McGraw Hill

NUMBER WORLDS®

Research & Efficacy

Research-Proven
Math Intervention for
Grades PreK–8

Table of Contents

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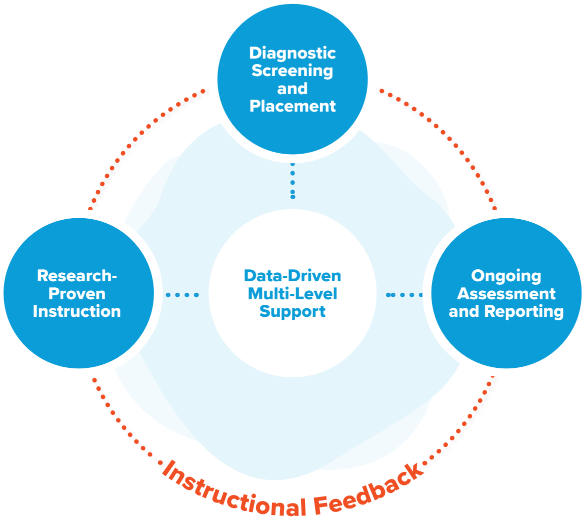
Achieving Math Proficiency

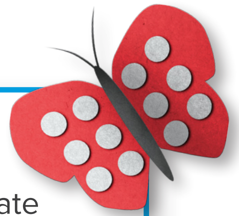
Math Intervention for Grades PreK–8

College and career readiness today requires a solid mathematical foundation. Moving students from passive receivers of information about numbers to active participants in gaining computational fluency and conceptual understanding drives math achievement. To do this, especially for students who have fallen behind, educators need to develop mathematical literacy by engaging them in hands-on learning experiences and concept application.

Number Worlds® is a research- and standards-based intervention math curriculum intended to develop student math proficiency for all PreK–8 students. Based on findings from field tests, effectiveness studies, educational research, and research around how children learn, *Number Worlds* and *Building Blocks™* are proven as effective curriculum intervention solutions to bring students struggling in mathematics up to par with their peers in math literacy and fluency.

Number Worlds supports Response to Intervention (RTI) and helps schools meet their academic objectives. RTI encourages working with at-risk students early on, and *Number Worlds* is the only math intervention curriculum with a built-in prevention program for Grades PreK–1.





Number Worlds helps struggling students accelerate math success by enabling educators to:

PREPARE students to meet rigorous state standards with proven curriculum and enhanced planning tools.

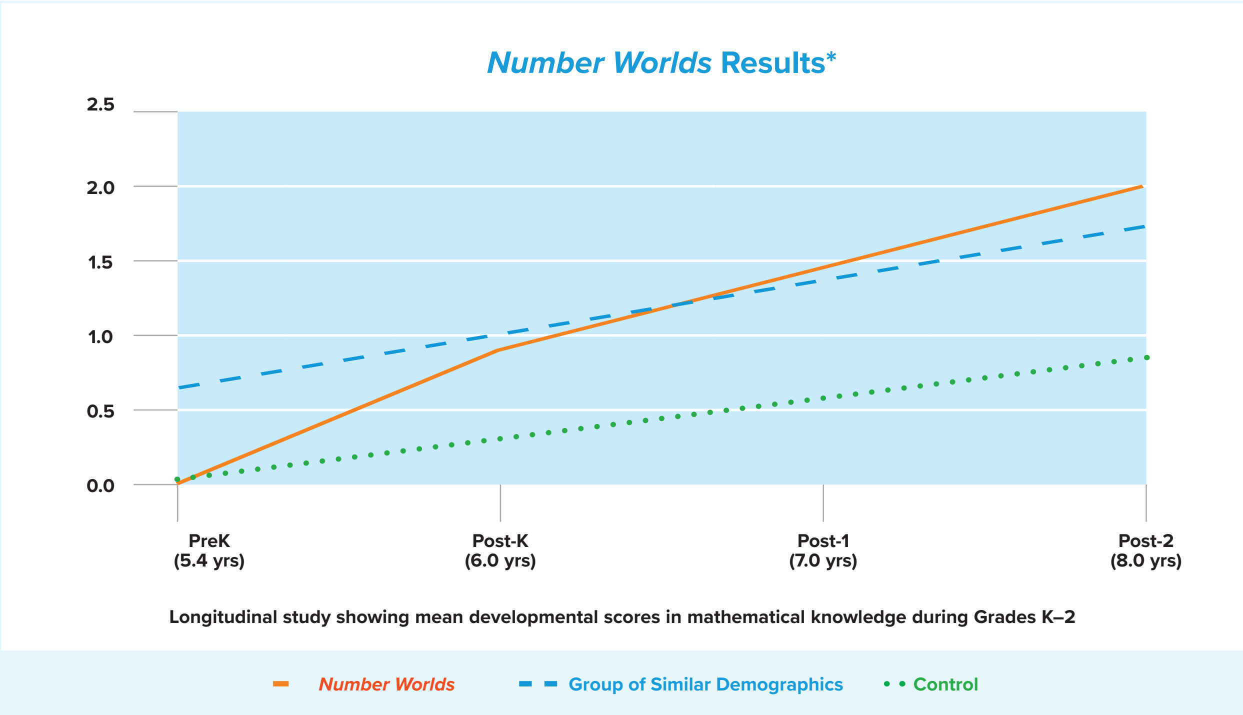
ENGAGE students with interactive games, embedded activities, digital resources, and project-based learning.

ASSESS student achievement with dynamic digital assessment and reporting tools.

Research Results

Results with *Number Worlds®*

Rigorous field testing shows that students who began at a disadvantage surpassed the performance of students who began on-level with their peers, simply with the help of the *Number Worlds* program. A longitudinal study measuring the progress of three groups of children from the beginning of Kindergarten to the end of Grade 2 demonstrates the program's efficacy.



*Griffin, Sharon. Fostering the Development of Whole-Number Sense. How Students Learn: Mathematics in the Classroom. Washington, D.C.: The National Academies Press, 2005.

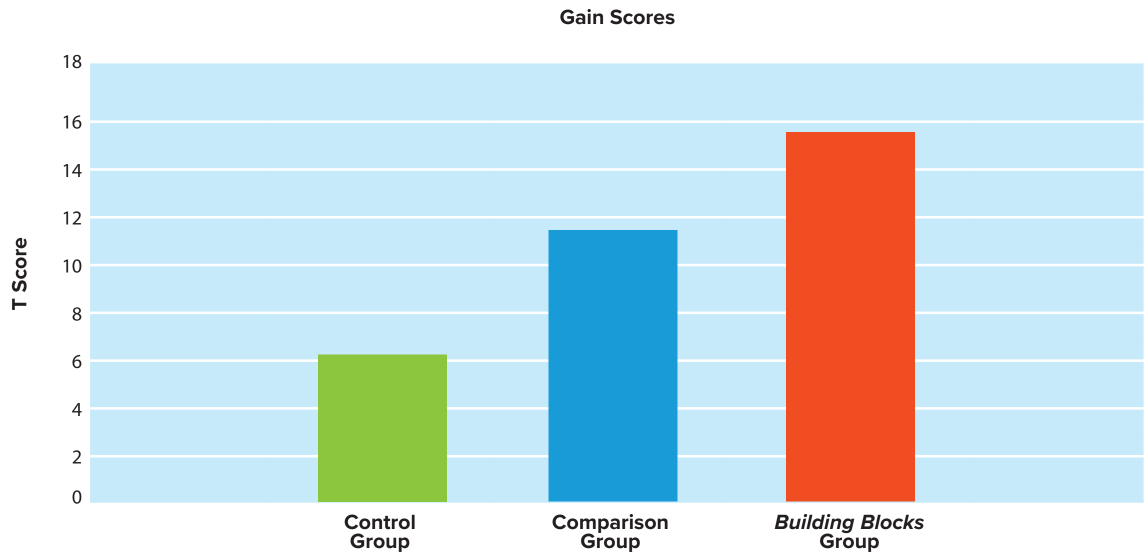
> *"We love using the* Number Worlds *program because it effectively differentiates instruction to meet the individual academic needs of our students."*
>
> **–Erika, Intervention Teacher, Massachusetts**

Results with *Building Blocks*™

Building Blocks Adaptive software, embedded in the *Number Worlds*® program, is based on National Science Foundation-funded research. *Building Blocks* includes online activities and an adaptive assignment engine that guides children through research-based learning trajectories.

In research studies, *Building Blocks* Adaptive software was shown to increase children's knowledge of essential mathematical concepts and skills. One key study tested *Building Blocks* against a comparable math program and a no-treatment control group. All classrooms were randomly assigned—the "gold standard" of scientific evaluation.

Building Blocks* Results

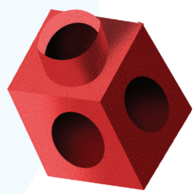


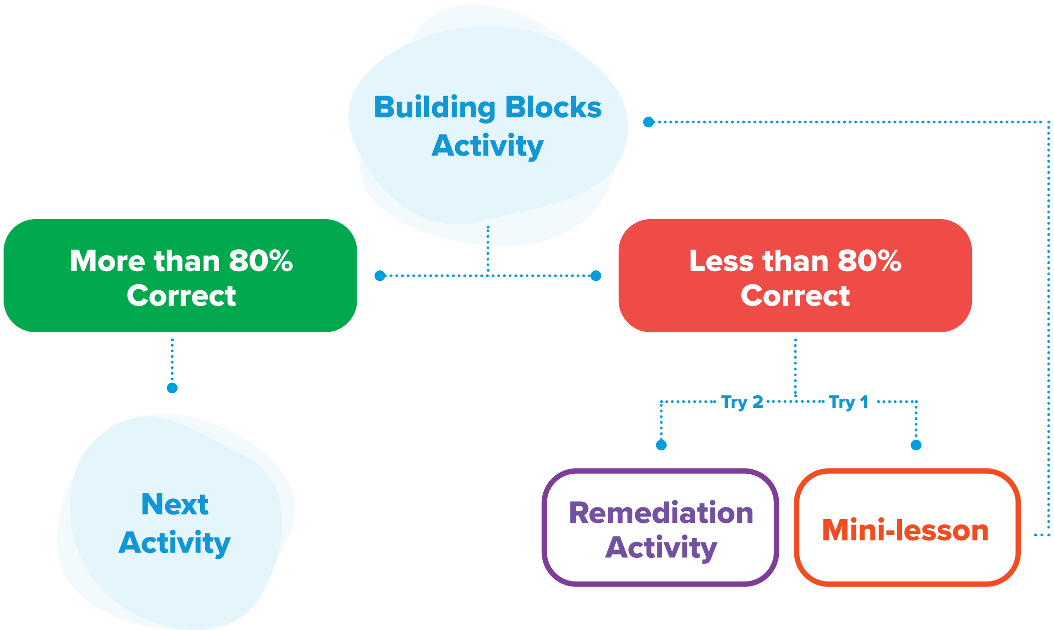
*Source: Clements, Douglas H., Julie Sarama, and Ann-Marie DiBiase eds. Engaging Young Children in Mathematics: Standards for Early Childhood Mathematics Education. Mahwah, NJ: Lawrence Erlbaum Associates, 2004.

Building Blocks ADAPTIVE

An Adaptive, Personalized Learning Experience

Building Blocks™ Adaptive is a collection of nearly 200 fully-animated activities for Grades PreK–8 providing conceptual development, practice, and remediation within a wide variety of mathematical topics. Activities are sequenced along research-based developmental paths, called learning trajectories, to help students move through stages of understanding. The program is adaptive and personalized so that all students can follow unique learning paths based on their performance and individual learning needs. Detailed progress reports give teachers the feedback they need to monitor the progress of every student and every class.





INDIVIDUALIZED LEARNING.
The entire *Building Blocks*™ curriculum enables students to reach their individual goals. Using a placement test, the program places them at their zone of proximal development , so they can each progress along the sequence at their own pace.

BUILT FOR FLEXIBILITY.
Building Blocks can be used by students and accessed by teachers at any time, from anywhere. It can be used as a center activity during the school day, or it can be accessed by students at home for additional practice.

DYNAMIC PROGRESS MONITORING AND REPORTING.
The *Building Blocks* reporting system provides a Student Progress Report and Learning Trajectory Report to enable teachers and administrators to track progress.

ADAPTIVE TECHNOLOGY.
Building Blocks automatically moves students forward along the learning trajectories when they successfully complete activities; it provides mini-lessons when students need instructional support, and moves students backward in a trajectory if they need additional pre-skill support.

Program Levels

Number Worlds® was designed with prevention levels (A–C) and intervention levels (D–J) to support Grades PreK–8.

Prevention: Levels A–C

Prepare PreK–1 students with the foundational skills and concepts necessary to be successful with more complex mathematics in the future. Each prevention level consists of 32 weeks of daily instruction including lessons on time and money.

Level A Building Foundations for PreK	**Level B** Grade K Key Standards	**Level C** Grade 1 Key Standards
Students acquire well-developed counting and quantity schemas.	Students develop a well-consolidated central conceptual structure for single-digit numbers.	Students link their central conceptual structure of the number to the formal number system.

Intervention: Levels D–J

Help students in Grades 2–8 learn the foundational skills and concepts needed to master key mathematical standards. Designed for flexibility, the five six-week units can be taught in any order or in isolation. A placement test enables the teacher to identify student needs.

	Level D Grade 2	**Level E** Grade 3	**Level F** Grade 4	**Level G** Grade 5	**Level H** Grade 6	**Level I** Grade 7	**Level J** Grade 8
Unit **1**	Number Sense Within 100	Number Sense	Number Sense	Number Sense	Number Sense	Number Sense	Number Sense
Unit **2**	Number Sense to 1,000	Addition	Addition & Subtraction	Multiplication & Division	Operations Sense	Operations Sense	Operations Sense
Unit **3**	Addition	Subtraction	Multiplication	Operations With Decimals	Algebra	Algebra	Algebra
Unit **4**	Subtraction	Multiplication & Division	Division	Operations With Fractions	Statistical Analysis	Statistical Analysis	Statistical Analysis
Unit **5**	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement

The Power of Project-Based Learning

Educators are entering an era of fast-paced, information-heavy learning environments and workplaces. Students need to be prepared for the new, knowledge-based economy where STEM-based information industries replace traditional jobs and industries (Molnar, 1997). Inquiry-Based Learning (IBL) is well-placed to support educators reaching out to today's learner. Through inquiry, students build the habits of mind that prepare them to compete with their peers locally and globally.

Unfortunately, students often lack the schema and experience with inquiry to allow them to successfully participate in and learn through IBL (de Jong, 2006). There is an increasingly powerful collection of research supporting the implementation of IBL in learning environments across the curriculum in order to give students the tools and thought processes they need to develop inquiring minds—with the power to solve creatively, actively, and intelligently even the toughest problems. In practice, IBL suggests student-centered learning that transforms the student from a passive receiver to an active participant in the learning process (Creedy et al., 1992).

At its heart, project-based learning is a specific form of IBL and one of the most basic approaches to learning, having been around in some form or another throughout history. According to Blumenfeld, et al. (1991), inquiry-based learning manifests itself through researching and responding to open-ended questions that are generated by the learners themselves. In project-based learning (PBL), students are given a little more guidance and structure around the research question and then provided the opportunity to explore, negotiate, and define a solution for the problem (Hmelo-Silver & Barrows, 2008).

"There is a lot of positive energy around Number Worlds® *in my district. The teachers, principals, and office staff especially like the detailed teacher guides, online resources, placement assessments, and ongoing progress monitoring.*

—Brenda, Special Education Instructional Specialist, Maryland

Students are required to build knowledge, filling gaps in current knowledge to successfully resolve the problem at hand. This instructional approach was first used heavily in schools of medicine but steadily made its way into STEM eeducation, beginning over 50 years ago. PBL expands upon the idea that instruction should be built around a larger task or problem placed in an authentic setting where students are given responsibility and ownership of resolving the task. The problem should be challenging, set in a structure that allows for investigation, and isn't fully resolved without reflection and discussion of the issues at hand (Savery & Duffy, 1995).

In *Number Worlds®*, PBL lessons were developed around the major principles identified by Barrows & Kelson (1993).

1. Problem solving with ill-formed problems offering many possible solutions
2. Goal of functional knowledge with cognitive flexibility
3. Self-directed learning
4. Collaboration
5. Taking ownership with active, engaged learning
6. Building a habit of reflection and self-appraisal in all learning experiences

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Building Number Sense with *Number Worlds*

What is number sense? We all know number sense when we see it but, if asked to define what it is and what it consists of, most of us, including the teachers among us, would have a much more difficult time. Yet this is precisely what we need to know to teach number sense effectively. Consider the answers three kindergarten children provide when asked the following question from the Number Knowledge Test (Griffin & Case, 1997):

Which is bigger: seven or nine?

Brie responds quickly, saying "Nine. Well you go, 'seven' (pause) 'eight,' 'nine' (putting up two fingers while saying the last two numbers). That means nine has two more than seven. So it's bigger."

Leah says hesitantly, "Nine? Because nine's a big number."

Caitlin looks genuinely perplexed, as if the question was not a sensible thing to ask and says, "I don't know."

Principle 1: Build upon children's current knowledge

Each new idea that is presented to children must connect to their existing knowledge if it is going to make any sense at all. Children must also be allowed to use their existing knowledge to construct new knowledge that is within reach—that is one step beyond where they are now—and a set of bridging contexts and other instructional supports should be in place to enable them to do so.

In the examples of children's thinking presented earlier, three different levels of knowledge are apparent. Brie appears to have acquired the knowledge network that underlies number sense and to be ready, therefore, to move on to the next developmental level: to connect this set of understandings to the written numerals (i.e., the formal symbols) associated with each counting word. Leah appears to have some understanding of some of the components of this network (i.e., that numbers have magnitude) and to be ready to use this understanding as a base to acquire the remaining understandings (e.g., that a number's magnitude and its position in the counting sequence are directly related).

Caitlin demonstrates little understanding of any element of this knowledge network, and therefore, she might benefit from exposure to activities that will help her acquire the "precursor" knowledge needed to build this network, namely knowledge of counting (e.g., the one-to-one correspondence rule) and knowledge of quantity (e.g., an intuitive understanding of relative amount).

To meet these individual needs, teachers need (a) a way to assess children's current knowledge, (b) activities that are multi-leveled so children with different entering knowledge can benefit from exposure to them, and (c) activities that are carefully sequenced and that span several developmental levels so children with different entering knowledge can be exposed to activities that are appropriate for their level of understanding. These are all available in the *Number Worlds*® program and are illustrated in various sections of this paper.

Principle 2: Follow the natural developmental progression when selecting new knowledge to be taught

Researchers who have investigated the manner in which children construct number knowledge between the ages of 3 and 9 years have identified a common progression that most, if not all, children follow (see Griffin, 2002; Griffin and Case, 1997 for a summary of this research). A mathematics program that provides opportunities for children to use their current knowledge to construct new knowledge that is a natural next step, and that fits their spontaneous development, will have the best chance of helping children maximize their progress in mathematics learning and development.

Because there are limits in development on the complexity of information children can handle at any particular age/stage (see Case, 1992), it makes no sense to attempt to speed up the developmental process by accelerating children through the curriculum. However, for children who are at an age when they should have acquired the developmental milestones but for some reason haven't, exposure to a curriculum that will give them ample opportunities to do so makes tremendous sense. It will enable them to catch up to their peers and thus benefit from the formal mathematics instruction that is provided in school.

Children who are developing normally also benefit from opportunities to broaden and deepen the knowledge networks they are constructing, to strengthen these understandings, and to use them in a variety of contexts.

Principle 3: Teach computational fluency as well as conceptual understanding

Because computational fluency and conceptual understanding have been found to go hand in hand in children's mathematical development (see Griffin, 2003; Griffin et al., 1994), opportunities to acquire computational fluency as well as conceptual understanding are built into every *Number Worlds*® activity. This is nicely illustrated in the following activity, drawn from the program.

The Dragon Quest Game that was developed for the Grade 1 program teaches a sophisticated set of understandings. Children are introduced to the Phase 1 activity by being told a story about a fire-breathing dragon that has been terrorizing the village where children live. The children playing the game are heroes who have been chosen to seek out the dragon and put out his fire. To extinguish this dragon's fire (as opposed to the other, more powerful dragons they will encounter in later phases), a hero will need at least 10 pails of water. If a hero enters into the dragon's area with fewer than 10 pails of water, they will become the dragon's prisoner and can be rescued only by one of the other players.

To play the game, children take turns rolling a die and moving their playing piece along the colored game board. If they land on a well pile (indicated by a star), they can pick a card from the facedown deck of cards, which illustrates with images and symbols (e.g., +4) a certain number of pails of water. Children are encouraged to add up their pails of water as they receive them through a variety of strategies, ranging from mental math (which is encouraged) to the use of tokens to keep track of the quantity accumulated. The first child to reach the dragon's lair with at least 10 pails of water can put out the dragon's fire and free any teammates who have become prisoners.

As children play this game and talk about their progress, they have ample opportunity to connect numbers to several different quantity representations (e.g., dot patterns on the die, distance of their pawn along the path, sets of buckets illustrated on the cards, and written numerals on the cards) and to acquire an appreciation of numerical magnitude across these contexts. With repeated play, they also become capable of performing a series of successive addition operations in their heads and of expanding the well pile. When they are required to submit formal proof to the mayor of the village that they have amassed sufficient pails of water to put out the dragon's fire, they become capable of writing a series of formal expressions to record the number of pails received and spilled over the course of the game. In contexts such as these, children receive ample opportunity to use the formal symbol system in increasingly efficient ways to make sense of quantitative problems they encounter in the course of their own activity.

Principle 4: Provide plenty of opportunity for hands-on exploration, problem-solving, and communication

Like the Dragon Quest Game that was just described, many of the activities created for the *Number Worlds*® program are set in game formats for hands-on exploration of number concepts, problem-solving, and communication. Communication is explicitly encouraged in a set of question prompts that are included with each small-group game (e.g., How far are you now? How many more buckets do you need to put out the dragon's fire? How do you know?), as well as in a more general set of dialogue prompts that are included in the teacher's guide. A wrap-up session at the end of each math lesson provides opportunities for children to discuss what they learned during game play each day, share their knowledge with their peers, and make their reasoning explicit.

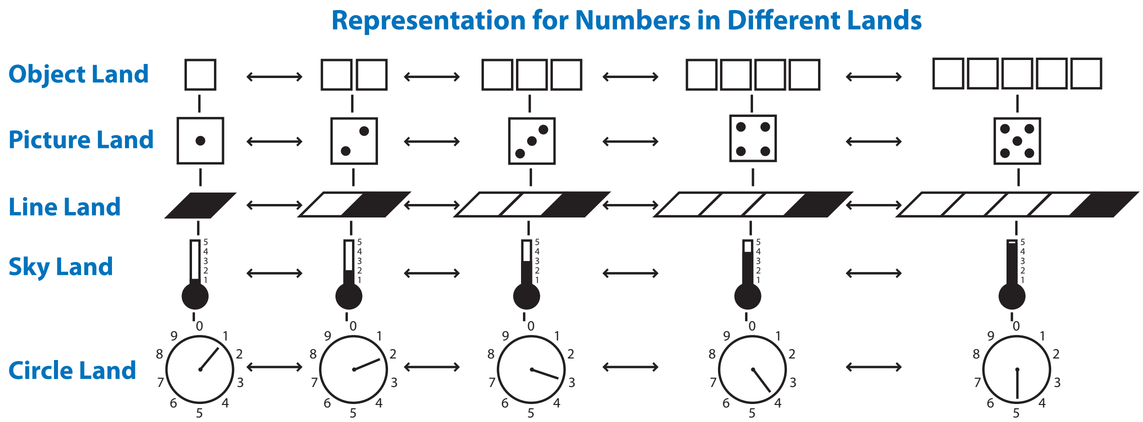
In the whole-group games and activities that were developed for the warm-up portion of each math lesson, children have ample opportunity to count (e.g., up from 1 and down from 10) and to solve mental math problems in a variety of contexts. In addition to developing computational fluency, these activities expose children to the language of mathematics and give them practice using it. Although this is valuable for all children, it is especially useful for ESL children, who may know how to count in their native language, but not yet in English. Allowing children to take turns in these activities and to perform individually gives teachers opportunities to assess each child's current level of functioning, which is important for instructional planning, and gives children opportunities to learn from each other.

Principle 5: Expose children to the major ways number is represented and talked about in developed societies

Number is represented in our culture in five major ways: as a group of objects, a dot-set pattern, a position on a line, a position on a scale (e.g., a thermometer), and a point on a dial. In each of these contexts, number is also talked about in different ways, with a larger number (and quantity) described as "more" in the world of dot-sets, as "further along" in the world of paths and lines, as "higher up" in the world of scale measures, and as "further around" in the world of dials.

Children who are familiar with these forms of representation and the language used to talk about number in these contexts have a much easier time making sense of the number problems they encounter inside and outside of school.

In the *Number Worlds®* program, children are systematically exposed to these forms of representations as they explore five different "lands" (See Figure 1). Learning activities developed for each land share a particular form of number representation while they simultaneously address specific knowledge goals for each grade level. Many games expose children to multiple representations of number.



Discussion

Children who have been exposed to the *Number Worlds* program do very well on number questions like the one presented in the introduction and on the Number Knowledge Test (Griffin & Case, 1997) from which this question was drawn. In several evaluation studies conducted with children from low-income communities, children who received the *Number Worlds* program made significant gains in conceptual knowledge of number and in number sense when compared to matched control groups who received readiness training of a different sort. These gains enabled them to perform as well as groups of children from China and Japan on a computation test administered at the end of Grade 1, and to keep pace with their more advantaged peers (and even outperform them on some measures) as they progressed through the first few years of formal schooling (Griffin & Case, 1997).

Teachers also report positive gains from using the *Number Worlds* program and from exposure to the instructional principles on which it is based. Many claim that their teaching of all subjects has been transformed.

They now facilitate discussion rather than dominate it; they pay much more attention to what children say and do, and they now allow children to take more responsibility for their own learning, with positive and surprising results. Above all, they now look forward to teaching math, and they and their students are eager to do more of it.

Building Blocks™ : Learning Trajectories

Children follow natural developmental progressions in learning, developing mathematical ideas in their own way. Curriculum research has revealed sequences of activities that are effective in guiding children through these levels of thinking. These developmental paths are the basis for *Building Blocks* learning trajectories. Learning trajectories have three parts: a mathematical goal, a developmental path through which children develop to reach that goal, and a set of activities matched to each of those levels that help children develop the next level. Thus, each learning trajectory has levels of understanding, each more sophisticated than the last, with tasks that promote growth from one level to the next. The *Building Blocks* learning trajectories give simple labels, descriptions, and examples of each level. Complete learning trajectories describe the goals of learning, the thinking and learning processes of children at various levels, and the learning activities in which they might engage. Detailed developmental level tables can be found in the *Number Worlds*® Teacher Implementation Guide.

Frequently Asked Questions

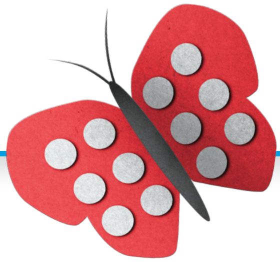
1. Why use learning trajectories?
Learning trajectories allow teachers to build students' mathematical knowledge along natural developmental pathways to help them move through stages of understanding. Because all activities in *Building Blocks Adaptive* are tied to a specific learning trajectory level, we know that students will be able to successfully work within their developmental capacities.

2. When are children "at" a level?
Children are at a certain level when most of their behaviors reflect the thinking—ideas and skills—of that level. As students learn new skills, they may continue to show behaviors from the previous trajectory level as well as some from the next level.

3. Can children work at more than one level at the same time?
Yes, although most children work mainly at one level or in transition between two levels (naturally, if they are tired or distracted, they may operate at a much lower level). Levels are not "absolute stages." They are "benchmarks" of complex growth that represent distinct ways of thinking.

4. How do these developmental levels support teaching and learning?
The levels help teachers, as well as curriculum developers, assess, teach, and sequence activities. Teachers who understand learning trajectories and the developmental levels that are at their foundation are more effective and efficient. Through planned teaching and also encouraging informal, incidental mathematics, teachers help children learn at an appropriate and deep level.

5. Why do the learning trajectories keep changing?
Learning trajectories are living things! That is, we are always learning more and adding new ideas as we continue our research and deepen our understanding of how children naturally develop math skills. Changes to the learning trajectories build on new information from our research and wisdom of expert practice. We have found that children who are provided high-quality mathematics experiences are capable of developing to levels one or more years beyond their peers.



Each column in the developmental level tables, such as “Counting,” represents a main developmental progression that underlies the learning trajectory for that topic. For some topics, there are “subtrajectories”—strands within the topic. In most cases, the names make this clear. For example, in Comparing and Ordering, some levels are about the “Comparer” levels, and others about building a “Mental Number Line.” Similarly, the related subtrajectories of “Composition” and “Decomposition” are easy to distinguish. Sometimes, for clarification, subtrajectories are indicated with a note in italics after the title. For example, in “Shapes ,” Parts and Representing are subtrajectories within the Shapes trajectory.

Building Blocks Adaptive™

Building Blocks, an integral part of the *Number Worlds®* program, is the result of National Science Foundation-funded research.

The *Building Blocks* program is designed to:

- Build upon young children's experiences with mathematics with activities that integrate different ways to explore and represent mathematics.
- Involve children in "doing mathematics".
- Establish a strong mathematical foundation.
- Develop a strong conceptual framework.
- Emphasize children's mathematical thinking and reasoning abilities.

Building Blocks increases students' attention and motivation with visual displays, animated graphics, virtual manipulatives, immediate feedback, and individualized tasks.

Learning Trajectories

Building Blocks™ Adaptive includes research-based computer tools with activities and a management system that guides students through research-based learning trajectories. Learning trajectories allow teachers to build the mathematical thinking of children as it develops naturally. Research has identified developmental paths, or sequences of activities, that effectively guide students through mathematical levels of thinking. *Building Blocks* learning trajectories are built upon these developmental paths. Learning trajectories have been identified for the following mathematical skills:

- Counting
- Subitizing
- Comparing Numbers
- Adding/Subtracting
- Composing Numbers
- Multiplying/Dividing
- Fractions
- Patterns, Structure, and Algebraic Thinking
- 2D Shapes
- Composing 2D Shapes
- Disembedding 2D Shapes
- 3D Shapes
- Composing 3D Shapes
- Spatial Visualization and Imagery
- Spatial Orientation
- Measurement: Length
- Measurement: Area
- Measurement: Volume
- Measurement: Angle and Turn
- Classification and Data Analysis

Each *Building Blocks* level provides a natural developmental building block to the next level. The levels help teachers and curriculum developers assess, teach, and sequence activities. Through planned teaching and by encouraging informal, incidental mathematics, teachers can help their students learn at appropriate levels.

References

Barrows, H. S, & Kelson, A.M. (1993). Problem-based learning: A total approach to education. Monograph. Springfield, IL: Southern Illinois University School of Medicine.

Blumenfeld, P. C., Soloway, E., Marx, R., Krajcik, J. S., Guzdial, M., & Palincsar, A. (1991). Motivating project-based learning: Sustaining the doing, supporting the learning. Educational Psychologist, 26, 369-398.

Boston, M., & Wolf, M. K. (2005). Assessing academic rigor in mathematics instruction: The development of the Instructional Quality Assessment toolkit. Regents of the University of California.

Case, R. (1992). The mind's staircase: Exploring the conceptual underpinnings of children's thought and knowledge. Hillsdale, NJ: Erlbaum.

Case, R., & Griffin, S. (1990). Child cognitive development: The role of central conceptual structures in the development of scientific and social thought. In E.A. Hauert (Ed.), Developmental psychology: Cognitive, perceptuo-motor, and neurological perspectives (pp. 193–230). North-Holland: Elsevier.

Clements, D. H., Sarama, J., & DiBiase, A.-M. (Eds.). (2004). Engaging Young Children in Mathematics: Standards for Early Childhood Mathematics Education. Mahwah, NJ: Lawrence Erlbaum Associates.

Clements, D. H., & Sarama, J. (in press). "Early Childhood Mathematics Learning." In F. K. Lester, Jr. (Ed.), Second Handbook of Research on Mathematics Teaching and Learning. New York: Information Age Publishing.

Creedy, D., Horsfall, J., Hand, B., (1992). Problem-based learning in nurse education: an Australian view. Journal of Advanced Nursing, 17, 727-733.

de Jong, T. (2006). Technological advances in inquiry learning. Science, 312(5773), 532-533.

Griffin, S., Case, R., & Siegler, R. (1994). Rightstart: Providing the central conceptual prerequisites for first formal learning of arithmetic to students at-risk for school failure. In K. McGilly (Ed.), Classroom lessons: Integrating cognitive theory and classroom practice (pp. 24-49). Cambridge, MA: Bradford Books MIT Press.

Griffin, S., Case, R., & Capodilupo, A. (1995). Teaching for understanding: The importance of central conceptual structures in the elementary mathematics curriculum. In A. McKeough, I. Lupert, & A. Marini (Eds.), Teaching for transfer: Fostering generalization in learning (pp. 121–151). Hillsdale, NJ: Erlbaum.

Griffin, S., & Case, R. (1996). Evaluating the breadth and depth of training effects when central conceptual structures are taught. Society for research in child development monographs, 61/83-102.

Griffin, S. (1997). Number Worlds: Grade one level. Durham, NH: Number Worlds Alliance Inc.

Griffin, S. (1997). Number Worlds: Kindergarten level. Durham, NH: Number Worlds Alliance Inc.

Griffin, S., & Case, R. (1997). Re-thinking the primary school math curriculum: An approach based on cognitive science. Issues in Education, 3, 1–49.

Griffin, S. (1998). Number Worlds: Grade two level. Durham, NH: Number Worlds Alliance Inc.

Griffin, S. (2000). Number Worlds: Preschool level. Durham, NH: Number Worlds Alliance Inc.

Griffin, S. (2002). The development of math competence in the preschool and early school years: Cognitive foundations and instructional strategies. In J. Royer (Ed.), Mathematical cognition (pp. 1–32). Greenwich, CT: Information Age Publishing.

Griffin, S. (2003). Laying the foundation for computational fluency in early childhood. Teaching children mathematics, 9, 306–309.

Griffin, S. (2004). Building number sense with *Number Worlds*: A mathematics program for young children. Early childhood research quarterly, 19(1), 173-180.

Hmelo-Silver, C. E. & Barrows, H. S. (2008). Facilitating collaborative knowledge building. Cognition and Instruction, 26(1), 48-94. Retrieved from EBSCOhost

Molnar, A. R. (1997). Computers in education: A brief history. T.H.E. Journal. 24(11), 63-68.

National Governors Association Center for Best Practices, C. O. C. S. S. O. (2013). "K–8 Publishers' Criteria for the Common Core State Standards for Mathematics. Retrieved from http://www.corestandards.org/resources

Ottmar, E. R., Decker, L. E., Cameron, C. E., Curby, T. W., & Rimm-Kaufman, S. E. (2013). Classroom instructional quality, exposure to mathematics instruction and mathematics achievement in fifth grade. Learning Environments Research, 1-20.

Savery, J. R. & Duffy, T. M. (1995). Problem based learning: An instructional model and its constructivist framework. Educational Technology, 35, 31-38.

Silk, E. M., Higashi, R., Shoop, R., & Schunn, C. D. (2010). Designing technology activities that teach mathematics. The Technology Teacher, 69(4), 21-27.

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