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Geometry

Illustrative Mathematics CERTIFIED

Lesson Sampler

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Geometry

Illustrative Mathematics is a problem-based core curriculum designed to address content and practice standards to foster learning for all. Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Teachers can shift their instruction and facilitate student learning with high-leverage routines to guide learners to understand and make connections between concepts and procedures.


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What is a Problem-based Curriculum?

In a problem-based curriculum, students work on carefully crafted and sequenced mathematics problems during most of the instructional time. Teachers help students understand the problems and guide discussions to be sure that the mathematical takeaways are clear to all. In the process, students explain their ideas and reasoning and learn to communicate mathematical ideas. The goal is to give students just enough background and tools to solve initial problems successfully, and then set them to increasingly sophisticated problems as their expertise increases.

The value of a problem-based approach is that students spend most of their time in math class doing mathematics: making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers. They go on to interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding.

Design Principles

Balancing Conceptual Understanding, Procedural Fluency, and Applications

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand, and be able to do, the mathematics.

Mathematical Practices are the Verbs of Math Class

In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

Build on What Students Know

New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them (NRC 2001) . In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students' learning.

Good Instruction Starts with Explicit Learning Goals

Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students' understanding. Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.

Different Learning Goals Require a Variety of Types of Tasks and Instructional Moves

The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- provide experience with a new context
- introduce a new concept and associated language
- introduce a new representation
- formalize the definition of a term for an idea previously encountered informally
- identify and resolve common mistakes and misconceptions
- practice using mathematical language
- work toward mastery of a concept or procedure
- provide an opportunity to apply mathematics to a modeling or other application problem

Each and Every Student Should Have Access to the Mathematical Work

With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional tool boxes should include knowledge of and skill in implementing supports for different learners. This curriculum incorporates extensive tools for specifically supporting English Language Learners and Students with Disabilities

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Instructional Model

Learning Goals and Targets

Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson. Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word "Let's." They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson. Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

Lesson Structure

1. INTRODUCE

Warm Up

Warm Up activities either:

- give students an opportunity to strengthen their number sense and procedural fluency.
- make deeper connections.
- encourage flexible thinking.

or:

- remind students of a context they have seen before.
- get them thinking about where the previous lesson left off.
- preview a calculation that will happen in the lesson.

2. EXPLORE AND DEVELOP

Classroom Activities

A sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class.

Each classroom activity has three phases.

The Launch

The teacher makes sure that students understand the context and what the problem is asking them to do.

Practice Problems

Each lesson includes an associated set of practice problems that may be assigned as homework or for extra practice in class. They can be collected and scored or used for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The design of practice problem sets looks different from many other curricula, but every choice was intentional, based on learning research, and meant to efficiently facilitate learning. The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Our approach emphasizes distributed practice rather than massed practice

Mathematical Modeling Prompts

Mathematics is a tool for understanding the world better and making decisions. School mathematics instruction often neglects giving students opportunities to understand this, and reduces mathematics to disconnected rules for moving symbols around on paper. Mathematical modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (NGA 2010). This mathematics will remain important beyond high school in students' lives and education after high school (NCEE 2013).

- Modeling Prompts can be thought of as a project or assignment. They are meant to be launched in class by a teacher, but but can be worked on independently or in small groups by students in or out of class.. We built in maximum flexibility for a teacher to implement these in a way that will work for them.
- The purpose of mathematical modeling is for students to understand that they can use math to better understand things they are interested in in the world.
- Mathematical modeling is different from solving word problems. There should be room to interpret the problem and a range of acceptable assumptions and answers. Modeling requires genuine choices to be made by the modeler.
- Modeling with mathematics is not a solitary activity and students should have support from their teacher and classmate while assessments focus on providing feedback that helps students improve their modeling skills.

Student Work Time
Students work individually, with a partner, or in small groups.

Activity Synthesis
The teacher orchestrates some time for students to synthesize what they have learned and situate the new learning within previous understanding.

National Governors Association Center for Best Practices (2010). Common Core State Standards for Mathematics. NCEE (2013). What Does It Really Mean to Be College and Work Ready? Retrieved November 20, 2017 from http://ncee.org/college-and-work-ready/

3. SYNTHESIZE

Lesson Synthesis

Students incorporate new insights gained during the activities into their big-picture understanding.

Cool Down

A task to be given to students at the end of the lesson. Students are meant to work on the Cool Down for about 5 minutes independently and turn it in.

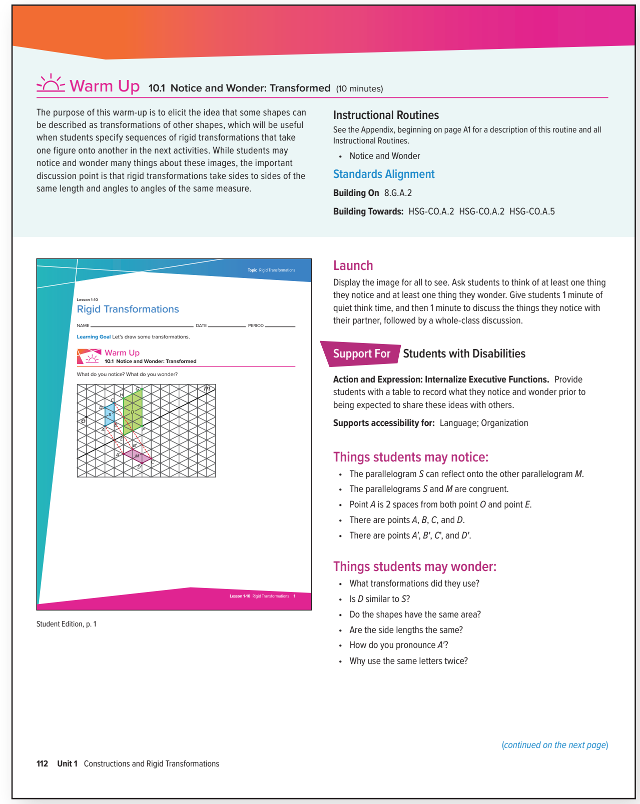
Instructional Routines

The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, yet others to applying mathematics to a real-world problem. These aspects of mathematical proficiency are interwoven. These materials include a small set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

Like any routine in life, these routines give structure to time and interactions. They are a good idea for the same reason all routines are a good idea: they let people know what to expect, and they make people comfortable.

Why are routines in general good for learning academic content? One reason is that students and the teacher have done these interactions before, in a particular order, and so they don't have to spend much mental energy on classroom choreography. They know what to do when, who is expected to talk when, and when they are expected to write something down. The structure of the routine frees them up to focus on the academic task at hand. Furthermore, a well-designed routine opens up conversations and thinking about mathematics that might not happen by themselves.

- Analyze It
- Anticipate, Monitor, Select, Sequence, Connect
- Aspects of Mathematical Modeling
- Card Sort
- Construct It
- Draw It
- Estimation
- Extend It
- Fit It
- Graph It
- Math Talk
- Notice and Wonder
- Poll the Class
- Take Turns
- Think Pair Share
- Which One Doesn't Belong?



Warm Up 10.1 Notice and Wonder: Transformed (10 minutes)

The purpose of this warm-up is to elicit the idea that some shapes can be described as transformations of other shapes, which will be useful when students specify sequences of rigid transformations that take one figure onto another in the next activities. While students may notice and wonder many things about these images, the important discussion point is that rigid transformations take sides to sides of the same length and angles to angles of the same measure.

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Notice and Wonder

Standards Alignment

Building On 8.G.A.2

Building Towards: HSG-CO.A.2 HSG-CO.A.2 HSG-CO.A.5

Topic Rigid Transformations

Lesson 1-10

Rigid Transformations

NAME ______ DATE ______ PERIOD ______

Learning Goal Let's draw some transformations.

Warm Up 10.1 Notice and Wonder: Transformed

What do you notice? What do you wonder?

Lesson 1-10 Rigid Transformations 1

Student Edition, p. 1

Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Support For Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a table to record what they notice and wonder prior to being expected to share these ideas with others.

Supports accessibility for: Language; Organization

Things students may notice:

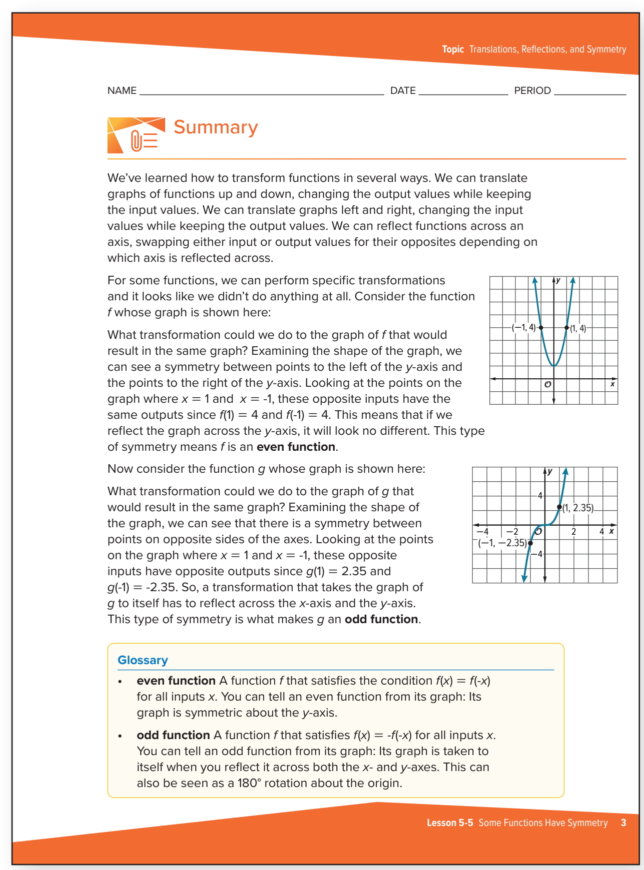
- The parallelogram S can reflect onto the other parallelogram M.
- The parallelograms S and M are congruent.
- Point A is 2 spaces from both point O and point E.
- There are points A, B, C, and D.
- There are points A', B', C', and D'.

Things students may wonder:

- What transformations did they use?
- Is D similar to S?
- Do the shapes have the same area?
- Are the side lengths the same?
- How do you pronounce A'?
- Why use the same letters twice?

(continued on the next page)

112 Unit 1 Constructions and Rigid Transformations



Topic Translations, Reflections, and Symmetry

NAME ______ DATE ______ PERIOD ______

Summary

We've learned how to transform functions in several ways. We can translate graphs of functions up and down, changing the output values while keeping the input values. We can translate graphs left and right, changing the input values while keeping the output values. We can reflect functions across an axis, swapping either input or output values for their opposites depending on which axis is reflected across.

For some functions, we can perform specific transformations and it looks like we didn't do anything at all. Consider the function f whose graph is shown here:

What transformation could we do to the graph of f that would result in the same graph? Examining the shape of the graph, we can see a symmetry between points to the left of the y-axis and the points to the right of the y-axis. Looking at the points on the graph where $x = 1$ and $x = -1$, these opposite inputs have the same outputs since $f(1) = 4$ and $f(-1) = 4$. This means that if we reflect the graph across the y-axis, it will look no different. This type of symmetry means f is an **even function**.

Now consider the function g whose graph is shown here:

What transformation could we do to the graph of g that would result in the same graph? Examining the shape of the graph, we can see that there is a symmetry between points on opposite sides of the axes. Looking at the points on the graph where $x = 1$ and $x = -1$, these opposite inputs have opposite outputs since $g(1) = 2.35$ and $g(-1) = -2.35$. So, a transformation that takes the graph of g to itself has to reflect across the x-axis and the y-axis. This type of symmetry is what makes g an **odd function**.

Glossary

- **even function** A function f that satisfies the condition $f(x) = f(-x)$ for all inputs x. You can tell an even function from its graph: Its graph is symmetric about the y-axis.
- **odd function** A function f that satisfies $f(x) = -f(-x)$ for all inputs x. You can tell an odd function from its graph: Its graph is taken to itself when you reflect it across both the x- and y-axes. This can also be seen as a 180° rotation about the origin.

Lesson 5-5 Some Functions Have Symmetry 3

How to Assess Progress

Illustrative Mathematics contains many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

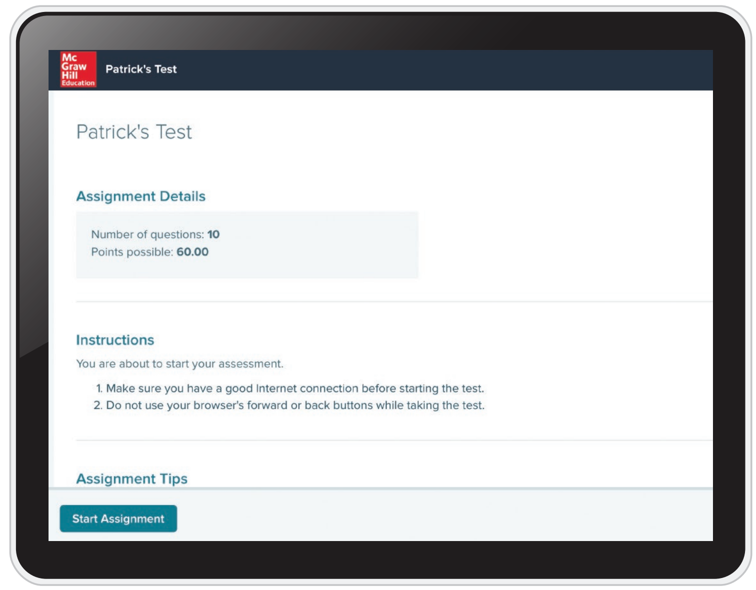
- Each unit begins with a diagnostic assessment ("Check Your Readiness") of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.

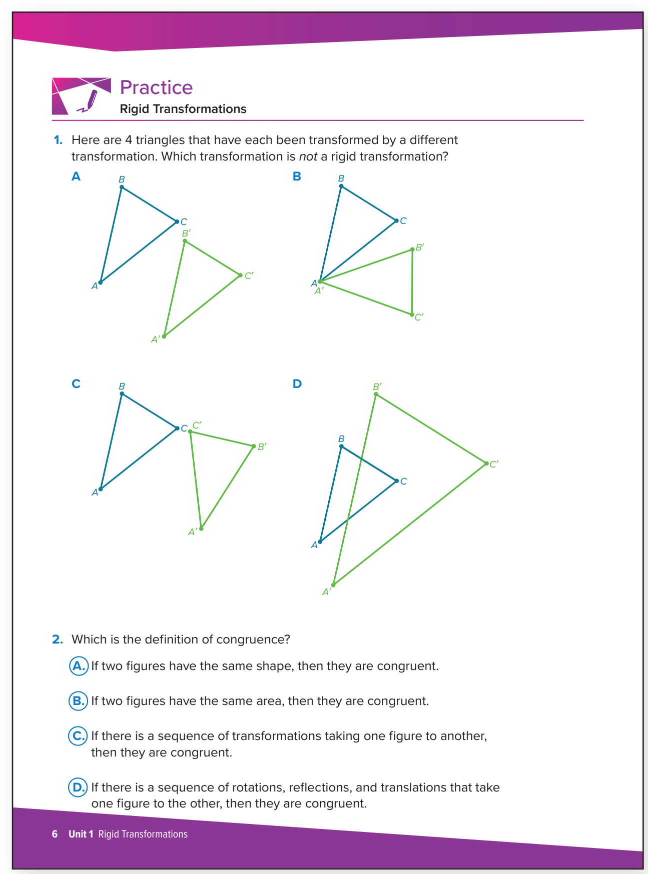
- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students' thinking.

- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day's lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.

- A set of cumulative practice problems is provided for each lesson that can be used for homework or in-class practice. The teacher can choose to collect and grade these or simply provide feedback to students.

- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.





Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. *Illustrative Mathematics* empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

Supporting English-language Learners

Illustrative Mathematics builds on foundational principles for supporting language development for all students. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

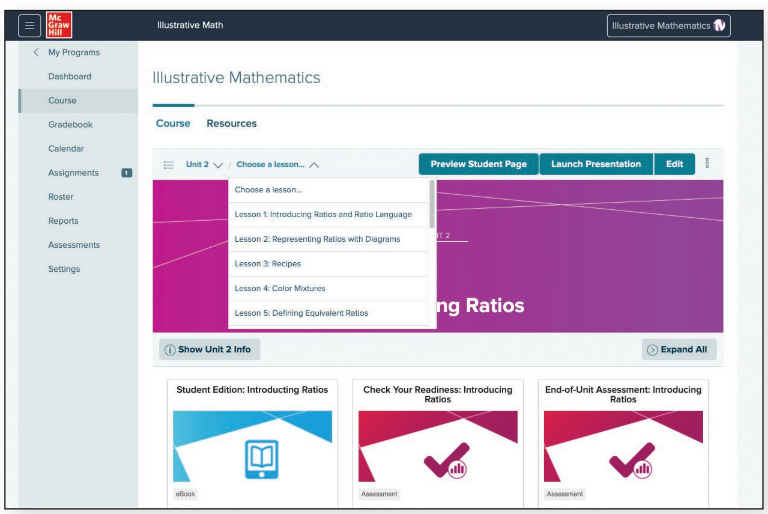
Aguirre, J.M. & Bunch, G. C. (2012). What's language got to do with it?: Identifying language demands in mathematics instruction for English Language Learners. In S. Celedón-Pattichis & N.

Digital

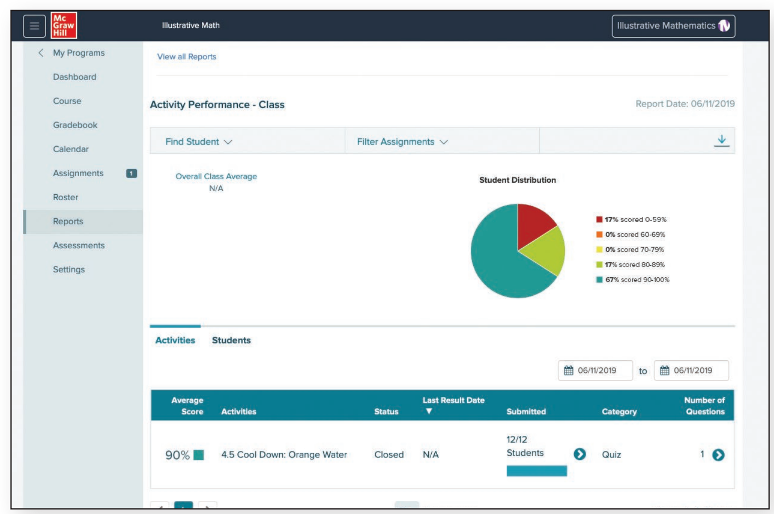
McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

Online resources offer:

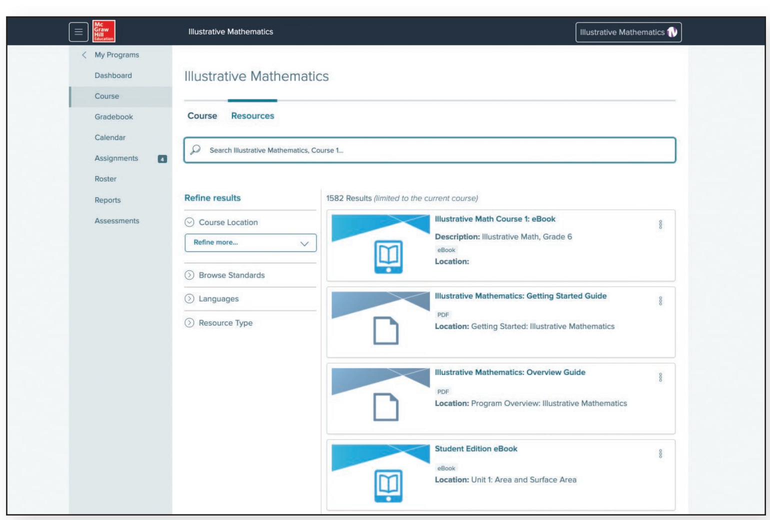
- customizable content
- the ability to add resources
- auto-scoring of student practice work
- on-going student assessments
- classroom performance reporting



Launch Presentations Digital versions of lessons to present content.



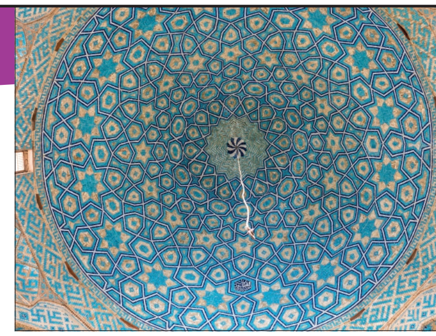
Reports Review the performance of individual students, classrooms, and grade levels.



Access Resources Point-of-use access to resources such as assessments, eBooks, and course guides.

Unit 1

Constructions and Rigid Transformations



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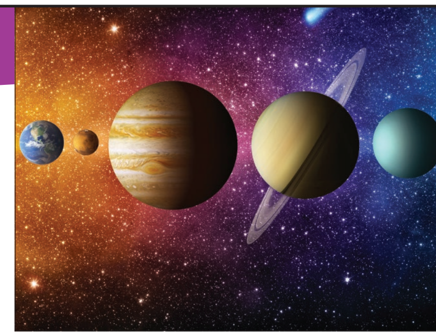
Unit 2

Congruence

Unit 3

Similarity



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Unit 4

Right Triangle Trigonometry

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Angles and Steepness

Defining Trigonometric Ratios

Unit 5

Solid Geometry

MaxyM/Shutterstock

Unit 6

Coordinate Geometry

Unit 7

Circles

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Unit 8

Conditional Probability

Unit 1

Constructions and Rigid Transformations

Prior Work

In Grade 8, students determine the angle-preserving and length-preserving properties of rigid transformations experimentally, mostly with the help of a coordinate grid. Students have previously studied angle properties, including the Triangle Angle Sum Theorem, but no formal proofs have been required. In this unit, students create rigid motions using construction tools with no coordinate grid. This leads to more rigorous definitions of rotations, reflections, and translations. Students begin to explain and prove angle relationships like the Triangle Angle Sum Theorem using these rigorous definitions and a few assertions.

Work in This Unit

In previous courses, students developed their understanding of the concept of functions. In this unit, the concept of a transformation is made somewhat more formal using the language of functions. While students do not use function notation, they do move away from describing transformations as "moves" that act on figures and towards describing them as taking points in the plane as inputs and producing points in the plane as outputs.

Constructions play a significant role in the logical foundation of geometry. A focus of this unit is for students to explore properties of shapes in the plane without the aid of given measurements. At this point, students have worked so much with numbers, equations, variables, coordinate grids, and other quantifiable structures, that it may come as a surprise just how far they can push concepts in geometry without measuring distances or angles. Constructions are used throughout several lessons to introduce students to reasoning about distances, generating conjectures, and attending to the level of precision required to define rigid motions later in the unit. The definition of a circle is an important foundation for concepts in this unit and throughout the course.

Then, students learn rigorous definitions of rigid motions without reference to a coordinate grid. In subsequent units, they use those definitions to prove theorems. To prepare students for future congruence proofs, students start to come up with a systematic, point-by-point sequence of transformations that will work to take *any* pair of congruent polygons onto one another. This point-by-point perspective also illustrates the transition from thinking about transformations as "moves" on the grid to thinking about transformations as functions that take points as inputs and produce points as outputs. Students also examine the rigid transformations that take some shapes to themselves, otherwise known as symmetries. The concept of transformations as functions is developed further in a later unit that explores coordinate geometry.

In the final lessons of the unit, students learn ways to express their reasoning more formally. Students create conjectures about angle relationships and prove them using what they know about rigid transformations. As a tool for communicating more precisely, students begin to label and mark figures to indicate congruence. In the culminating lesson of the study of constructions, students build on their experiences with perpendicular bisectors to answer questions about allocating resources in a real-world situation.

Students have the opportunity to choose appropriate tools in nearly ever lesson as they select among the options in their geometry toolkit as well as dynamic geometry software. **MP5**

For this reason, this math practice is only highlighted in lessons where it's particularly salient.

Unit Planner

Lessons		Days	Standards
Check Your Readiness Assessment		1	
Topic **Constructions**			
Lesson 1-1	Build It	1	HSN-Q.A.1, HSG-CO.D.12, HSG-CO.D.13
Lesson 1-2	Constructing Patterns	1	HSG-CO.A.1, HSG-CO.D.12, HSG-CO.D.13
Lesson 1-3	Construction Techniques 1: Perpendicular Bisectors	1	HSG-CO.A.1, HSG-CO.A.4, HSG-CO.C.9, HSG-CO.D.12
Lesson 1-4	Construction Techniques 2: Equilateral Triangles	1	HSG-CO.A.3, HSG-CO.C, HSG-CO.D.12, HSG-CO.D.13
Lesson 1-5	Construction Techniques 3: Perpendicular Lines and Angle Bisectors	1	HSG-CO.C.9, HSG-CO.D.12, HSG-CO.D.13
Lesson 1-6	Construction Techniques 4: Parallel and Perpendicular Lines	1	HSG-CO.A.1, HSG-CO.D.12, HSG-CO.A.2, HSG-CO.A.4
Lesson 1-7	Construction Techniques 5: Squares	1	HSG-CO.A.3, HSG-CO.D.13
Lesson 1-8**	Using Technology for Constructions	1	HSG-CO.A.1, HSG-CO.D.12, HSG-CO.D.13
Lesson 1-9*	Speedy Delivery	1	HSG-CO.D.12, HSG-MG.A.3, HSN-Q.A.2, HSN-Q.A.3, HSG-MG.A.3
Mid-Unit Assessment		1	
Topic **Rigid Transformations**			
Lesson 1-10	Rigid Transformations	1	HSG-CO.A.2, HSG-CO.A.5
Lesson 1-11	Defining Reflections	1	HSG-CO.A.2, HSG-CO.A.4, HSG-CO.C.10, HSG-CO.C.9
Lesson 1-12	Defining Translations	1	HSG-CO.A.4, HSG-CO.C
Lesson 1-13	Incorporating Rotations	1	HSG-CO.A.1, HSG-CO.A.2, HSG-CO.A.5, HSN-Q.A.3, HSG-CO.B.6
Lesson 1-14	Defining Rotations	1	HSN-Q.A.3, HSG-CO.A.4
Lesson 1-15	Symmetry	1	HSG-CO.A.2, HSG-CO.A.3
Lesson 1-16	More Symmetry	1	HSG-CO.A.3
Lesson 1-17	Working with Rigid Transformations	1	HSG-CO.A.2, HSG-CO.A.5
Lesson 1-18**	Practicing Point by Point Transformations	1	HSG-CO.A.2, HSG-CO.A.5
Topic **Evidence and Proof**			
Lesson 1-19	Evidence, Angles, and Proof	1	HSG-CO.C.9
Lesson 1-20	Transformations, Transversals, and Proof	1	HSG-CO.A.1, HSG-CO.A.2, HSG-CO.C.9, HSG-CO.C.10
Lesson 1-21	One Hundred and Eighty	1	HSG-CO.A.2, HSG-CO.C.10, HSG-CO.C.9
Topic **Designs**			
Lesson 1-22**	Now What Can You Build?	1	HSG-CO.D.12, HSG-CO.D.13
End of Unit Assessment		1	
	TOTAL	25	

**indicates Lessons in which there are optional activities*
***indicates optional Lessons*

Required Materials

- [ ] Copies of blackline master (Lessons 9, 15, 16, 20, 22,)
- [ ] Dynamic geometry software (Lessons 8, 9)
- [ ] Geometry toolkits (Lessons 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22)
- [ ] Masking tape (Lesson 3)
- [ ] Measuring tapes (Lesson 3)
- [ ] Pre-printed cards, cut from copies of the blackline master (Lessons 14, 17)
- [ ] Pre-printed slips, cut from copies of the blackline master (Lesson 11)
- [ ] Protractors (Lessons 13, 14)
- [ ] Sticky notes (Lessons 15, 16)
- [ ] Tools for creating a visual display (Lessons 15, 16)

Blackline Activity Masters

- [ ] Reference Chart (All Lessons)
- [ ] Tesselations (Lesson 9, Activity 4)
- [ ] Info Gap: Reflections (Lesson 11, Activity 2)
- [ ] Info Gap: Rotations (Lesson 14, Activity 2)
- [ ] Shapes (Lesson 15, Activity 2)
- [ ] Card Sort: How Did This Get There? (Lesson 17, Activity 2)
- [ ] Designs (Lesson 22, Activity 2)

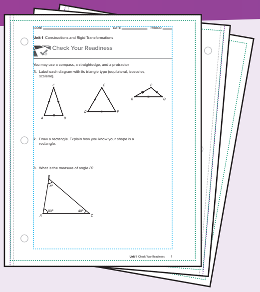
Pre-Unit Diagnostic Assessment

The pre-unit diagnostic assessment, *Check Your Readiness,* evaluate students' proficiency with prerequisite concepts and skills that they need to be successful in the unit. The item descriptions below offer guidance for students who may answer items incorrectly.

The assessment also may include problems that assess what students already know of the upcoming unit's key ideas, which you can use to pace or tune instruction. In rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.

Materials Students will not need any materials, but they might want tracing paper, protractor, compass, straightedge, graph paper, and a calculator (four-function is sufficient).

Available as a digital assessment or a printable assessment.



1. Item Description

This item tests whether a student recalls the definitions of triangle classifications by side length. This understanding is a prerequisite for success in Lesson 4. The item assumes students are familiar with the tick-mark notation for congruent sides, though the side lengths are sufficiently different. Understanding the tick marks is not required.

First Appearance of Skill or Concept: Lesson 4

2. Item Description

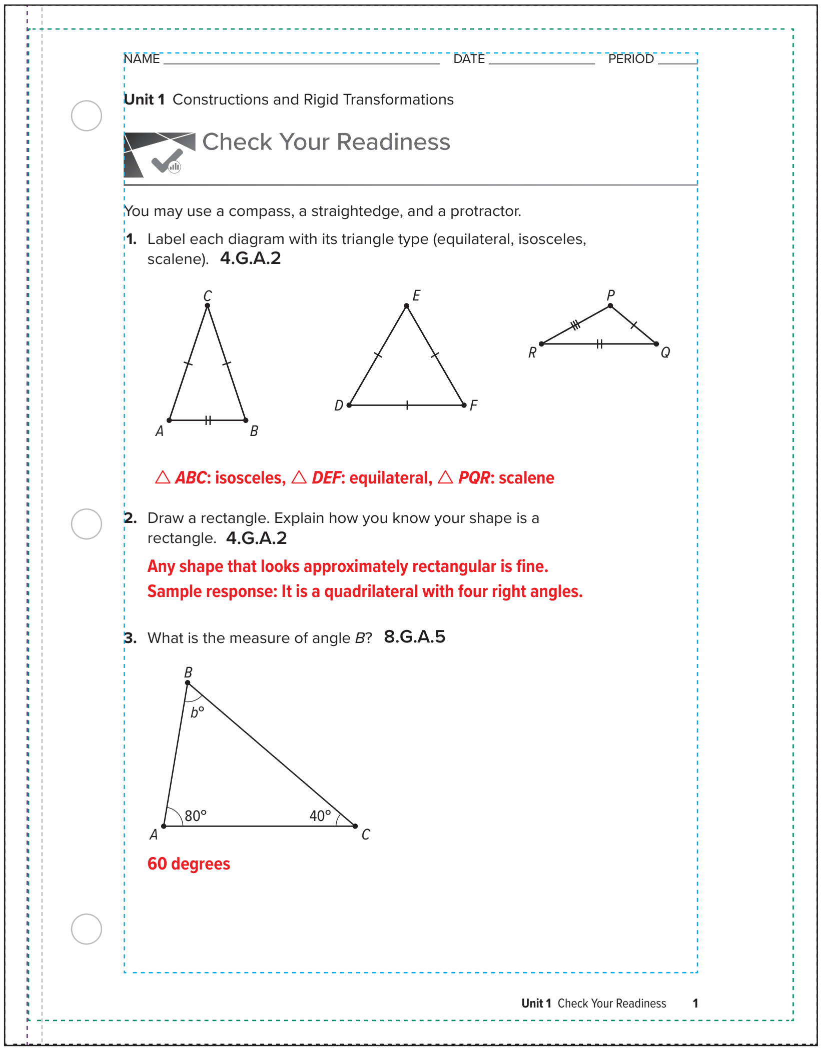
Students often have trouble defining shapes as precisely as is needed at the high school level. Watch out for students who provide extraneous information, such as two short sides and two long sides. Understanding these precise definitions is a prerequisite for success in Lesson 7.

First Appearance of Skill or Concept: Lesson 7

3. Item Description

Students will prove the Triangle Angle Sum theorem in Lesson 21, but the expectation is that they will already know and be able to use it.

First Appearance of Skill or Concept: Lesson 21



NAME ____________ DATE ________ PERIOD ______

Unit 1 Constructions and Rigid Transformations

Check Your Readiness

You may use a compass, a straightedge, and a protractor.

1. Label each diagram with its triangle type (equilateral, isosceles, scalene). **4.G.A.2**

△ ***ABC***: isosceles, △ ***DEF***: equilateral, △ ***PQR***: scalene

2. Draw a rectangle. Explain how you know your shape is a rectangle. **4.G.A.2**

Any shape that looks approximately rectangular is fine.
Sample response: It is a quadrilateral with four right angles.

3. What is the measure of angle *B*? **8.G.A.5**

60 degrees

Unit 1 Check Your Readiness 1

4. Item Description and Analysis

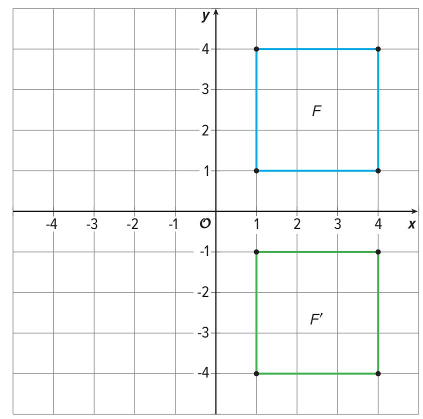
Students should be comfortable describing transformations on the grid. During this unit, they will extend that understanding to the plane without the structure of a grid. If students miss the question, then you need to pay particular attention to the isometric grid transformations starting in Lesson 10.

Answer Choice	***Students selecting this choice...***
B	if they mix up the *x*-axis and the *y*-axis.
C	if they don't recognize the symmetry of the figures.

First Appearance of Skill or Concept: Lesson 10

NAME ______________________________ DATE ______________ PERIOD ______

4. Select **all** transformations that take figure *F* to figure *F′*. **8.G.A.2**



- **A.** Reflect figure *F* across the *x*-axis.
- **B.** Reflect figure *F* across the *y*-axis.
- **C.** Rotate figure *F* 90 degrees clockwise around the origin.
- **D.** Rotate figure *F* 180 degrees counterclockwise around the origin.
- **E.** Translate figure *F* so that (4,1) goes to (4,-1).
- **F.** Translate figure *F* so that (1,4) goes to (1,-1).

2 **Unit 1** Check Your Readiness

5. Item Description

If students miss the question, then you need to pay particular attention to the introduction of symmetry in Lesson 15.

A and D would only work if the figure was a square.

E takes *AD* to *BC,* but it's not a symmetry.

First Appearance of Skill or Concept: Lesson 15

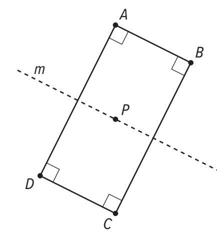
6. Item Description

Students should connect rigid transformations and congruence based on their middle school experience. If students miss the question or have errors in their reasoning, then you need to pay particular attention to the math talk within Lesson 6.

First Appearance of Skill or Concept: Lesson 6

NAME ______________ DATE ______________ PERIOD ______

5. Select **all** transformations that take rectangle *ABCD* onto itself. **HSG-CO.A.3**



(A.) Rotate by 90 degrees clockwise using center *P*.

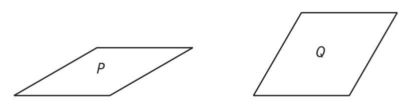
(B.) Rotate by 180 degrees clockwise using center *P*.

(C.) Reflect across line *m*.

(D.) Reflect across diagonal *AC*.

(E.) Translate by the directed line segment from *A* to *B*.

6. Is there a rigid transformation taking Rhombus *P* to Rhombus *Q*? Explain how you know. **8.G.A.2**



No, Rhombus *P* is not congruent to Rhombus *Q*.

Unit 1 Check Your Readiness 3

7. Item Description

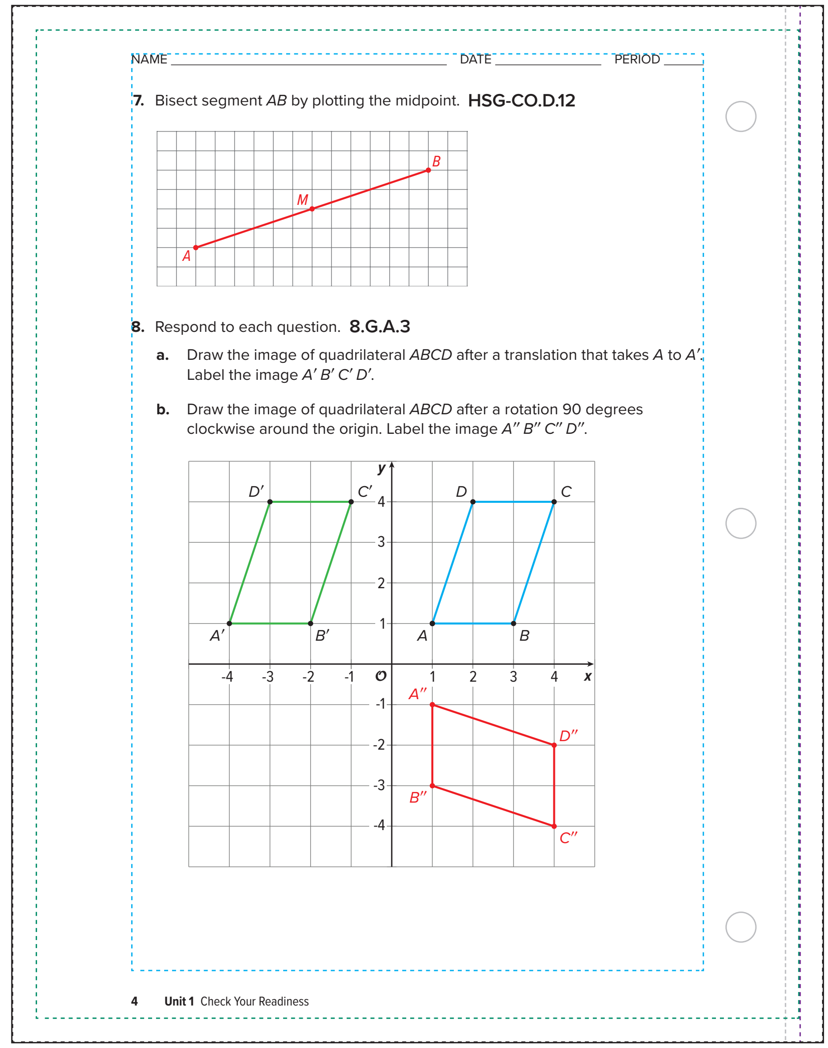
Students will be finding midpoints for a variety of constructions, this item assesses if they are prepared for the tagged standard. If students miss the question, pay particular attention to some of the bisection constructions in Lesson 1.

First Appearance of Skill or Concept: Lesson 1

8. Item Description

Students should be comfortable drawing transformations on the grid. During this unit, they will extend that understanding to the plane without the structure of a grid. If students miss the first part of the question, then you need to pay particular attention to the isometric grid translations in Lesson 10. If they miss the second part of the question, then you need to pay particular attention to the isometric grid rotations in Lesson 13.

First Appearance of Skill or Concept: Lesson 13



NAME ______________ DATE ______________ PERIOD ______

7. Bisect segment *AB* by plotting the midpoint. **HSG-CO.D.12**

8. Respond to each question. **8.G.A.3**

a. Draw the image of quadrilateral *ABCD* after a translation that takes *A* to *A′*. Label the image *A′ B′ C′ D′*.

b. Draw the image of quadrilateral *ABCD* after a rotation 90 degrees clockwise around the origin. Label the image *A″ B″ C″ D″*.

4 **Unit 1** Check Your Readiness

9. Item Description

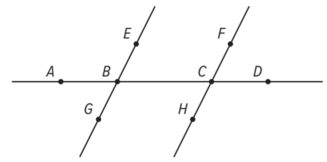
Students should be comfortable identifying congruent and supplementary angles formed by parallel lines and a transversal. In Lesson 20, they will prove these relationships.

Students will likely write $FCD + HCD = 180$, which provides an opportunity to discuss notation. Technically, that equation is meaningless, but we know they mean the measures of the angles formed by those points. There will be times in class when jotting notes using informal notation is fine. During some assignments, students will practice precise communication, and that won't be sufficient.

First Appearance of Skill or Concept: Lesson 20

NAME ______________ DATE ________ PERIOD ____

9. Lines *EG* and *FH* are parallel. **8.G.A.5**



a. Name all the angles congruent to angle *ABE*.

$\angle BCF$, $\angle GBC$, $\angle HCD$ (or equivalent)

b. Write an equation that represents the relationship between angle *FCD* and angle *HCD*.

$m\angle FCD + m\angle HCD = 180$ (or equivalent)

Lesson 1-10

Rigid Transformations

Goals (Teacher-Facing)

- Comprehend that rigid transformations produce congruent figures by preserving distance and angles.
- Draw the result of a transformation (in written language) of a given figure.
- Explain (orally and in writing) a sequence of transformations to take a given figure onto another.

Student Learning Goals

Let's draw some transformations.

Learning Targets

- Given a figure and the description of a transformation, I can draw the figure's image after the transformation.
- I can describe the sequence of transformations necessary to take a figure onto another figure.
- I know that rigid transformations result in congruent figures.

Required Materials

- Geometry toolkits

Required Preparation

Each student will need 4 different colors for What's the Same?, so be sure the geometry toolkits have enough colored pencils.

Create a display of the reference chart for all to see. It should remain posted for the rest of the year.

Before this lesson, the reference chart will be blank. The blank reference chart is included as a blackline master, as well as a teacher copy of a completed version. The purpose of the reference chart is to be a resource for students to reference as they make formal arguments. Students will continue adding to it throughout the course. Every claim they make needs to be supported by referencing assertions, definitions, or theorems from the reference chart.

If you teach multiple sections of this course, consider hiding entries on the class reference chart and revealing them at the appropriate time rather than making multiple displays.

Lesson Pacing

	Pacing (min)
Warm Up 10.1 Notice and Wonder: Transformed	10
Activity 10.2 What's the Same?	15
Activity 10.3 Does Order Matter?	10
Lesson Synthesis	5
Cool Down 10.4 How Will That Get There?	5
TOTAL	**45**

Standards Alignment

Building On

8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Addressing

HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

HSG-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Building Towards

HSG-CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

HSG-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Lesson Narrative

This lesson builds on prior knowledge about congruence to reinforce the idea that the rigid motions, **translations, reflections,** and **rotations** preserve distances and angles. These motions and the sequences of the motions, called **rigid transformations,** affect the entire plane, but students generally focus on a single figure and its image (the result of a transformation). Students also recall that the definition of **congruent** is any two figures where there is a sequence of translations, rotations, and reflections that takes the first figure onto the second. In this lesson, students study transformations on a grid, then in subsequent lessons in this unit, students learn precise definitions for rigid motions that apply off the grid. The study of coordinate geometry is reserved for a subsequent unit. Students work on an isometric grid to push them toward needing precise definitions.

During this lesson, students focus on translations and reflections. While students may mention them, rotations do not get defined until a subsequent lesson. Students determine that different sequences of rigid motions can result in the same image, and when sequences are reordered, they sometimes don't result in the same image. This idea builds toward the concept of transformations as functions while preparing students to reason about sequences of transformations in addition to single motions. Students make arguments and critique the arguments of others when they compare strategies for finding sequences of rigid transformations that take one figure onto another. MP3

At this point, students take the distance between a point and a line as the distance along the perpendicular as a definition. In a subsequent unit, students will prove that the shortest path between a point and a line is along the perpendicular.

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems. We recommend making technology available.

Warm Up 10.1 Notice and Wonder: Transformed (10 minutes)

The purpose of this warm up is to elicit the idea that some shapes can be described as transformations of other shapes, which will be useful when students specify sequences of rigid transformations that take one figure onto another in the next activities. While students may notice and wonder many things about these images, the important discussion point is that rigid transformations take sides to sides of the same length and angles to angles of the same measure.

Instructional Routines

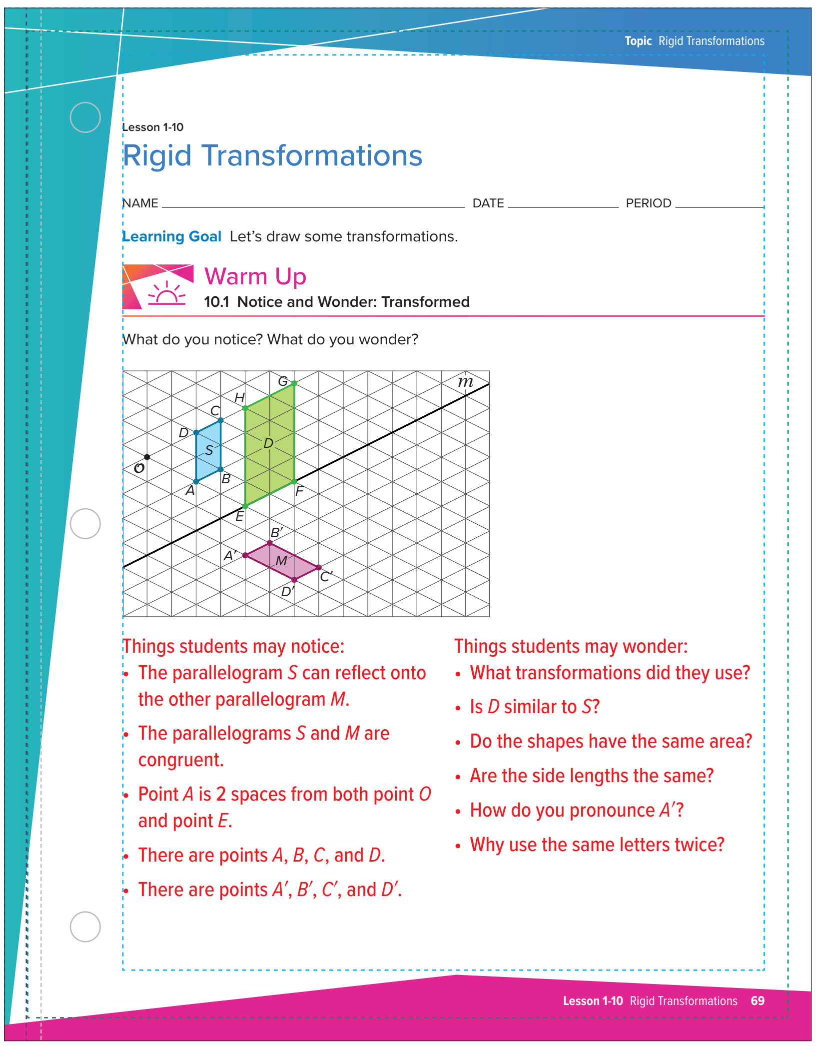
See the Appendix, beginning on page A1, for a description of this routine and all Instructional Routines.

- Notice and Wonder

Standards Alignment

Building On 8.G.A.2

Building Towards HSG-CO.A.2, HSG-CO.A.5


Topic Rigid Transformations

Lesson 1-10

Rigid Transformations

NAME ______ DATE ______ PERIOD ______

Learning Goal Let's draw some transformations.

Warm Up

10.1 Notice and Wonder: Transformed

What do you notice? What do you wonder?

Things students may notice:
- The parallelogram *S* can reflect onto the other parallelogram *M*.
- The parallelograms *S* and *M* are congruent.
- Point *A* is 2 spaces from both point *O* and point *E*.
- There are points *A*, *B*, *C*, and *D*.
- There are points *A′*, *B′*, *C′*, and *D′*.

Things students may wonder:
- What transformations did they use?
- Is *D* similar to *S*?
- Do the shapes have the same area?
- Are the side lengths the same?
- How do you pronounce *A′*?
- Why use the same letters twice?

Lesson 1-10 Rigid Transformations 69

Student Edition, p. 69

Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Support For Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a table to record what they notice and wonder prior to being expected to share these ideas with others.

Supports accessibility for: Language; Organization

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After all responses have been recorded without commentary or editing, ask students, Is there anything on this list that you are wondering about now? Encourage students to respectfully disagree, ask for clarification, and point out contradicting information.

If sequences of rigid transformations or corresponding measurements do not come up during the conversation, ask students to discuss this idea. Reinforce that because the sizes, shapes, and angles did not change from Figure *S* to Figure *M*, that transformation is called a **rigid transformation.** But the transformation from Figure *S* to Figure *D* is not a rigid transformation because the size changed.

If the difference between *A* and *A′* does not come up during the conversation, ask students to discuss this idea and tell them that *A′* is pronounced "*A* prime." Explain that *ABCD* is called the original figure and *A′ B′ C′ D′* is called the **image** of the transformation

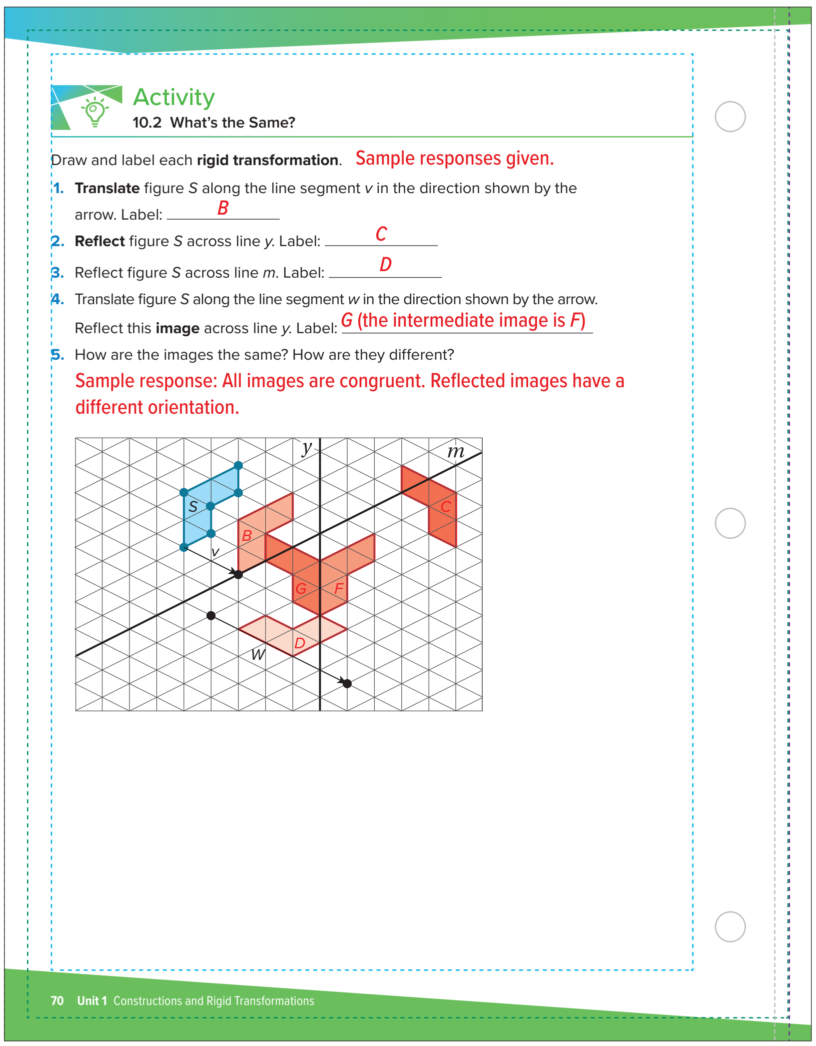
Activity 10.2 What's the Same? (15 minutes)

The purpose of this activity is to activate students' prior knowledge of rigid transformations. Students also see that the image of a polygon is determined by the images of each of its vertices. Students build toward the concept that transformations are functions that take points as inputs and produce points as outputs so that distances and angles are preserved.

Standards Alignment

Building On 8.G.A.2

Addressing HSG-CO.A.2, HSG-CO.A.5



Activity
10.2 What's the Same?

Draw and label each **rigid transformation**. Sample responses given.

1. **Translate** figure *S* along the line segment *v* in the direction shown by the arrow. Label: *B*
2. **Reflect** figure *S* across line *y*. Label: *C*
3. Reflect figure *S* across line *m*. Label: *D*
4. Translate figure *S* along the line segment *w* in the direction shown by the arrow. Reflect this **image** across line *y*. Label: *G* (the intermediate image is *F*)
5. How are the images the same? How are they different?
Sample response: All images are congruent. Reflected images have a different orientation.

70 Unit 1 Constructions and Rigid Transformations

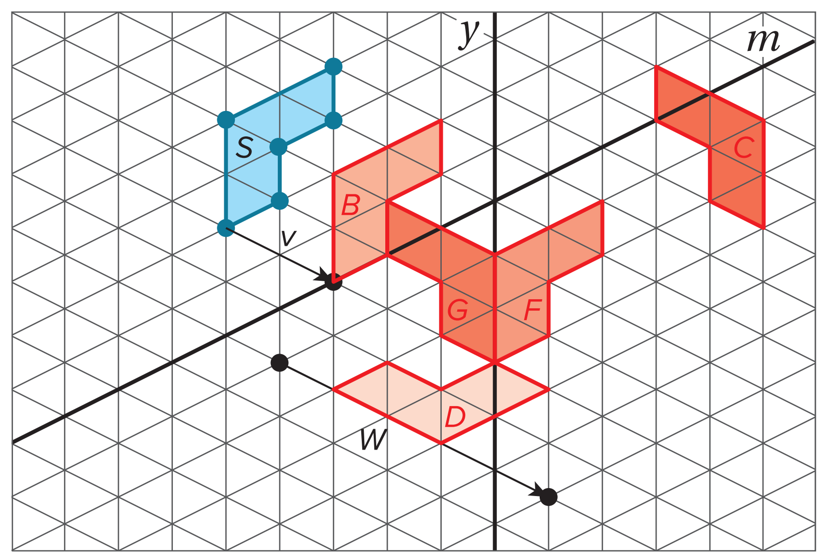
Student Edition, p. 70

Launch

Give students 3 minutes of quiet time to work, then pause for a brief whole-class discussion.

Invite a student to demonstrate how to use tracing paper to translate. Recommend that students start with the edges of the tracing paper parallel to the sides of the paper so they can see if they accidentally tilt the tracing paper as they translate. Invite students to define **translation**. (A translation has a distance and a direction. It moves every point in a figure the given distance in the given direction.)

Display the image from the warm up with construction marks



Invite students to define **reflection.** (Every point of the figure ends up on the other side of the line of reflection and the same distance from the line.) If not mentioned by students, point out that the construction marks are lines perpendicular to the line of reflection.

(*continued on the next page*)

Support For Students with Disabilities

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch, take time to review the following terms from previous lessons that students will need to access for this activity: rigid transformation, image, translation, reflection, and congruent.

Supports accessibility for: Memory; Language

Anticipated Misconceptions

Some students may have trouble reflecting on the isometric grid. Ask these students to use tracing paper to fold across the line of reflection to find the image.

If students are stuck as to how to translate when *w* isn't connected to a vertex of the figure, remind students that *w* is telling them the direction and the distance, but the location doesn't matter.

Activity Synthesis

The important idea for discussion is that rigid transformations preserve distances and angles. Display a student's work for all to see and ask:

- **For the reflection of *S* across line *y*, how do the side lengths of *S* compare to the corresponding side lengths in its image?** The lengths are equal.
- **What is the measures of the angle in the upper left corner of *S*? How does this compare to the corresponding angle measure in any of the images of *S*?** This angle and all of its images measure 120 degrees.

Activity 10.3 Does Order Matter? (10 minutes)

The purpose of this activity is to observe that the order of the transformations in a sequence of transformations can have an effect on the image.

Monitor for students who define a sequence that works even when the order is reversed to compare to students whose sequences don't work when the order is reversed during the discussion.

Making dynamic geometry software available gives students an opportunity to choose appropriate tools strategically. **MP5**

Instructional Routines

See the Appendix, beginning on page A1, for a description of these routines and all Instructional Routines.

- Draw It
- Think pair share

Standards Alignment

Building On 8.G.A.2

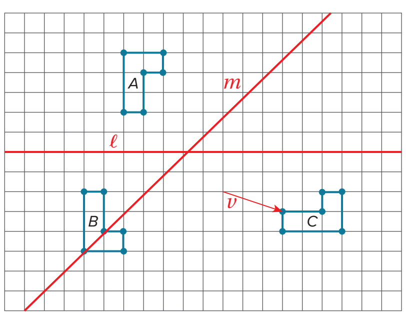
Addressing HSG-CO.A.2, HSG-CO.A.5

Topic Rigid Transformations

NAME ______ DATE ______ PERIOD ______

Activity
10.3 Does Order Matter?

Here are 3 **congruent** L shapes on a grid.



1. Describe a sequence of transformations that will take Figure *A* onto Figure *B*.
   Sample response: Reflect across line ℓ (drawn into diagram), then translate left 2 units.
2. If you reverse the order of your sequence, will the reverse sequence still take *A* onto *B*?
   Sample response: Yes
3. Describe a sequence of transformations that will take Figure *A* onto Figure *C*.
   Sample response: Reflect across line m (drawn into diagram), then translate by v (drawn into diagram).
4. If you reverse the order of your sequence, will the reverse sequence still take *A* onto *C*?
   Sample response: No

Lesson 1-10 Rigid Transformations 71

Student Edition, p. 71

Launch

Arrange students in groups of 2. Remind students that if there is a sequence of rigid transformations that takes one figure onto another, the figures are called **congruent.** Tell students there are many possible answers for the questions. After quiet work time, ask students to compare their responses to their partner's and decide if they are both correct, even if they are different. Follow with a whole-class discussion.

If students are struggling after several minutes, invite students to share what rigid motions they will need. (Reflection because Figure *B* is an L shape and Figure *C* can be rotated to look like an L, but Figure *A* cannot be.) Suggest that they start each sequence with a reflection, then use a translation.

Support For English Language Learners

Reading, Writing: MLR3 Clarify, Critique, Correct. Present a first draft of a description of a sequence of transformations that will take Figure *A* onto Figure *B*: I flipped it over and slid it around. Prompt discussion by asking, What were the steps the author took? Ask students to clarify and correct the statement. Improved statements should include some of the following: a directed distance for a translation, specifying which point is taken to which point as a result of a translation, the words "translation" and "reflection," the line over which Figure A is being reflected. This will help students develop descriptions of transformations.

Design Principle(s): Maximize meta-awareness; Optimize output (for explanation)

(*continued on the next page*)

Are you ready for more?

1. Construct some examples of sequences of two rigid transformations that take Figure *A* to a new Figure *D* where reversing the order of the sequence also takes Figure *A* to Figure *D*.

   Sample response: Translate 5 units to the left. Then translate 2 units to the right.

2. Make some conjectures about when reversing the order of a sequence of two rigid transformations still takes a figure to the same place.

   Sample responses: If each transformation is a translation; if each transformation is a rotation through the same point; if each transformation is a reflection and the two reflection lines are perpendicular; if one transformation is a reflection in a line and the other is translation by a directed segment in a direction parallel to the line.

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Student Edition, p. 72

Support For Students with Disabilities

Engagement: Internalize Self-Regulation. Demonstrate giving and receiving constructive feedback. Use a structured process and display sentence frames to support productive feedback. For example, This method works/doesn't work because..., Another strategy would be _________ because..., and Is there another way to say/do...?

Supports accessibility for: Social-emotional skills; Organization; Language

Anticipated Misconceptions

Reversing the sequence means using the same steps, but step 2 becomes step 1. For example, 1) reflect across line ℓ then 2) translate left 2 units would become 1) translate left 2 units then 2) reflect across line ℓ.

Activity Synthesis

Select previously identified students to share their responses.

Highlight that reversing the steps in a sequence of transformations sometimes results in the same transformation, and sometimes it results in a different transformation.

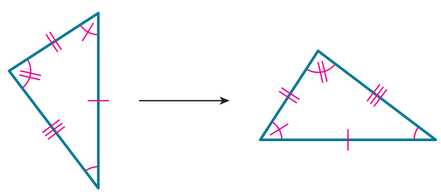
Lesson Synthesis (5 minutes)

Ask students, What important ideas did you learn about **rigid transformations?** The result is called an **image.** The image is **congruent** to the original figure.

Display the blank reference chart for all to see and give 1 copy of the blank reference chart blackline master to each student. Explain that in order to write convincing arguments, they need to support their statements with facts. The reference chart is a way to keep track of those facts for future reference when they are trying to prove new facts. Ask students to add an assertion and a definition to their reference charts as you add them to the class reference chart:

A rigid transformation is a translation, reflection, rotation, or any sequence of the three. Rigid transformations take lines to lines, angles to angles of the same measure, and segments to segments of the same length.

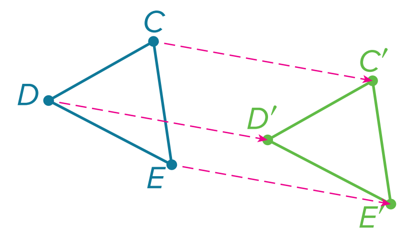
(Assertion)



One figure is **congruent** to another if there is a sequence of translations, rotations, and reflections that takes the first figure exactly onto the second figure. The second figure is called the image of the rigid transformation.

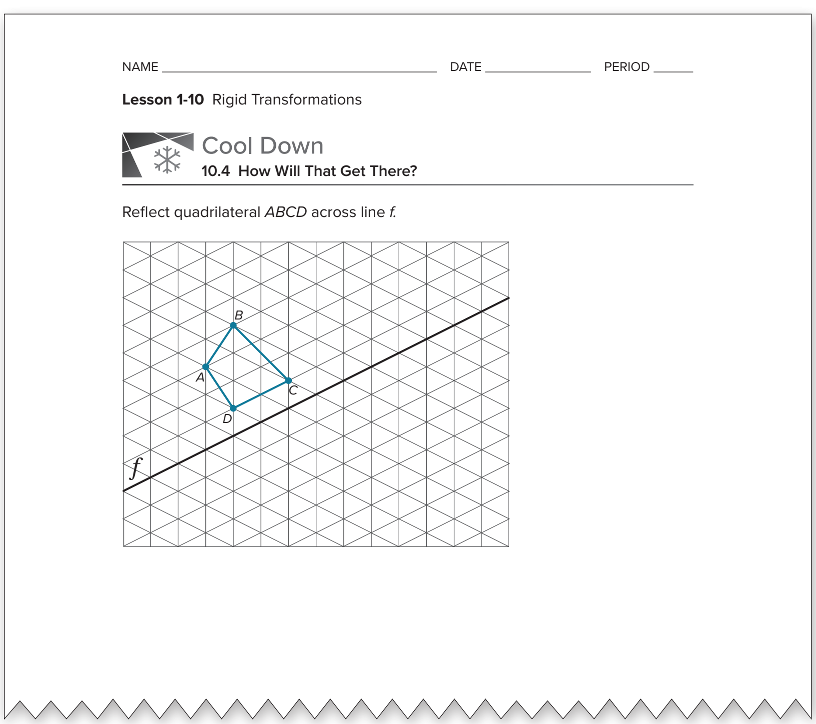
(Definition)

$\triangle EDC \cong \triangle E'D'C'$



Each entry in the reference chart includes a statement, a diagram and a type. The types will be assertions, definitions, and theorems. Explain that an assertion is an observation that seems to be true, but is not proven. The fact that rigid transformations always take lines to lines, angles to angles of the same measure, and segments to segments of the same length seems to be true, but there is no way to prove or disprove this. So, moving forward, they can assert that rigid transformations have these properties and find out what follows from that starting point. Remind students that earlier, they conjectured that the perpendicular bisector of a segment is the set of points that are the same distance away from each endpoint. They *could* write this as an assertion based on their experiments, but they will see in the next unit that it's actually possible to prove that it's true based on the assertion they just made about rigid transformations. Geometry is generally more interesting when you try to connect as many ideas as possible to just a few starting assertions.

Cool Down 10.4 How Will That Get There? (5 minutes)



NAME ______________ DATE ________ PERIOD ______

Lesson 1-10 Rigid Transformations

Cool Down
10.4 How Will That Get There?

Reflect quadrilateral *ABCD* across line *f*.

Printable Cool Down

Standards Alignment

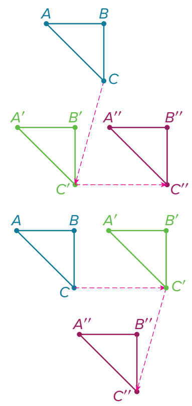
Building On 8.G.A.2

Addressing HSG-CO.A.5

NAME ______________________________ DATE _______________ PERIOD _________

Summary
Rigid Transformations

A figure is called **congruent** to another figure if there is a sequence of translations, rotations, and reflections that takes one of the figures onto the other. This is because translations, rotations, and reflections are rigid motions. Any sequence of rigid motions is called a **rigid transformation**. A rigid transformation is a transformation that doesn't change measurements on any figure. With a rigid transformation, figures like polygons have corresponding sides of the same length and corresponding angles of the same measure.



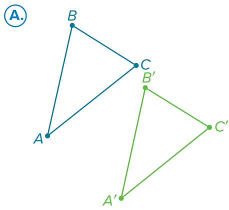
The result of any transformation is called the **image**. The points in the original figure are the inputs for the transformation sequence and are named with capital letters. The points in the image are the outputs and are named with capital letters and an apostrophe, which is referred to as "prime."

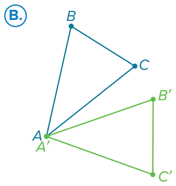
There are many ways to show that 2 figures are congruent since many sequences of transformations take a figure to the same image. However, order matters in a set of instructions. Sometimes we can switch 2 steps in a sequence and get the same output, but other times, switching 2 steps results in a different image. These 2 sequences of transformations both have the points A, B, and C as inputs and points A'', B'', and C'' as outputs. Each step in the sequences of rigid transformations creates a triangle that is congruent to triangle ABC.

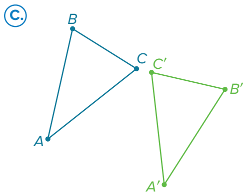
Glossary
- **congruent**
- **image**
- **reflect**
- **rigid transformation**
- **translate**

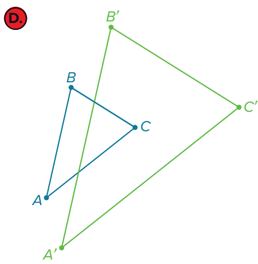
Practice
Rigid Transformations

1. Here are 4 triangles that have each been transformed by a different transformation. Which transformation is *not* a rigid transformation?

A.


B.


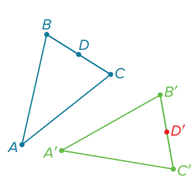
C.


D. (answer)


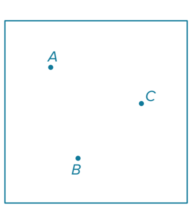
NAME ______________________________ DATE _______________ PERIOD _________

2. What is the definition of congruence?
   - A. If two figures have the same shape, then they are congruent.
   - B. If two figures have the same area, then they are congruent.
   - C. If there is a sequence of transformations taking one figure to another, then they are congruent.
   - D. (answer) If there is a sequence of rotations, reflections, and translations that take one figure to the other, then they are congruent.

3. There is a sequence of rigid transformations that takes A to A', B to B', and C to C'. The same sequence takes D to D'. Draw and label D':

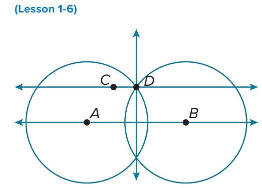


4. Three schools are located at points A, B, and C. The school district wants to locate its new stadium at a location that will be roughly the same distance from all 3 schools. Where should they build the stadium? Explain or show your reasoning. (Lesson 1-9)



Sample response: Build the stadium as close as possible to the intersection of the perpendicular bisectors of the sides of triangle ABC. Draw triangle ABC. Construct the perpendicular bisector of each side of the triangle. These lines intersect in a point that is the same distance away from each vertex of the triangle.

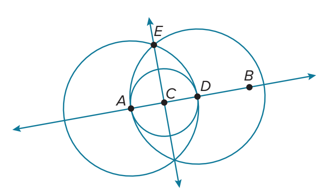
5. To construct a line passing through point C that is parallel to the line AB, Han constructed the perpendicular bisector of AB and then drew line CD. (Lesson 1-6)



Is CD guaranteed to be parallel to AB? Explain how you know.

No. Sample response: Han's lines only look parallel. Nothing about this construction guarantees the lines won't intersect. One way to construct a line parallel to AB, is to start with a line through C that is perpendicular to AB. Then a line perpendicular to that line is guaranteed to be perpendicular to line AB.

6. This diagram is a straightedge and compass construction of a line perpendicular to line AB passing through point C. Select **all** the statements that must be true. (Lesson 1-5)



- A. $AD = BD$
- B. $EC = AD$
- C. (answer) $AC = DC$
- D. (answer) $EA = ED$
- E. $ED = DB$
- F. $CB = AD$

Summary and Practice

Student Edition

Lesson 1-10

Rigid Transformations

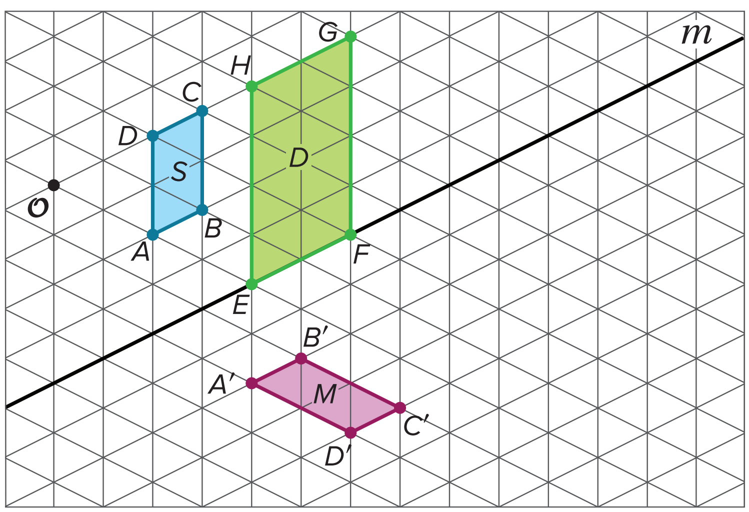
NAME ______________________ DATE ______________ PERIOD ______________

Learning Goal Let's draw some transformations.

Warm Up

10.1 Notice and Wonder: Transformed

What do you notice? What do you wonder?

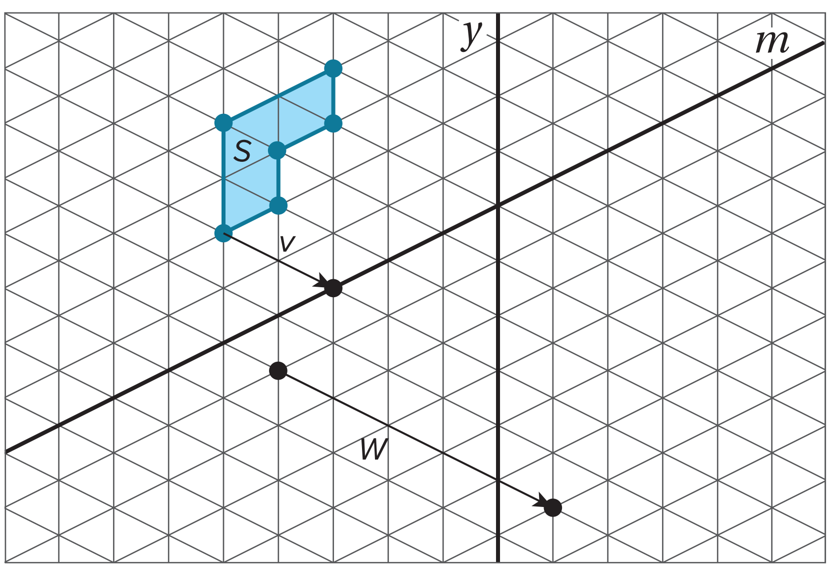


Activity

10.2 What's the Same?

Draw and label each **rigid transformation**.

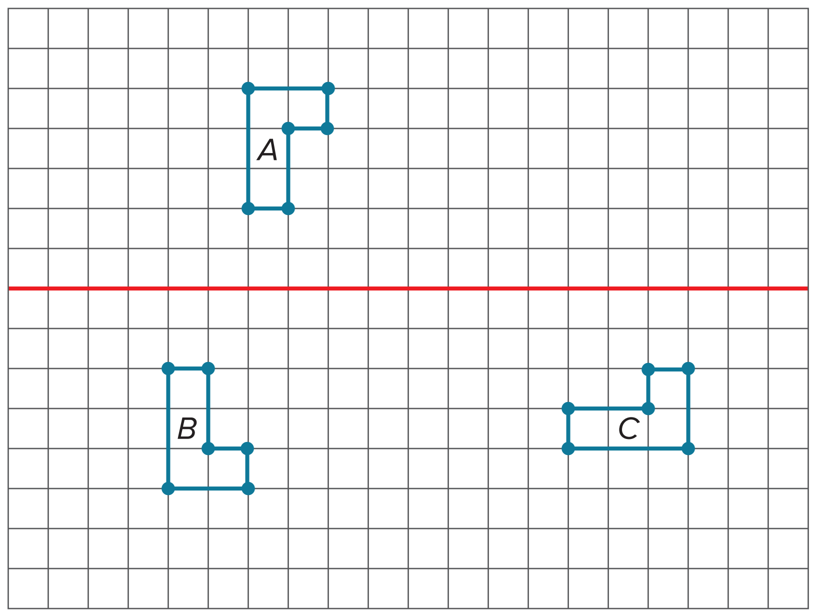
1. **Translate** figure *S* along the line segment *v* in the direction shown by the arrow. Label: ______________
2. **Reflect** figure *S* across line *y*. Label: ______________
3. Reflect figure *S* across line *m*. Label: ______________
4. Translate figure *S* along the line segment *w* in the direction shown by the arrow. Reflect this **image** across line *y*. Label: ______________________________
5. How are the images the same? How are they different?



NAME ______________________ DATE ____________ PERIOD ____________

Activity
10.3 Does Order Matter?

Here are 3 **congruent** L shapes on a grid.



1. Describe a sequence of transformations that will take Figure *A* onto Figure *B*.

2. If you reverse the order of your sequence, will the reverse sequence still take *A* onto *B*?

3. Describe a sequence of transformations that will take Figure *A* onto Figure *C*.

4. If you reverse the order of your sequence, will the reverse sequence still take *A* onto *C*?

Are you ready for more?

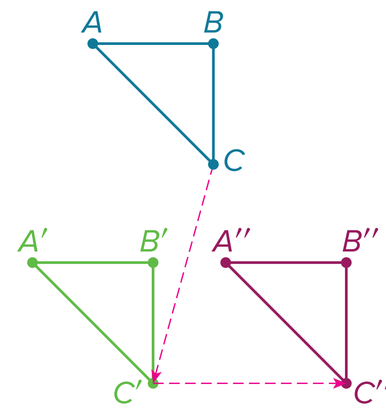
1. Construct some examples of sequences of two rigid transformations that take Figure *A* to a new Figure *D* where reversing the order of the sequence also takes Figure *A* to Figure *D*.

2. Make some conjectures about when reversing the order of a sequence of two rigid transformations still takes a figure to the same place.

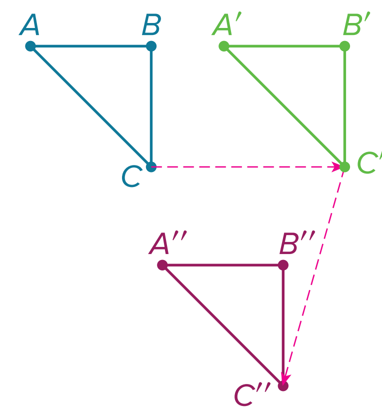
NAME ______________________ DATE ____________ PERIOD ________

Summary

Rigid Transformations



A figure is called **congruent** to another figure if there is a sequence of translations, rotations, and reflections that takes one of the figures onto the other. This is because translations, rotations, and reflections are rigid motions. Any sequence of rigid motions is called a **rigid transformation**. A rigid transformation is a transformation that doesn't change measurements on any figure. With a rigid transformation, figures like polygons have corresponding sides of the same length and corresponding angles of the same measure.



The result of any transformation is called the **image**. The points in the original figure are the inputs for the transformation sequence and are named with capital letters. The points in the image are the outputs and are named with capital letters and an apostrophe, which is referred to as "prime."

There are many ways to show that 2 figures are congruent since many sequences of transformations take a figure to the same image. However, order matters in a set of instructions. Sometimes we can switch 2 steps in a sequence and get the same output, but other times, switching 2 steps results in a different image. These 2 sequences of transformations both have the points *A*, *B*, and *C* as inputs and points *A″*, *B″*, and *C″* as outputs. Each step in the sequences of rigid transformations creates a triangle that is congruent to triangle *ABC*.

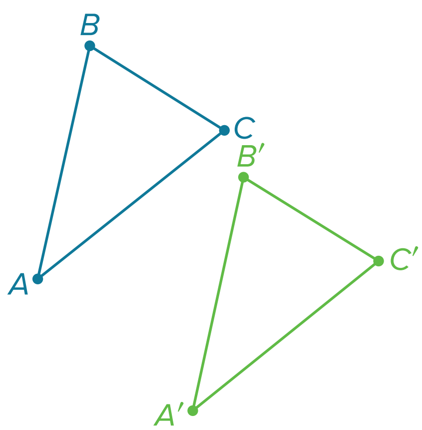
Glossary

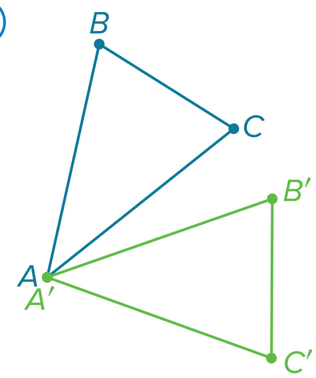
congruent
image
reflect
rigid transformation
translate

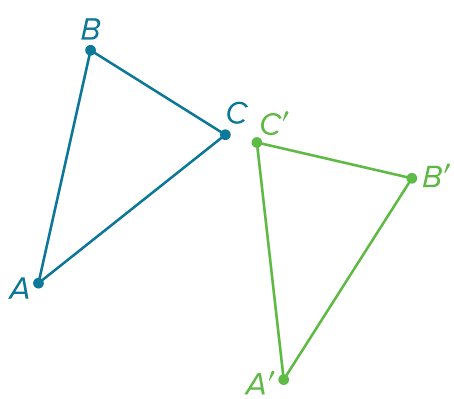
Practice

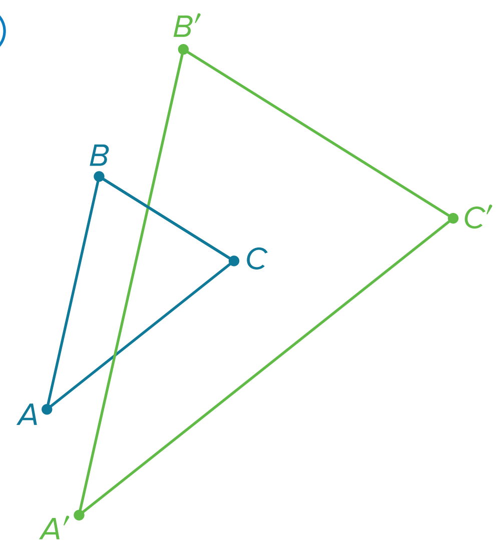
Rigid Transformations

1. Here are 4 triangles that have each been transformed by a different transformation. Which transformation is *not* a rigid transformation?

A. 

B. 

C. 

D. 

NAME ______________________ DATE ____________ PERIOD ____________

2. What is the definition of congruence?

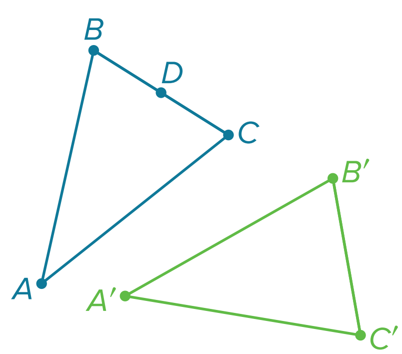
A. If two figures have the same shape, then they are congruent.

B. If two figures have the same area, then they are congruent.

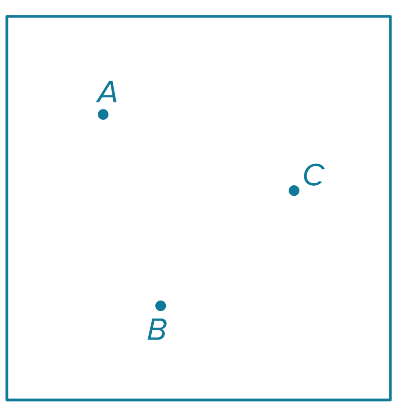
C. If there is a sequence of transformations taking one figure to another, then they are congruent.

D. If there is a sequence of rotations, reflections, and translations that take one figure to the other, then they are congruent.

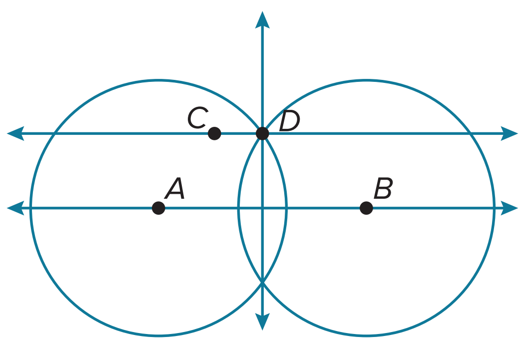
3. There is a sequence of rigid transformations that takes A to A', B to B', and C to C'. The same sequence takes D to D'. Draw and label D':



4. Three schools are located at points A, B, and C. The school district wants to locate its new stadium at a location that will be roughly the same distance from all 3 schools. Where should they build the stadium? Explain or show your reasoning. **(Lesson 1-9)**

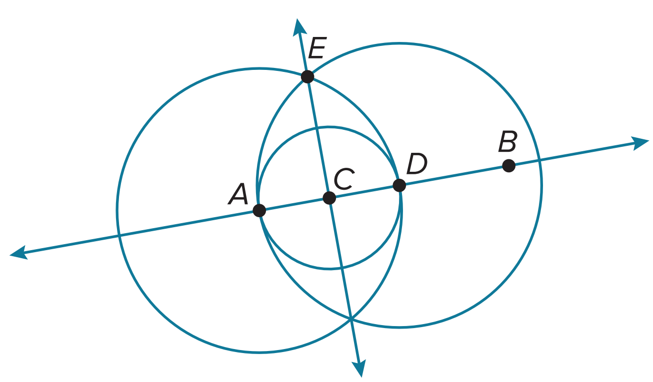


5. To construct a line passing through point *C* that is parallel to the line *AB*, Han constructed the perpendicular bisector of *AB* and then drew line *CD*. **(Lesson 1-6)**



Is *CD* guaranteed to be parallel to *AB*? Explain how you know.

6. This diagram is a straightedge and compass construction of a line perpendicular to line *AB* passing through point *C*. Select **all** the statements that must be true. **(Lesson 1-5)**



A. $AD = BD$

B. $EC = AD$

C. $AC = DC$

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E. $ED = DB$

F. $CB = AD$

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