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**iM Illustrative Mathematics™**  
**Course 2**



Lesson  
Sampler

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## Course 2

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To accommodate different implementations of the program, course numbers rather than grade levels are referenced on the covers of McGraw-Hill *Illustrative Mathematics* materials. However, grade levels are referenced in the materials as this is how *Illustrative Mathematics* was originally written.

Course 1 = Grade 6, Course 2 = Grade 7, Course 3 = Grade 8

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*Illustrative Mathematics* is a problem-based core curriculum designed to address content and practice standards to foster learning for all. Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Teachers can shift their instruction and facilitate student learning with high-leverage routines to guide learners to understand and make connections between concepts and procedures.

## What is a Problem-based Curriculum?

In a problem-based curriculum, students work on carefully crafted and sequenced mathematics problems during most of the instructional time. Teachers help students understand the problems and guide discussions to be sure that the mathematical takeaways are clear to all. In the process, students explain their ideas and reasoning and learn to communicate mathematical ideas. The goal is to give students just enough background and tools to solve initial problems successfully, and then set them to increasingly sophisticated problems as their expertise increases.

The value of a problem-based approach is that students spend most of their time in math class doing mathematics: making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers. They go on to interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding.







*“Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience . . .”<sup>1</sup>*

## Creating a World Where Learners Know, Use, and Enjoy Mathematics

Decades of research shows that students learn best when they are given a chance to start work on a problem before being shown a solution method. This gives students the chance to build conceptual understanding that can cement procedural skills by tying them together. It allows students to develop strategies for tackling non-routine problems and to engage in productive struggle.

*Illustrative Mathematics* is a problem-based curriculum designed to address content and practice standards to foster learning for all. Students are encouraged to take an active role to see what they can figure out before having things explained to them or being told what to do.

<sup>1</sup>Hiebert, J., et. al. (1996). Problem solving as a basis for reform in curriculum and instruction: the case of mathematics. *Educational Researcher* 25(4), 12-21. doi.org/10.3102/0013189X025004012

# The Highest Rated Curriculum. A Partner You Know and Trust.

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- Personalized service and support from a local McGraw-Hill sales representative
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## Supporting the *Illustrative Mathematics* Mission

As an IM Certified™ Partner, McGraw-Hill is committed to providing the support needed to successfully implement *Illustrative Mathematics*. A portion of every purchase is earmarked toward supporting the continued development of high-quality math curriculum.

## Perfect Scores from EdReports

Grade 6	Grade 7	Grade 8
MEETS EXPECTATIONS	MEETS EXPECTATIONS	MEETS EXPECTATIONS
FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14
RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES Score: 18/18

# Design Principles

## **Balancing Conceptual Understanding, Procedural Fluency, and Applications**

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand, and be able to do, the mathematics.

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## **Mathematical Practices are the Verbs of Math Class**

In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

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## **Build on What Students Know**

New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them (NRC 2001). In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students' learning.

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## **Good Instruction Starts with Explicit Learning Goals**

Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students' understanding. Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.



## Different Learning Goals Require a Variety of Types of Tasks and Instructional Moves

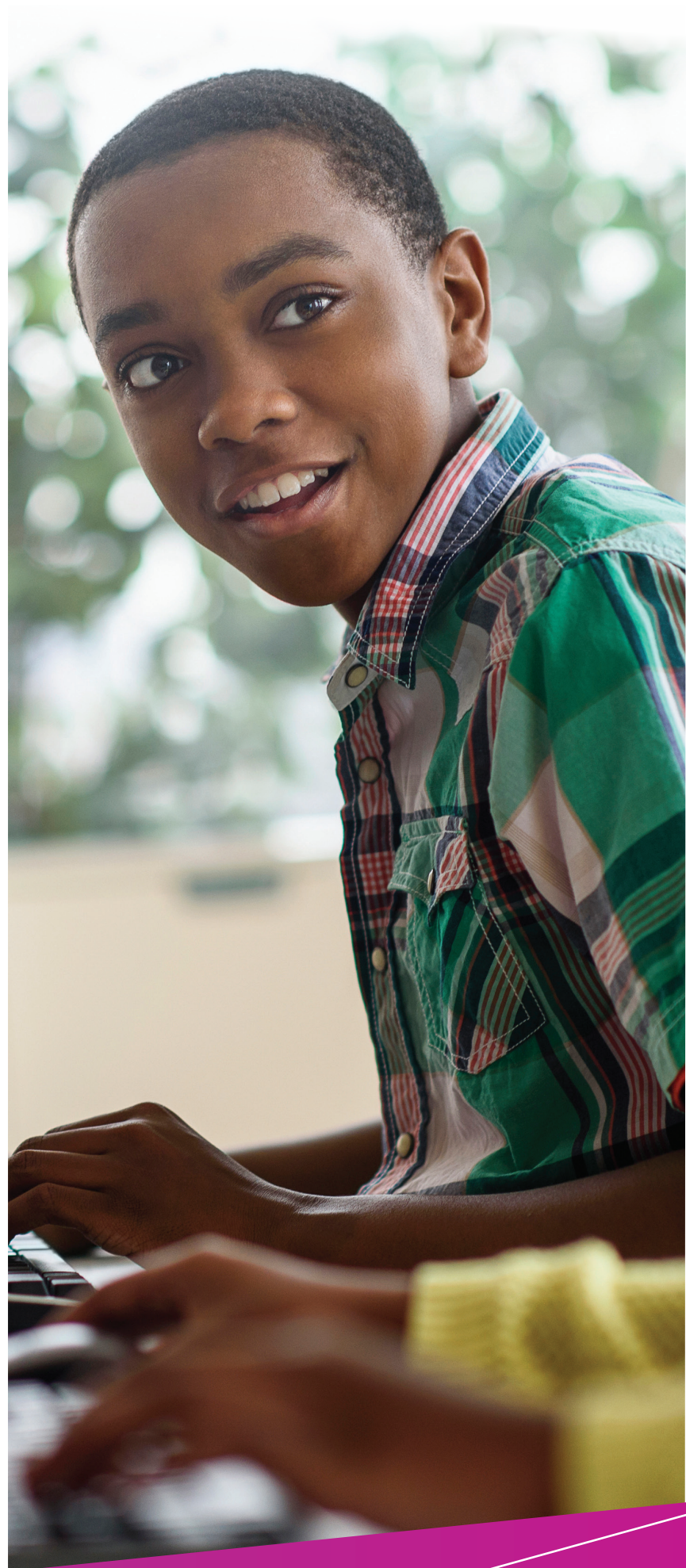
The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- provide experience with a new context
- introduce a new concept and associated language
- introduce a new representation
- formalize the definition of a term for an idea previously encountered informally
- identify and resolve common mistakes and misconceptions
- practice using mathematical language
- work toward mastery of a concept or procedure
- provide an opportunity to apply mathematics to a modeling or other application problem

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## Each and Every Student Should Have Access to the Mathematical Work

With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional tool boxes should include knowledge of and skill in implementing supports for different learners. This curriculum incorporates extensive tools for specifically supporting English Language Learners and Students with Disabilities





# Instructional Model

## Learning Goals and Targets

### Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson. Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word “Let’s.” They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

### Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson. Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

## Lesson Structure

1. INTRODUCE	2. EXPLORE AND DEVELOP
<p><b>Warm Up</b></p> <p>Warm Up activities either:</p> <ul style="list-style-type: none"><li>■ give students an opportunity to strengthen their number sense and procedural fluency.</li><li>■ make deeper connections.</li><li>■ encourage flexible thinking.</li></ul> <p>or:</p> <ul style="list-style-type: none"><li>■ remind students of a context they have seen before.</li><li>■ get them thinking about where the previous lesson left off.</li><li>■ preview a calculation that will happen in the lesson.</li></ul>	<p><b>Classroom Activities</b></p> <p>A sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class.</p> <p><b>Each classroom activity has three phases.</b></p> <p><b>The Launch</b></p> <p>The teacher makes sure that students understand the context and what the problem is asking them to do.</p>

## Practice Problems

Each lesson includes an associated set of practice problems that may be assigned as homework or for extra practice in class. They can be collected and scored or used for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The design of practice problem sets looks different from many other curricula, but every choice was intentional, based on learning research, and meant to efficiently facilitate learning. The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Our approach emphasizes distributed practice rather than massed practice

### 3. SYNTHESIZE

#### Student Work Time

Students work individually, with a partner, or in small groups.

#### Activity Synthesis

The teacher orchestrates some time for students to synthesize what they have learned and situate the new learning within previous understanding.

#### Lesson Synthesis

Students incorporate new insights gained during the activities into their big-picture understanding.

#### Cool Down

A task to be given to students at the end of the lesson. Students are meant to work on the Cool Down for about 5 minutes independently and turn it in.

# Instructional Routines

Plans include a set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

Like any routine in life, these routines give structure to time and interactions. They are a good idea for the same reason all routines are a good idea: they let people know what to expect, and they make people comfortable.

Why are routines in general good for learning academic content? One reason is that students and the teacher have done these interactions before, in a particular order, and so they don't have to spend much mental energy on classroom choreography. They know what to do when, who is expected to talk when, and when they are expected to write something down. The structure of the routine frees them up to focus on the academic task at hand. Furthermore, a well-designed routine opens up conversations and thinking about mathematics that might not happen by themselves.

- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- Notice and Wonder
- Number Talk
- Poll the Class
- Take Turns
- Think Pair Share
- True or False
- Which One Doesn't Belong?

Topic: Let's Put It to Work

Lesson 1-19

Designing a Tent

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

Learning Goal: Let's design some tents.

Activity

19.1 Tent Design—Part 1

Have you ever been camping?  
You might know that sleeping bags are all about the same size, but tents come in a variety of shapes and sizes.  
Your task is to design a tent to accommodate up to four people and estimate the amount of fabric needed to make your tent. Your design and estimate must be based on the information given and have mathematical justification.  
First, look at these examples of tents, the average specifications of a camping tent, and standard sleeping bag measurements. Talk to a partner about:

- Similarities and differences among the tents
- Information that will be important in your designing process
- The pros and cons of the various designs

Tent Styles

Lesson 1-19: Designing a Tent 133

Activity

1.2 More Orange, Green, or Blue?

Your teacher will assign you to look at Pattern A or Pattern B.  
In your pattern, which shape covers more of the plane: blue rhombuses, orange trapezoids, or green triangles? Explain how you know.

Pattern A

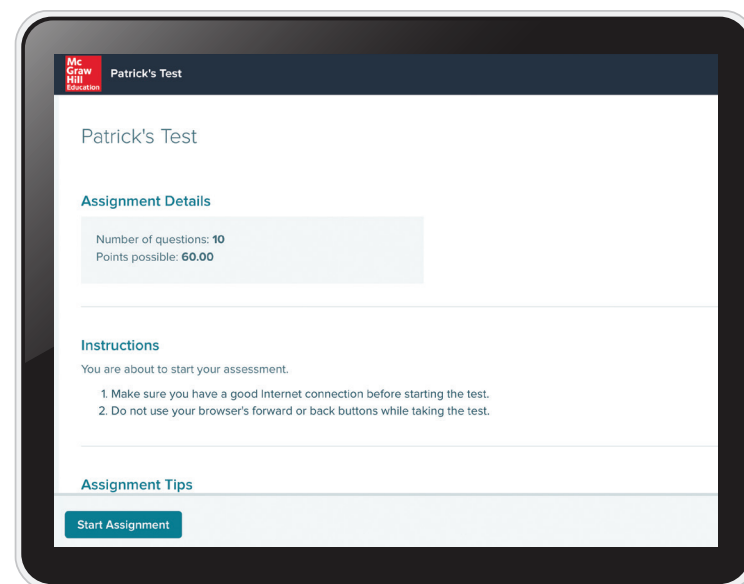
Pattern B

4 • Unit 1 Area and Surface Area

## How to Assess Progress

*Illustrative Mathematics* contains many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

- Each unit begins with a diagnostic assessment (“Check Your Readiness”) of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.
- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students’ thinking.
- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day’s lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.
- A set of cumulative practice problems is provided for each lesson that can be used for homework or in-class practice. The teacher can choose to collect and grade these or simply provide feedback to students.
- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.



**Are you ready for more?**

Rearrange the triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.

**Activity**  
3.3 Off the Grid

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.

Figure A: A right triangle with a vertical leg of 3 cm and a horizontal leg of 5 cm. The hypotenuse is the outer boundary, and the area inside is shaded blue.

Figure B: A square with side length 4 cm. Inside, a smaller square is formed by connecting the midpoints of the sides. The four right triangles at the corners are shaded orange.

Figure C: A square with side length 5 cm. Inside, a smaller square is formed by connecting the midpoints of the sides. The four right triangles at the corners are shaded green.

**Summary**  
Reasoning to Find Area

There are different strategies we can use to find the area of a region. We can:

1. Decompose it into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.

The diagram shows a blue T-shaped region on a grid. An arrow points to the same region decomposed into a central square and four right triangles, illustrating the strategy of decomposition.

18 Unit 1 Area and Surface Area





## Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. *Illustrative Mathematics* empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

## Supporting English-language Learners

*Illustrative Mathematics* builds on foundational principles for supporting language development for all students. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

Aguirre, J.M. & Bunch, G. C. (2012). What's language got to do with it?: Identifying language demands in mathematics instruction for English Language Learners. In S. Celedón-Pattichis & N.

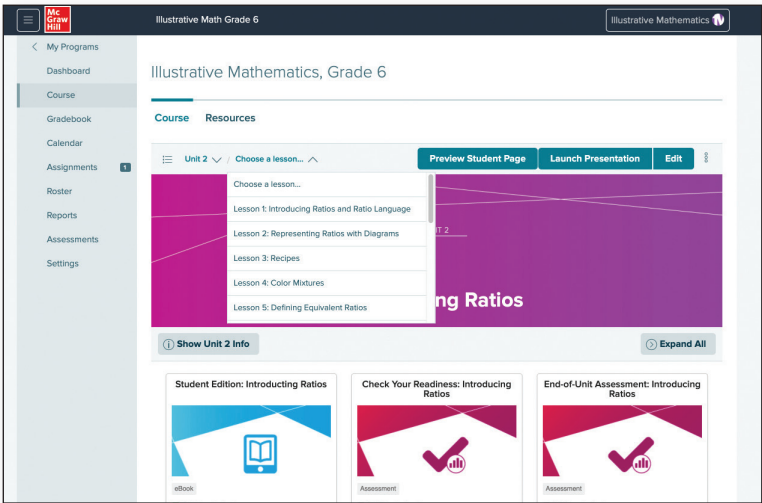


## Digital

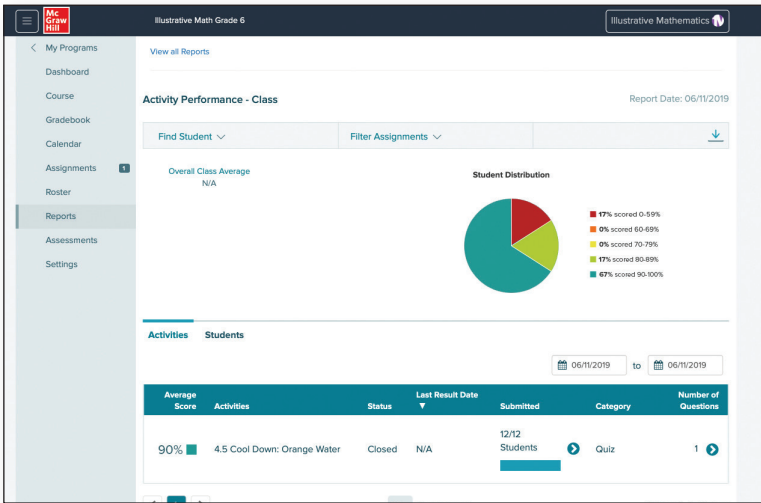
McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

Online resources offer:

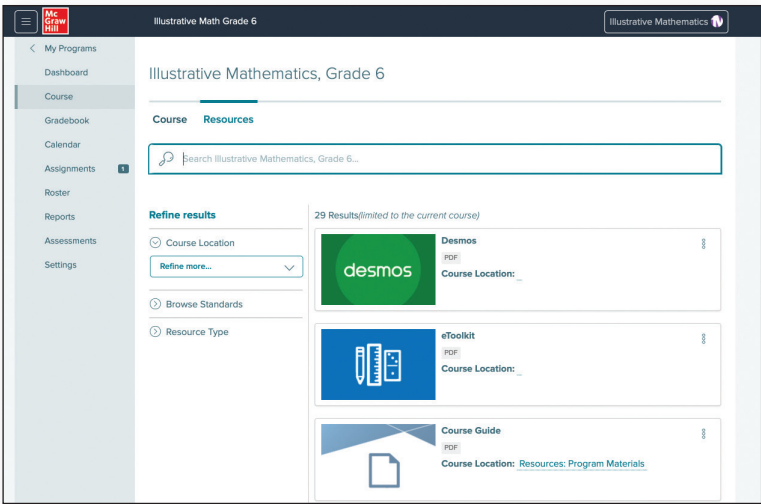
- customizable content
- the ability to add resources
- auto-scoring of student practice work
- on-going student assessments
- classroom performance reporting



**Launch Presentations** Digital versions of lessons to present content.



**Reports** Review the performance of individual students, classrooms, and grade levels.



**Access Resources** Point-of-use access to resources such as assessments, eBooks, and course guides.

## Unit 1

# Scale Drawings



## Scaled Copies

**Lesson 1-1** What Are Scaled Copies?

**1-2** Corresponding Parts and Scale Factors

**1-3** Making Scaled Copies..

**1-4** Scaled Relationships

**1-5** The Size of the Scale Factor

**1-6** Scaling and Area

## Scale Drawings

**1-7** Scale Drawings

**1-8** Scale Drawings and Maps

**1-9** Creating Scale Drawings

**1-10** Changing Scales in Scale Drawings

**1-11** Scales without Units

**1-12** Units in Scale Drawings

## Let's Put It to Work

**1-13** Draw It to Scale

## Unit 2

# Introducing Proportional Relationships



## Representing Proportional Relationships with Tables

**Lesson 2-1** One of These Things is Not Like the Others

**2-2** Introducing Proportional Relationships with Tables

**2-3** More about Constant of Proportionality

## Representing Proportional Relationships with Equations

**2-4** Proportional Relationships and Equations

**2-5** Two Equations for Each Relationship

**2-6** Using Equations to Solve Problems

## Comparing Proportional and Nonproportional Relationships

**2-7** Comparing Relationships with Tables

**2-8** Comparing Relationships with Equations

**2-9** Solving Problems about Proportional Relationships.

## Representing Proportional Relationships with Graphs

**2-10** Introducing Graphs of Proportional Relationships

**2-11** Interpreting Graphs of Proportional Relationships

**2-12** Using Graphs to Compare Relationships

**2-13** Two Graphs for Each Relationship

## Let's Put It to Work

**2-14** Four Representations

**2-15** Using Water Efficiently

## Unit 3

# Measuring Circles

## Circumference of a Circle

**Lesson 3-1** How Well Can You Measure?

**3-2** Exploring Circles

**3-3** Exploring Circumference

**3-4** Applying Circumference

**3-5** Circumference and Wheels

## Area of a Circle

**3-6** Estimating Areas

**3-7** Exploring the Area of a Circle

**3-8** Relating Area to Circumference

**3-9** Applying Area of Circles

## Let's Put it to Work

**3-10** Distinguishing Circumference and Area

**3-11** Stained-Glass Windows



## Unit 4

# Proportional Relationships and Percentages



## Proportional Relationships with Fractions

### Lesson 4-1 Lots of Flags

4-2 Ratios and Rates with Fractions

4-3 Revisiting Proportional Relationships

4-4 Half as Much Again

4-5 Say It with Decimals

## Percent Increase and Decrease

4-6 Increasing and Decreasing

4-7 One Hundred Percent

4-8 Percent Increase and Decrease with Equations

4-9 More and Less Than 1%

## Applying Percentages

4-10 Tax and Tip

4-11 Percentage Contexts

4-12 Finding the Percentage

4-13 Measurement Error

4-14 Percent Error

4-15 Error Intervals

## Let's Put It to Work

4-16 Posing Percentage Problems



Unit 5

# Rational Number Arithmetic



## Interpreting Negative Numbers

**Lesson 5-1** Interpreting Negative Numbers

## Adding and Subtracting Rational Numbers

**5-2** Changing Temperatures

**5-3** Changing Elevation

**5-4** Money and Debts

**5-5** Representing Subtraction

**5-6** Subtracting Rational Numbers

**5-7** Adding and Subtracting to Solve Problems

## Multiplying and Dividing Rational Numbers

**5-8** Position, Speed, and Direction

**5-9** Multiplying Rational Numbers

**5-10** Multiply!

**5-11** Dividing Rational Numbers

**5-12** Negative Rates

## Four Operations with Rational Numbers

**5-13** Expressions with Rational Numbers

**5-14** Solving Problems with Rational Numbers

## Solving Equations When There are Negative Numbers

**5-15** Solving Equations with Rational Numbers

**5-16** Representing Contexts with Equations

## Let's Put It to Work

**5-17** The Stock Market

## Unit 6

# Expressions, Equations, and Inequalities



## Representing Situations of the Form $px + q = r$ and $p(x + q) = r$

### Lesson 6-1 Relationships between Quantities

- 6-2 Reasoning about Contexts with Tape Diagrams (Part 1)
- 6-3 Reasoning about Contexts with Tape Diagrams (Part 2)
- 6-4 Reasoning about Equations and Tape Diagrams (Part 1)
- 6-5 Reasoning about Equations and Tape Diagrams (Part 2)
- 6-6 Distinguishing between Two Types of Situations

## Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ and Problems that Lead to Those Equations

- 6-7 Reasoning about Solving Equations (Part 1)
- 6-8 Reasoning about Solving Equations (Part 2)
- 6-9 Dealing with Negative Numbers
- 6-10 Different Options for Solving One Equation
- 6-11 Using Equations to Solve Problems
- 6-12 Solving Problems about Percent Increase or Decrease

## Inequalities

- 6-13 Reintroducing Inequalities
- 6-14 Finding Solutions to Inequalities in Context
- 6-15 Efficiently Solving Inequalities
- 6-16 Interpreting Inequalities
- 6-17 Modeling with Inequalities

## Writing Equivalent Expressions

- 6-18 Subtraction in Equivalent Expressions
- 6-19 Expanding and Factoring.
- 6-20 Combining Like Terms (Part 1)
- 6-21 Combining Like Terms (Part 2)
- 6-22 Combining Like Terms (Part 3)

## Let's Put It to Work

- 6-23 Culminating Lesson

## Unit 7

# Angles, Triangles, and Prisms



## Angle Relationships

**Lesson 7-1** Relationships of Angles

**7-2** Adjacent Angles

**7-3** Nonadjacent Angles

**7-4** Solving for Unknown Angles

**7-5** Using Equations to Solve for Unknown Angles

## Drawing Polygons with Given Conditions

**7-6** Building Polygons (Part 1)

**7-7** Building Polygons (Part 2)

**7-8** Triangles with Three Common Measures

**7-9** Drawing Triangles (Part 1)

**7-10** Drawing Triangles (Part 2)

## Solid Geometry

**7-11** Slicing Solids

**7-12** Volume of Right Prisms

**7-13** Decomposing Bases for Area

**7-14** Surface Area of Right Prisms

**7-15** Distinguishing Volume and Surface Area

**7-16** Applying Volume and Surface Area

## Let's Put It to Work

**7-17** Building Prisms

## Unit 8

# Probability and Sampling



## Probabilities of Single-Step Events

### Lesson 8-1 Mystery Bags

- 8-2 Chance Experiments
- 8-3 What are Probabilities?
- 8-4 Estimating Probabilities through Repeated Experiments
- 8-5 More Estimating Probabilities
- 8-6 Estimating Probabilities using Simulation

## Probabilities of Multi-Step Events

- 8-7 Simulating Multi-Step Experiments
- 8-8 Keeping Track of all Possible Outcomes
- 8-9 Multi-Step Experiments
- 8-10 Designing Simulations

## Sampling

- 8-11 Comparing Groups
- 8-12 Larger Populations
- 8-13 What Makes a Good Sample?
- 8-14 Sampling in a Fair Way

## Using Samples

- 8-15 Estimating Population Measures of Center
- 8-16 Estimating Population Proportions
- 8-17 More about Sampling Variability
- 8-18 Comparing Populations using Samples
- 8-19 Comparing Populations with Friends

## Let's Put It to Work

- 8-20 Memory Test

## Unit 9

# Putting It All Together



## Running a Restaurant

### Lesson 9-1 Planning Recipes

#### 9-2 Costs of Running a Restaurant

#### 9-3 More Costs of Running a Restaurant

#### 9-4 Restaurant Floor Plan

## How Crowded is This Neighborhood?

#### 9-5 How Crowded is This Neighborhood?

#### 9-6 Fermi Problems

#### 9-7 More Expressions and Equations

#### 9-8 Measurement Error (Part 1)

#### 9-9 Measurement Error (Part 2)

## Designing a Course

#### 9-10 Measuring Long Distances over Uneven Terrain

#### 9-11 Bundling a Trundle Wheel

#### 9-12 Using a Trundle Wheel to Measure Distances

#### 9-13 Designing a 5K Course



# Scale Drawings

## Prior Work

### Geometry and Geometric Measurement

Work with scale drawings in Grade 7 draws on earlier work with geometry and geometric measurement.

- Students began to learn about two- and three-dimensional shapes in **kindergarten**, and continued this work in **Grades 1 and 2**, composing, decomposing, and identifying shapes.
  - Students' work with geometric measurement began with length and continued with area.
  - Students learned to “structure two-dimensional space,” that is, to see a rectangle with whole-number side lengths as an array of unit squares, or rows or columns of unit squares.
- **In Grade 3**, students distinguished between perimeter and area. They connected rectangle area with multiplication, understanding why (for whole-number side lengths) multiplying the side lengths of a rectangle yields the number of unit squares that tile the rectangle. They used area diagrams to represent instances of the distributive property.
- **In Grade 4**, students applied area and perimeter formulas for rectangles to solve real-world and mathematical problems, and learned to use protractors.
- **In Grade 5**, students extended the formula for the area of a rectangle to include rectangles with fractional side lengths.
- **In Grade 6**, students built on their knowledge of geometry and geometric measurement to produce formulas for the areas of parallelograms and triangles, using these formulas to find surface areas of polyhedra.

## Work in This Unit

In this unit, students study scaled copies of pictures and plane figures, then apply what they have learned to scale drawings, e.g., maps and floor plans. This provides geometric preparation for Grade 7 work on proportional relationships as well as Grade 8 work on dilations and similarity.

### Coherence

Students begin by looking at copies of a picture, some of which are to scale, and some of which are not. They use their own words to describe what differentiates scaled and non-scaled copies of a picture. As the unit progresses, students learn that all lengths in a scaled copy are multiplied by a scale factor and all angles stay the same. They draw scaled copies of figures. They learn that if the scale factor is greater than 1, the copy will be larger, and if the scale factor is less than 1, the copy will be smaller. They study how area changes in scaled copies of an image.

Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can be used with scale drawings. They interpret and draw maps and floor plans. They work with scales that involve units (e.g., “1 cm represents 10 km”), and scales that do not include units (e.g., “the scale is 1 to 100”). They learn to express scales with units as scales without units, and vice versa. They understand that actual lengths are products of a scale factor and corresponding lengths in the scale drawing, thus lengths in the drawing are the product of the actual lengths and the reciprocal of that scale factor. They study the relationship between regions and lengths in scale

drawings. Throughout the unit, they discuss their mathematical ideas and respond to the ideas of others. **MP3** **MP6**

In the culminating lesson of this unit, students make a floor plan of their classroom or some other room or space at their school. This is an opportunity for them to apply what they have learned in the unit to everyday life. **MP4**

### Geometry Toolkit

In the unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, centimeter ruler, protractor (clear protractors with no holes that show radial lines are recommended), and an index card to use as a straightedge or to mark right angles. Providing students with these toolkits gives opportunities for students to develop abilities to select appropriate tools and use them strategically to solve problems.

**MP5**

Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

## Scaling in Grade 7

Note that the study of scaled copies is limited to pairs of figures that have the same rotation and mirror orientation (i.e. that are not rotations or reflections of each other), because the unit focuses on scaling, scale

factors, and scale drawings. In Grade 8, students will extend their knowledge of scaled copies when they study translations, rotations, reflections, and dilations.

## Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent	Generalize	Explain
<ul style="list-style-type: none"><li>a scaled copy for a given scale factor (Lessons 3 and 5)</li><li>distances using different scales (Lesson 11)</li><li>relevant features of a classroom with a scale drawing (Lesson 13)</li></ul>	<ul style="list-style-type: none"><li>about corresponding distances and angles in scaled copies (Lesson 4)</li><li>about scale factors greater than, less than, and equal to 1 (Lesson 5)</li><li>about scale factors and area (Lesson 6 and 10)</li><li>about scale factors with and without units (Lesson 12)</li></ul>	<ul style="list-style-type: none"><li>how to use scale drawings to find actual distances (Lesson 7 and 11)</li><li>how to use scale drawings to find actual distances, speed, and elapsed time (Lesson 8)</li><li>how to use scale drawings to find actual areas (Lesson 12)</li></ul>

In addition, students are expected to describe features of scaled copies, justify and critique reasoning about scaled copies, and compare how different scales affect drawings. Over the course of the unit, teachers can support students’ mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

(continued on the next page)

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students’ use of a new term in the lessons that follow where it was first introduced.

	New Terminology			
Lesson	Receptive		Productive	
1-1	scaled copy polygon	original		
1-2	corresponding figure	scale factor segment		
1-4	quadrilateral distance	measurement	corresponding original	scale factor
1-5	reciprocal			
1-6	area two-dimensional	one-dimensional	squared	
1-7	scale drawing represent three-dimensional	scale actual	scaled copy	
1-8	estimate constant speed	travel	scale	
1-9	floorplan			
1-10	appropriate	dimension	actual	represent
1-11	scale without units _____ to _____		scale drawing	
1-12	equivalent scales		_____ to _____	

	Lessons	Days	Standards
	Check Your Readiness Assessment	1	
Topic	Scaled Copies		
	Lesson 1-1 What are Scaled Copies?	1	7.G.A.1
	Lesson 1-2 Corresponding Parts and Scale Factors	1	7.G.A.1, 7.RP.A.2
	Lesson 1-3* Making Scaled Copies	1	7.G.A.1, 7.RP.A.2
	Lesson 1-4* Scaled Relationships	1	7.G.A.1
	Lesson 1-5* The Size of the Scale Factor	1	7.G.A.1, 7.RP.A.2
	Lesson 1-6* Scaling and Area	1	7.G.A.1, 7.G.B.4, 7.G.B.6, 7.RP.A.2.a
Topic	Scale Drawings		
	Lesson 1-7 Scale Drawings	1	7.G.A.1
	Lesson 1-8* Scale Drawings and Maps	1	7.G.A.1, 7.RP.A, 7.RP.A.2.b
	Lesson 1-9 Creating Scale Drawings	1	7.G.A.1
	Lesson 1-10 Changing Scales in Scale Drawings	1	7.G.A.1, 7.G.B.6, 7.RP.A, 7.RP.A.3
	Lesson 1-11 Scales without Units	1	7.G.A.1
	Lesson 1-12* Units in Scale Drawings	1	7.G.A.1
Topic	Let's Put It To Work		
	Lesson 1-13* Draw It to Scale	1	7.G.A.1
	End-of-Unit Assessment	1	
	*indicates Lessons in which there are optional activities	TOTAL	13-15

Required Materials

- ☐ blank paper (Lesson 13)
- ☐ Geometry Toolkit (Lessons 2-10, 12)

☐ tracing paper

☐ graph paper

☐ colored pencils

☐ scissors

☐ index card

☐ ruler

☐ protractor
- ☐ graph paper (Lesson 13)
- ☐ measuring tools (Lesson 13)
- ☐ metric and customary conversion charts (Lesson 12)

- ☐ pattern blocks (Lesson 6)
- ☐ rulers (Lesson 11)
- ### Blackline Activity Masters
- ☐ 7.1.1.3, Pairs of Scaled Polygons, one per pair (Lesson 1, Activity 1.3)
- ☐ 7.1.5.2, Card Sort: Scaled Copies, one per group (Lesson 5, Activity 5.2)
- ☐ 7.1.5.3, Scaling a Puzzle, one per group (Lesson 5, Activity 5.3)
- ☐ 7.1.6.3, Area of Scaled Parallelograms and Triangles, one per pair (Lesson 6, Activity 6.3)

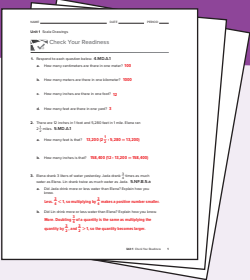
- ☐ 7.1.7.2, Sizing Up a Basketball Court, one per student (Lesson 7, Activity 7.2)
- ☐ 7.1.10.2, Same Plot, Different Drawings, one per group (Lesson 10, Activity 10.2)
- ☐ 7.1.11.2, Apollo Lunar Module, one per student (Lesson 11, Activity 11.2)
- ☐ 7.1.12.2, Card Sort: Scales, one per group (Lesson 12, Activity 12.2)
- ☐ 7.1.12.4, Pondering Pools, one per pair (Lesson 12, Activity 12.4)

# Pre-Unit Diagnostic Assessment

The pre-unit diagnostic assessment, *Check Your Readiness*, evaluates students' proficiency with prerequisite concepts and skills that they need to be successful in the unit. The item descriptions below offer guidance for students who may answer items incorrectly. The assessment also may include problems that assess what students already know of the upcoming unit's key ideas, which you can use to pace or tune instruction. In rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.



Available as a digital assessment or a printable assessment.



## 1. Item Description

Throughout Unit 1, students work with length and area in a variety of contexts.

**First Appearance of Skill or Concept:** Lesson 7

- Beginning in Lesson 7, students need to know how to convert units fluently and efficiently, and prior to Lesson 7, some students may use approaches that involve unit conversion.

**If most students struggle with this item...**

- Plan to use this problem to create an anchor chart early in the unit for students to use as they think about appropriate scales to use throughout this unit.

## 2. Item Description

Students perform multiple unit conversions, working with fractions and large numbers.

**First Appearance of Skill or Concept:** Lesson 11

**If most students struggle with this item...**

- Plan to practice unit conversions before doing Lesson 11.
- In Grade 6 Unit 3 Lessons 3 and 4, you will find activities to support conversions with fractions and large numbers.

## 3. Item Description

The language of scaling appears in grade 5. While the scaling of this unit is geometric in nature, the language and thought process associated with this grade 5 standard is very helpful for students.


**First Appearance of Skill or Concept:** Lesson 2

**If most students struggle with this item...**

- Plan to emphasize scaling language when launching and synthesizing Lesson 2 Activity 3, *Scaled Triangles*.
- As students are examining the triangles, they will encounter fractional multipliers  $2$ ,  $\frac{1}{2}$ , and  $\frac{2}{3}$ .
- MLR2** *Collect and Display* can be used to highlight student language such as “twice as big” and “half the size” and connect that language to scale factors.

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**Unit 1** Scale Drawings

 **Check Your Readiness**

1. Respond to each question below. **4.MD.A.1**

a. How many centimeters are there in one meter? **100**

b. How many meters are there in one kilometer? **1000**

c. How many inches are there in one foot? **12**

d. How many feet are there in one yard? **3**

2. There are 12 inches in 1 foot and 5,280 feet in 1 mile. Elena ran  $2\frac{1}{2}$  miles. **5.MD.A.1**

a. How many feet is that?  **$13,200$  ( $2\frac{1}{2} \cdot 5,280 = 13,200$ )**

b. How many inches is that?  **$158,400$  ( $12 \cdot 13,200 = 158,400$ )**

3. Elena drank 3 liters of water yesterday. Jada drank  $\frac{3}{4}$  times as much water as Elena. Lin drank twice as much water as Jada. **5.NF.B.5.a**

a. Did Jada drink more or less water than Elena? Explain how you know.  
**Less.  $\frac{3}{4} < 1$ , so multiplying by  $\frac{3}{4}$  makes a positive number smaller.**

b. Did Lin drink more or less water than Elena? Explain how you know.  
**More. Doubling  $\frac{3}{4}$  of a quantity is the same as multiplying the quantity by  $\frac{3}{2}$ , and  $\frac{3}{2} > 1$ , so the quantity becomes larger.**

Unit 1 Check Your Readiness 1

## 4. Item Description

One important use of scale models is to help students make calculations about the object represented. Part of these calculations use the scale while other parts involve finding the lengths or areas of the model. This task makes sure students can effectively calculate the areas of complex shapes.

**First Appearance of Skill or Concept:** Lesson 6

**If most students struggle with this item...**

- Plan to do Lesson 6 Activity 3, *Area of Scaled Parallelograms and Triangles*, focusing students on attending to the base and height of their polygon.
- During the activity launch, include a discussion on how students can determine the area of figures if they aren't sure of a formula.
- During the launch, review the pre-unit diagnostic item featuring students who decomposed and rearranged as well as students who found parallelograms and triangles and can speak to how they used the base and height.
- If students need more support with calculating the area of complex shapes, refer to Grade 6 Unit 1.

## 5. Item Description

Students have been calculating perimeter and area of figures since elementary school. In Grade 6, they begin to use variables and exponential notation in these formulas.

This exercise checks students' understanding of perimeter and area using variables.

**First Appearance of Skill or Concept:** Lesson 6

**If most students struggle with this item...**

- Plan to use this problem to review area and perimeter. Grade 6, Unit 1, Lessons 5 and 6 could also be used as review.

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

4. Each small square in the graph paper represents 1 square unit. Find the area of each figure. Explain your reasoning. **6.G.A.1**

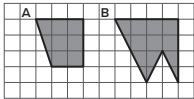


Figure A has an area of  $7\frac{1}{2}$  square miles. It can be divided into a 2-unit-by-3-unit rectangle (with an area of 6 square units) and a 1-unit-by-3-unit triangle (with an area of  $1\frac{1}{2}$  square units).

Figure B has an area of 10 square units. It can be surrounded by a 4-unit-by-4-unit square with two triangles removed. Those triangles have areas of 4 and 2. The area of Figure B is 10 square units since  $16 - 4 - 2 = 10$ .

5. This rectangle has side lengths  $r$  and  $s$ . **6.EE.A.2.c**



For each expression, say whether it gives the *perimeter* of the rectangle, the *area* of the rectangle, or *neither*.

- a.  $r + s$  **neither (it's half the perimeter)**
- b.  $r \cdot s$  **area**
- c.  $2r + 2s$  **perimeter**
- d.  $r^2 + s^2$  **neither**



6. Item Description

Students scale figures up and down, which is similar to scaling recipes up and down. This item checks that students are comfortable representing scaling with a table.

First Appearance of Skill or Concept: Lesson 2

If most students struggle with this item...

- Plan to do Lesson 2, Activity 3, *Scaled Triangles* with extra attention to question 3 and the activity synthesis.
- The synthesis uses the table of scaled copies to begin thinking about and articulating scale factor.

7. Item Description

Some students may have seen or worked with scale drawings before. This item checks whether or not they can reproduce a drawing at given scale on a grid.

First Appearance of Skill or Concept: Lesson 1

If most students do well with this item...

It may be possible to skip Lesson 3, Optional Activity 2.

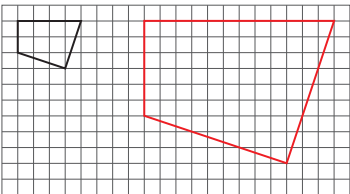
NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

6. A recipe for 1 loaf of bread calls for 2 cups of flour, 12 tablespoons of water, and 1 teaspoon of salt. The recipe can be scaled up to make multiple loaves of bread. Complete the table that shows the quantities to use for multiple loaves of bread. **6.RP.A.3.a**

Number of Loaves	Cups of Flour	Tablespoons of Water	Teaspoons of Salt
1	2	12	1
2	4	24	2
4	8	48	4
3	6	36	3

7. Here is a polygon. **7.G.A.1**

Draw a scaled copy of the polygon with scale factor 3.



# What Are Scaled Copies?

## Goals

- Describe (orally) characteristics of scaled and non-scaled copies.
- Identify scaled copies of a figure and justify (orally and in writing) that the copy is a scaled copy.

## Student Learning Goals

Let’s explore scaled copies.

## Learning Targets

- I can describe some characteristics of a scaled copy.
- I can tell whether or not a figure is a scaled copy of another figure.

## Required Materials

- pre-printed slips, cut from copies of the blackline master 7.1.1.3, *Pairs of Scaled Polygons*

## Required Preparation

You will need the *Pairs of Scaled Polygons* blackline master for this lesson. Print and cut slips A–J for the *Pairs of Scaled Polygons* activity. Prepare 1 copy for every 2 students. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

## Lesson Narrative

This lesson introduces students to the idea of a **scaled copy** of a picture or a figure. Students learn to distinguish scaled copies from those that are not—first informally, and later, with increasing precision. They may start by saying that scaled copies have the same shape as the original figure, or that they do not appear to be distorted in any way, though they may have a different size. Next, they notice that the lengths of segments in a scaled copy vary from the lengths in the original figure in a uniform way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. Students work toward articulating the characteristics of scaled copies quantitatively (e.g., “all the segments are twice as long,” “all the lengths have shrunk by one third,” or “all the segments are one-fourth the size of the segments in the original”), articulating the relationships carefully along the way. **MP6**

The lesson is designed to be accessible to all students regardless of prior knowledge, and to encourage students to make sense of problems and persevere in solving them from the very beginning of the course. **MP1**

## Instructional Routines

- **MLR1** Stronger and Clearer Each Time
- **MLR8** Discussion Supports
- Take Turns
- Think Pair Share

## Lesson Pacing

	Pacing (min)
<b>Warm Up 1.1</b> Printing Portraits	10
<b>Activity 1.2</b> Scaling F	10
<b>Activity 1.3</b> Pairs of Scaled Polygons	15
<b>Lesson Synthesis</b>	5-10
<b>Cool Down 1.4</b> Scaling L	5
<b>TOTAL</b>	<b>45-50</b>

## Standards Alignment

### Addressing

**7.G.A.1** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.



## Warm Up 1.1 Printing Portraits (10 minutes)

### Standards Alignment

Addressing: 7.G.A.1

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

This opening task introduces the term **scaled copy**. It prompts students to observe several copies of a picture, visually distinguish scaled and unscaled copies, and articulate the differences in their own words. Besides allowing students to have a mathematical conversation about properties of figures, it provides an accessible entry into the concept and gives an opportunity to hear the language and ideas students associate with scaled figures.

Students are likely to have some intuition about the term “to scale,” either from previous work in grade 6 (e.g., scaling a recipe, or scaling a quantity up or down on a double number line) or from outside the classroom. This intuition can help them identify scaled copies.

Expect them to use adjectives such as “stretched,” “squished,” “skewed,” “reduced,” etc., in imprecise ways. This is fine, as students’ intuitive definition of scaled copies will be refined over the course of the lesson. As students discuss, note the range of descriptions used. Monitor for students whose descriptions are particularly supportive of the idea that lengths in a scaled copy are found by multiplying the original lengths by the same value. Invite them to share their responses later.

### Instructional Routines

#### • Think Pair Share

- **What** Students have quiet time to think about a problem and work on it individually, and then time to share their response or their progress with a partner. Once these partner conversations have taken place, some students are selected to share their thoughts with the class.
- **Why** This is a teaching routine useful in many contexts whose purpose is to give all students enough time to think about a prompt and form a response before they are expected to try to verbalize their thinking. First they have an opportunity to share their thinking in a low-stakes way with one partner, so that when they share with the class they can feel calm and confident, as well as say something meaningful that might advance everyone’s understanding. Additionally, the teacher has an opportunity to eavesdrop on the partner conversations so that they can purposefully select students to share with the class.

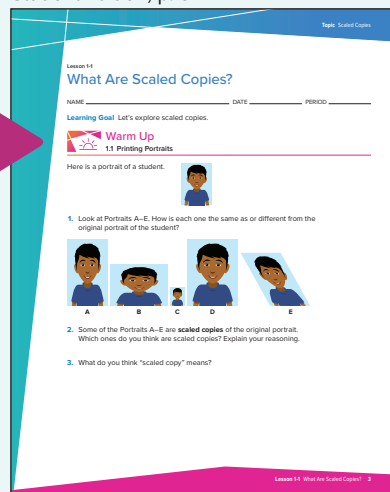
### Launch

**Print Activity:** Arrange students in groups of 2. Give students 2–3 minutes of quiet think time and a minute to share their response with their partner.

**Digital Activity:** Have students work in groups of 2–3 to complete the activity. They should have quiet time in addition to share time, while solving the problem and developing language to describe scaling.

*Go online to find the digital version of this activity.*

*(continued on the next page)*

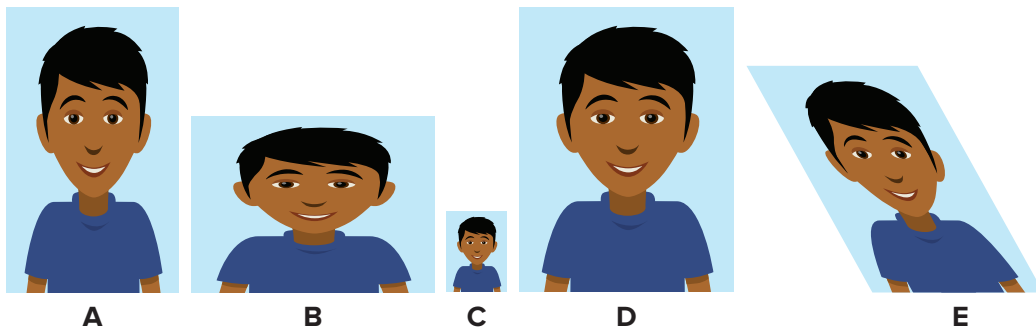


## Student Task Statement

Here is a portrait of a student.



1. Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?



Answers vary. Sample response:

- Similarities: Pictures A–E are all based on the same original portrait. They all show the same boy wearing a blue shirt and brown hair. They all have the same white background.
  - Differences: They all have different sizes; some have different shapes. Pictures A, B, and E have been stretched or somehow distorted. C and D are not stretched or distorted but are each of a different size than the original.
2. Some of the Portraits A–E are **scaled copies** of the original portrait. Which ones do you think are scaled copies? Explain your reasoning. **C and D are scaled copies.** Sample explanation:
    - A, B, and E are not scaled copies because they have changed in shape compared to the original portrait. Portrait A is stretched vertically, so the vertical side is now much longer than the horizontal side. B is stretched out sideways, so the horizontal sides are now longer than the vertical. E seems to have its upper left and lower right corners stretched out in opposite directions. The portrait is no longer a rectangle.
    - C is a smaller copy and D is a larger copy of the original, but their shapes remain the same.
  3. What do you think “scaled copy” means? Answers vary. Sample definitions:
    - A scaled copy is a copy of a picture that changes in size but does not change in shape.
    - A scaled copy is a duplicate of a picture with no parts of it distorted, though it could be larger, smaller, or the same size.
    - A scaled copy is a copy of a picture that has been enlarged or reduced in size but nothing else changes.

(continued on the next page)

## Activity Synthesis

Select a few students to share their observations. Record and display students' explanations for the second question. Consider organizing the observations in terms of how certain pictures are or are not distorted. For example, students may say that C and D are scaled copies because each is a larger or smaller version of the picture, but the face (or the sleeve, or the outline of the picture) has not changed in shape. They may say that A, B, and E are not scaled copies because something other than size has changed. If not already mentioned in the discussion, guide students in seeing features of C and D that distinguish them from A, B, and E.

Invite a couple of students to share their working definition of scaled copies. Some of the students' descriptions may not be completely accurate. That is appropriate for this lesson, as the goal is to build on and refine this language over the course of the next few lessons until students have a more precise notion of what it means for a picture or figure to be a scaled copy.



## Activity 1.2 Scaling F (10 minutes)

### Standards Alignment

Addressing: 7.G.A.1

This task enables students to describe more precisely the characteristics of scaled copies and to refine the meaning of the term. Students observe copies of a line drawing on a grid and notice how the lengths of line segments and the angles formed by them compare to those in the original drawing.

Students engage in analyzing structure in multiple ways in this task. Identifying distinguishing features of the scaled copies means finding similarities and differences in the shapes. In addition, the fact that corresponding parts increase by the *same* scale factor is a vital structural property of scaled copies.

For the first question, expect students to explain their choices of scaled copies in intuitive, qualitative terms. For the second question, students should begin to distinguish scaled and unscaled copies in more specific and quantifiable ways. If it does not occur to students to look at lengths of segments, suggest they do so.

As students work, monitor for students who notice the following aspects of the figures. Students are not expected to use these mathematical terms at this point, however.

- The original drawing of the letter F and its scaled copies have equivalent width-to-height ratios.
- We can use a scale factor (or a multiplier) to compare the lengths of different figures and see if they are scaled copies of the original.
- The original figure and scaled copies have corresponding angles that have the same measure.

*(continued on the next page)*

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

## • Instructional Routines

### • Mathematical Language Routines

- **MLR1 Stronger and Clearer Each Time** To provide a structured and interactive opportunity for students to revise and refine both their ideas and their verbal and written output. This routine provides a purpose for student conversation as well as fortifies output. The main idea is to have students think or write individually about a response, use a structured pairing strategy to have multiple opportunities to refine and clarify the response through conversation, and then finally revise their original written response. Throughout this process, students should be pressed for details, and encouraged to press each other for details.

### • Think Pair Share

## Launch

Keep students in the same groups. Give them 3–4 minutes of quiet work time, and then 1–2 minutes to share their responses with their partner. Tell students that how they decide whether each of the seven drawings is a scaled copy may be very different than how their partner decides. Encourage students to listen carefully to each other’s approach and to be prepared to share their strategies. Use gestures to elicit from students the words “horizontal” and “vertical” and ask groups to agree internally on common terms to refer to the parts of the F (e.g., “horizontal stems”).

## Support For Students with Disabilities

**Engagement: Internalize Self Regulation** Display sentence frames to support small group discussion. For example, “That could/couldn’t be true because...,” “We can agree that...,” and “Is there another way to say/do...?”

**Supports accessibility for:** Social-emotional skills; Organization; Language

## Support For English Language Learners

### Speaking

**MLR1 Stronger and Clearer Each Time** This is the first time Math Language Routine 1 is suggested as a support in this course. In this routine, students are given a thought-provoking question or prompt and asked to create a first draft response in writing. Students meet with 2–3 partners to share and refine their response through conversation. While meeting, listeners ask questions such as, “What did you mean by . . .?” and “Can you say that another way?” Finally, students write a second draft of their response reflecting ideas from partners, and improvements on their initial ideas. The purpose of this routine is to provide a structured and interactive opportunity for students to revise and refine their ideas through verbal and written means.

**Design Principle(s):** Optimize output (for explanation)

*(continued on the next page)*



**How It Happens:**

1. Use this routine to provide students a structured opportunity to refine their explanations for the first question: “Identify all the drawings that are scaled copies of the original letter F drawing. Explain how you know.” Allow students 2–3 minutes to individually create first draft responses in writing.
2. Invite students to meet with 2–3 other partners for feedback.
  - Instruct the speaker to begin by sharing their ideas without looking at their written draft, if possible. Provide the listener with these prompts for feedback that will help their partner strengthen their ideas and clarify their language: “What do you mean when you say....?”, “Can you describe that another way?”, “How do you know that \_ is a scaled copy?”, “Could you justify that differently?” Be sure to have the partners switch roles. Allow 1–2 minutes to discuss.
3. Signal for students to move on to their next partner and repeat this structured meeting.
4. Close the partner conversations and invite students to revise and refine their writing in a second draft.
  - Provide these sentence frames to help students organize their thoughts in a clear, precise way: “Drawing \_ is a scaled copy of the original, and I know this because....”, “When I look at the lengths, I notice that....”, and “When I look at the angles, I notice that....”
  - Here is an example of a second draft:
  - “Drawing 7 is a scaled copy of the original, and I know this because it is enlarged evenly in both the horizontal and vertical directions. It does not seem lopsided or stretched differently in one direction. When I look at the length of the top segment, it is 3 times as large as the original one, and the other segments do the same thing. Also, when I look at the angles, I notice that they are all right angles in both the original and scaled copy.”
5. If time allows, have students compare their first and second drafts. If not, have the students move on by working on the following problems.

**Anticipated Misconceptions**

Students may make decisions by “eyeballing” rather than observing side lengths and angles. Encourage them to look for quantifiable evidence and notice lengths and angles.

Some may think vertices must land at intersections of grid lines (e.g., they may say Drawing 4 is not a scaled copy because the endpoints of the shorter horizontal segment are not on grid crossings). Address this during the whole-class discussion, after students have a chance to share their observations about segment lengths.

*(continued on the next page)*

**Activity**  
1.2 Scaling F

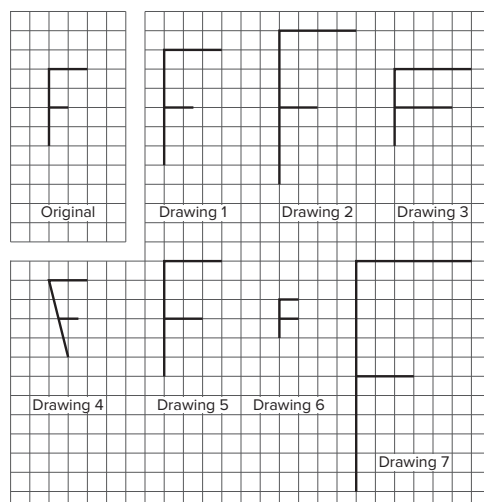
Here is an original drawing of the letter F and some other drawings.

1. Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know.
2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?
3. On the grid, draw a different scaled copy of the original letter F.

Unit 1 Scale Drawings

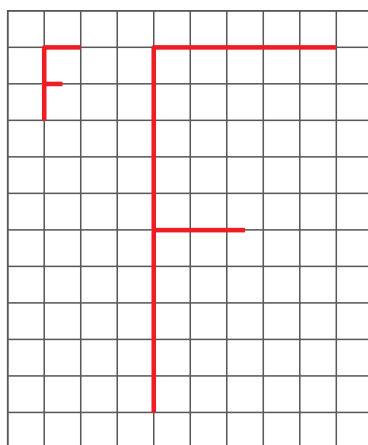
## Student Task Statement

Here is an original drawing of the letter F and some other drawings.



1. Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know. **Drawings 1, 2, and 7 are scaled copies of the original drawing. Explanations vary. Sample explanation: I know because they are not stretched differently in one direction. They are enlarged evenly in both vertical and horizontal directions.**
2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice? **Answers vary. Sample responses:**
  - In the scaled copies, every segment is the same number of times as long as the matching segment in the original drawing.
  - In the scaled copies, all segments keep the same relationships as in the original. The original drawing of F is 4 units tall. Its top horizontal segment is 2 units wide and the shorter horizontal segment is 1 unit. In Drawing 1, the F is 6 units tall and 3 units wide; in Drawing 2, it is 8 units tall and 4 units wide, and in Drawing 7, it is 8 units tall and 4 units wide. In each scaled copy, the width is half of the height, just as in the original drawing of F, and the shorter horizontal segment is half of the longer one.
3. On the grid, draw a different scaled copy of the original letter F.

**Drawings vary. Sample response:**



(continued on the next page)

## Activity Synthesis

Display the seven copies of the letter F for all to see. For each copy, ask students to indicate whether they think each one is a scaled copy of the original F. Record and display the results for all to see. For contested drawings, ask 1–2 students to briefly say why they ruled these out.

Discuss the identified scaled and unscaled copies.

- **What features do the scaled copies have in common?** Be sure to invite students who were thinking along the lines of scale factors and angle measures to share.
- **How do the other copies fail to show these features?** Sometimes lengths of sides in the copy use different multipliers for different sides. Sometimes the angles in the copy do not match the angles in the original.

If there is a misconception that scaled copies must have vertices on intersections of grid lines, use Drawing 1 (or a relevant drawing by a student) to discuss how that is not the case.

Some students may not be familiar with words such as “twice,” “double,” or “triple.” Clarify the meanings by saying “two times as long” or “three times as long.”



## Activity 1.3 Pairs of Scaled Polygons (15 minutes)

### Standards Alignment

Addressing: 7.G.A.1

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

In this activity, students hone their understanding of scaled copies by working with more complex figures. Students work with a partner to match pairs of polygons that are scaled copies. The polygons appear comparable to one another, so students need to look very closely at all side lengths of the polygons to tell if they are scaled copies.

As students confer with one another, notice how they go about looking for a match. Monitor for students who use precise language to articulate their reasoning (e.g., “The top side of A is half the length of the top side of G, but the vertical sides of A are a third of the lengths of those in G.”). **MP6**

You will need the *Pairs of Scaled Polygons* blackline master for this activity.

### Instructional Routines

#### Mathematical Language Routines

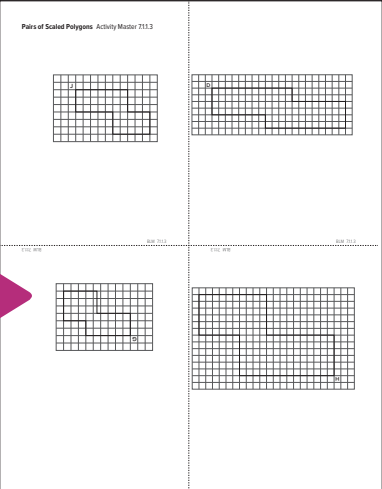
- **MLR8 Discussion Supports** To support rich discussions about mathematical ideas, representations, contexts, and strategies. The examples provided can be combined and used together with any of the other routines. They include multi-modal strategies for helping students comprehend complex language and ideas, and can be used to make classroom communication accessible, to foster meta-awareness of language, and to demonstrate strategies students can use to enhance their own communication and construction of ideas.

#### Take Turns

- **What** Students work with a partner or small group. They take turns in the work of the activity, whether it be spotting matches, explaining, justifying, agreeing or disagreeing, or asking clarifying questions. If they disagree, they are expected to support their case and listen to their partner’s arguments. The first few times students engage in these activities, the teacher should demonstrate, with a partner, how the discussion is expected to go. Once students are familiar with these structures, less set-up will be necessary. While students are working, the teacher can ask students to restate their question more clearly or paraphrase what their partner said.
- **Why** Building in an expectation, through the routine, that students explain the rationale for their choices and listen to another’s rationale deepens the understanding that can be achieved through these activities. Specifying that students take turns deciding, explaining, and listening limits the phenomenon where one student takes over and the other does not participate. Taking turns can also give students more opportunities to construct logical arguments and critique others’ reasoning. **MP3**

(continued on the next page)

BLM 7.1.1.3 Pairs of Scaled Polygons



Launch

Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the cards and place them face-up. Tell them that each polygon has one and only one match (i.e., for each polygon, there is one and only one scaled copy of the polygon). Select two cards and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 2. Give each group a set of 10 slips cut from the blackline master. Encourage students to refer to a running list of statements and diagrams to refine their language and explanations of how they know one figure is a scaled copy of the other.

Support For Students with Disabilities

**Representation: Internalize Comprehension** Provide a range of examples and counterexamples. During the demonstration of how to set up and do the matching activity, select two cards that do not match, and invite students to come up with a shared justification. Display sentence frames such as: “Figure \_\_\_\_ and figure \_\_\_\_ do/do not match because . . .”

**Supports accessibility for:** Conceptual processing

Support For English Language Learners

Speaking

**MLR8 Discussion Supports** Use this routine to support small-group discussion. As students take turns finding a match of two polygons that are scaled copies of one another and explaining their reasoning to their partner, display the following sentence frames for all to see: “\_\_\_\_ matches \_\_\_\_ because . . .” and “I noticed \_\_\_\_, so I matched . . .” Encourage students to challenge each other when they disagree with the sentence frames “I agree because . . .”, and “I disagree because . . .” This will help students clarify their reasoning about scaled copies of polygons.

**Design Principle(s):** Support sense-making; Optimize output (for explanation)

Anticipated Misconceptions

Some students may think a figure has more than one match. Remind them that there is only one scaled copy for each polygon and ask them to recheck all the side lengths.

Some students may think that vertices must land at intersections of grid lines and conclude that, e.g., G cannot be a copy of F because not all vertices on F are on such intersections. Ask them to consider how a 1-unit-long segment would change if scaled to be half its original size. Where must one or both of its vertices land?

Student Task Statement

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

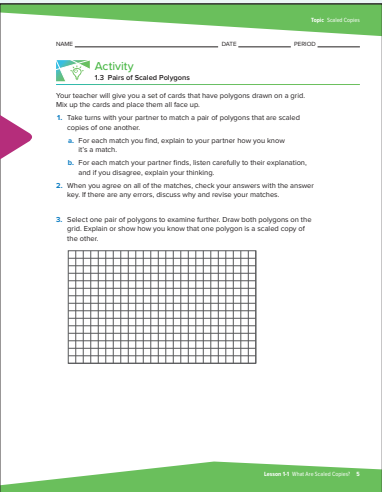
1. Take turns with your partner to match a pair of polygons that are scaled copies of one another.
  - a. For each match you find, explain to your partner how you know it’s a match.
  - b. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.

The following polygons are scaled versions of one another:

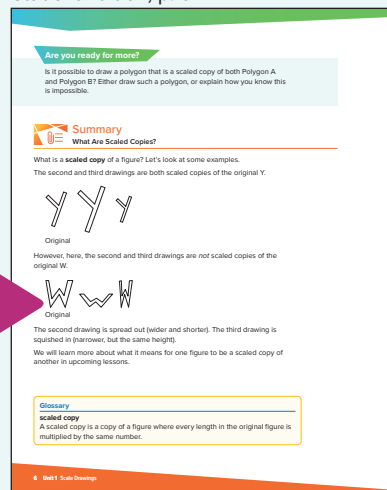
- A and C
- B and D
- E and I
- F and G
- H and J

(continued on the next page)

Student Edition, p. 5



## Student Edition, p. 6



- When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches. **No answer needed.**
- Select one pair of polygons to examine further. Draw both polygons on the grid. Explain or show how you know that one polygon is a scaled copy of the other. **Answers vary. Sample explanation for A and C: All the side lengths in C are twice as long as the lengths of the matching sides in A.**

### Are you ready for more?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible. **It's impossible to draw a polygon that is a scaled copy of both Polygon A and Polygon B. Sample explanations:**

- If I draw a polygon that is a scaled copy of A, all the side lengths would be the same number of times larger or smaller than A, but they won't be the same number of times larger or smaller than B.
- A and B are not scaled copies of each other, so if I draw a scaled copy of one, it will not be a scaled copy of the other.

### Activity Synthesis

The purpose of this discussion is to draw out concrete methods for deciding whether or not two polygons are scaled copies of one another, and in particular, to understand that just eyeballing to see whether they look roughly the same is not enough to determine that they are scaled copies.

Display the image of all the polygons. Ask students to share their pairings and guide a discussion about how students went about finding the scaled copies. Ask questions such as:

- When you look at another polygon, what exactly did you check or look for?** **General shape, side lengths**
- How many sides did you compare before you decided that the polygon was or was not a scaled copy?** **Two sides can be enough to tell that polygons are not scaled copies; all sides are needed to make sure a polygon is a scaled copy.**
- Did anyone check the angles of the polygons? Why or why not?** **No; the sides of the polygons all follow grid lines.**

If students do not agree about some pairings after the discussion, ask the groups to explain their case and discuss which of the pairings is correct. Highlight the use of quantitative descriptors such as “half as long” or “three times as long” in the discussion. Ensure that students see that when a figure is a scaled copy of another, all of its segments are the same number of times as long as the corresponding segments in the other.



## Lesson Synthesis What are Scaled Copies?

In this lesson, we encountered copies of a figure that are both scaled and not scaled. We saw different versions of a portrait of a student and of a letter F, as well as a variety of polygons that had some things in common.

In each case, we decided that some were scaled copies of one another and some were not. Consider asking students:

- What is a scaled copy?
- What are some characteristics of scaled copies? How are they different from figures that are not scaled copies?
- What specific information did you look for when determining if something was a scaled copy of an original?

While initial answers need not be particularly precise at this stage of the unit (for example, “scaled copies look the same but are a different size”), guide the discussion toward making careful statements that one could test. The lengths of segments in a scaled copy are related to the lengths in the original figure in a consistent way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. We might say, “All the segments are twice as long,” or “All the segments are one-third the size of the segments in the original.”



## Cool Down 1.4 Scaling L (5 minutes)

### Printable Cool Down

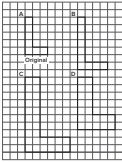
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**Lesson 1.4 What Are Scaled Copies?**

**Cool Down**

**1.4 Scaling L**

Are any of the figures B, C, or D scaled copies of figure A? Explain how you know.



Lesson 1.4 What Are Scaled Copies? 1

### Student Task Statement

Are any of the figures B, C, or D scaled copies of figure A? Explain how you know. **Only figure C is a scaled copy of figure A. Sample explanation: In figure C, the length of each segment of the letter L is twice the length of the matching segment in A. In B, none of the segments are double the length. In figure D, some segments are double in length and some are not. So the block letters in B and D are not enlarged evenly.**

### Standards Alignment

Addressing: 7.G.A.1



### Are you ready for more?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.



### Summary

#### What Are Scaled Copies?

What is a **scaled copy** of a figure? Let's look at some examples.

The second and third drawings are both scaled copies of the original Y.



Original

However, here, the second and third drawings are *not* scaled copies of the original W.



Original

The second drawing is spread out (wider and shorter). The third drawing is squished in (narrower, but the same height).

We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

#### Glossary

##### scaled copy

A scaled copy is a copy of a figure where every length in the original figure is multiplied by the same number.

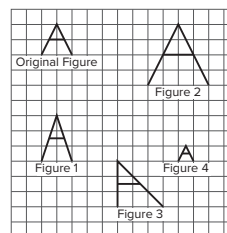
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### Practice

#### What Are Scaled Copies?

- Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.



Figures 2 and 4 are scaled copies. Sample explanations: The original A fits inside a square. The horizontal segment is halfway up the height of the square. The tip of the A is at the midpoint of the horizontal side of the square; Figure 1 is inside a rectangle, not a square, so it is not a scaled copy. Figure 3 fits inside a square but the shape is different than the original letter A, since one of the legs of the A in Figure 3 is now vertical, so it also is not a scaled copy; Figure 2 is twice as high and twice as wide as the original A, and Figure 4 is half as tall and half as wide, but in both figures the locations of the horizontal segment and the tip of the letter A still match the original.

- Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.



Figure A



Figure B

Do you agree with Tyler? Explain your reasoning.

**No.** For the smaller figure to be a scaled copy, the figure would have to be half as wide as well.

- Here is a picture of a set of billiard balls in a rack.



Here are some copies of the picture. Select **all** the pictures that are scaled copies of the original picture.



A



C



B



D

- Complete each equation with a number that makes it true.

a.  $5 \cdot \underline{3} = 15$

b.  $4 \cdot \underline{8} = 32$

c.  $6 \cdot \underline{1.5, \frac{3}{2}, \text{or equivalent}} = 9$

d.  $12 \cdot \underline{0.25, \frac{1}{4}, \text{or equivalent}} = 3$

Student Edition

## Lesson 1-1

# What Are Scaled Copies?

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**Learning Goal** Let's explore scaled copies.

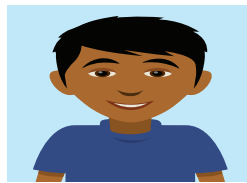
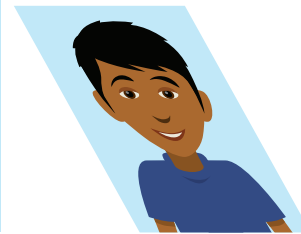
## Warm Up

### 1.1 Printing Portraits

Here is a portrait of a student.



1. Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?

**A****B****C****D****E**

2. Some of the Portraits A–E are **scaled copies** of the original portrait. Which ones do you think are scaled copies? Explain your reasoning.
3. What do you think “scaled copy” means?

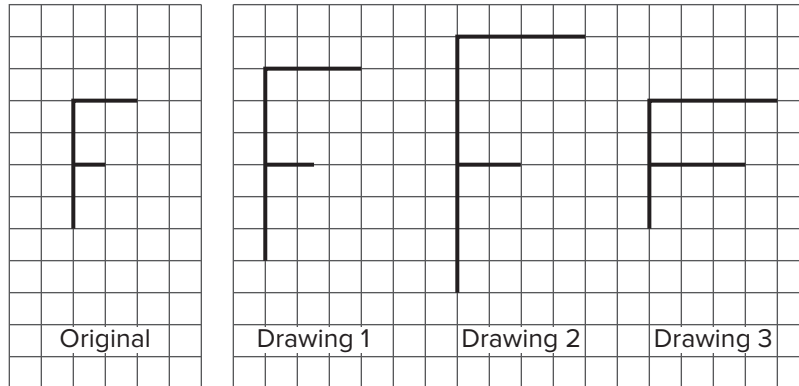


## Activity

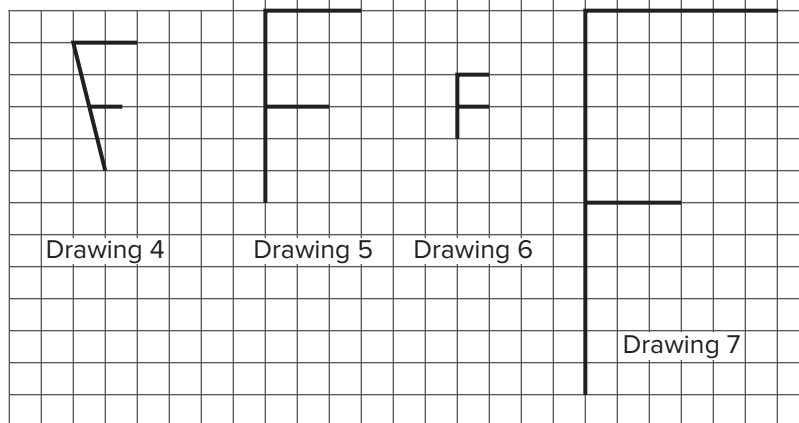
### 1.2 Scaling F

Here is an original drawing of the letter F and some other drawings.

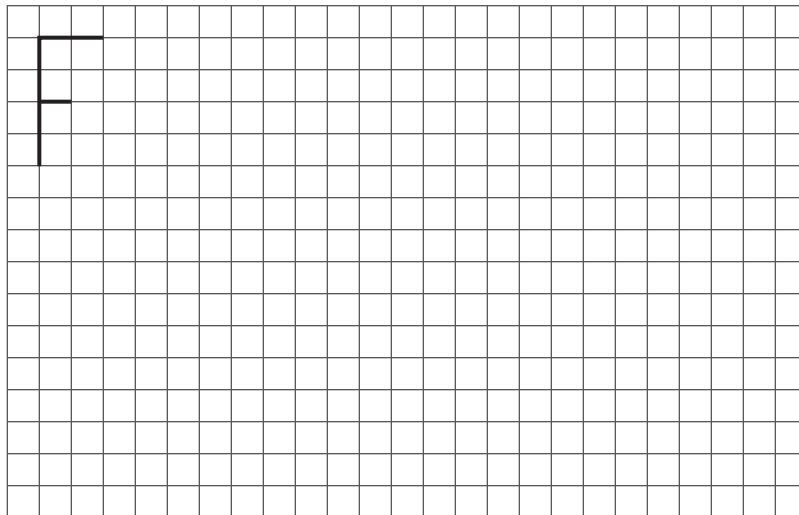
1. Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know.



2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?



3. On the grid, draw a different scaled copy of the original letter F.



NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

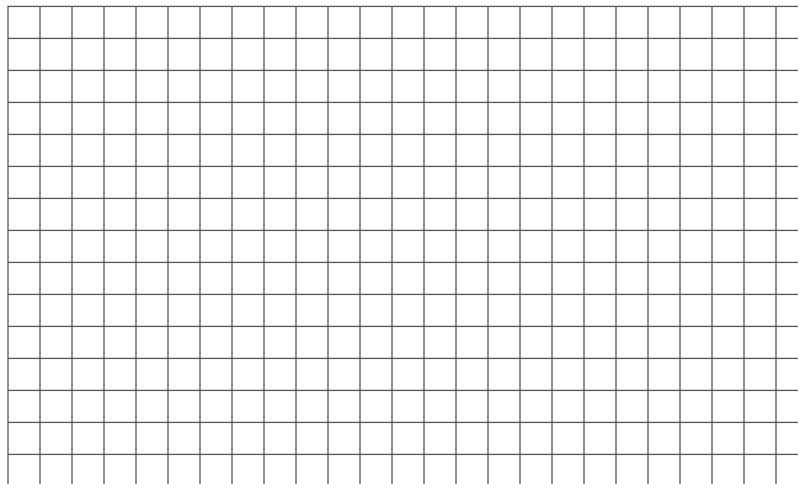


## Activity

### 1.3 Pairs of Scaled Polygons

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

1. Take turns with your partner to match a pair of polygons that are scaled copies of one another.
  - a. For each match you find, explain to your partner how you know it's a match.
  - b. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.
2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.
3. Select one pair of polygons to examine further. Draw both polygons on the grid. Explain or show how you know that one polygon is a scaled copy of the other.





### Are you ready for more?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.

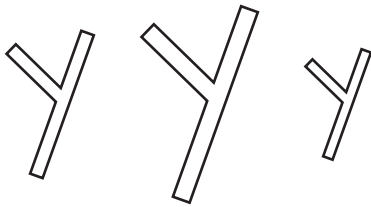


## Summary

### What Are Scaled Copies?

What is a **scaled copy** of a figure? Let's look at some examples.

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We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

### Glossary

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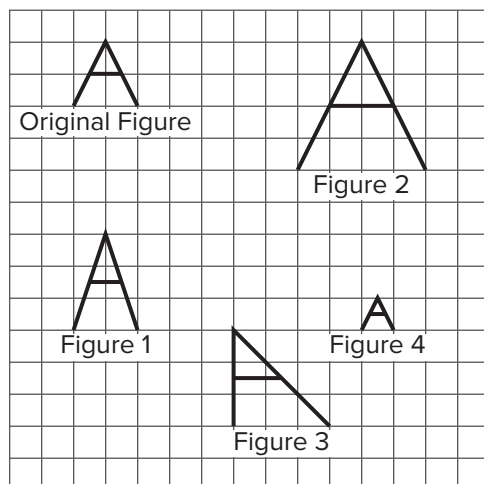
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## Practice

### What Are Scaled Copies?

- Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.



- Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.

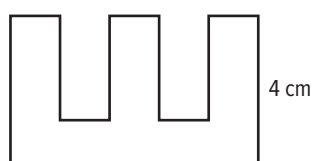


Figure A

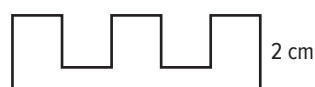


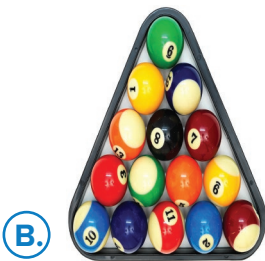
Figure B

Do you agree with Tyler? Explain your reasoning.

3. Here is a picture of a set of billiard balls in a rack.



Here are some copies of the picture. Select **all** the pictures that are scaled copies of the original picture.



4. Complete each equation with a number that makes it true.

a.  $5 \cdot \underline{\hspace{1cm}} = 15$

b.  $4 \cdot \underline{\hspace{1cm}} = 32$

c.  $6 \cdot \underline{\hspace{1cm}} = 9$

d.  $12 \cdot \underline{\hspace{1cm}} = 3$



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