 2018; Poon, 2018; Klemencic, 2015).

Student agency, or agency that is directed toward achieving learning outcome goals and academic success in school, requires an acknowledgment of the structures and practices of the classroom. Tension can arise, however, when a student's desire to leverage agency in their own ways diverges from classroom and school-sanctioned ways of leveraging agency (Adie, Willis, & Van der Kleij, 2018). Research demonstrates that, in order to encourage sanctioned ways of enacting student agency, it is important that students be "positioned as knowledgeable leaders in the classroom and teachers work alongside their students to engage in flexible and adaptive teaching. Such contexts can provide rich learning spaces for students and teachers" (Vaughn, 2019, p.13).

Student Agency in *Reveal Math*

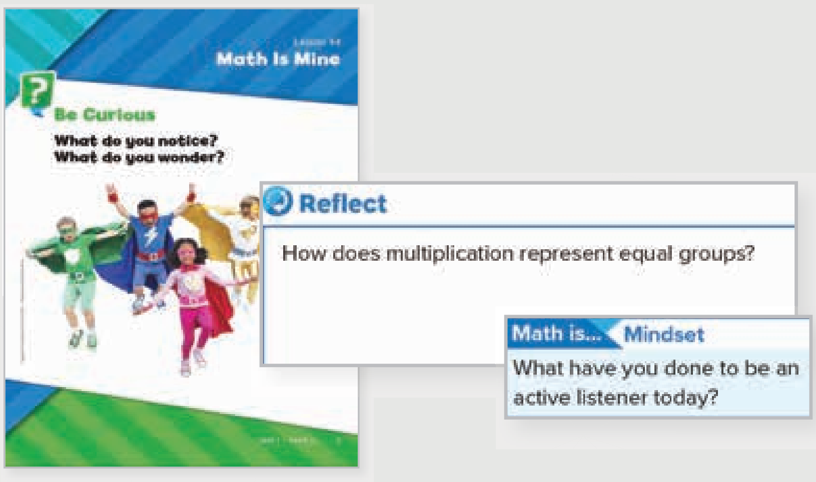
Reveal Math champions student agency by encouraging growth mindset, metacognition, productive struggle, and social and emotional learning. Briefly, growth mindset is the belief that abilities can be improved with effort. Research has shown that students with growth mindsets outperform those with fixed mindsets. In a study that followed 373 students transitioning to 7th grade, the research team monitored their math grades over the following two years. Their analysis showed significant improvement for students with growth mindset (Blackwell, Trzesniewski & Dweck, 2007). Practices around metacognition, productive struggle, and social and emotional learning objectives are integrated into lessons and provide teacher guidance and support to help students build these competencies. It is critical to understand teacher implementation of these competencies and their impact on student behavior and learning outcomes.

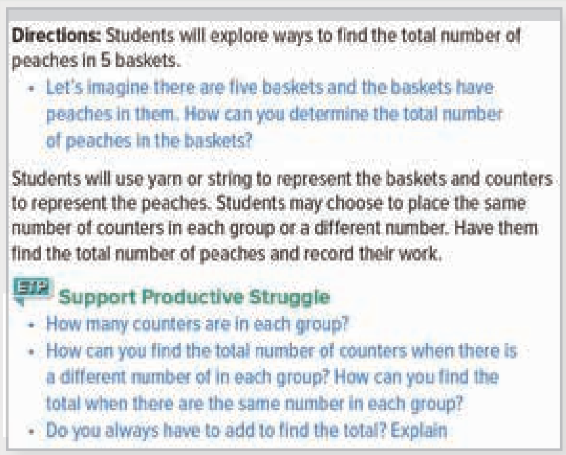
Explore how *Reveal Math* fosters student agency by promoting the following:

Social and Emotional Learning—The process by which students develop the set of skills, knowledge, and behaviors involved in understanding and regulating emotions, approaching challenges, and building positive relationships with others.

Metacognition and Reflection—The process by which students contemplate how they think through and approach problems.

Productive Struggle—The process by which students attempt to work through new or unfamiliar concepts, thereby building deeper mathematical understanding.




Directions: Students will explore ways to find the total number of peaches in 5 baskets.

- Let's imagine there are five baskets and the baskets have peaches in them. How can you determine the total number of peaches in the baskets?

Students will use yarn or string to represent the baskets and counters to represent the peaches. Students may choose to place the same number of counters in each group or a different number. Have them find the total number of peaches and record their work.

ETP **Support Productive Struggle**

- How many counters are in each group?
- How can you find the total number of counters when there is a different number of in each group? How can you find the total when there are the same number in each group?
- Do you always have to add to find the total? Explain

Social and Emotional Learning

When children learn and teachers teach, there is more that happens than just the transfer of content knowledge and information. Schools are dynamic and social environments in which both learners and teachers continuously interact, make decisions, and adapt to new circumstances. Developing the skills to successfully navigate school (and later, work and community) environments is a continuous and complex process that requires careful instruction and ongoing support for positive social, emotional, and behavioral skill development. A commonly used term for the development of these specific sets of skills is **Social and Emotional Learning**, or **SEL**.

The Collaborative for Academic, Social, and Emotional Learning (CASEL), a leading organization in the field, defines SEL as "the process through which children and adults understand and manage emotions, set and achieve positive goals, feel and show empathy for others, establish and maintain positive relationships, and make responsible decisions" (CASEL, 2017). Decades of research across a wide spectrum of educational settings have demonstrated that when educators support SEL, both in and out of the classroom, the positive benefits not only promote student success during the school years but also later in post-secondary education, the workforce, and beyond.

Additional research on the economic value of SEL integration into education has demonstrated that the benefits of such integration outweigh the initial investment costs, with a reported 11:1 return for every dollar spent on SEL instruction (Dusenbury & Weissberg, 2017). Meta-analyses (analyses of multiple research studies) have shown that this high return on investment is due to the significant improvements in outcomes across several factors, ranging from academic achievement to reductions in bullying and improved workforce readiness (Durlak, et al., 2011; Taylor, Oberle, Durlak, & Weissberg, 2017).

Social and Emotional Learning in Mathematics

Attending to students' social and emotional learning, specifically in mathematics learning contexts, has been shown to help students improve their math self-efficacy and attitudes toward math (Jones, Jones, & Vermette, 2009). Jones et al. (2009) indicate that when teachers create a socially and emotionally supportive learning environment, there is a positive impact on student attitudes, behaviors, and academic performance. An SEL-conducive climate makes space for students to work with a diverse group of individuals (DeLay et al., 2016; Jones et al., 2009) and allows time for reflection.

Elements of Social and Emotional Learning Instruction

- **Self-Awareness:** The ability to identify one's own emotions, thoughts, strengths, and weaknesses, as well as the development of a sense of self-confidence.
- **Self-Management:** The ability to regulate one's own behaviors, emotions, thoughts, and motivations, as well as the ability to set appropriate goals.
- **Social Awareness:** The ability to understand and empathize with the perspectives and norms of others, including those with backgrounds different from one's own.
- **Relationship Skills:** The skills involved in communicating clearly, listening well, cooperating with others, resisting inappropriate social pressure, negotiating conflict constructively, and seeking and offering help when needed.
- **Responsible Decision-Making:** The practice of making constructive choices about personal behavior and social interactions based on ethical standards, safety concerns, and social norms.

Social and Emotional Learning in *Reveal Math*

Social and emotional learning objectives are integrated into every *Reveal Math* lesson, including strategies and techniques to help teachers and students build their social and emotional competencies. Math Is... Mindset prompts appear in the student and teacher materials, keeping social and emotional learning at the top of students' minds as they interact and discuss throughout the lesson.

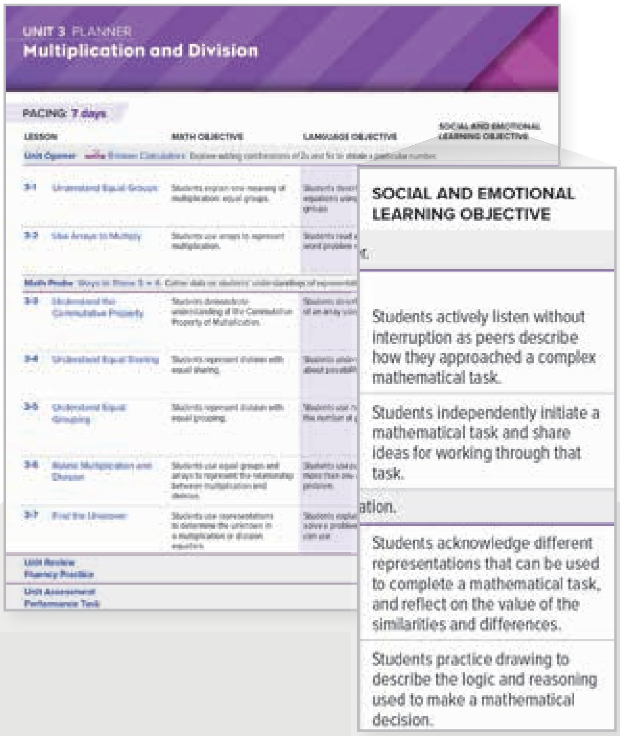

Math is... Mindset

- What can you do to be an active listener?

SEL Relationship Skills: Effective Communication

As students engage in collaborative discourse around the *Notice & Wonder*™, encourage them to actively and respectfully listen to one another. Invite students to think about and share what active listening looks and sounds like. As students discuss what they noticed and wondered, encourage classmates to listen as well as provide thoughtful feedback. Capitalize on opportunities to also model these behaviors when students are speaking.

Effective communication includes active listening. Remind students that an active listener gives full attention to the speaker by looking at the speaker and providing thoughtful feedback to the speaker. As students discuss what they noticed and wondered, remind classmates to listen actively and as appropriate, provide thoughtful feedback.

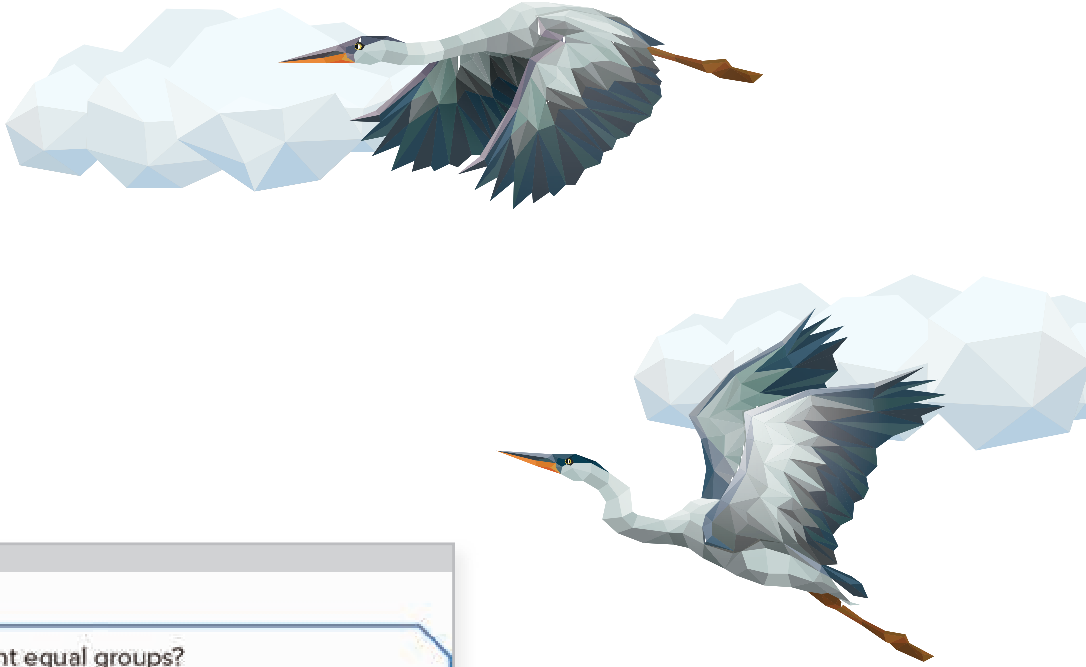


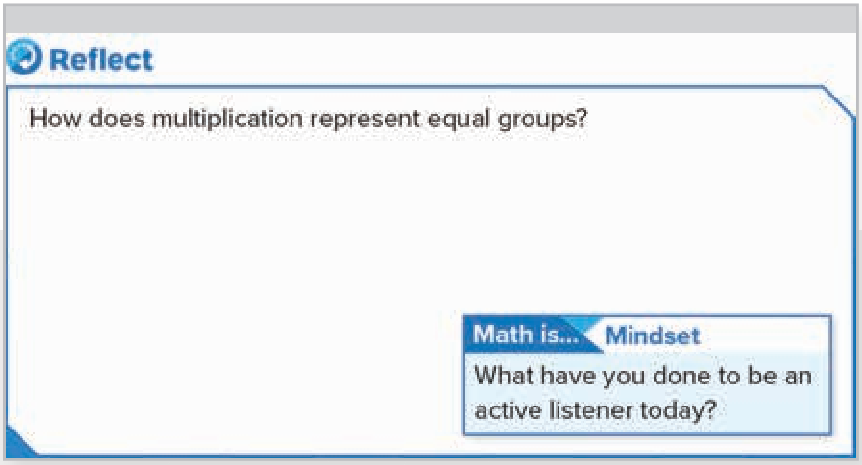
Integrate

Highlighted social and emotional learning strategies support students as they build proficiency and familiarity with concepts and skills. These include questions for teachers to help guide discussions and develop students' ability to justify their thinking. Other exercises present students with several ways that fictional students solved a problem, asking them to take on others' perspectives and explain how problems can be solved using multiple approaches.

Instruct

The teacher edition of *Reveal Math* presents opportunities for teachers to provide explicit guidance and instruction in SEL competencies. *Reveal Math* provides teachers with support to encourage students to understand their strengths, stay motivated, be persistent, and develop organizational thoughts and strategies.





Reflect

Students reflect on their learning and think metacognitively at key points in lessons. For example, students might be instructed to write through their frustrations and brainstorm coping strategies. Teachers can encourage students to think back on their learning and ask questions pertaining to how they feel about the topic and the knowledge they've obtained as part of the self-management SEL competency.

Metacognition and Reflection

Metacognition refers to an individual's knowledge concerning cognitive processes and regulation of these processes in relation to cognitive objectives (Desoete & De Craene, 2019; Flavell, 1976; Jin & Kim, 2018). In other words, metacognition is the process of thinking about thinking. John Dewey proposed that reflective thinking, which is key to metacognition, comes about in moments of confusion, wonder, and curiosity (1910, p.6). Such an awareness of one's own thinking can improve the learning process. Reflection additionally helps to promote social and emotional learning: students benefit from reflecting on what they think and how they feel about what they have learned.

Indeed, strategic metacognitive engagement has been shown to aid in performance in the classroom and overall academic achievement. For example, in one study, students' problem-solving processes were qualitatively shown to be supported by engaging in metacognitive regulation—the active monitoring and controlling of cognitive processes (Jin & Kim, 2018). Students were able to help monitor and adjust each other's thinking through their conversations. As students commented, "This makes no sense" or "I don't understand this," other students would respond with, "Let's try to think of this another way." Desoete and De Craene (2019) noted that metacognitive skills were associated with mathematical accuracy.

Metacognition in Mathematics

Integrating metacognitive practices into the mathematics learning process can promote knowledge acquisition, retention, and application. At the conceptual development stage, when students are first encountering new ideas and skills, they benefit from thinking about the relationships between pre-existing knowledge and new information in order to build understanding (Mevarech & Kramarski, 2003). According to Gray (1991), "Metacognition as a component of mathematics instruction involves active learning to help students become aware of, reflect upon, and consciously direct their thinking and problem-solving efforts" (p. 24). It is important to note here that metacognitive skill development is critical for all learners, including those with learning disabilities. For example, Desoete and De Craene (2019) found that metacognitive activities can help students with learning disabilities build computational accuracy and mathematical reasoning.

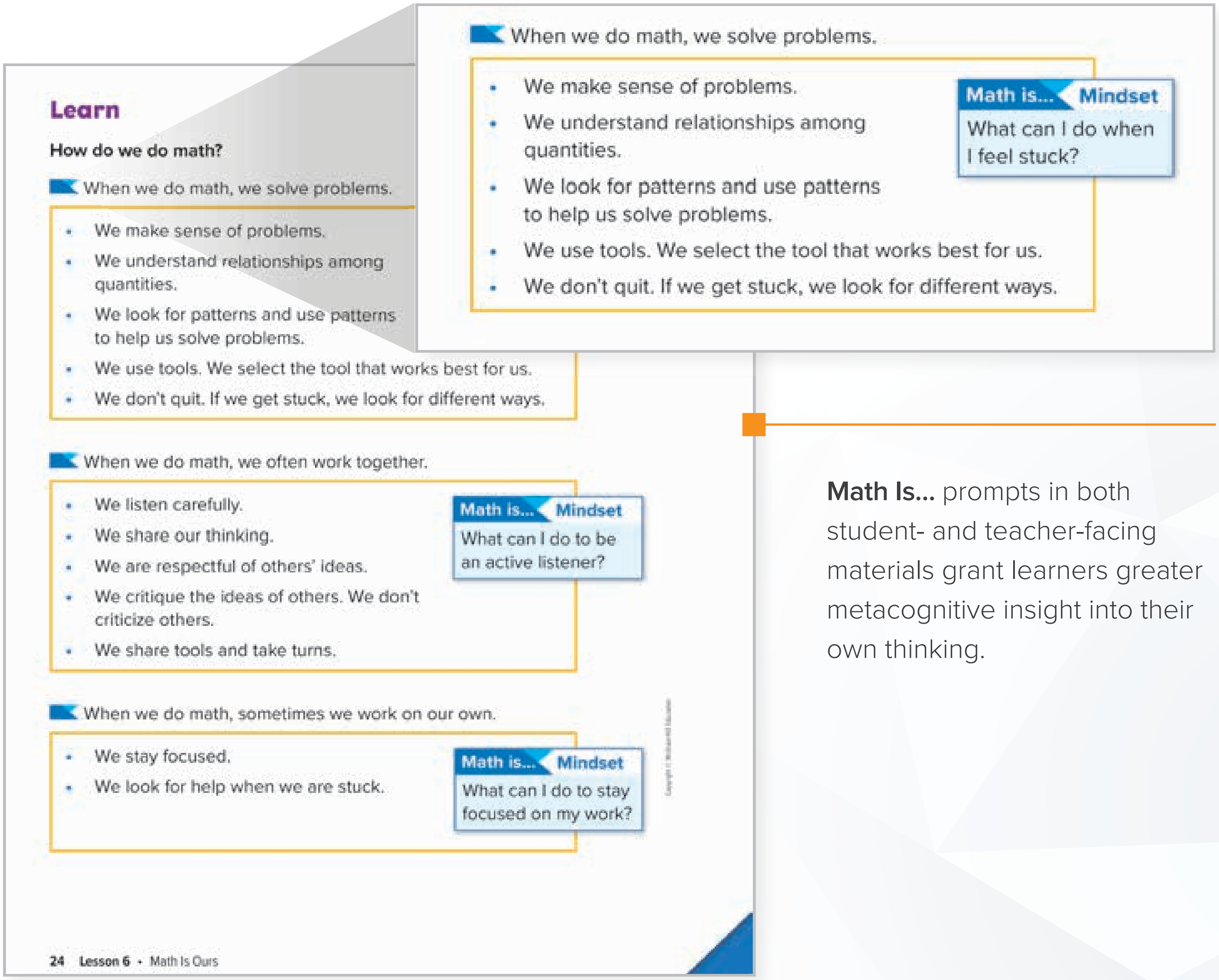
There are several practical methods that students can use to reflect on their learning and engage in overall metacognition. Verbalizing and writing the steps to solve a problem is one method that helps students reflect on, monitor, and evaluate their problem-solving abilities and strategies. This has been shown to increase conceptual understanding and provide students the opportunity to evaluate their learning (Gray, 1991; Martin et al., 2017). Another method involves writing about their thinking, which contributes to their mathematical learning (Martin, Polly, & Kissel, 2017). For example, students may write math journal entries to think about what they have learned and what they may not yet understand.

"Metacognition as a component of mathematics instruction involves active learning to help students become aware of, reflect upon, and consciously direct their thinking and problem-solving efforts."

-Susan Gray, 1991

Metacognition and Reflection in *Reveal Math*

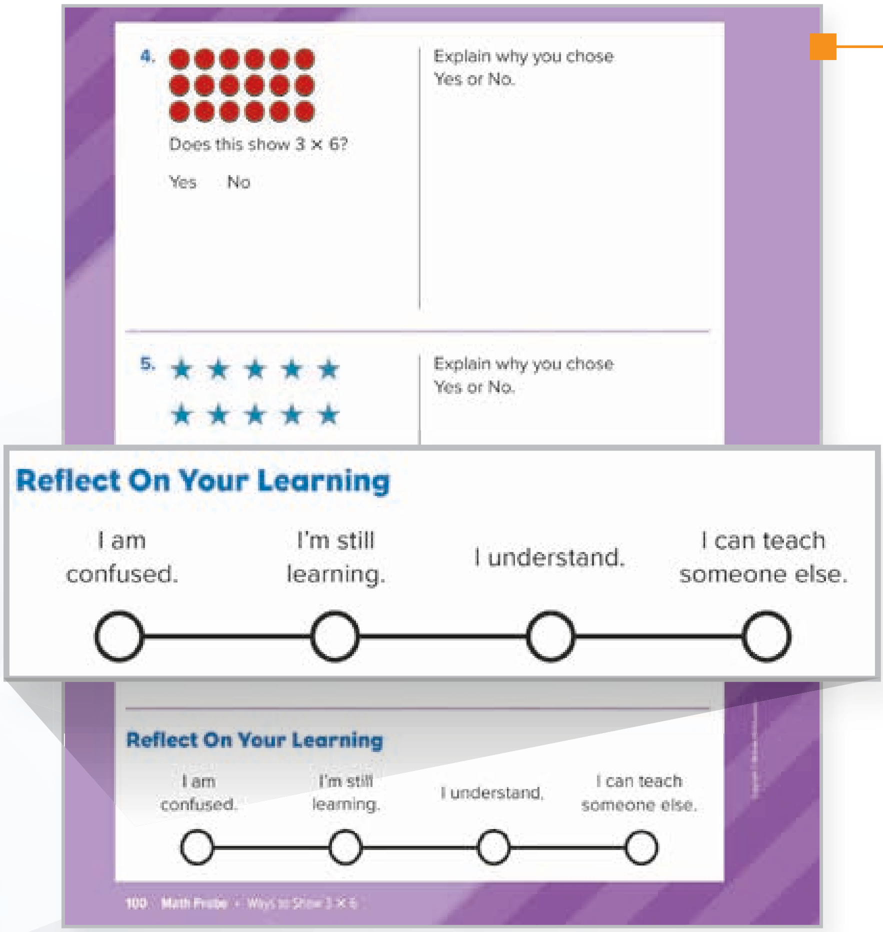
Teachers can best support students by utilizing prompts that encourage reflection—that ask students to justify their reasoning and choice of strategy or elaborate on their high-level thought processes (Booth et al., 2017; Hattie, 2017, p. 152). *Reveal Math* provides additional metacognitive prompts in both student- and teacher-facing materials. For example, Math is... Mindset prompts, found at critical points throughout each lesson, encourage students to think about their own thinking in relation to the information presented or a mathematical problem. These prompts guide students to plan and set goals before solving a problem, a process that helps build key metacognitive skills. The activities also promote reflection, prompting students to bridge their prior knowledge with new information presented in the lesson.



Math Is... prompts in both student- and teacher-facing materials grant learners greater metacognitive insight into their own thinking.

Additionally, Metacognitive Checks appear within each unit's Math Probe, as well as at the end of each lesson as part of the formative assessment Exit Ticket or Lesson Quiz*, which assess students' understanding of the lesson concepts. Here, students are prompted to "Reflect On Your Learning," which allows them to consider how well they understand the lesson content and engage in thinking about their own thinking and how they feel about their learning.

This approach connects intuition, modeling, and conceptual representation—the intersection of which fosters deeper mathematical learning (Hattie, 2017, p. 136). Metacognition also empowers students to take greater control of their education, building from the support of a teacher's modeling and moving toward the ability to practice skills and concepts independently. An added benefit to this approach is that when teachers use strong focusing questions, they are also modeling how to ask clarifying questions in a way that will serve students better in later phases of learning, when they ask themselves those clarifying questions.



Metacognitive Checks within the Math Probes contain Reflect on Your Learning prompts that call for students to evaluate their understanding of the material.

**Lesson Quiz only in 6–8. 6–8 lessons are divided into two sessions, the first culminating in the Exit Ticket to inform instruction and the second ending in the Lesson Quiz to inform differentiation. K–5 lessons are one session culminating in the Exit Ticket.*

Productive Struggle

When defining productive struggle, it is important to note what it does not entail. Productive struggle should not result in unnecessary frustration derived from overly difficult tasks or challenges that are not mathematically appropriate or useful (Hiebert and Grouws, 2007; Warshauer, 2014). The goal of productive struggle is to allow students to engage in math thinking that causes some cognitive dissonance or disequilibrium but lies within the students' current ability to reason.

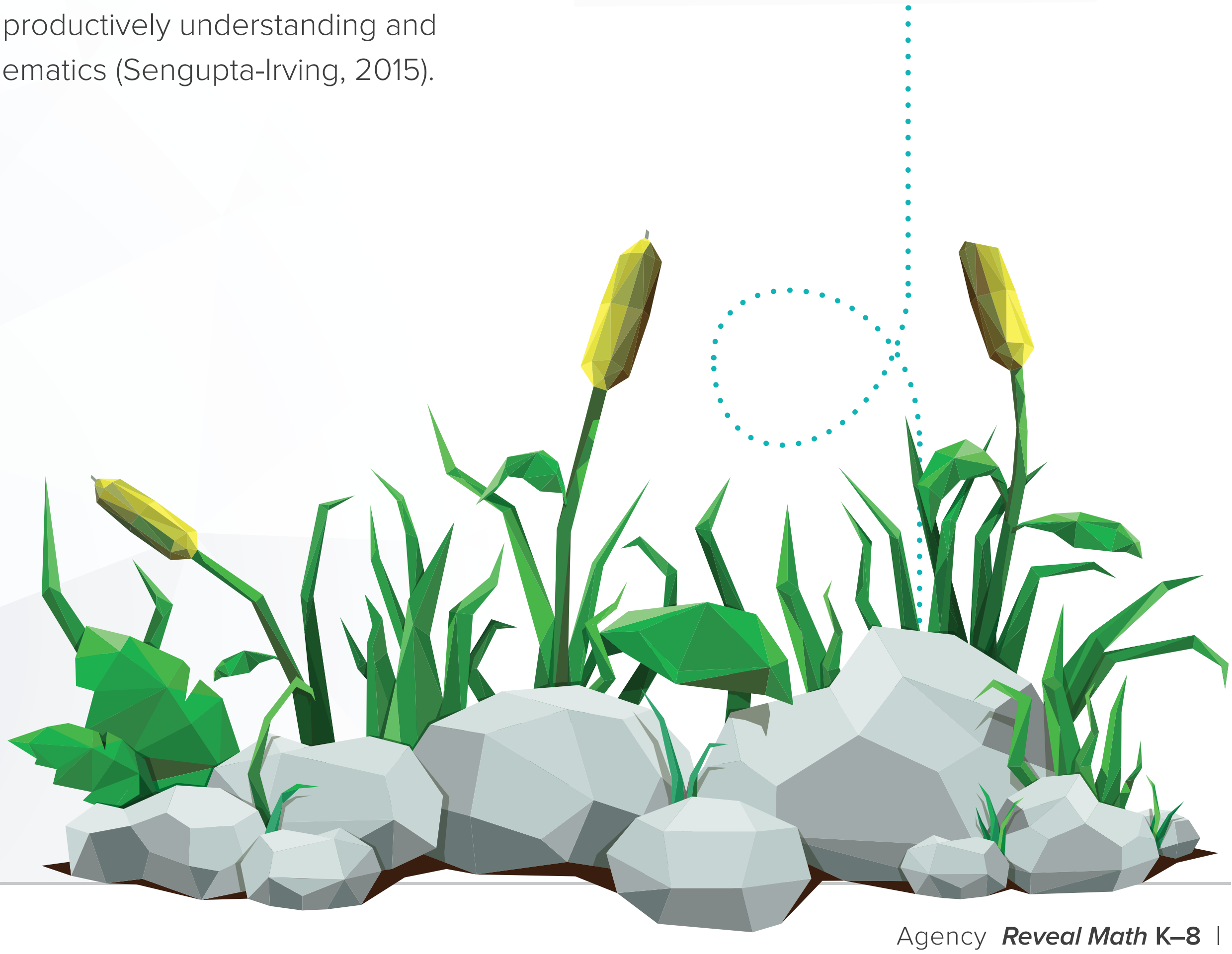
Research identifies productive struggle as an essential component of effective mathematics classrooms resulting from opportunities for students to attempt solving problems that target new and unfamiliar concepts (Boaler & Dweck, 2016; Preiss & Sternberg, 2010; Warshauer, 2014; Hattie, 2017, p. 117). Drawing on the idea that students need to engage in thinking that has some perplexity, confusion, or doubt (Dewey, 1933), Hiebert and Grouws (2007) describe productive struggle as "the intellectual effort students expend to make sense of mathematical concepts, to figure out something that is not immediately apparent" (p. 387).

However, research on productive struggle points to a certain tension between what we know is best for learners and our natural inclination to reduce discomfort and difficulty for them (Seeley, 2016, p. 22). When teachers perceive students' struggles, or their inability to answer a problem correctly at first, as a reason to "rescue" them by explaining the solution, they ultimately detract from the opportunity for students to tighten their grasp of new mathematical ideas through experiential learning (NCTM 2014, p. 48).

Research has shown that "students' struggles with learning mathematics are often viewed as a problem and cast in a negative light in mathematics classrooms" (Hiebert & Wearne, 1993; Borasi, 1996), which harms both engagement and learning outcomes. Allowing productive struggle to become an integral part of learning environments normalizes it, thereby transforming the occasional mistake into a natural and positive part of progressing toward understanding.

> *"Students' struggles with learning mathematics are often viewed as a problem and cast in a negative light in mathematics classrooms."*
>
> **–James Hiebert and Diana Wearne, 1993**

In a math classroom, how students choose to participate and the learning opportunities they are afforded directly impact how students will exercise their agency toward engaging with and doing math (Sengupta-Irving, 2015). Through productive struggle, students learn to reason about math, fail and make mistakes, and debate ideas and solutions, all of which can allow the expression of student agency toward the goal of productively understanding and doing mathematics (Sengupta-Irving, 2015).



Productive Struggle in *Reveal Math*

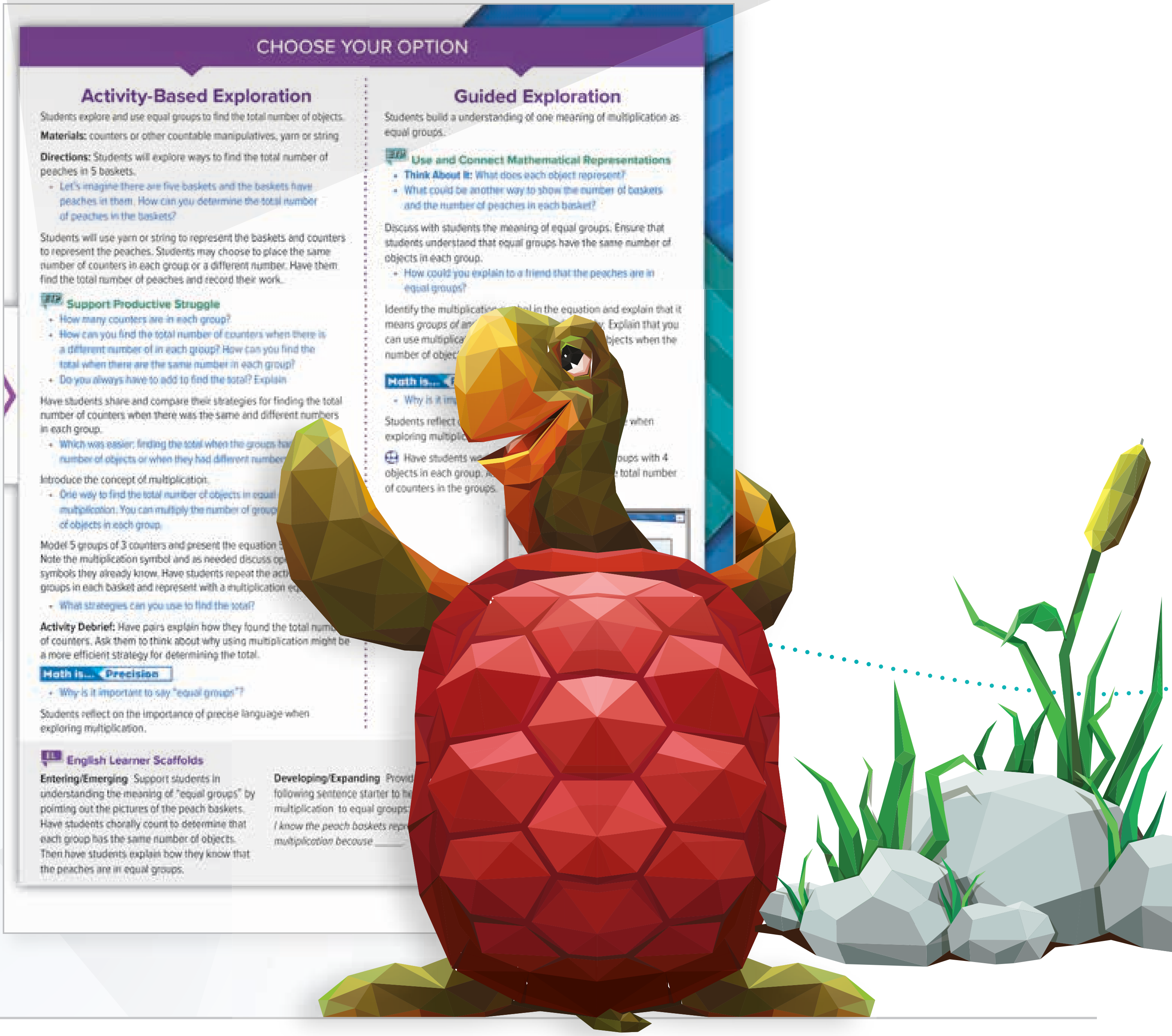
Among the hallmarks of productive struggle in practice is the notion that student-centered activities should be built into the curriculum rather than appearing as optional or time-permitting only (Cowen, 2016). The instructional model of *Reveal Math* incorporates this idea with the Explore and Develop activity-based exploration in each lesson, essentially posing a problem to students before teaching specific methods to solve (Boaler & Dweck, 2016, p. 81). During these activities, students are introduced to a new concept by starting with a rich task that has multiple points of entry (Hattie, 2017)—or, put differently, a task that has a "low floor and high ceiling" for students with varied abilities (Boaler & Dweck, 2016, pp. 84–85).

The activity encourages individual learners to suggest strategies for working through the problem before teachers formally introduce procedures, formulas, and new concepts. Critically, during the activity-based explorations, students must engage in productive struggle while working toward solutions—drawing on their intuitions and existing knowledge and taking opportunities to hypothesize about the nature of the problem. Seely (2016) posits, "When students have some time to explore and even struggle with a problem, our role as teacher becomes one of facilitating and stimulating conversation among students to ensure that they uncover and discuss the important mathematical ideas that lie within the problem" (p.33).

This instructional method is designed to maximize engagement and set the stage for new concepts, vocabulary, and procedures appearing later in the lesson. The *Reveal Math* Teacher Edition also provides teachers with scaffolded questions to guide students who might feel discouraged. These purposeful questions are located at the point-of-use and help teachers find ways to alleviate frustration while still allowing students to explore and find their own paths through the problem.

Support Productive Struggle

- How many counters are in each group?
- How can you find the total number of counters when there is a different number of in each group? How can you find the total when there are the same number in each group?
- Do you always have to add to find the total? Explain



03 | Discourse

Classroom Discourse and Language

Classroom discourse is an important avenue for learning and an underutilized tool in mathematics classrooms. Discourse encompasses interactions between members of the community and their attempts to develop shared meanings using a variety of tools, language, and norms (Bennett, 2014; Hicks, 1995; Lampert, Rittenhouse, & Crumbaugh, 1996; Moschkovich, 2012; Sherin, 2002; Yackel & Cobb, 1996). As Sherin (2002) explains, "discourse is the process of how individuals communicate" (p.206)—a key aspect of learning.

In mathematics classrooms, "discourse requires students to evaluate and interpret the perspectives, ideas, and mathematical arguments of others as well as construct valid arguments of their own" (Bennett, 2014, p.20). Discursive practices in the classroom help students build shared knowledge about mathematical ideas, language, representations, and symbols, so they may all participate in and learn mathematics. As described by Steele and Raith (2017), "Mathematical discourse should build on and honor student thinking, provide students with opportunities to share ideas, clarify understandings, develop convincing arguments, and advance the mathematical learning of the entire class" (p.123). Building a community of learners through a deliberate focus on classroom culture and norms allows the classroom community to socialize into new ways of interacting through discourse (Bennett, 2014; Lampert & Cobb, 2003).

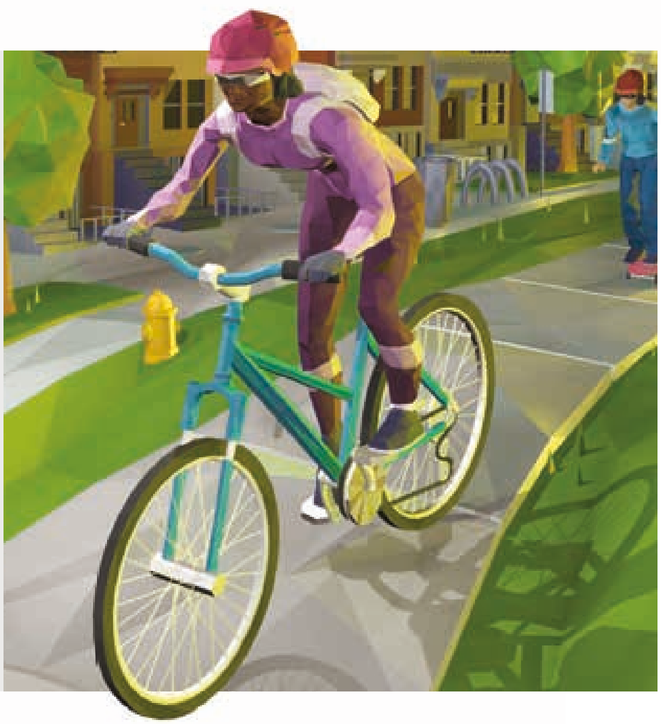
Classroom Discourse Promoting Equity

Discourse has also been shown to support equitable classroom environments because it can serve as a lever in advancing the mathematical learning of all members of the classroom community (NCTM, 2014). Moschkovich (2012) posits, “Classroom practices that support mathematical reasoning and broaden participation provide opportunities for students to use multiple semiotic resources to participate in, combine, and value multiple mathematical discourse practices. Equitable classroom practices also honor student resources, in particular the ‘repertoires of practice’ among students from nondominant communities” (p. 16). Using classroom discourse to share ideas, clarify understandings, construct arguments, develop language, and learn to see the perspective of others allows all students to participate, feel safe, and be empowered to take control of their learning (NCTM, 2014, p. 29).

While discourse involves more than language, Wagner, Herbel-Eisenmann, and Choppin (2012) point out that “language exemplifies and creates culture, and consequently, the language of instruction privileges culture associated with that language” (p. 2). By paying close attention to discourse practices in the classroom, teachers can surface the cultural knowledge and skills that inform the ways students use language in academic talk. In mathematics, students need to contend not only with new concepts and procedures that make up the school mathematics landscape, but also the mathematics vocabulary that accompanies such learning. As students engage in the discourse of the mathematics classroom, they begin to add the formal math language into their personal and informal language. Thus, teachers must leverage students’ informal language as a bridge to the more formal aspects of mathematics terminology and ideas (Lampert & Cobb, 2003).

“Mathematical discourse should build on and honor student thinking, provide students with opportunities to share ideas, clarify understandings, develop convincing arguments, and advance the mathematical learning of the entire class.”

-Cory Bennett, 2014



Classroom Discourse and Language in *Reveal Math*

Reveal Math was developed around the belief that mathematics is not just a series of operations, but a way of communicating and thinking. Teachers will find language supports embedded at the unit and lesson levels to help all students build a shared language with which to communicate effectively about math. For example, the Language of Math prompts promote the development of key vocabulary terms that support how we talk about and think about math in the context of the lesson content.

The instructional design of *Reveal Math* keeps the teacher as the facilitator and encourages rich class discussion, participation, and reasoning from the very beginning of the lesson. Every lesson launches with a Be Curious activity consisting of a sense-making routine. Designed to develop students' ability to make sense of a situation, the activity guides a classroom discussion where students engage in collaborative conversations to connect and apply mathematics. Teachers additionally lead whole-group discussions to connect concepts to strategies and procedures using examples during the **Explore and Develop** instructional moment.

To promote effective reflection and collaboration, teachers best support students by utilizing prompts that encourage students to justify their reasoning or choice of strategy—or at a minimum, to elaborate on their thought process (Booth et al., 2017; Hattie, 2017, p. 152). *Reveal Math* provides Math Is... prompts in both student- and teacher-facing materials, affording learners greater metacognitive insight into their own thinking. Further, it aligns with a movement away from the teacher as the sole authority in the classroom, and towards a more effective and engaging mode of student-driven learning. Additionally, the instructional design of *Reveal Math* integrates all eight of NCTM's Effective Mathematics Teaching Practices throughout the lesson, providing teachers with open-ended questions to support meaningful classroom discussions.

Patterns can help you solve problems.

It is always true that when you subtract 9 from a number you will have one less than when you subtract 8.

$18 - 8 = 10$ $18 - 9 = 9$

Math is... Generalizations
Is this always true? Does this always work?

I can use this pattern to solve other problems.
- When I take one more away there will be one less.
- $10 - 3 = 7$ $10 - 4 = 6$
- 4 is one more than 3.
- 6 is one less than 7.

Math is... Generalizations
Can I use this strategy in other situations?

Work Together

What patterns do you see? How can the patterns help you solve the equations?

$14 - 7 = 7$	$14 - 5 = 9$	$14 - 3 = 11$
$13 - 7 =$ ____	$13 - 5 =$ ____	$13 - 3 =$ ____
$12 - 7 =$ ____	$12 - 5 =$ ____	$12 - 3 =$ ____
$11 - 7 =$ ____	$11 - 5 =$ ____	$11 - 3 =$ ____

Patterns can help me solve a problem.

- If I subtract 9 from a number, it will be one less than if I subtract 8.

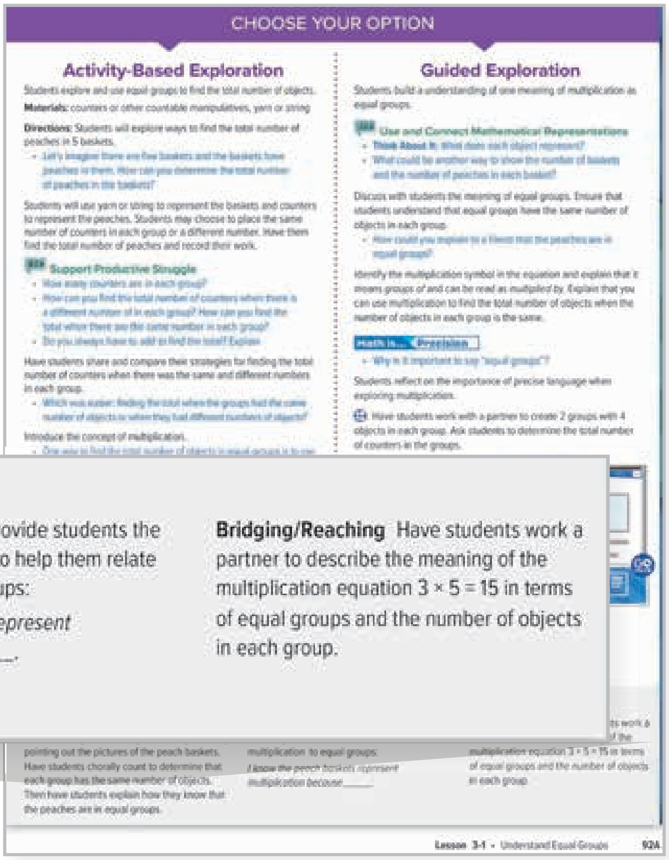
$15 - 8 = 7$ $15 - 9 = 6$

$14 - 8 = 6$ $14 - 9 = 5$

Math is... Patterns
How can the pattern help me solve the problem?

Both **Explore and Develop** pathways—Guided or Activity-Based Exploration—allow students to discuss and consider multiple representations, strategies, and procedures when solving problems. To support this discourse, ample resources are meaningfully integrated into the lesson through **Math Is...** prompts.

English Learner Scaffolds provide teachers with point-of-use practices to help EL students cultivate meaning of math vocabulary as well as ideas and concepts in context. Three levels of specialized instruction—Entering/Emerging, Developing/Expanding, and Bridging/Reaching—support varied learning needs.



English Learner Scaffolds

Entering/Emerging Support students in understanding the meaning of "equal groups" by pointing out the pictures of the peach baskets. Have students chorally count to determine that each group has the same number of objects. Then have students explain how they know that the peaches are in equal groups.

Developing/Expanding Provide students the following sentence starter to help them relate multiplication to equal groups:
I know the peach baskets represent multiplication because ____.

Bridging/Reaching Have students work a partner to describe the meaning of the multiplication equation $3 \times 5 = 15$ in terms of equal groups and the number of objects in each group.

04 | Sense-Making

Sense-Making

The work of a mathematician is to solve problems. Developing problem-solving skills—such as the ability to understand the problem context, make sense of the issue at hand, and find or create the tools needed to solve the problem—is a vital part of school mathematics. This is because ultimately, the ability to reason with and make sense of fundamental standards, skills, and facts is what makes them useful and usable (Ball & Bass, 2003).

What does it mean to make sense of mathematics? Schoenfeld (1992) describes what it means to think mathematically as "...developing a mathematical point of view... and developing competence with tools of the trade, and using those tools in the service of the goal of understanding structure" (p.1). Put another way, "Understanding is the key to becoming a mathematician. Understanding what a problem is asking, understanding how to come up with a strategy to solve the problem, and understanding enough to write or draw in detail how to solve the problem... is crucial to becoming a competent and confident mathematician" (Ostrow, 1999, p. 4). It is important for students to make sense of mathematical ideas themselves as they work toward mathematical proficiency. However, Ball and Bass (2003) point out that "making sense refers to making mathematical ideas sensible, or perceptible, and allows for understanding based only on personal conviction" (p. 29).

Individual sense-making is just a first step in developing mathematical understanding and reasoning; opening up individual thinking to discussion and critique in a community of learners allows for the development of a collective set of practices and norms that is the backbone of mathematical reasoning (Yackel & Hanna, 2003). The mathematics classroom, then, becomes the community of practice within which students "develop the appropriate mathematical habits and dispositions of interpretation and sense-making" (Schoenfeld, 1992, p.13).

Developing Sense-Making in the Classroom

How can teachers create a community of practice steeped in the idea that sense-making is crucial to learning mathematics with understanding? Research has shown practices teachers can employ that support the building of sense-making and reasoning in a classroom community.

In a study of fourth- and fifth-grade teachers and students, Kazemi and Stipek (2001) found that while all teachers enacted social norms that created a safe classroom environment for mathematical thinking and learning, only two of the teachers employed socio-mathematical norms in ways that supported deeper sense-making and reasoning. In those classrooms, students were required to use mathematical thinking —not just procedural steps—to justify answers, demonstrate an understanding of the relationship between strategies, use errors to reconceptualize a problem, and work collaboratively to understand problems through arguments and mathematical justifications (Kazemi & Stipek, 2001, p. 78).

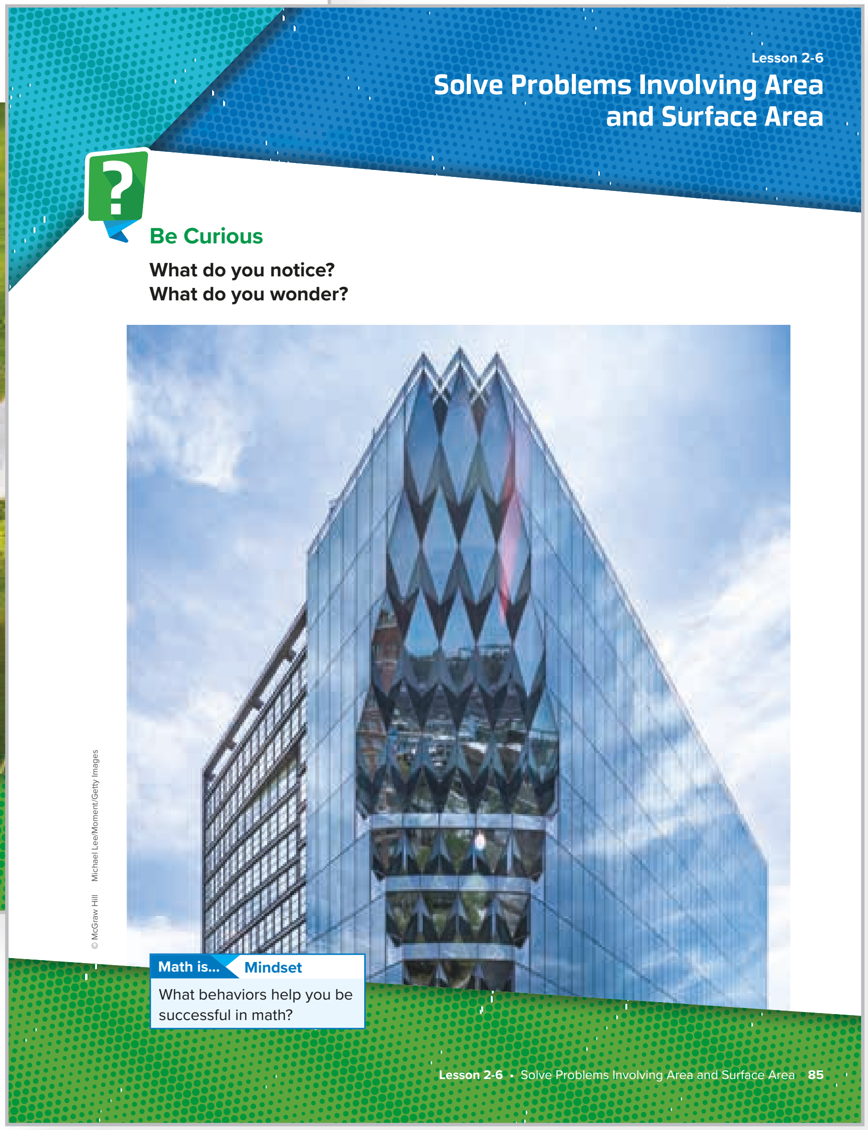
By focusing on more specific socio-mathematical practices and norms, teachers can create a community of practice that immerses students in math classrooms designed to make sense-making and reasoning a consistent and regular part of learning mathematics. In a fraction-sense intervention study focused on understanding and reasoning about fraction meanings and relationships, students in the intervention outperformed control students on measures of fraction magnitude, fraction concepts, and fraction arithmetic, demonstrating the improved academic achievement of students engaged in sense-making as a regular part of learning math (Dyson et al., 2018).

Sense-Making in *Reveal Math*

Reveal Math supports teachers in their efforts to develop an approach that allows students to make sense of problems and develop problem-solving skills. Drawing on the work from *Reveal Math* author Annie Fetter, *Reveal Math* incorporates sense-making routines into every lesson launch.

A **Be Curious** activity launches every lesson and is designed to encourage students' curiosity and ideas while they observe a situation, problem, or phenomenon. Students apply previously learned problem-solving strategies or knowledge to make sense of the problem at hand or to wonder about how they should approach the situation. Built to respect and welcome all ideas, the exercise permits students to discuss what they notice about the problem and what they don't know or understand. The focus is to engage the classroom community in making sense of the problem and context and encourage curiosity about the mathematics. In order to help students be curious, notice, and wonder about math problems, Be Curious moments in *Reveal Math* employ four practices, suggested by Fetter, to encourage sense-making: get rid of the question; get rid of the question and the numbers; give students the answer; ask about ideas, not answers.

These practices allow students to find entry into problems by connecting the knowledge they currently hold to the problem-solving discussion. Students have the opportunity to determine what they know and what they still need to know.



Engaging in **Be Curious** moments as a consistent part of math teaching and learning goes a long way toward developing a mathematics community that supports thinking, reasoning, and communicating and sets the stage for using sense-making throughout all components of the *Reveal Math* program.

05 | Fluency

Mathematical Fluency

There is a good deal of research on mathematical fluency explaining what it is and the role it plays in students' mathematical learning. There is some theoretical debate surrounding what fluency means and how it can be measured: Biancarosa and Shanley (2016) state that it should be treated as "a holistic description of a skilled performance" (p. 14). In other words, it is not one specific skill, nor is it simply about speed. Baroody (2011, as cited in Clarke, Nelson, & Shanley, 2016) defined fluency as the quick, accurate recall of facts and procedures, and the ability to use them efficiently (p. 71). Carr et al. (2011) similarly describe it as both the retrieval of math facts as well as the ability to quickly compute answers to more complex problems.

The repeated themes in fluency research seem to relate to accuracy and speed (Rhymer, Dittmer, Skinner, & Jackson, 2000) as well as efficiency. Efficiency and speed are somewhat related in that, as students develop and use more efficient strategies to solve problems, they are likely to increase their speed. Thus, for the purposes of this paper, fluency refers to the accuracy and speed at which a student computes a mathematical computation.

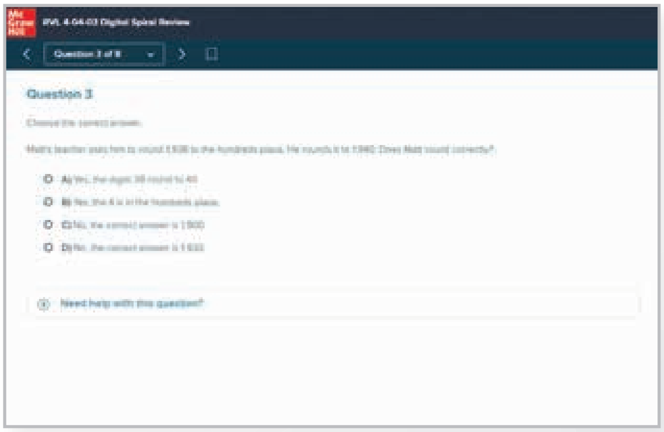
Fluency, not to be confused with automaticity, involves the application of automatic computation. For example, multi-digit addition or long division requires the application of memorized computations while fluently carrying out the procedure (Hasselbring & Bausch, 2017). This means that although automatic recall of math facts is important, students must also be able to quickly and accurately conduct procedures to be fluent in more complex mathematical computations. In this way, fluency also takes into account the relationships among conceptual understanding, procedural knowledge, and basic fact recall (Clarke, Nelson, & Shanley, 2016).

Fluency in *Reveal Math*

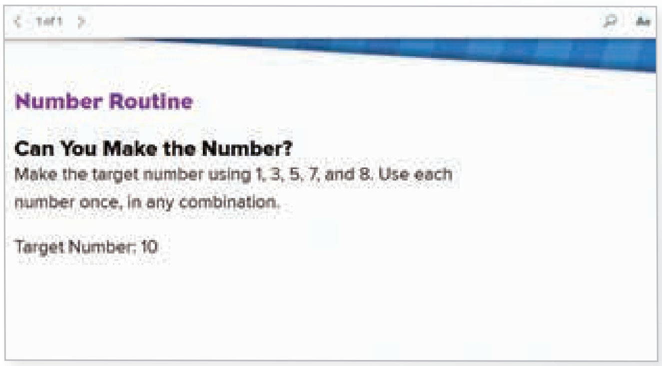
Each lesson in *Reveal Math* begins with a **Number Routine**. These short activities are designed to help students activate their prior knowledge and to practice skills that will be needed for the new mathematical content. Often, these activities include problems that aid in increasing students' computational accuracy and speed.

At the end of each lesson, students are provided practice problems. By completing these problems and receiving immediate feedback, students can work on their computational speed and accuracy and become more efficient with corresponding mathematical tasks. Additionally, engaging Digital Games and Spiral Review practice offer further opportunities to build fluency while providing immediate feedback.

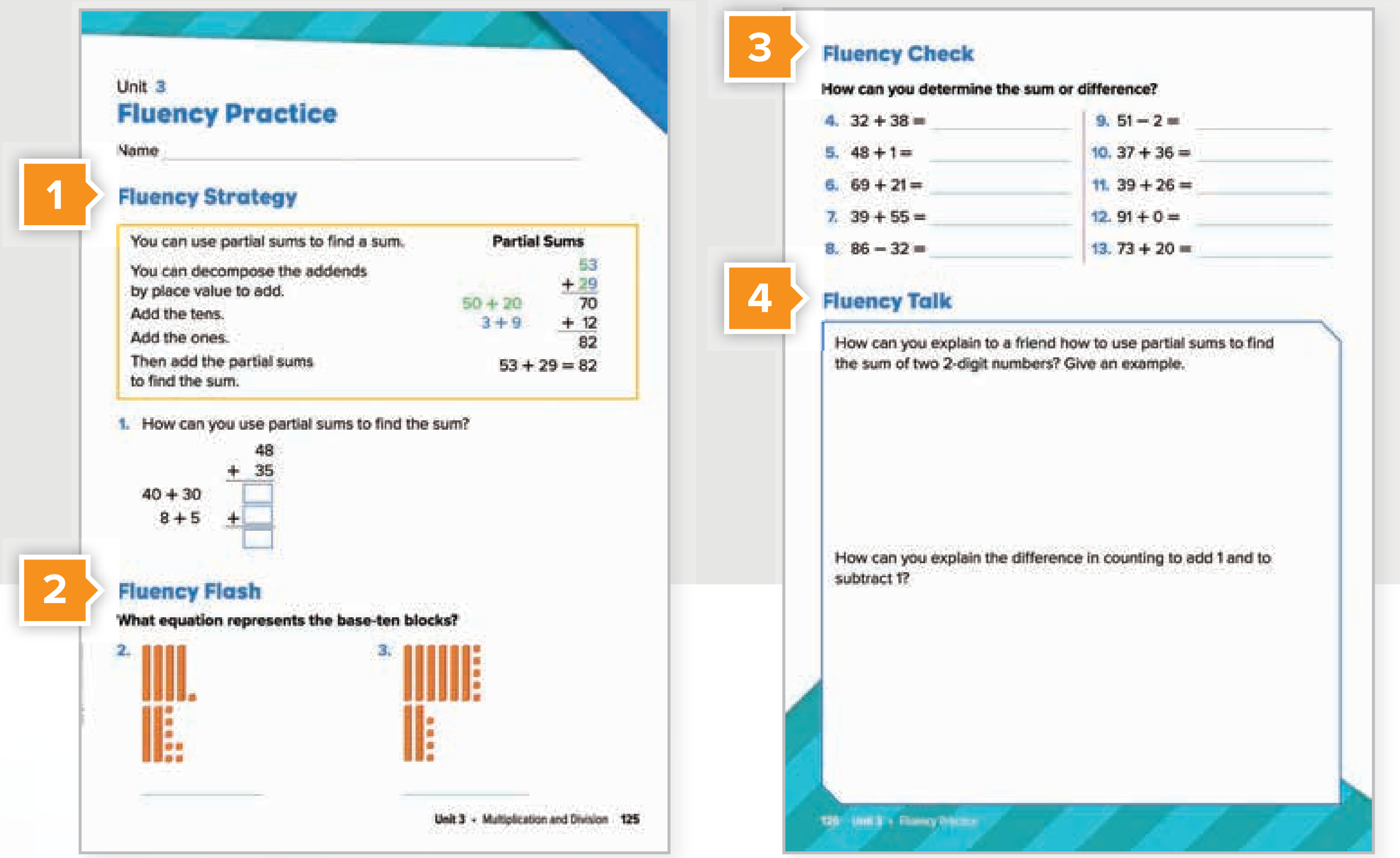
Digital Games



Spiral Review



Number Routines



At the end of each unit, students are provided "Fluency Practice." These pages contain the following sections:

1. **Fluency Strategy:** Students are presented a strategy to help them recall their prior learning. They are asked questions related to the strategy, which promote conceptual understanding of the strategy and computational skill.
2. **Fluency Flash*:** Students are provided one or two quick problems that may involve mathematical models and asked to write or solve a problem. These types of problems also aid in developing conceptual understanding.
3. **Fluency Check:** Students complete practice problems designed to increase speed and accuracy with specific computational skills.
4. **Fluency Talk:** Students are given a prompt and space to write about their strategies and explain their thinking

*K–5 only.

06 Instructional Routines

Instructional Routines

Most teachers establish classroom routines during the first few days of school. These routines can help students understand expected behaviors and reduce the cognitive demands of learning new concepts (Leinhardt, Weidman, & Hammond, 1987). Well-practiced and understood classroom routines allow students to remain concentrated on learning without diverting attention to more general rules and activities (Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010; Leinhardt et al., 1987). As defined by Leinhardt and colleagues (1987), "Routines... are fluid, paired, scripted segments of behavior that help movement toward a shared goal. Routines can have explicit descriptors, can be modeled or, more commonly, can simply evolve through shared exchange of cues" (p. 136). When implementing routines, it is critical that students are aware of and involved in the learning process, with clear roles and expectations (Bulgren & Scanlon, 1998). These can be placed into categories of routines, including management, instructional support, and teacher-student exchange (Leinhardt & Steele, 2005).

Of importance to curriculum programs are routines that support instruction. According to Yinger (1979), "Instructional routines are methods and procedures established by the teacher to carry out specific instructional moves" (p. 166). Instructional moves are steps a teacher takes to conduct and carry out activities. Yinger gives examples such as, "giving instructions, questioning, presenting information, monitoring, evaluating student performance, and offering feedback" (p. 165). Critical aspects of instructional routines include frequency of use, closeness to classroom practice, positive impacts to the learning of all students, and the ability to teach the routines in multiple settings (Hiebert & Morris, 2012).

For example, several different routines can be used to support instructional dialogue, a practice in which “an explanation is co-constructed by the teacher and students in the class during an instructional conversation” (Lampert et al. 2010, p. 131).

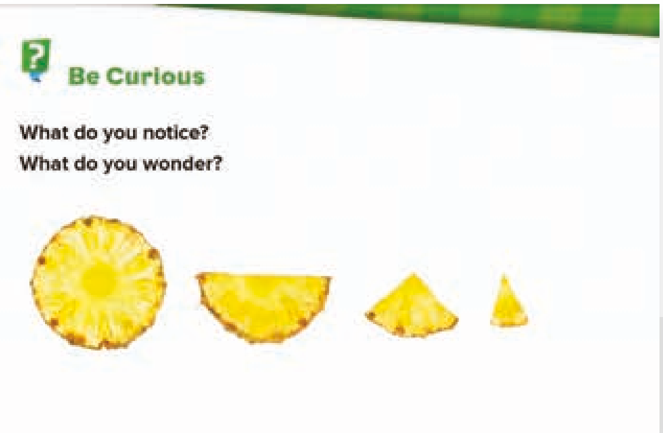
The ability to facilitate meaningful discussions using routines takes training and understanding by the teachers. In their work with instructional dialogue, Leinhardt and Steele (2005) used what they call “exchange” routines to help when explanations are necessary. These include the call-on routine, the revise routine, and the clarification routine. The call-on routine involves an “open invitation to discussion” (2005, p. 143), often followed by prompts to further explain, clarify, or give other comments. The revise routine can be coupled with the call-on routine, wherein students can revise or expand on previous statements. The clarification routine could be used when there is confusion surrounding a topic or idea. Each of these routines has its own established rules so that every student can feel protected and willing to share and contribute to the dialogue.

Instructional Routines in *Reveal Math*

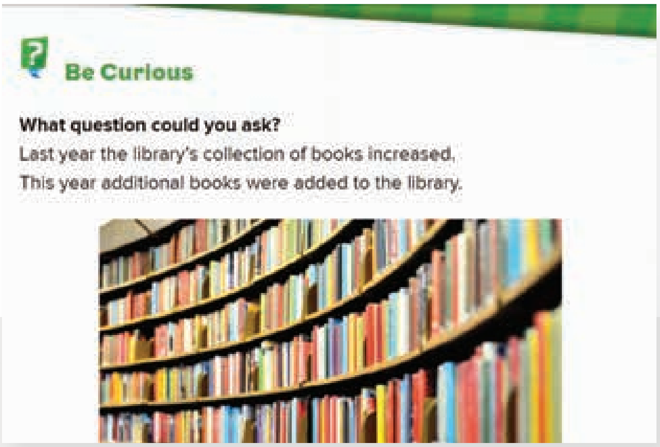
Reveal Math provides three types of instructional routines throughout the program: Sense-Making Routines, Number Routines, and Math Language Routines. By establishing these routines early on and adapting them as students progress to the next grade level, teachers can help reduce cognitive load, and students can focus on their mathematical thinking and learning.

Sense-Making Routines

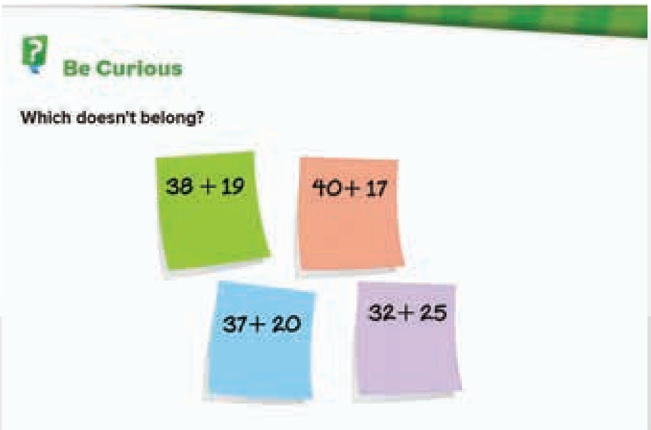
In order to become problem solvers in mathematics, students must understand the problem context, make sense of the issue at hand, and then find the tools needed to solve the problem. As such, *Reveal Math* includes the following sense-making routines:



Notice and Wonder™—Developed at the Math Forum, the Notice and Wonder routine has teachers present students with an image or problem scenario without providing any questions, data, or answers. Students write or draw the things they notice and wonder. The teacher can then engage students in a class discussion about their observations and record their comments on the board. The teacher should allow for students' comments to be non-mathematical in nature but should eventually want to steer the conversation to focus on mathematical wonderings.



Numberless Word Problems—This routine requires students to look for relationships among the objects given in the problem and to discuss the things they notice. It begins by presenting a problem or image without any numbers. Teachers serve an important role in guiding students in this routine as they may be confused by the lack of numbers. The teacher should help broaden student thinking about problems, beyond solving and numbers, and assist as students make connections to other students' thinking and strategies.

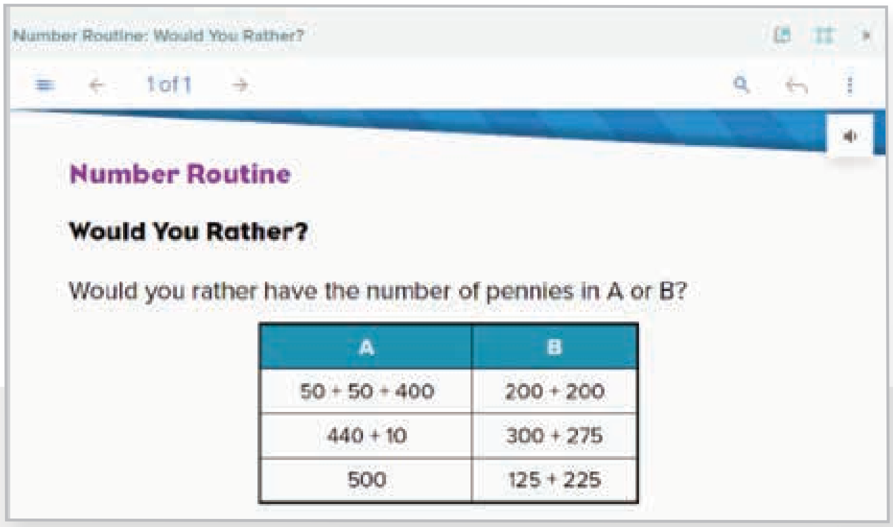


Which Doesn't Belong?—In this routine, students look for similarities and differences among numbers, images, or terms, and determine which one doesn't belong with the others in the group. The teacher begins by presenting three to no more than six numbers or images with attributes, such as color or size, and then gives students time to think about the similarities and differences to determine which one doesn't belong. The teacher should encourage students to find more than one solution.

Is It Always True?—In this routine, students are presented with one or more images or situations and think about the relationships among the objects in the image. Students consider whether the relationships always hold true or whether they are unique to this particular image or situation.

Number Routines

The purpose of number routines is to help students develop a better sense of numbers and how they function. Without a solid foundation in number sense, it is difficult to learn and understand geometry and statistics, for example. Using number routines, students are able to make more sense of math rather than simply following rigid sets of rules (Shumway, 2011). *Reveal Math* includes the following number for K–5:



About How Much?—Students build estimating skills by explaining their strategies and then comparing and analyzing their estimates to the actual value.

Break Apart/Decompose It—Students build flexibility with numbers by decomposing them, sharing their thinking, and discussing patterns.

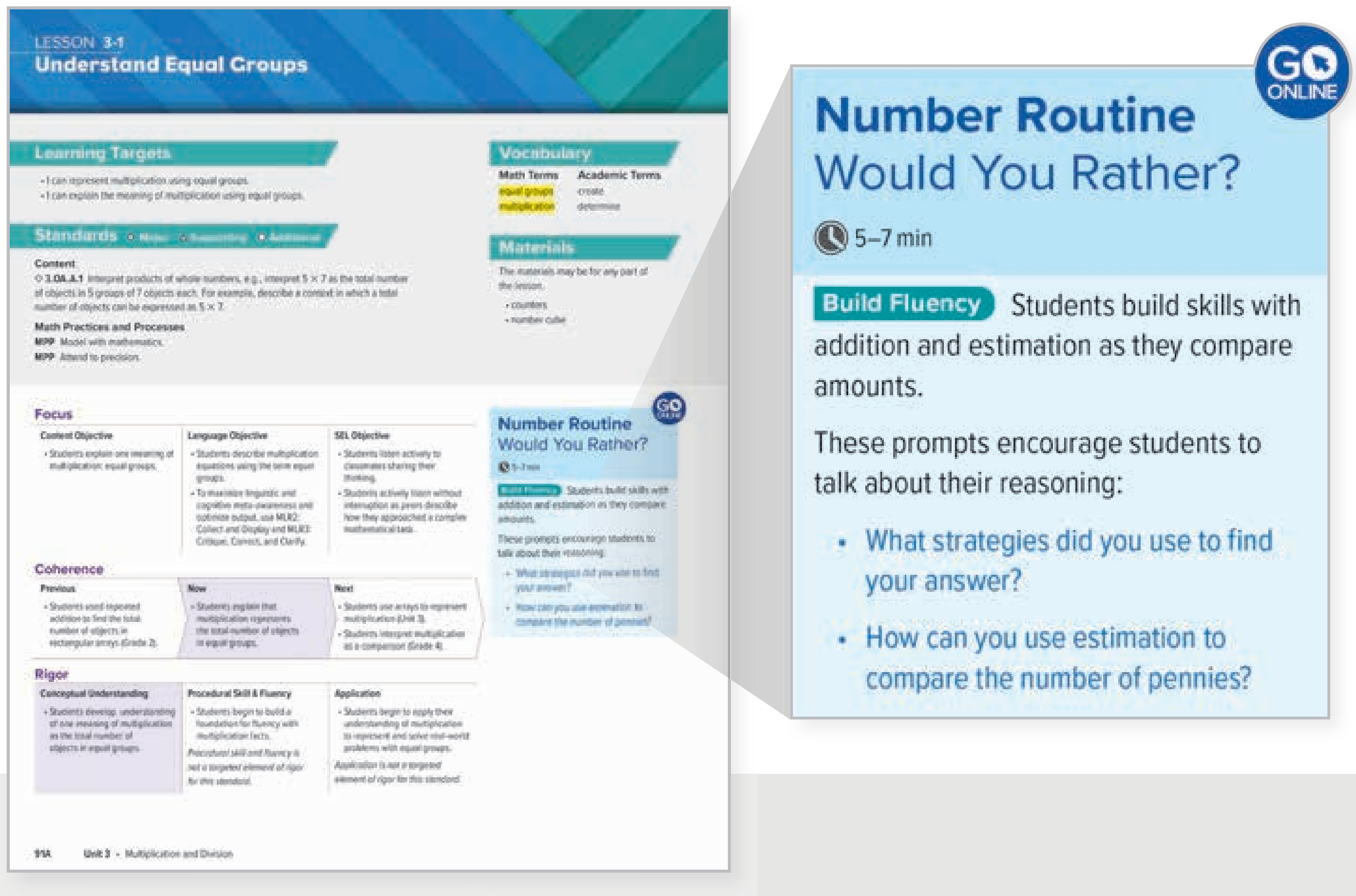
Can You Make the Number?—Students build flexibility and efficiency with operations by building expressions with a value for the given target number.

Find the Pattern, Make a Pattern—Students build efficiency by determining the rules for a given pattern, and then continue the pattern or create a new pattern.

Find the Missing Values—Students build their identification of patterns and efficiency with solving equations by analyzing a series of equations, looking for patterns, and finding missing values.

Greater Than or Less Than—Students build place value sense, estimations skills, and comparison skills by estimating or evaluating the value of an expression and comparing it to a target benchmark number.

Let's Count—Students build proficiency with skip counting by counting forward or backward using a given counting interval.



Mystery Number—Students build mathematical reasoning and thinking by looking at clues one at a time, proposing possible solutions, and eliminating solutions that are no longer viable.

What Did You See?—Students build visual discrimination, quantitative reasoning, and mathematical discourse by viewing images and then describing and discussing what they saw.

What's Another Way to Write It?—Students build number sense by writing alternative expressions to a given expression and looking at relationships among the different expressions.

Where Does It Go?—Students build estimating skills by placing a target number on a number line and justify their reasoning.

Which Benchmark Is It Closest To?—Students enhance rounding and reasoning skills by determining which benchmark a given number is closest to and explain their reasoning.

Would You Rather?—Students build number sense and enhance decision-making by choosing between two options, both of which require mental math, and then give the rationale for their choice.

Kindergarten also includes the following number routines: Counting Things, Start and Stop, The Counting Path, and The Match. **Grades 6–8** includes the following number routines: About Between, or Exact; About How Many?; Five Breaks; Give Me Five; If I Know This...; In My Head?; Is It Reasonable?; It's About; More or Less Than...; Number Strings Matrix; Or You Could...; and This or That?

Math Language Routines

Mathematical Language Routines (MLRs) are structured but adaptable formats for amplifying, assessing, and developing students' mathematical language. These routines were developed by the Stanford University UL/SCALE team based on a framework with four design principles:

Design Principle 1: Support sense-making

> MLR **Collect and Display**
> As you discuss the questions with the students, listen and write key words on the board that students use, such as *groups, objects, number of,* and *multiplication.* Display the words and phrases for student reference. Use the student-generated expressions to help them make connections between student language and math vocabulary. Update the collection with new understandings as the lesson progresses.

Design Principle 2: Optimize output

> MLR **Critique, Correct, and Clarify**
> On the board write, *There are 5 groups with 3 objects in each group.* Pair students to discuss whether this statement about the baskets of peaches is correct. Ask them to identify any mistakes and to make changes. Have students write a new, correct version of the sentence.

The eight* Math Language Routines align with one or more of the four design principles.

Stronger and Clearer Each Time—Students revise and refine their ideas and their verbal and written output.

Collect and Display—Words and phrases spoken by students are collected into a reference for them to refer to later.

Critique, Correct, and Clarify—Students analyze, reflect on, and develop a piece of writing that is not their own.

Information Gap—Students communicate with partners or team members to convey missing pieces of necessary information.

**6–8 has one additional routine, Numbered Heads Together.*

Design Principle 3: Cultivate conversation

MLR **Stronger and Clearer Each Time**

Have students justify why Greta does or does not have enough eggs. Have them first verbalize their ideas, and then share any information they know to justify it. Encourage students to question each other, clarify, and revise as needed.

Design Principle 4: Maximize meta-awareness

MLR **Co-Craft Questions**

Have students work alone to write questions about the problem and how to solve it. Then pair students to compare their questions. Elicit questions and use them to discuss the problem and strategies for solving it in more detail.

Co-Craft Questions and Problems—Students use conversation skills to generate, choose, and improve questions and problems before producing answers.

Three Reads—Students reflect on the ways mathematical questions are presented.

Compare and Connect—Students identify, compare, and contrast different mathematical approaches, representations, concepts, examples, and language.

Discussion Supports—Students have supported discussions about mathematical ideas, representations, contexts, and strategies.

Conclusion

The foundation on which *Reveal Math* has been built reflects decades of work by educators, mathematicians, and researchers. Based on that work, the entire *Reveal Math* program is designed to spark curiosity, make connections among math concepts, build and support mathematical communication, encourage collaboration, and instill confidence in students.

A strong foundation begins with creating and supporting an equitable classroom. This involves classroom experiences designed to meet the specific needs of each and every learner. Furthermore, the integration of social and emotional learning into the mathematics curriculum can also lead to greater learning outcomes. Part of social and emotional learning involves the act of reflecting on one's own learning and thoughts. Opportunities for students to engage in metacognition are woven throughout the program at key moments as well as at the beginning and end of each lesson. Supports for students to engage in classroom discourse are included at key points in the curriculum as well.

A strong mathematical foundation begins with research in the field of mathematics education. This is also imperative for student success. Mathematical sense-making, using problem-solving skills to make sense of problems and situations, is a focus of the *Reveal Math* program. Students also have opportunities to engage in productive struggle as they explore and develop new mathematical concepts. Furthermore, to support new learning and concept development, students need to have a solid footing in accurately completing mathematical operations and procedures. The ability to quickly and efficiently execute these procedures provides students greater cognitive capacity to focus on new ideas. As such, each unit in each grade includes fluency practice.

Lastly, teachers are given supports throughout the *Reveal Math* Teacher Edition with included instructional routines. These are explained and described in the Math Is... Unit at the beginning of the course and then utilized throughout each course. This includes routines related to pedagogical content knowledge. Predictability in classroom and instructional routines allows students to focus on new learning, which can lead to greater success.

The *Reveal Math* program is designed to encourage and support teachers and students in their daily mathematical routines. This math curriculum guides students as they develop life skills that will benefit them not only in their educational careers but throughout their lives.

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