McGraw-Hill Illustrative Mathematics Course 3



Lesson Sampler



McGraw-Hill Illustrative Mathematics Course 3

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To accommodate different implementations of the program, course numbers rather than grade levels are referenced on the covers of McGraw-Hill *Illustrative Mathematics* materials. However, grade levels are referenced in the materials as this is how Illustrative Mathematics was originally written.

Course 1 = Grade 6, Course 2 = Grade 7, Course 3 = Grade 8

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Illustrative Mathematics is a problem-based core curriculum designed to address content and practice standards to foster learning for all. Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Teachers can shift their instruction and facilitate student learning with high-leverage routines to guide learners to understand and make connections between concepts and procedures.



What is a Problem-based Curriculum?

In a problem-based curriculum, students work on carefully crafted and sequenced mathematics problems during most of the instructional time. Teachers help students understand the problems and guide discussions to be sure that the mathematical takeaways are clear to all. In the process, students explain their ideas and reasoning and learn to communicate mathematical ideas. The goal is to give students just enough background and tools to solve initial problems successfully, and then set them to increasingly sophisticated problems as their expertise increases.

The value of a problem-based approach is that students spend most of their time in math class doing mathematics: making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers. They go on to interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding.



"Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience . . ."¹

Creating a World Where Learners Know, Use, and Enjoy Mathematics

Decades of research shows that students learn best when they are given a chance to start work on a problem before being shown a solution method. This gives students the chance to build conceptual understanding that can cement procedural skills by tying them together. It allows students to develop strategies for tackling non-routine problems and to engage in productive struggle.

Illustrative Mathematics is a problem-based curriculum designed to address content and practice standards to foster learning for all. Students are encouraged to take an active role to see what they can figure out before having things explained to them or being told what to do.

¹Hiebert, J., et. al. (1996). Problem solving as a basis for reform in curriculum and instruction: the case of mathematics. Educational Researcher 25(4), 12-21. doi.org/10.3102/0013189X025004012

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As an IM Certified[™] Partner, McGraw-Hill is committed to providing the support needed to successfully implement *Illustrative Mathematics*. A portion of every purchase is earmarked toward supporting the continued development of high-quality math curriculum.

Perfect Scores from EdReports

Grade 6	Grade 7	Grade 8
MEETS EXPECTATIONS	MEETS EXPECTATIONS	MEETS EXPECTATIONS
FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14
RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES

Design Principles

Balancing Conceptual Understanding, Procedural Fluency, and Applications

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand, and be able to do, the mathematics.

Mathematical Practices are the Verbs of Math Class

In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

Build on What Students Know

New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them (NRC 2001). In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students' learning.

Good Instruction Starts with Explicit Learning Goals

Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students' understanding. Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.

Different Learning Goals Require a Variety of Types of Tasks and Instructional Moves

The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- provide experience with a new context
- introduce a new concept and associated language
- introduce a new representation
- formalize the definition of a term for an idea previously encountered informally
- identify and resolve common mistakes and misconceptions
- practice using mathematical language
- work toward mastery of a concept or procedure
- provide an opportunity to apply mathematics to a modeling or other application problem

Each and Every Student Should Have Access to the Mathematical Work

With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional tool boxes should include knowledge of and skill in implementing supports for different learners. This curriculum incorporates extensive tools for specifically supporting English Language Learners and Students with Disabilities



Instructional Model

Learning Goals and Targets

Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson. Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word "Let's." They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson. Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

Lesson Structure

1. INTRODUCE

Warm Up

Warm Up activities either:

- give students an opportunity to strengthen their number sense and procedural fluency.
- make deeper connections.
- encourage flexible thinking.

or:

- remind students of a context they have seen before.
- get them thinking about where the previous lesson left off.
- preview a calculation that will happen in the lesson.

2. EXPLORE AND DEVELOP

Classroom Activities

A sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class.

Each classroom activity has three phases.

The Launch

The teacher makes sure that students understand the context and what the problem is asking them to do.

Practice Problems

Each lesson includes an associated set of practice problems that may be assigned as homework or for extra practice in class. They can be collected and scored or used for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The design of practice problem sets looks different from many other curricula, but every choice was intentional, based on learning research, and meant to efficiently facilitate learning. The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Our approach emphasizes distributed practice rather than massed practice

3. SYNTHESIZE

Student Work Time

Students work individually, with a partner, or in small groups.

Activity Synthesis

The teacher orchestrates some time for students to synthesize what they have learned and situate the new learning within previous understanding.

Lesson Synthesis

Students incorporate new insights gained during the activities into their big-picture understanding.

Cool Down

A task to be given to students at the end of the lesson. Students are meant to work on the Cool Down for about 5 minutes independently and turn it in.

Instructional Routines

Plans include a set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

Like any routine in life, these routines give structure to time and interactions. They are a good idea for the same reason all routines are a good idea: they let people know what to expect, and they make people comfortable.

Why are routines in general good for learning academic content? One reason is that students and the teacher have done these interactions before, in a particular order, and so they don't have to spend much mental energy on classroom choreography. They know what to do when, who is expected to talk when, and when they are expected to write something down. The structure of the routine frees them up to focus on the academic task at hand. Furthermore, a welldesigned routine opens up conversations and thinking about mathematics that might not happen by themselves.

- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- Notice and Wonder
- Number Talk
- Poll the Class
- Take Turns
- Think Pair Share
- True or False
- Which One Doesn't Belong?





How to Assess Progress

Illustrative Mathematics contains many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

- Each unit begins with a diagnostic assessment ("Check Your Readiness") of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.
- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students' thinking.
- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day's lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.
- A set of cumulative practice problems is provided for each lesson that can be used for homework or inclass practice. The teacher can choose to collect and grade these or simply provide feedback to students.
- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.

Patrick's Test			
Assignment Details			
Number of questions: 10 Points possible: 60.00			
Instructions			
You are about to start your as 1. Make sure you have a g 2. Do not use your browse	sessment. ood Internet connection before st r's forward or back buttons while t	arting the test. aking the test.	
Assignment Tips			
Start Assignment			





Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. *Illustrative Mathematics* empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

Supporting English-language Learners

Illustrative Mathematics builds on foundational principles for supporting language development for all students. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

Aguirre, J.M. & Bunch, G. C. (2012). What's language got to do with it?: Identifying language demands in mathematics instruction for English Language Learners. In S. Celedón-Pattichis & N.

Digital

McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

Online resources offer:

- customizable content
- the ability to add resources
- auto-scoring of student practice work
- on-going student assessments
- classroom performance reporting

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Illustrative Mathematics, Gr	ade 6		
😑 Unit 2 👽 / Choose a lesson 🔨	Preview Student	Page Launch Presentation	Edit
Choose a lesson			
Lesson 1: Introducing Ratios an	nd Ratio Language		
Lesson 2: Representing Ratios	with Diagrams		
Lesson 3: Recipes			
Lesson 4: Color Mixtures Lesson 5: Defining Equivalent	Ratios ng Rati	os	
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Student Edition: Introducting Ratios	Check Your Readiness: Introducin Ratios	ng End-of-Unit Assessment Ratios	Introducing
	Illustrative Mathematics, Gr Course Resources Course Resources Choose a lesson (Lesson 1: Introducting Ratios Lesson 2: Representing Ratios Lesson 3: Recipes Lesson 4: Color Mixtures Lesson 5: Defining Equivalent Student Edition: Introducting Ratios Student Edition: Introducting Ratios	Illustrative Mathematics, Grade 6	Illustrative Mathematics, Grade 6



Launch Presentations Digital versions of lessons to present content.

Reports Review the performance of individual students, classrooms, and grade levels.

	Illustrative Math Grade 6		Illustrative Mathematics 💎
 My Programs Dashboard Course Gradebook 	Illustrative Mathematic	s, Grade 6	
Calendar Assignments Roster		s, Grade 6	
Reports Assessments Settings	Refine results O Course Location Refine more O Browse Standards	29 Results/imited to the current course) desmos Pori Course Locat	§
	③ Resource Type	eToolkit For Course Locat	ion: _
		Course Guide For Course Locat	e g

Access Resources Point-of-use access to resources such as assessments, eBooks, and course guides.



Unit 1

Rigid Transformations and Congruence

Rigid Transformations

Lesson 1-1 Moving in the Plane

- **1-2** Naming the Moves
- 1-3 Grid Moves
- 1-4 Making the Moves
- 1-5 Coordinate Moves
- **1-6** Describing Transformations

Properties of Rigid Transformations

- 1-7 No Bending or Stretching
- 1-8 Rotation Patterns
- 1-9 Moves in Parallel
- 1-10 Composing Figures

Congruence

- **1-11** What Is the Same?
- 1-12 Congruent Polygons
- 1-13 Congruence

Angles in a Triangle

- **1-14** Alternate Interior Angles.
- 1-15 Adding the Angles in a Triangle
- 1-16 Parallel Lines and the Angles in a Triangle

Let's Put It to Work

1-17 Rotate and Tessellate



Dilations, Similarity, and Introducing Slope

Dilations

Lesson 2-1 Projecting and Scaling

- 2-2 Circular Grid
- 2-3 Dilations with No Grid
- 2-4 Dilations on a Square Grid
- 2-5 More Dilations

Similarity

- 2-6 Similarity
- 2-7 Similar Polygons
- 2-8 Similar Triangles
- 2-9 Side Length Quotients in Similar Triangles

Slope

- 2-10 Meet Slope
- 2-11 Writing Equations for Lines
- 2-12 Using Equations for Lines

Let's Put It to Work

2-13 The Shadow Knows



Linear Relationships

Proportional Relationships

Lesson 3-1 Understanding Proportional Relationships

- **3-2** Graphs of Proportional Relationships
- **3-3** Representing Proportional Relationships
- **3-4** Comparing Proportional Relationships

Representing Linear Relationships

- **3-5** Introduction to Linear Relationships
- 3-6 More Linear Relationships
- **3-7** Representations of Linear Relationships
- **3-8** Translating to y = mx + b

Finding Slopes

3-9 Slopes Don't Have to be Positive

- 3-10 Calculating Slope
- 3-11 Equations of All Kinds of Lines

Linear Equations

3-12 Solutions to Linear Equations

3-13 More Solutions to Linear Equations

Let's Put It to Work

3-14 Using Linear Relations to Solve Problems



Linear Equations and Linear Systems

Puzzle Problems

Lesson 4-1 Number Puzzles.

Linear Equations in One Variables

- 4-2 Keeping the Equation Balanced
- 4-3 Balanced Moves
- 4-4 More Balanced Moves
- 4-5 Solving Any Linear Equation
- 4-6 Strategic Solving
- 4-7 All, Some, or No Solutions
- 4-8 How Many Solutions?
- 4-9 When Are They the Same?

Systems of Linear Equations

- 4-10 On or Off the Line?
- 4-11 On Both of the Lines
- 4-12 Systems of Equations
- 4-13 Solving Systems of Equations
- 4-14 Solving More Systems
- 4-15 Writing Systems of Equations

Let's Put It to Work

4-16 Solving Problems with Systems of Equations



Unit 5

Functions and Volume

Inputs and Outputs

Lesson 5-1 Inputs and Outputs

5-2 Introduction to Functions

Representing and Interpreting Functions

- **5-3** Equations for Functions
- 5-4 Tables, Equations, and Graphs of Functions
- 5-5 More Graphs of Functions
- 5-6 Even More Graphs of Functions
- 5-7 Connecting Representations of Functions

Linear Functions and Rates of Change

- 5-8 Linear Functions
- 5-9 Linear Models
- 5-10 Piecewise Linear Functions

Cylinders and Cones

- **5-11** Filling Containers
- 5-12 How Much Will Fit?.
- 5-13 The Volume of a Cylinder
- **5-14** Finding Cylinder Dimensions
- 5-15 The Volume of a Cone
- 5-16 Finding Cone Dimensions

Dimensions and Spheres

- 5-17 Scaling One Dimension
- 5-18 Scaling Two Dimensions
- 5-19 Estimating a Hemisphere
- 5-20 The Volume of a Sphere
- 5-21 Cylinders, Cones, and Spheres

Let's Put It to Work

5-22 Volume as a Function of



Unit 6 Associations in Data

Does This Predict That?

Lesson 6-1 Organizing Data

6-2 Plotting Data

Associations in Numerical Data

- 6-3 What a Point in a Scatter Plot Means
- 6-4 Fitting a Line to Data
- 6-5 Describing Trends in Scatter Plots
- 6-6 The Slope of a Fitted Line
- 6-7 Observing More Patterns in Scatter Plots
- 6-8 Analyzing Bivariate Data

Associations in Categorical Data

- 6-9 Looking for Associations
- 6-10 Using Data Displays to Find Associations

Let's Put It to Work

6-11 Gone in 30 Seconds



Exponents and Scientific Notation

Exponent Review

Lesson 7-1 Exponent Review

Exponent Rules

- 7-2 Multiplying Powers of Ten
- 7-3 Powers of Powers of Ten
- 7-4 Dividing Powers of Ten
- 7-5 Negative Exponents with Powers of Ten
- 7-6 What about Other Bases?
- 7-7 Practice with Rational Bases
- 7-8 Combining Bases

Scientific Notation

- 7-9 Describing Large and Small Numbers using Powers of Ten
- 7-10 Representing Large Numbers on the Number Line
- 7-11 Representing Small Numbers on the Number Line
- 7-12 Applications of Arithmetic with Powers of Ten
- 7-13 Definition of Scientific Notation
- 7-14 Multiplying, Dividing, and Estimating with Scientific Notation
- 7-15 Adding and Subtracting with Scientific Notation

Let's Put It to Work

7-16 Is a Smartphone Smart Enough to Go to the Moon?

Unit 8

Pythagorean Theorem and Irrational Numbers



Lesson 8-1 The Areas of Squares and Their Side Lengths

Side Lengths and Areas of Squares

- 8-2 Side Lengths and Areas
- 8-3 Rational and Irrational Numbers
- 8-4 Square Roots on the Number Line
- 8-5 Reasoning about Square Roots

The Pythagorean Theorem

- 8-6 Finding Side Lengths of Triangles
- 8-7 A Proof of the Pythagorean Theorem
- 8-8 Finding Unknown Side Lengths
- 8-9 The Converse
- 8-10 Applications of the Pythagorean Theorem
- 8-11 Finding Distances in the Coordinate Plane

Side Lengths and Volumes of Cubes

- 8-12 Edge Lengths and Volumes
- 8-13 Cube Roots

Decimal Representations of Rational

and Irrational Numbers

- 8-14 Decimal Representations of Rational Numbers
- 8-15 Infinite Decimal Expansions

Let's Put It to Work

8-16 Culminating Lesson





Unit 9 Putting It All Together

Putting It All Together

Lesson 9-1 Tessellations of the Plane

- 9-2 Regular Tessellations
- 9-3 Tessellating Polygons
- 9-4 What Influences Temperature?
- 9-5 Plotting the Weather
- 9-6 Using and Interpreting a Mathematical Model

Linear Relationships

Prior Work

Proportional Relationships and Geometry

Work with linear relationships in Grade 8 builds on earlier work with rates and proportional relationships in Grade 7, and Grade 8 work with geometry.

• At the end of the previous unit **on dilations**, students learned the terms "slope" and "slope triangle," used the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope, and found an equation for a line with a positive slope and vertical intercept.

Work in This Unit

In this unit, students gain experience with linear relationships and their representations as graphs, tables, and equations through activities designed and sequenced to allow them to make sense of problems and persevere in solving them. **MP1**

Because of this dependency, this unit and the previous one should be done in order.

Coherence

The unit begins by revisiting different representations of proportional relationships (graphs, tables, and equations), and the role of the constant of proportionality in each representation and how it may be interpreted in context. MP2

Next, students analyze the relationship between number of cups in a given stack of cups and the height of the stack—a relationship that is linear but not proportional—in order to answer the question "How many cups are needed to get to a height of 50 cm?" They are not asked to solve this problem in a specific way, giving them an opportunity to choose and use strategically representations that appeared earlier in this unit (table, equation, graph) or in the previous unit (equation, graph). MP5

Students are introduced to "rate of change" as a way to describe the rate per 1 in a linear relationship and note that its numerical value is the same as that of the slope of the line that represents the relationship. Students analyze another linear relationship (height of water in a cylinder vs number of cubes in the cylinder) and establish a way to compute the slope of a line from any two distinct points on the line via repeated reasoning. **MP8**

They learn a third way to obtain an equation for a linear relationship by viewing the graph of a line in the coordinate plane as the vertical translation of a proportional relationship.

So far, the unit has involved only lines with positive slopes and *y*-intercepts. Students next consider the graph of a line with a negative y-intercept and equations that might represent it. They consider situations represented by linear relationships with negative rates of change, graph these, and interpret coordinates of points on the graphs in context. **MP4 MP2**

The unit concludes with two lessons that involve graphing two equations in two unknowns and finding and interpreting their solutions. **MP2**

Doing this involves considering correspondences among different representations, in particular, what it means for a pair of values to be a solution for an equation and the correspondence between coordinates of points on a graph and solutions of an equation. **MP1**

Geometry Toolkit

In this unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to select appropriate tools and use them strategically to solve problems. **MP5**

Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

On using the terms ratio, rate, and proportion

In these materials, a *quantity* is a measurement that is or can be specified by a number and a unit, e.g., 4 oranges, 4 centimeters, "my height in feet," or "my height" (with the understanding that a unit of measurement will need to be chosen). The term *ratio* is used to mean an association between two or more quantities and the fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are identified as "unit rates" for the ratio *a* : *b*. The word "per" is used with students in interpreting a unit rate in context, as in "\$3 per ounce," and "at the same rate" is used to signify a situation characterized by equivalent ratios. In Grades 6–8, students write rates without abbreviated units, for example as "3 miles per hour" or "3 miles in every 1 hour." Use of notation for derived units such as fractions $\frac{\text{mi}}{\text{hr}}$ waits for high school—except for the special cases of area and volume. Students have worked with area since Grade 3 and volume since Grade 5. Before Grade 6, they have learned the meanings of such things as sq cm and cu cm. After students learn exponent notation in Grade 6, they also use cm² and cm³.

A proportional relationship is a collection of equivalent ratios. In high school—after their study of ratios, rates, and proportional relationships—students discard the term "unit rate," referring to *a* to *b*, *a* : *b*, and $\frac{a}{b}$ as "ratios."

A proportional relationship between two quantities represented by *a* and *b* is associated with two constants of proportionality: $\frac{a}{b}$ and $\frac{b}{a}$. Throughout the unit, the convention is if *a* and *b* are represented by columns in a table and the column for *a* is to the left of the column for *b*, then $\frac{b}{a}$ is the constant of proportionality for the relationship represented by the table.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as describing, generalizing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent	Generalize	Explain
 situations involving proportional relationships (Lesson 1) 	 categories for graphs (Lesson 2) about equations and linear relationships 	 how to graph proportional relationships (Lesson 3)
 constants of proportionality in different ways (Lesson 3) 	(Lesson 7)in order to make predictions about the slope	 how to use a graph to determine information about a linear situation (Lessons 5 and 6)
 slope using expressions (Lesson 7) 	of lines (Lesson 10)	how to graph linear relationships (Lesson 10)
 linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 8) 		 how slope relates to changes in a situation (Lesson 11)
 situations using negative slopes and slopes of zero (Lesson 9) 		
 situations by graphing lines and writing equations (Lesson 12) 		
 situations involving linear relationships (Lesson 14) 		

In addition, students are expected to describe observations about the equation of a translated line and describe features of an equation that could make one variable easier or harder to solve for than the other. Students will also have opportunities to use language to interpret situations involving proportional relationships, interpret graphs using different scales, interpret slopes and intercepts of linear graphs, justify reasoning about linear relationships, justify correspondences between different representations, and justify which equations correspond to graphs of horizontal and vertical lines.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

	Nev	v Terminology
Lesson	Receptive	Productive
3-1	represent scale label	constant of proportionality
3-2	equation	
3-3	rate of change	equation
3-5	linear relationship constant rate	slope
3-6	vertical intercept <i>y</i> -intercept	
3-7	initial (value or amount)	constant rate
3-8	relate	
3-9	horizontal intercept <i>x</i> -intercept	
3-10	intersection point	rate of change vertical intercept y-intercept
3-11	constraint	horizontal line vertical line
3-12	solution to an equation with two variables variable combination set of solutions	

	Lessons		Days	Standards
	Check Your Readiness Assessment		1	
Торіс	Proportional Relationships			
	Lesson 3-1 Understanding Proportional Relationships		1	8.EE.B, 8.EE.B.5
	Lesson 3-2 Graphs of Proportional Relationships		1	8.EE.B, 8.EE.B.5
	Lesson 3-3 Representing Proportional Relationships		1	8.EE.B, 8.EE.B.5
	Lesson 3-4 Comparing Proportional Relationships		1	8.EE.B, 8.EE.B.5
Торіс	Representing Linear Relationships			
	Lesson 3-5 Introduction to Linear Relationships		1	8.EE.B
	Lesson 3-6 More Linear Relationships		1	8.EE.B, 8.EE.B.5
	Lesson 3-7 Representations of Linear Relationships		1	8.EE.B, 8.EE.B.6
	Lesson 3-8 Translating to $y = mx + b$		1	8.EE.B, 8.G.A.1
Торіс	Finding Slopes			
	Lesson 3-9 Slopes Don't Have to be Positive		1	8.EE.B
	Lesson 3-10 Calculating Slope		1	8.EE.B, 8.EE.B.6
	Lesson 3-11 Equations of All Kinds of Lines		1	8.EE.B, 8.EE.B.6
Торіс	Linear Equations			
	Lesson 3-12 Solutions to Linear Equations		1	8.EE.B, 8.EE.C
	Lesson 3-13 More Solutions to Linear Equations		1	8.EE.C, 8.EE.C.8.a
Торіс	Let's Put It To Work			
	Lesson 3-14 Using Linear Relations to Solve Problems		2	8.EE.B.6, 8.EE.C.8.a
	End-of-Unit Assessment		1	
		TOTAL	16	

Required Materials

Geometry Toolkit (Lessons 8, 9, 12)	graph paper (Lessons 3, 5, 10)	8.3.2.2, Card Sort: Proportional Relationships, one per pair
tracing paper	rulers (Lessons 5, 6)	 (Lesson 2, Activity 2.2)
graph paper	straightedges (Lessons 2, 3, 10)	8.3.3.3, <i>Info Gap: Proportional Relationships</i> , one per pair (Lesson 3, Activity 3.3)
colored pencils	string (Lesson 11)	8362 Slopes Vertical Intercents and Graphs one per pair
scissors	tools for creating a visual display	 (Lesson 6, Activity 6.2)
index card	(Lesson 4)	8.3.8.3, Translating a Line, one per pair
ruler		(Lesson 8, Activity 8.3)
protractor		8.3.10.3, <i>Making Designs</i> , one per pair (Lesson 10, Activity 10.3)
		8.3.13.3, I'll Take an X, Please, one per pair

Blackline Activity Masters

(Lesson 13, Activity 13.3)

Pre-Unit Diagnostic Assessment

The pre-unit diagnostic assessment, *Check Your Readiness,* evaluates students' proficiency with prerequisite concepts and skills that they need to be successful in the unit. The item descriptions below offer guidance for students who may answer items incorrectly.

The assessment also may include problems that assess what students already know of the upcoming unit's key ideas, which you can use to pace or tune instruction. In rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.

Materials: Students will need a straight edge for this assessment.







1. Item Description

In this unit, students review their work with proportional relationships as a lead-in to linear equations.

First Appearance of Skill or Concept: Lesson 3

If most students struggle with this item....

- Plan to use this problem or a similar one as an additional warm-up activity.
- While they are working use *Collect and Display* as a way to gather and show the student discourse. **MLR2**
- Note any words or phrases that can be added to a visual display for students to use throughout the unit.
- In addition, during Lessons 1 and 2 plan to stress multiple ways we can tell a relationship is proportional, such as finding a constant of proportionality, and how we can do that using coordinates of points on the graph.

2. Item Description

Students move from scale factors to genuine proportional relationships to prepare for linear relationships.

First Appearance of Skill or Concept: Lesson 1

If most students struggle with this item....

• Before beginning Lesson 1, do Grade 6 Unit 3 Lesson 7 Activity 3, *Making Bracelets*, to practice the concept of generating equivalent ratios.

3. Item Description

In 7th Grade, students learned to write equations to describe proportional relationships. The graphs of these equations are lines through the origin.

In this unit, students will write equations for proportional relationships as well as other linear relationships.

First Appearance of Skill or Concept: Lesson 1

If most students struggle with this item....

• Plan to do Lesson 1 Activity 3, *Moving Twice as Fast*. During the Activity Synthesis spend some extra time discussing the third question and sharing their equations.

NAME _					DATE _				_
Unit 3	l inear Rel	ationships							
V	Che	CK YOUR	R	eadine	SS				
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						Uni	t 3 Check You	r Readiness	1

4. Item Description

Students will need to be familiar with the coordinate plane for their work with graphing lines.

Do not be concerned if students do not come up with an equation for the set of all points with *x*-coordinate 3. That is something they will learn to do in this unit.

First Appearance of Skill or Concept: Lesson 1

If most students struggle with this item....

- Plan to pause students as they are working on Lesson 1 Activity 2 Question 4 to ensure that they can plot and mark points once they have identified the bug's location at the given time.
- If students need additional practice, refer to 6th Grade, Unit 7, Lesson 11, Activity 1.
- If students struggle with describing the location of all points with x-coordinate 3, they will have additional opportunities to learn this concept beginning in Lesson 11. Follow the suggestions in the launch of Activity 2.

5. Item Description

In this unit, students are presented with various forms of linear equations and various ways of thinking about those forms.

One interpretation of the form y = mx + b is to consider it a vertical translation of the line y = mx.

First Appearance of Skill or Concept: Lesson 8

If most students struggle with this item....

- Plan to use Activity 1 in Lesson 8 to review translations.
- If students need additional practice recalling translations, especially translations of lines, refer to Unit 1 Lesson 9, *Moves in Parallel*.

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4. Ref	er to	the	coc	ordir	nate	pla	ane	sho	own	. 8	.EE	E.B								
a.	Plot	t an	d lal	oel 3	3 di	fere	ent	poi	nts	wit	h x	-co	ord	lina	ate	3.				
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b.	Ske	tch	or c	lesc	ribe	e all	poi	nts	int	he	pla	ane	wit	th >	(-C(oroc	dinate	3.		
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6. Item Description

Another interpretation of the form y = mx + b is to start with a given amount and thereafter increase the amount at a constant rate.

Students are asked to engage in repeated reasoning in anticipation of this way of thinking.

First Appearance of Skill or Concept: Lesson 2

If most students struggle with this item....

- Plan to review it with students before beginning Lesson 2 Activity 2.
- Be sure to amplify vocabulary such as "constant of proportionality" and "unit rate" throughout this lesson.

NA	ME _		DATE	PERIOD	
6.	A st	ore sells ice cream with assorted toppi an ice cream, plus 50 cents per ounce	ngs. They ch of toppings.	arge \$3.00 7.EE.B.3	
	a.	How much does an ice cream cost wit	h 4 ounces o	f toppings?	
		\$5			
	b.	How much does an ice cream cost wit	h 11 ounces c	of toppings?	
		\$8.50			
	c.	If Elena's ice cream cost \$1.50 more the how much more did Elena's toppings	aan Jada's ice weigh? <mark>3 our</mark>	e cream, I ces	
			Un	it 3 Check Your Readiness	3
			011	Check four reduitess	5

Lesson 3-1

Understanding Proportional Relationships

Goals

- Comprehend that for the equation of a proportional relationship given by y = kx, k represents the constant of proportionality.
- Create graphs and equations of proportional relationships in context, including an appropriate scale.
- Interpret diagrams or graphs of proportional relationships in context.

Student Learning Goals

Let's study some graphs.

Learning Targets

- I can graph a proportional relationship from a story.
- I can use the constant of proportionality to compare the pace of different animals.

Standards Alignment

Building On

7.RP.A.2 Recognize and represent proportional relationships between quantities.

Addressing

8.EE.B Understand the connections between proportional relationships, lines, and linear equations.

Building Toward

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Lesson Narrative

This lesson is the first of four where students work with proportional relationships from a Grade 8 perspective. Embedded alongside their work with proportional relationships, students learn about graphing from a blank set of axes. Attending to precision in labeling axes, choosing an appropriate scale, and drawing lines are skills students work with in this lesson and refine over the course of this unit and in units that follow. MP6

The purpose of this lesson is to get students thinking about what makes a "good" graph by first considering what are the components of a graph (e.g., labels, scale) and then adding scale to graphs of the pace of two bugs. Students also graph a line based on a verbal description of a relationship and compare the newly graphed line to already graphed proportional relationships.

This lesson includes graphs with elapsed time in seconds on the vertical axis and distance traveled in centimeters on the horizontal axis. It is common for people to believe that time is always the independent variable, and should therefore always be on the horizontal axis. This is a really powerful heuristic. The problem is, it isn't true.

In general, a context that involves a relationship between two quantities does not dictate which quantity is the independent variable and which is the dependent variable: that is a choice made by the modeler. Consider this situation: A runner is traveling one mile every 10 minutes. There is more than one way to represent this situation.

- We can say the number of miles traveled, *d*, depends on the number of minutes that have passed, *t*, and write *d* = 0.1*t*. This way of expressing the relationship might be more useful for questions like, "How far does the runner travel in 35 minutes?"
- We can also say that the number of minutes that have passed, *t*, depends on the number of miles traveled, *d*, and write *t* = 10*d*. This way of expressing the relationship might be more useful for questions like, "How long does it take the runner to travel 2 miles?"

These are both linear relationships. The rate of change in the first corresponds to speed (0.1 miles per minute), and the rate of change in the second corresponds to pace (10 minutes per mile). Both have meaning, and both could be of interest. It is up to the modeler to decide what kinds of questions she wants to answer about the context and which way of expressing the relationship will be most useful in answering those questions.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3 Clarify, Critique, Correct
- MLR5 Co-Craft Questions
- Notice and Wonder

Lesson Pacing

	Pacing (min)
Warm Up 1.1 Notice and Wonder: Two Graphs	5
Activity 1.2 Moving Through Representations	15
Activity 1.3 Moving Twice as Fast	15
Lesson Synthesis	5-10
Cool Down 1.4 Turtle Race	5
TOTAL	45-50



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Building Toward: 8.EE.B.5

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.



Warm Up 1.1 Notice and Wonder: Two Graphs (5 minutes)

The purpose of this Warm Up is to get a conversation started about what features a graph needs. In the following activities, students will put these ideas to use by adding scale to some axes with two proportional relationships graphed on it.

Instructional Routines

Notice and Wonder MP1

Launch

Tell students they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and ask students to give a signal when they have noticed or wondered about something.

Student Task Statement



What do you notice? What do you wonder?

Things students may notice:

- · The second set of axes are not labeled
- If the first graph is about speed, then f is twice as fast as g.
- Graph g is something going a speed of 2 centimeters every second
- Graph *f* is something going a pace of about 0.25 seconds per 1 centimeter.

Things students may wonder:

- What do the two points mean?
- Why does one graph show two lines while the other only has one?
- What do g and f represent?

Activity Synthesis

Invite students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the missing labels are not mentioned, make sure to bring them up.

Activity 1.2 Moving Through Representations (15 minutes)

Standards Alignment

Addressing: 8.EE.B Building Toward: 8.EE.B.5

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines. In this activity, students investigate the paces of two different bugs. Using the chart at the start of the activity, students answer questions about pace, decide on a scale for the axes, and mark and label the time needed to travel 1 cm for each bug (unit rate).

Identify students who use different scales on the axes to share during the Activity Synthesis. For example, some students may count by 1s on the distance axis while others may count by 0.5s.

Instructional Routines

- Mathematical Language Routines
 - MLR5 Co-Craft Questions

Launch

Arrange students in groups of 2. Before students start working, ensure that they understand that each bug's position is measured at the front of their head. So for example, in the second diagram, the ladybug has moved 4 centimeters and the ant has moved 6 centimeters.

Ask students to review the images and the first problem in the activity and give a signal when they have finished. Invite students to share their ideas about which bug is represented by line *u* and which bug is represented by line *v*. (The ladybug is *u*, the ant is *v*.) If not brought up in students' explanations, draw attention to how the graph shows the *pace* of the two bugs—that is, the graph shows how much time it takes to go a certain distance, which is different than a graph of speed, which shows how much distance you go for a certain amount of time.

Give students work time to complete the remaining problems with their partner followed by a whole-class discussion.

Support For Students with Disabilities

Action and Expression: Develop Expression and Communication Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "Based on the diagrams, line *u* represents _____ because...."

Supports accessibility for: Language; Organization

Support For English Language Learners

Writing, Speaking

MLR5 Co-Craft Questions Use this routine to help students interpret the first image, and to increase awareness of language used to make comparisons about speed and pace. Display only the prompt and images (without the line graphs). Invite students to write possible mathematical questions about the situation. When students share their questions with the class, highlight those that wonder about distance, time and the meaning of tick marks in the diagrams. Reveal the graph and ask students to work on the questions that follow. This helps students produce the language of mathematical questions about different representations for speed.

Design Principle(s): Optimize output; Cultivate conversation

Anticipated Misconceptions

Students might confuse pace with speed and interpret a steeper line as meaning the ladybug is moving faster. Monitor students to ensure that they attend to the time and distance on the tick mark diagrams and plot points as (*distance, time*) with time on the *y*-axis and distance on the *x*-axis. Reinforce language of how many seconds per a given interval of distance. Make explicit that twice as fast means half the pace.

Student Task Statement

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times. Each tick mark represents 1 centimeter.



- Lines u and v also show the positions of the two bugs. Which line shows the ladybug's movement? Which line shows the ant's movement? Explain your reasoning. ladybug: line u ant: line v
- How long does it take the ladybug to travel 12 cm? The ant? ladybug: 6 seconds ant: 4 seconds





Distance Traveled (cm)

- 3. Scale the vertical and horizontal axes by labeling each grid line with a number. You will need to use the time and distance information shown in the tick-mark diagrams. See graph below.
- 4. Mark and label the point on line *u* and the point on line *v* that represent the time and position of each bug after traveling 1 cm. See graph below.



(continued on the next page)





Standards Alignment

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Addressing: 8.EE.B Building Toward: 8.EE.B.5

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

Are you ready for more?

- **1.** How fast is each bug traveling?
 - The ladybug is traveling at 2 cm/sec and the ant is traveling at 3 cm/sec.
- Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning.
 - No, the ant is always half as much again as far from the start as the ladybug.

Activity Synthesis

Display the images from the problem for all to see. Begin the discussion by inviting students to share their solutions for how long it takes each bug to travel 12 cm. Encourage students to reference one or both of the images as they explain their thinking.

Ask previously selected students to share their graphs with added scale and how they decided on what scale to use. If possible, display these graphs for all to see. There are many correct ways to choose a scale for this situation, though some may have made it difficult for students to plot the answer to the final problem. If this happened, highlight these graphs and encourage students to read all problems when they are making decisions about how to construct a graph. Since this activity had a problem asking for information about 1 cm, it makes sense to count by 1s (or even something smaller!) on the distance axis.

Activity 1.3 Moving Twice as Fast (15 minutes)

In this activity, students use the representations from the previous activity and add a third bug that is moving twice as fast as the ladybug. Students are also asked to write equations for all three bugs. An important aspect of this activity is students making connections between the different representations.

Monitor for students using different strategies to write their equations. For example, some students may reason from the unit rates they can see on their graphs and write equations in the form of y = kx, where k is the unit rate (constant of proportionality). Others may write equations of the form $\frac{y}{x} = \frac{b}{a}$, where (a, b) is a point on the line. Select several of these students to share during the discussion.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Mathematical Language Routines
 - MLR3 Clarify, Critique, Correct

Launch

Keep students in the same groups. Give 5–7 minutes work time followed by a whole-class discussion.

Students with Disabilities Support For

Action and Expression: Internalize Executive Functions Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, show only one question at a time, pausing to check for understanding before moving on.

Supports accessibility for: Organization; Attention

English Language Learners Support For

Speaking

MLR3 Clarify, Critique, and Correct For the first question "Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug," display an incomplete

statement like, "I looked at how far the ladybug went and made my bug go farther" or a flawed statement like "I put my bug 2 tick marks ahead of the ant." Invite students to discuss with a partner possible ways to correct or clarify each statement. This will give students an opportunity to use language to clarify their understanding of proportionality.

Design Principle(s): Maximize meta-awareness

Student Task Statement

Refer to the tick-mark diagrams and graph in the earlier activity when needed.

1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug.



- 2. Plot this bug's positions on the coordinate axes with lines *u* and *v*, and connect them with a line. See graph. Line *n* represents a bug moving double the ladybug's distance in the same amount of time.
- 3. Write an equation for each of the three lines. Answers vary. Possible response: Equations are ladybug: $y = \frac{1}{2}x$, ant: $y = \frac{1}{3}x$, new bug: $y = \frac{1}{4}x$ (twice as fast as ladybug), where x represents the distance traveled and y represents elapsed time.





(continued on the next page)

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Activity Synthesis

Display both images from the previous task for all to see. Invite previously selected students to share their equations for each bug. Sequence students so that the most common strategies are first. Record the different equations created for each bug and display these for all to see.

As students share their reasoning about the equation for the third bug, highlight strategies that support using the equation (original is k and new one is $\frac{1}{2}k$) and graph (less steep, still constant proportionality, half point values). If no students write an equation of the form y = kx, do so and remind students of the usefulness of k, the constant of proportionality, when reasoning about proportional relationships.

Consider asking the following questions to help students make connections between the different representations:

- What features of the tick-mark diagrams, lines, and equations can you identify that would allow someone to figure out which bug is moving faster? The tick-mark diagrams give the coordinates of points that will go on the graph because they show how far each bug has gone after each amount of time. We can see the positions of the bugs on the tick-mark diagrams so we know which is faster. The graph shows how far they went for any amount of time and the slope helps to show which is faster. Both help compare the movements of the two or three bugs.
- The tick-mark diagrams show some of the bugs' movements, but not all of them. How can we use the graphs of the lines to get more complete information? The tick-mark diagrams only show time every 2 seconds. On the graph we can see the bugs' positions at any point in time.
- Are you convinced that your graph (or equation) supports the fact that the new bug is going twice as fast as the ladybug?

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Lesson Synthesis Understanding Proportional Relationships

Display a scaled graph of the two bugs for all to see. Remind students that line u is the ladybug and that line v is the ant.



Ask students:

- What would the graph of a bug going 3 times faster than the ant look like? It would go through the points (0, 0), $(1, \frac{1}{9})$, and (9, 1).
- What would an equation showing the relationship between the bugs' distance and time look like? Since it is going 3 times faster and goes through the point (9, 1), it has constant of proportionality of $\frac{1}{9}$, which means one equation is $y = \frac{1}{9}x$.
- If we wanted to scale the graph so we could see how long it takes the ladybug to travel 50 cm, what numbers could we use on the vertical axis? The ladybug travels 50 cm in 25 seconds, so the vertical axis would need to extend to at least that value.





Standards Alignment

Building On: 7.RP.A.2 Building Toward: 8.EE.B.5

Cool Down 1.4 Turtle Race (5 minutes)

Student Task Statement

This graph represents the positions of two turtles in a race.



- 1. On the same axes, draw a line for a third turtle that is going half as fast as the turtle described by line g. A line through (0, 0), (1, 1), (2, 2), etc.
- **2.** Explain how your line shows that the turtle is going half as fast. Looking at the values for 2 seconds, turtle *g* moves 4 cm and the third turtle moves only 2 cm. This third turtle covers half the distance in the same amount of time.



184 Unit 3 Linear Relationships

	Practice Understanding Proportion	onal Relationships	
1.	Priya jogs at a constant speed. Ti between her distance and time is Diego bikes at a constant speed Sketch a graph showing the relat Diego's distance and time.	he relationship s shown on the graph. twice as fast as Priya. ionship between	B C C C C C C C C C C C C C C C C C C C
2.	A you-pick blueberry farm offers for \$16.50. Sketch a graph of the relationship pounds of blueberries. A ray that passes through (0, 0) a	6 lbs of blueberries o between cost and and (6, 16.5)	120 96 94 96 96 96 96 96 96 96 96 96 96 96 96 96
3.	A line contains the points (-4, 1) a not each of these points is also o	nd (4, 6). Decide whethe on the line: (Lesson 2-12)	er or
	a. (0, 3.5) On the line	b. (12, 11) Not on the	line
	c. (80, 50) On the line	d. (-1, 2.875) On the line	•
4.	The points (2, -4), (x, y), A, and B Find an equation relating x and y $\frac{y+4}{x-2} = \frac{3}{4}$ (or equivalent)	all lie on the line. : (Lesson 2-11)	3 9 9 2 9 9 1 2 4 5 1 2 4 5 1 2 4 5 1 2 4 5 1 2 4 5 1 2 4 5 2 4 5 6 3 1 1 4 7 1 5 1 1
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Lesson 3-1 Understanding Proportional Relationships 185

Student Edition

		Topic Proportional Relationship
Lesson 3-1 Understanding Proportio	onal Re	elationships
NAME	DATE	PERIOD
Learning Goal Let's study some graphs.		
Warm Up 1.1 Notice and Wonder: Two Graphs		
What do you notice? What do you wonder?		
())))))))))))))		



Activity

1.2 Moving Through Representations

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times. Each tick mark represents 1 centimeter.



- the tick-mark diagrams.
- **4.** Mark and label the point on line *u* and the point on line *v* that represent the time and position of each bug after traveling 1 cm.

 Are you ready for more? 1. How fast is each bug traveling? 2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning. 2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning. 2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning. 2. Will there ever be a time when the ant is twice as fast as the ladybug. On each tickmark diagram, mark the position of this bug. 2. Plot this bug's positions on the coordinate axes with lines u and v, and contact there with a line. 	NAM	E D	ATE	PERIOD
 How fast is each bug traveling? Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning. Explain or show your reasoning. Activity 1.3 Moving Twice as Fast Refer to the tick-mark diagrams and graph in the earlier activity when needed. Imagine a bug that is moving twice as fast as the ladybug. On each tickmark diagram, mark the position of this bug. Plot this bug's positions on the coordinate axes with lines u and v, and access there with a line. 	4	Are you ready for more?		
 2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning. 2. Plot this bug's positions on the coordinate axes with lines u and v, and parameters there with a line. 	1	. How fast is each bug traveling?		
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Understanding Proportional Relationships

Graphing is a way to help us make sense of relationships. But the graph of a line on a coordinate axis without scale or labels isn't very helpful.

For example, let's say we know that on longer bike rides Kiran can ride 4 miles every 16 minutes and Mai can ride 4 miles every 12 minutes. Here are the graphs of these relationships.



Without labels we can't even tell which line is Kiran and which is Mai! Without labels and a scale on the axes, we can't use these graphs to answer questions like:

- 1. Which graph goes with which rider?
- 2. Who rides faster?
- **3.** If Kiran and Mai start a bike trip at the same time, how far are they after 24 minutes?
- 4. How long will it take each of them to reach the end of the 12-mile bike path?

NAME_____ DATE _____ PERIOD _____

Here are the same graphs, but now with labels and scale.

Revisiting the questions from earlier:

 Which graph goes with each rider? If Kiran rides 4 miles in 16 minutes, then the point (4, 16) is on his graph. If he rides for 1 mile, it will take 4 minutes. 10 miles will take 40 minutes. So, the upper graph represents Kiran's ride. Mai's points for the same distances are (1, 3), (4, 12), and (10, 30), so hers is the lower graph. (A letter next to each line would help us remember which is which!)



Topic Proportional Relationships

- Who rides faster? Mai rides faster because she can ride the same distance as Kiran in a shorter time.
- **3.** If Kiran and Mai start a bike trip at the same time, how far are they after 20 minutes?

The points on the graphs at height 20 are 5 miles for Kiran and a little less than 7 miles for Mai.

4. How long will it take each of them to reach the end of the 12-mile bike path? The points on the graphs at a horizontal distance of 12 are 36 minutes for Mai and 48 minutes for Kiran. (Kiran's time after 12 miles is almost off the grid!)

Glossary

constant of proportionality

In a proportional relationship, the values for one quantity are each multiplied by the same number to get the values for the other quantity. This number is called the constant of proportionality.

Practice Understanding Proportional Relationships 1. Priva jogs at a constant speed. The relationship between her distance and time is shown on the graph. Distance (miles) Diego bikes at a constant speed twice as fast as Priya. Sketch a graph showing the relationship between 4 Diego's distance and time. 2 О 0.2 0.4 0.6 Time (hours) 2. A you-pick blueberry farm offers 6 lbs of blueberries 120 108 for \$16.50.

Sketch a graph of the relationship between cost and pounds of blueberries.



x

1.0

0.8

- **3.** A line contains the points (-4, 1) and (4, 6). Decide whether or not each of these points is also on the line: (Lesson 2-12)
 - **a.** (0, 3.5) **b.** (12, 11)
 - **c.** (80, 50) **d.** (-1, 2.875)
- **4.** The points (2, -4), (*x*, *y*), *A*, and *B* all lie on the line. Find an equation relating *x* and *y*. (Lesson 2-11)



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