McGraw Hill Education

Applying Games and Manipulatives in Math Intervention Curriculum to Foster Enhanced Conceptual Understanding of Numbers

SRA Number Worlds™

Accelerate Math Success

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Summary

The SRA Number Worlds curriculum is often referred to as a “research-based” program to distinguish it from more traditional mathematics programs that are produced by educational publishing houses to teach learning goals established by the mathematics education community (e.g., NCTM 2000). In this paper, the author of SRA Number Worlds describes the very different roots of this curriculum and the ways that research in the cognitive sciences has shaped not only the learning goals of the program but also the manner in which these goals are taught.

The extensive use of games and manipulatives in the program to enhance math engagement and to teach number sense—as well as a variety of more specific math concepts and skills—is justified by research in the learning sciences as well as by common sense, always a useful touchstone when making any theoretically motivated curricular decisions. Although the original program has been expanded over the years to include lessons to teach Common Core State Standards that were not addressed in the original program, the primary and central focus of the program has remained true to its proven foundation since its inception in 1988. It is this focus that is the subject of the present paper.

The birth of SRA Number Worlds within a cognitive science research program

In 1988, the James S. McDonnell Foundation launched a new research program, titled "Cognitive Science for Educational Practice", in an effort to stimulate new approaches to the teaching of science, mathematics and reading and to improve the achievement of American students in each of these content domains. Cognitive science research teams who had spent years studying how children's thinking and learning develops in one of these content areas and who could propose an educational application that was based on this research were asked to apply for a 3-year grant. The present author and her colleague, Robbie Case, received one of 10 grants that were awarded. Based on the success of the first 3-year grant, the author received two subsequent 3-year grants to extend the SRA Number Worlds program to higher grade levels, to continue to assess its effectiveness, and to develop methods to enhance the knowledge and effectiveness of mathematics teachers.

Over the course of 10 years, research teams who had been awarded grants in the first, second, and/or third phase of this program met frequently to describe the educational applications they were developing, the instructional approaches they were using, and the research that supported these applications and approaches. The discussion that followed each of these presentations provided intense and stimulating learning opportunities for all present and could be described as a hotbed for knowledge development, given that the research teams involved were among the top cognitive science researchers in North America who also had an interest in education. The SRA Number Worlds program benefitted greatly from the ideas shared and the feedback provided by this group of scientists.

Learning goals of SRA Number Worlds: Teaching the Central Conceptual Structures for Number

The learning goals of the SRA Number Worlds program were quite straightforward, from a cognitive science perspective. Building on Piaget's (1950) theory of intellectual development and refinements to this theory proposed by Case (1992), Griffin, Case & Siegler (1994) were able to construct a detailed portrait of the knowledge structure that children who are successful in school math have available at the age of 5–6 years. This knowledge structure (depicted in figure 1 and described further in the following section) was called a "mental number line structure" or, more formally, a "central conceptual structure for number" because it was believed to: (a) define the knowledge that enables children to demonstrate number sense, (b) provide a foundation for all higher learning of mathematics, and (c) enable children to solve a broad range of quantitative problems, such as time and money problems as well as the more traditional arithmetic problems that children encounter in the first few years of school.

The thinking was that, if children who are successful in school mathematics have managed to construct this knowledge structure before the start of first grade and children who struggle with school math provide no evidence of having this knowledge structure available, it might be a good idea to create a math program to teach this knowledge to children who have not yet acquired it. The kindergarten level of the SRA Number Worlds program was created in 1988 to see whether this deep, foundational knowledge could be taught and whether, if acquired, it would have the effects (e.g., ensuring successful learning of arithmetic in school; enabling students to solve time and money problems that were not taught in the program) predicted by the theory.

Based on the success of this program in enabling hundreds of low-income students who started school without this knowledge to acquire it by the end of the kindergarten year (Griffin, Case & Siegler, 1994) and to achieve success in school mathematics in subsequent grades (Griffin, 2002), the program was expanded to teach higher-level conceptual structures that had also been identified in cognitive developmental research (Griffin & Case, 1997), and that were believed to underlie successful learning of math concepts (e.g. base-ten understandings; multiplication and division in grades 2–5; fraction, decimal and percent understandings in grades 5–8) at later ages and grade levels.

Although a primary focus in developing each level of the curricula was to teach number sense (as defined by the central conceptual structures for number) and to give students a solid conceptual understanding of the math concepts, skills, and problem-solving strategies expected at each grade level, the expanded program also addresses learning goals (e.g., for geometry, measurement and statistics) suggested in the Common Core State Standards that were not included in the original program. Teachers are responsible for ensuring that their students master high priority concepts listed in the grade level standards and users of the SRA Number Worlds curriculum can feel confident that lessons to teach these key standards for accelerating learning have been included.

In the following section, the author describes the 6-year-old central conceptual structure for number. The pre-K and kindergarten levels of the program (Levels A and B) were designed to teach this knowledge and several lessons in the Grade 1 (Level C) program were designed to ensure that this knowledge has been thoroughly mastered before moving on to teach foundational knowledge for the 8-year-old central conceptual structure that is taught in the Grade 2 (Level D) program. The choice to describe the least complex central conceptual structure was motivated by two considerations. First, this structure provides the foundation for all higher-order conceptual structures, which are built on this knowledge base in increasingly complex ways. Second, this structure is the easiest to describe because the structure at the next level up—the 8-year-old central conceptual structure—has already become so complex, through maturation and experience, that it requires three-dimensional modeling to depict all the concepts included in it and the inter-relationships among these concepts

The 6-year-old central conceptual structure

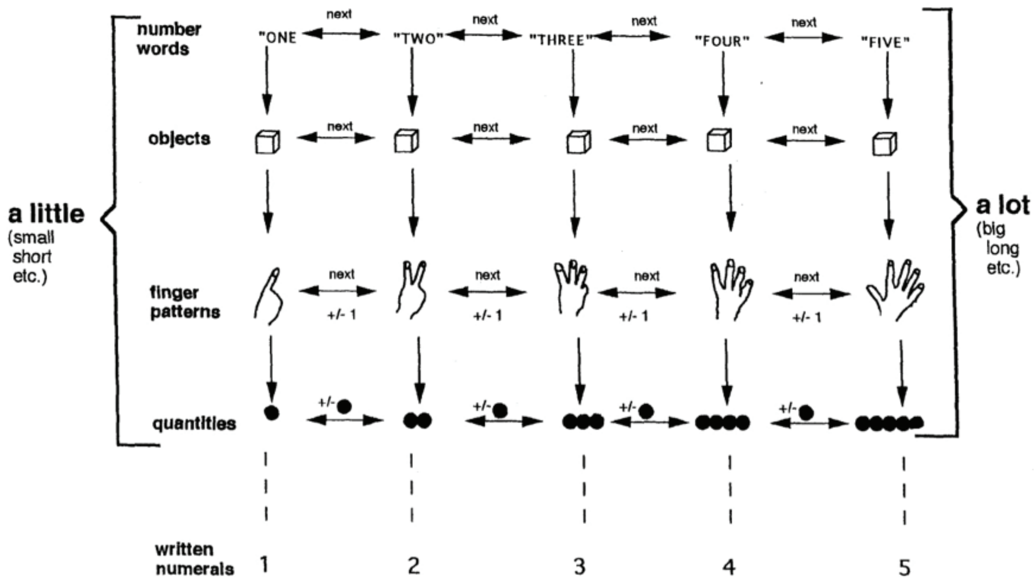

Figure 1: "Mental Counting Line" Central Conceptual Structure

This image depicts a tightly integrated set of interrelationships among concepts (i.e., a knowledge network), with the top lines indicating concepts that are mastered a few years earlier than the concepts indicated in the bottom lines. Starting with the top line, it suggests that:

- Children know the counting sequence, at least from one to ten, by heart. They know that these number words always occur in a fixed sequence and they can count as easily down from ten to one as they can count up from one to ten. They can also count up or down from any point in the sequence and tell you, for example, that if you start at "four," "five" is the next number up and "three" is the next down.
- Children know that, when you are counting a set of objects to determine how many are in the set, you must touch each object once, and only once, while counting. They also know that the last count word you say tells you how many are in the set.
- Children know that each count word is associated with a particular finger display. They know that one additional digit is raised for each count word you say next in the sequence when you are counting up and one finger is lowered for each count word you say next in the sequence when you are counting down. They can also create finger displays, at will, for any number in the one-to-ten sequence.

- Children know that each count word is associated with a quantity of a particular size. They know that the size of this quantity is increased by one each time you say the next number up in the counting sequence and is decreased by one each time you say the next number down. They also know that quantities can be represented in several different ways (i.e., as groups of objects; as dot-set patterns; as position on a horizontal line or path; as position on a vertical scale measure; as position on a circular dial) and they know, for example, that “five” is always “five” in each of these contexts even though it is represented very differently (e.g., as a dot-set pattern on a die or as distance along a segmented line). Finally, they know the language that is used to describe increases and decreases in quantity in each of these contexts. For example, “five” is “bigger” or “more” than “four” when describing groups of objects or dot-set patterns; it is “farther along” when describing position on a line; it is “higher up” when describing position on a scale measure; it is “farther around” when describing position on a dial. They also know that these words are equivalent and are often used interchangeably to describe magnitude changes.
- Children may also know the written numerals that are associated with each count word but, as indicated by the dotted line connecting this line to all upper lines in the figure, this knowledge is not an essential part of the conceptual structure.

Two important features of this structure have yet to be mentioned. The first is that, as illustrated by the vertical and horizontal lines and arrows that connect these concepts, children know that you can use the count sequence alone, in the absence of real quantities, to determine how many you will have if you add (or subtract) one or two to (or from) any quantity. All you need to do is count up (or down) from the initial quantity by the number you wish to add or subtract. The number you stop at will tell you the size of the new set. This knowledge gives children tremendous leverage. It enables them to solve addition and subtraction problems in their heads, without the use of concrete manipulatives. The second important feature of the structure, as indicated by the vertical lines on the outside edges, is that it enables children to use the counting numbers alone, in the absence of concrete objects, to make magnitude comparisons along several quantitative dimensions (e.g., length, weight, height, monetary value) because they know that numbers that are higher up in the sequence always indicate a larger quantity.

In the SRA Number Worlds kindergarten program, several lessons are devoted to teaching each set of concepts illustrated in this figure and helping students construct relationships among them. For students who start kindergarten with little of this knowledge in place, it can take a whole school year to enable them to construct all the knowledge this figure depicts, in a well-consolidated form, and to enable them to use it effectively to make magnitude predictions and assessments.

The Importance of games and manipulatives for conceptual development

The value of number line board games

How can this central conceptual knowledge be taught? As suggested by the title of this structure, it looks like an elaborated number line and indeed, Resnick (1983), summing up a decade of research, said that children who are successful in first grade arithmetic report having something like a mental number line inside their heads that they use to solve addition and subtraction problems. The author's first thought was, if this is how children represent the number system at 5–6 years, in the normal course of development, it makes sense to use number lines liberally, in all shapes and forms, to teach this knowledge to children who have not yet acquired it so that students can gain concrete exposure to the representation of number that we ultimately want them to construct in their heads. In 2014, number lines are common in all mathematics classrooms and the author thinks this is partly (or perhaps largely) due to SRA Number Worlds evaluation research that showed how effective this manipulative can be to teach conceptual knowledge of number. In 1988, when the program was first created and evaluated, number lines were far less common in classrooms, appearing, if at all, in a number line that was pasted on the wall at ceiling height, and referred to infrequently.

In the current SRA Number Worlds program, number lines are a much-used manipulative at all levels of the program, and in various forms. These include:

(a) step-by-step number lines from 1 to 10 and 1 to 20 that students can walk or hop along and acquire a physical sense that "9," for example, is bigger than "7" because it is farther along the line than "7;" it is closer to "10;" and you need to take two more steps to reach it if you are currently standing on "7."

(b) number line board games from 1 to 10, 1 to 25, and 1 to 100 that students move along to reach some goal, using people pawns to give them: (i) a sense that it is they, themselves, who are moving farther away from "0" with each roll of the die and each quantity added and (ii) a personal stake in answering questions about who is closer to the goal, how do you know, and how do you think that happened (Answer: He/she rolled more big numbers on the die).

(c) large Neighborhood number lines from 1 to 100 that depict numbered houses, that distinguish blocks of 10 with different colors and odd and even numbers with different roof profiles, and that students can move along to accomplish some task, such as: delivering packages to certain houses; verbalizing how many blocks of 10 and how many individual houses you will pass by to make this delivery; and earning $1 or $2 for each successful delivery.

(d) vertical number lines that depict bar graphs, elevators in a building, or thermometers that students can use to record specific quantities (e.g., the number of bean bags that went into the target on a particular throw) or that students can move up (or down) to reach some goal by mentally adding (or subtracting) quantities rolled on a die.

(e) magnetized rational number lines that extend from 0 to 1, that are segmented into 24 spaces, and that can be used with magnetic chips to create physical models of 5/8, 2/3, and 3/4, for example, in order to determine which of these numbers is larger or smaller.

Two important pieces of research, in addition to findings from the authors' evaluation of SRA Number Worlds, provide strong support for the use of number line board games to enhance mathematics learning. The first comes from research in cognitive neuroscience (Deheane, 1997), which revealed that magnitudes are coded in the brain by groups of neurons that are specifically tuned to detect certain numerosities and that are spatially distributed in the brain in a line-like fashion. These neurons, moreover, have been shown to be influenced by learning and experience. Thus, when we teach the number line conceptual structure, we are actually supporting the development of a brain structure that enables humans to make magnitude comparisons and to solve a variety of mathematical problems. The second important finding comes from research in mathematics learning. In a carefully controlled study, Booth and Siegler (2008) showed that students who used number lines during instruction made greater gains on standard math tests than students who were not given this opportunity.

As educators move into the Common Core State Standards (CCSS) era and its associated Standards of Mathematical Practices (National Governors Association, 2010), it is interesting to note that the number line is featured prominently in these standards and, indeed, that the SRA Number Worlds curriculum appears to have paved the way for these standards, in several respects, long before the Standards were created and published.

In the CCSS, as well as in the SRA Number Worlds program, the number line is used as a visual/physical model to represent the counting numbers and to provide a spatial model for magnitude comparisons. It constitutes an effective tool to develop estimation strategies; to solve a variety of addition, subtraction, multiplication and division problems; and to develop an understanding of the meaning of these operations (CaCCSS-M, 2010). In the CCSS, the number line is first explicitly addressed in second grade Measurement and Data. References to it occur throughout the elementary and middle school grades as well as in high school Statistics and Probability (NCDPI, 2010). Not only does the number line persist across grade levels but it is also present across mathematical domains (CaCCSS-M, 2010).

The value of concrete manipulatives

The author's second thought, when considering how to teach the central conceptual structure, was that quantity representations lie at the heart of this structure and, indeed, it is these quantity representations that give the count words and the written symbols—both abstract concepts and empty symbols without their quantity referents—their meaning. Thus, it made sense to adopt the following instructional principle to guide program development and teachers' use of the program:

When building new conceptual knowledge, instruction should always begin in the world of real quantities to give students opportunities to explore the concept in the physical world (e.g., with concrete manipulatives) before asking them to use number words and math talk to describe these quantities (or quantity transactions) orally and before asking them to take the final step and to use written symbols to describe these quantities (or quantity transactions) in writing.

This principle receives strong support from two of the leading thinkers in the 20th Century who devoted their lives to studying the development of children's thought and learning. Jean Piaget's (1950) theory states that cognitive development follows an invariant sequence and that thinking always starts at the sensorimotor level: An idea is first felt, seen, touched or acted upon in the physical world before it is abstracted and becomes a mental concept. John Dewey (1918) expressed a similar idea earlier in the century, suggesting that thought begins in action, and opportunities "to do" in the physical world should provide the cornerstone of education. More recently, Hiebert (1997) provides advice from several leading math education researchers who unanimously consider the use of manipulatives to be an essential ingredient in teaching math for understanding.

Concrete manipulatives are also an essential element in the Common Core State Standards and their value in providing the building blocks for math learning is evident in the concept of "rigor" associated with the standards (National Governors Association, 2013). Rigorous programs have been shown to provide students with the conceptual understanding, procedural skill and fluency and application strategies for learning in context that make them successful lifelong learners (Boston & Wolf, 2005). As noted in the previous paragraphs, SRA Number Worlds has always provided rigorous instruction that accelerates struggling learners and transitions them to active mathematicians in the classroom.

- **Conceptual Understanding** Researchers who have investigated the manner in which children construct number knowledge and conceptual understanding of content have proven that the "precursor" understandings identified in the central conceptual structure for number are required to allow students to build the conceptual understanding they need to handle increasingly complex information and topics (Griffin, 2002; Griffin and Case, 1997).

- **Procedural Skill and Fluency** Because computational fluency and conceptual understanding have been found to go hand in hand in children's mathematical development (Griffin, 2003; Griffin, Case & Siegler, 1994), opportunities to acquire computational fluency, as well as conceptual understanding, are built into every SRA Number Worlds lesson.

- **Application** Students are expected to use mathematics and choose the appropriate concepts for application. They need to apply mathematical concepts in real-world situations. Applications can be motivational and interesting, and there is a need for students at all levels to connect the mathematics they are learning to the world around them. This application is most evident in the Find the Math, Reflect and Project-Based Learning activities in each SRA Number Worlds lesson.

The value of games

The author's third thought, when considering how to teach conceptual knowledge of number, was that the learning experiences provided in the SRA Number Worlds curriculum should be fun and engaging for students to maximize their engagement in mathematics problem-solving. This seemed especially important for students who needed a prevention or intervention program because it seemed likely that these students might not have had successful math learning experiences in the past, might not have confidence in their math problem-solving ability, and might not be as eager to engage in mathematics as their more successful peers. For this reason, the author decided to use a game format for as many of the learning activities created for the program as possible and the author formulated the following instructional principle to guide program development:

To the greatest extent possible, activities created for the program should capture students emotions and imaginations as well as their minds.

Creating games to enhance learning came naturally because the author had previously spent five years creating or adapting games for education and assessing their effectiveness. The author had also written a master's thesis on this subject, titled "The Use of Games in Education to Teach Cognitive Skills." A game format also seemed to map nicely onto the definition of mathematics provided by Hiebert, et al. (1997), who suggest that mathematics is essentially problem solving, with a stated goal and a set of tools provided to reach this goal. Finally, while researching the early childhood experiences of Head Start children who lagged behind their middle income peers in number knowledge by about two years, Starkey and Klein (2007) found that board games were significantly less prevalent in the homes of the Head Start sample than in the homes of their middle income peers. It seems likely, therefore, that board games had contributed to the development of number knowledge in the middle class sample and providing these "middle class learning experiences" (i.e., board games) to children who lag behind in number knowledge might be a very effective way to teach this knowledge.

In addition to the three instructional principles just described, nine additional instructional principles were adopted from current research in the learning sciences (e.g., Bransford, Brown & Cocking, 1999) and used systematically to construct each activity used in the SRA Number Worlds curriculum as well as the sequence of activities across the program. These are described in Griffin (2002) and Griffin (2007) and readers who are interested in the instructional technology that shaped the SRA Number Worlds curriculum can consult these papers for further details.

What kinds of knowledge does SRA Number Worlds teach?

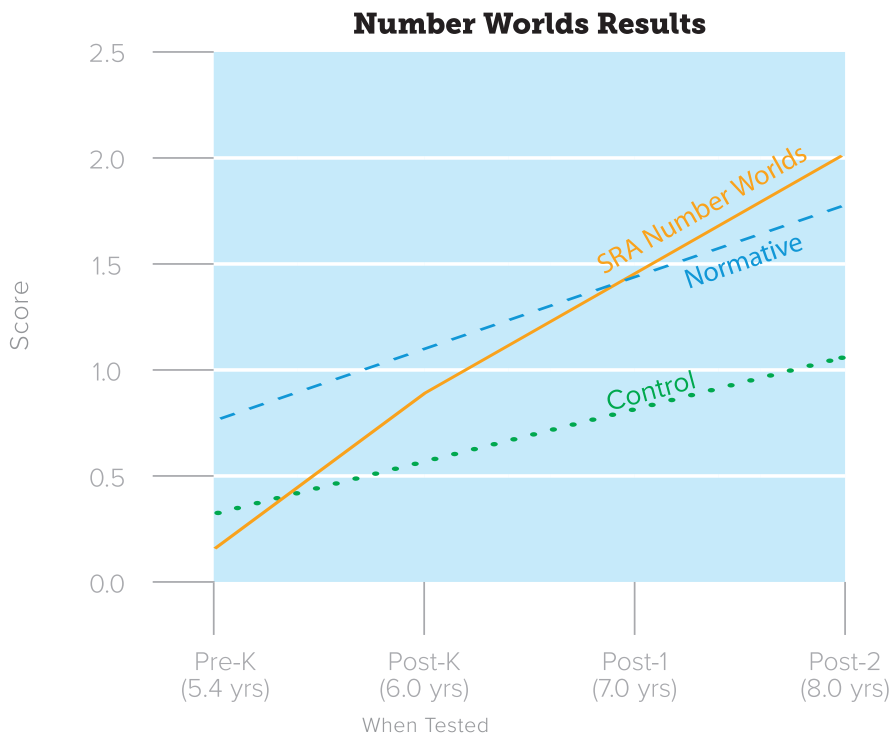

Figure 2: Longitudinal Study Showing Mean Developmental Scores in Mathematical Knowledge During Grades K to 2". Remove "Solid line . . .Control Group" verbiage to lines 2 and 3. Change labels on lines to "Number Worlds Group", "Normative Group", and "Control Group

Because of the strong focus on conceptual understanding in the SRA Number Worlds curriculum, it is not surprising that exposure to the program would produce strong gains in conceptual knowledge, as illustrated by the mean developmental level scores that three groups of students achieved on the Number Knowledge test at four time periods: the beginning of kindergarten, the end of kindergarten, the end of grade 1, and the end of grade 2 (See figure 2). As the figure suggests, the mean developmental level score of the Normative (middle income) group was close to 1.0 (i.e., the 6-year-old level) at the beginning of kindergarten, suggesting that students in this group were on track for successful learning of arithmetic. At each subsequent time period, their performance followed the expected pattern for successful learners and approached Level 2 (i.e., the 8-year-old level) at the end of grade 2. By contrast, the SRA Number Worlds group and the Control group, both drawn from low-income communities, started kindergarten one to two years below the Normative group. The SRA Number Worlds group caught up to the Normative group by the end of kindergarten and surpassed these students in number knowledge at the end of grade 2 whereas the Control group, who received a variety of other mathematics programs for this entire time period, lagged farther and farther behind as they progressed through these grade levels.

Whether or not students would make equally strong gains in their procedural knowledge was a question worth asking, the author thought, and the author assessed this as well in the same longitudinal study, in a variety of ways. The findings provided compelling evidence that gains in students' procedural knowledge matched or exceeded gains in their conceptual knowledge. For example, on the computation test that Stigler, Lee & Stevenson (1990) used for all their international comparisons, children exposed to the SRA Number Worlds curriculum performed as well as their Asian peers at the end of grade 1; significantly better than the control sample used for this study (who had received a variety of other mathematics programs); and significantly better, as well, than Stigler, Lee & Stevenson's U.S. sample.

Strategic knowledge is one of the hallmarks of number sense and this knowledge was also assessed in a number of ways, with students in the SRA Number Worlds samples showing greater mastery of this type of knowledge than students in the control samples (who had received a variety of other math programs). For example, on a test measuring flexibility in problem solving, students were asked to solve the following problem, mentally, at the end of grade 1: 3 + 5 + 2 – 1 = ?. They were then asked if they could solve this problem a different way and those who responded "Yes" were asked to do so. Students in the control groups typically solved the problem in a linear fashion, proceeding from left to right, and performing each operation as it appeared in the expression. The majority could not think of another way to do it. By contrast, children in the SRA Number Worlds sample often made problem-solving easier by making pairs of numbers (e.g., 3 + 2 = 5); by making "10" (e.g., 5 + 5 = 10); or by performing the subtraction operation first (e.g., 2 -1 = 1) thereby reducing the number of addends and subtrahends to three. The majority of these students could think of another way to solve this problem and demonstrated this capability.

Last, but by no means least, anecdotal data from a number of sources suggest that children receiving the SRA Number Worlds program demonstrate three changes in their attitudes toward math, which can be described as social-emotional knowledge. First, they report thinking that math is fun whereas previously it had seemed boring. Second, they report that they like doing math whereas previously it had seemed like hard work. Third, they often describe themselves as "good in math" whereas previously they had thought they might be stupid and/or unable to master this subject. Together, these three beliefs create a "circle of success," which will motivate them to continue their math engagement and their math learning well beyond their school-age years. This is perhaps the best reward of all for the developers of the SRA Number Worlds curriculum and for teachers who have used this curriculum in their classrooms.

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