McGraw-Hill

iM Illustrative Mathematics™

Course 1

Lesson Sampler

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Illustrative Mathematics™

Course 1

To accommodate different implementations of the program, course numbers rather than grade levels are referenced on the covers of McGraw-Hill *Illustrative Mathematics* materials. However, grade levels are referenced in the materials as this is how Illustrative Mathematics was originally written.

Course 1 = Grade 6, Course 2 = Grade 7, Course 3 = Grade 8

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Illustrative Mathematics is a problem-based core curriculum designed to address content and practice standards to foster learning for all. Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Teachers can shift their instruction and facilitate student learning with high-leverage routines to guide learners to understand and make connections between concepts and procedures.


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What is a Problem-based Curriculum?

In a problem-based curriculum, students work on carefully crafted and sequenced mathematics problems during most of the instructional time. Teachers help students understand the problems and guide discussions to be sure that the mathematical takeaways are clear to all. In the process, students explain their ideas and reasoning and learn to communicate mathematical ideas. The goal is to give students just enough background and tools to solve initial problems successfully, and then set them to increasingly sophisticated problems as their expertise increases.

The value of a problem-based approach is that students spend most of their time in math class doing mathematics: making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers. They go on to interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding.

"Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience . . ."[1]

Creating a World Where Learners Know, Use, and Enjoy Mathematics

Decades of research shows that students learn best when they are given a chance to start work on a problem before being shown a solution method. This gives students the chance to build conceptual understanding that can cement procedural skills by tying them together. It allows students to develop strategies for tackling non-routine problems and to engage in productive struggle.

Illustrative Mathematics is a problem-based curriculum designed to address content and practice standards to foster learning for all. Students are encouraged to take an active role to see what they can figure out before having things explained to them or being told what to do.

[1]Hiebert, J., et. al. (1996). Problem solving as a basis for reform in curriculum and instruction: the case of mathematics. Educational Researcher 25(4), 12-21. doi.org/10.3102/0013189X025004012

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As an IM Certified™ Partner, McGraw-Hill is committed to providing the support needed to successfully implement *Illustrative Mathematics.* A portion of every purchase is earmarked toward supporting the continued development of high-quality math curriculum.

Perfect Scores from EdReports

Grade 6	Grade 7	Grade 8
MEETS EXPECTATIONS	MEETS EXPECTATIONS	MEETS EXPECTATIONS
FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14
RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES Score: 18/18

Design Principles

Balancing Conceptual Understanding, Procedural Fluency, and Applications

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand, and be able to do, the mathematics.

Mathematical Practices are the Verbs of Math Class

In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

Build on What Students Know

New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them (NRC 2001) . In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students' learning.

Good Instruction Starts with Explicit Learning Goals

Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students' understanding. Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.

Different Learning Goals Require a Variety of Types of Tasks and Instructional Moves

The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- provide experience with a new context
- introduce a new concept and associated language
- introduce a new representation
- formalize the definition of a term for an idea previously encountered informally
- identify and resolve common mistakes and misconceptions
- practice using mathematical language
- work toward mastery of a concept or procedure
- provide an opportunity to apply mathematics to a modeling or other application problem

Each and Every Student Should Have Access to the Mathematical Work

With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional tool boxes should include knowledge of and skill in implementing supports for different learners. This curriculum incorporates extensive tools for specifically supporting English Language Learners and Students with Disabilities

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Instructional Model

Learning Goals and Targets

Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson. Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word “Let’s.” They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson. Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

Lesson Structure

1. INTRODUCE

Warm Up

Warm Up activities either:

- give students an opportunity to strengthen their number sense and procedural fluency.
- make deeper connections.
- encourage flexible thinking.

or:

- remind students of a context they have seen before.
- get them thinking about where the previous lesson left off.
- preview a calculation that will happen in the lesson.

2. EXPLORE AND DEVELOP

Classroom Activities

A sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class.

Each classroom activity has three phases.

The Launch

The teacher makes sure that students understand the context and what the problem is asking them to do.

Practice Problems

Each lesson includes an associated set of practice problems that may be assigned as homework or for extra practice in class. They can be collected and scored or used for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The design of practice problem sets looks different from many other curricula, but every choice was intentional, based on learning research, and meant to efficiently facilitate learning. The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Our approach emphasizes distributed practice rather than massed practice

Student Work Time
Students work individually, with a partner, or in small groups.

Activity Synthesis
The teacher orchestrates some time for students to synthesize what they have learned and situate the new learning within previous understanding.

3. SYNTHESIZE

Lesson Synthesis

Students incorporate new insights gained during the activities into their big-picture understanding.

Cool Down

A task to be given to students at the end of the lesson. Students are meant to work on the Cool Down for about 5 minutes independently and turn it in.

Instructional Routines

Plans include a set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

Like any routine in life, these routines give structure to time and interactions. They are a good idea for the same reason all routines are a good idea: they let people know what to expect, and they make people comfortable.

Why are routines in general good for learning academic content? One reason is that students and the teacher have done these interactions before, in a particular order, and so they don't have to spend much mental energy on classroom choreography. They know what to do when, who is expected to talk when, and when they are expected to write something down. The structure of the routine frees them up to focus on the academic task at hand. Furthermore, a well-designed routine opens up conversations and thinking about mathematics that might not happen by themselves.

- Algebra Talk
- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- Notice and Wonder
- Number Talk
- Poll the Class
- Take Turns
- Think Pair Share
- True or False
- Which One Doesn't Belong?


Topic Let's Put It to Work

Lesson 1-19

Designing a Tent

NAME ______ DATE ______ PERIOD ______

Learning Goal Let's design some tents.

Activity

19.1 Tent Design—Part 1

Have you ever been camping?

You might know that sleeping bags are all about the same size, but tents come in a variety of shapes and sizes.

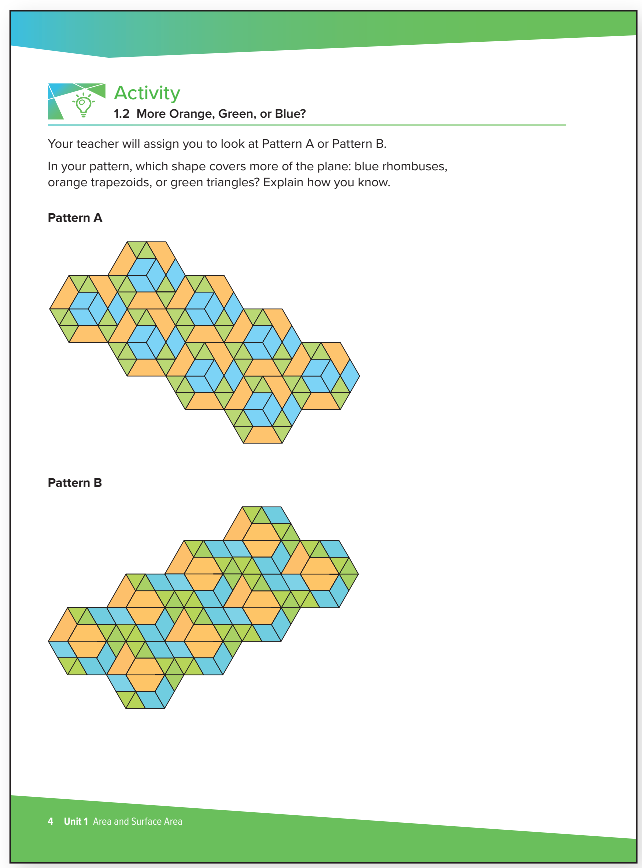
Your task is to design a tent to accommodate up to four people and estimate the amount of fabric needed to make your tent. Your design and estimate must be based on the information given and have mathematical justification.

First, look at these examples of tents, the average specifications of a camping tent, and standard sleeping bag measurements. Talk to a partner about:

- Similarities and differences among the tents
- Information that will be important in your designing process
- The pros and cons of the various designs

Tent Styles

Lesson 1-19 Designing a Tent **133**


Activity

1.2 More Orange, Green, or Blue?

Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, orange trapezoids, or green triangles? Explain how you know.

Pattern A

Pattern B

4 Unit 1 Area and Surface Area

How to Assess Progress

Illustrative Mathematics contains many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

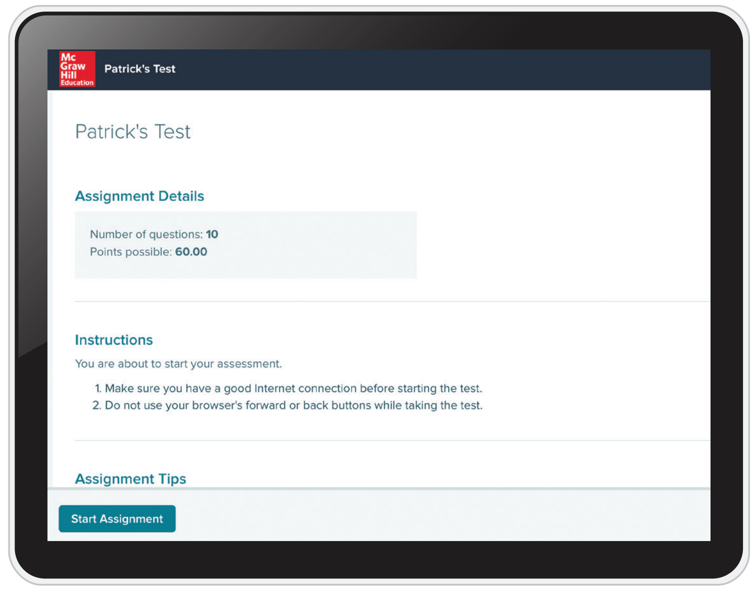
- Each unit begins with a diagnostic assessment ("Check Your Readiness") of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.

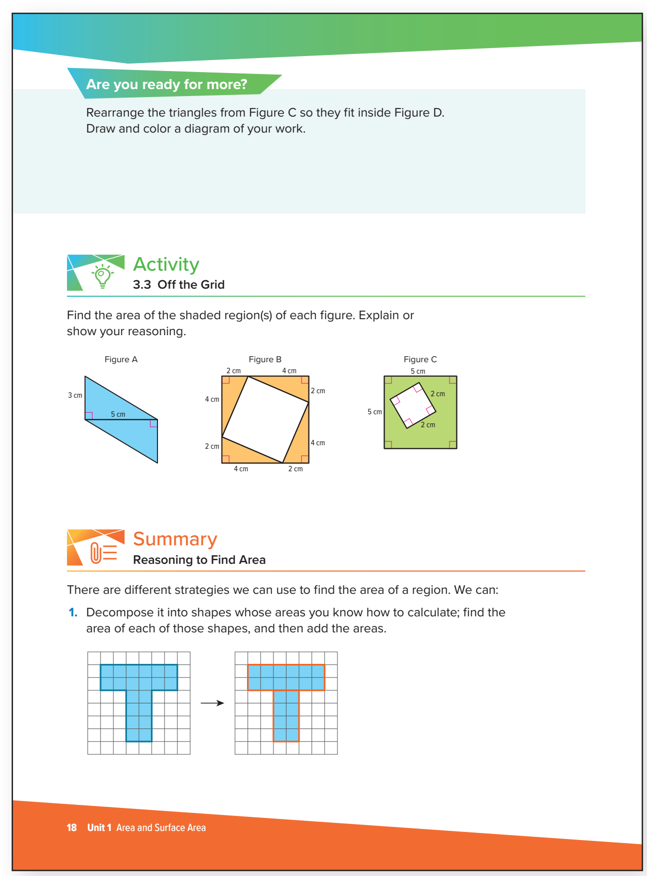
- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students' thinking.

- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day's lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.

- A set of cumulative practice problems is provided for each lesson that can be used for homework or in-class practice. The teacher can choose to collect and grade these or simply provide feedback to students.

- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.





Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. *Illustrative Mathematics* empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

Supporting English-language Learners

Illustrative Mathematics builds on foundational principles for supporting language development for all students. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

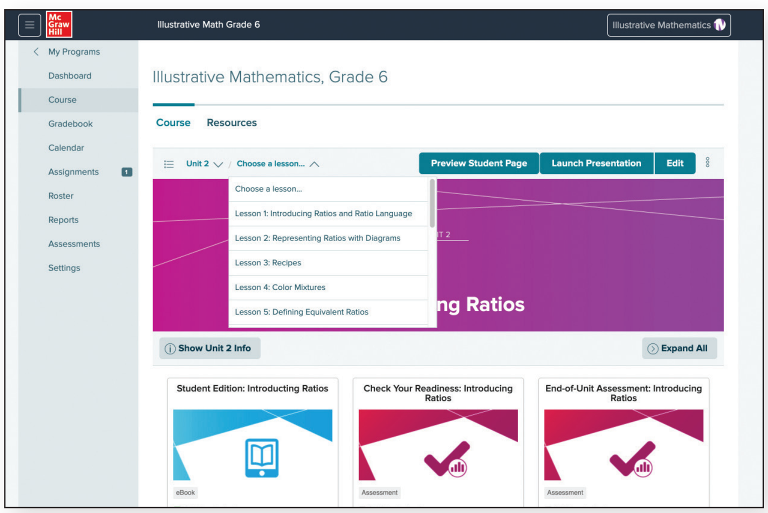
Aguirre, J.M. & Bunch, G. C. (2012). What's language got to do with it?: Identifying language demands in mathematics instruction for English Language Learners. In S. Celedón-Pattichis & N.

Digital

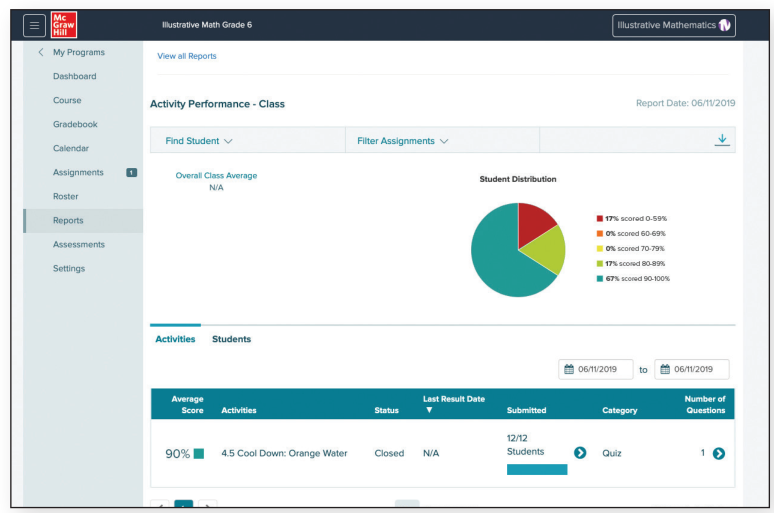
McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

Online resources offer:

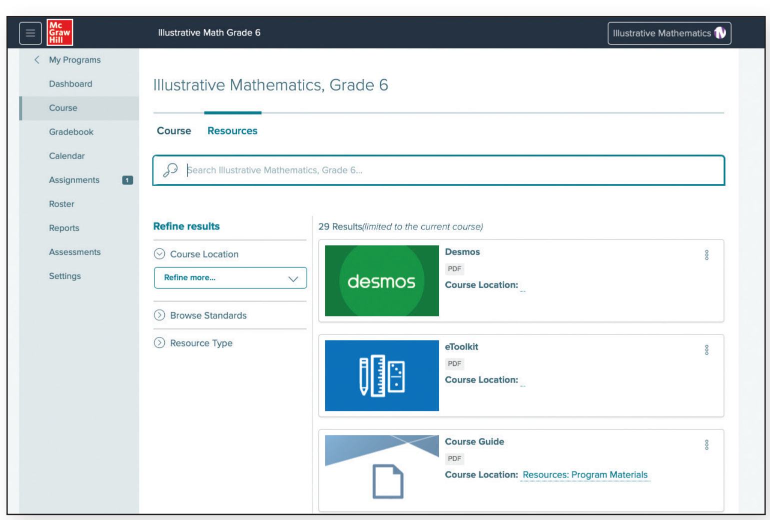
- customizable content
- the ability to add resources
- auto-scoring of student practice work
- on-going student assessments
- classroom performance reporting



Launch Presentations Digital versions of lessons to present content.



Reports Review the performance of individual students, classrooms, and grade levels.



Access Resources Point-of-use access to resources such as assessments, eBooks, and course guides.

Unit 1

Area and Surface Area

Unit 2

Introducing Ratios

Easy Production/Image Source

Unit 3

Unit Rates and Percentages

Unit 4

Dividing Fractions

Unit 5

Arithmetic in Base Ten

Unit 6

Expressions and Equations

Unit 7

Rational Numbers

Unit 8

Data Sets and Distributions

f11photo/Shutterstock

Unit 9

Putting It All Together

Unit 2

Introducing Ratios

Prior Work

Numbers and Operations

Work with ratios in Grade 6 draws on earlier work with numbers and operations.

- In elementary school, students worked to understand, represent, and solve arithmetic problems involving quantities with the same units.
- In Grade 4, students began to use two-column tables, e.g., to record conversions between measurements in inches and yards.
- In Grade 5, they began to plot points on the coordinate plane, building on their work with length and area.

Relationships Between Quantities

These early experiences were a brief introduction to two key representations used to study relationships between quantities, a major focus of work that begins in Grade 6 with the study of ratios.

- Starting in Grade 3, students worked with relationships that can be expressed in terms of ratios and rates (e.g., conversions between measurements in inches and in yards); however, they did not use these terms.
- In Grade 4, students studied multiplicative comparison.
- In Grade 5, they began to interpret multiplication as scaling, preparing them to think about simultaneously scaling two quantities by the same factor. They learned what it means to divide one whole number by another, so they are well equipped to consider the quotients $\frac{a}{b}$ and $\frac{b}{a}$ associated with a ratio $a : b$ for non-zero whole numbers a and b.

Work in This Unit

In this unit, students learn that a ratio is an association between two quantities, e.g., "1 teaspoon of drink mix to 2 cups of water." Students analyze contexts that are often expressed in terms of ratios, such as recipes, mixtures of different paint colors, constant speed (an association of time measurements with distance measurements), and uniform pricing (an association of item amounts with prices).

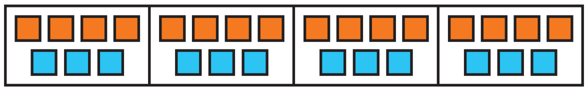
Guiding Principles

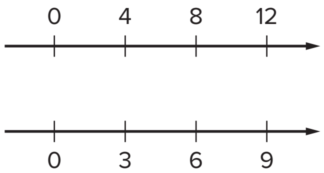
One of the principles that guided the development of these materials is that students should encounter examples of a mathematical concept in various contexts before the concept is named and studied as an object in its own right. The development of ratios, equivalent ratios, and unit rates in this unit and the next unit is in accordance with that principle.

- In this unit, equivalent ratios are first encountered in terms of multiple batches of a recipe and "equivalent" is first used to describe a perceivable sameness of two ratios, for example, two mixtures of drink mix and water taste the same or two mixtures of red and blue paint are the same shade of purple.
- Building on these experiences, students analyze situations involving both discrete and continuous quantities, and involving ratios of quantities with units that are the same and that are different.
- Several lessons later, equivalent acquires a more precise meaning. All ratios that are equivalent to $a : b$ can be made by multiplying both a and b by the same non-zero number (note that students are not yet considering negative numbers). MP6

Representations

This unit introduces *discrete diagrams* and *double number line diagrams*, representations that students use to support thinking about equivalent ratios before their work with tables of equivalent ratios.





Initially, discrete diagrams are used because they are similar to the kinds of diagrams students might have used to represent multiplication in earlier grades.

Next come double number line diagrams. These can be drawn more quickly than discrete diagrams, but are more similar to tables while allowing reasoning based on the length of intervals on the number lines.

After some work with double number line diagrams, students use tables to represent equivalent ratios.

(continued on the next page)

Because equivalent pairs of ratios can be written in any order in a table and there is no need to attend to the distance between values, tables are the most flexible and concise of the three representations for equivalent ratios, but they are also the most abstract. Use of tables to represent equivalent ratios is an important stepping stone toward use of tables to represent linear and other functional relationships in Grade 8 and beyond. Because of this, students should learn to use tables to solve all kinds of ratio problems, but they should always have the option of using discrete diagrams and double number line diagrams to support their thinking.

When a ratio involves two quantities with the same units, we can ask and answer questions about ratios of each quantity and the total of the two. Such ratios are sometimes called "part-part-whole" ratios and are often used to introduce ratio work. However, students often struggle with them so, in this unit, the study of part-part-whole ratios occurs at the end. (Note that tape diagrams are reserved for ratios in which all quantities have the same units.) The major use of part-part-whole ratios occurs with certain kinds of percentage problems, which comes in the next unit.

On using the terms *ratio*, *rate*, and *proportion*.

In these materials, a quantity is a measurement that is or can be specified by a number and a unit, e.g., 4 oranges, 4 centimeters, "my height in feet," or "my height" (with the understanding that a unit of measurement will need to be chosen).

The term *ratio* is used to mean an association between two or more quantities and the fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. Ratios of the form $1 : \frac{b}{a}$ and $\frac{a}{b} : 1$ (which are equivalent to $a : b$) are highlighted as useful but $\frac{a}{b}$ and $\frac{b}{a}$ are not identified as unit rates for the ratio $a : b$ until the next unit. However, the meanings of these fractions in contexts is very carefully developed. The word "per" is used with students in interpreting a unit rate in context, as in "$3 per ounce," and "at the same rate" is used to signify a situation characterized by equivalent ratios.

In the next unit, students learn the term *unit rate* and that if two ratios $a : b$ and $c : d$ are equivalent, then the unit rates $\frac{a}{b}$ and $\frac{c}{d}$ are equal.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as interpreting, explaining, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret	Explain	Compare
• ratio notation (Lesson 1) • different representations of ratios (Lesson 6) • situations involving equivalent ratios (Lesson 8) • situations with different rates (Lesson 9) • tables of equivalent ratios (Lesson 11 and 12) • questions about situations involving ratios (Lesson 17)	• features of ratio diagrams (Lesson 2) • reasoning about equivalence (Lesson 4) • reasoning about equivalent rates (Lesson 10) • reasoning with reference to tables (Lesson 14) • reasoning with reference to tape diagrams (Lesson 15)	• situations with and without equivalent ratios (Lessons 3) • representations of ratios (Lesson 6 and 13) • situations with different rates (Lesson 9 ans 12) • situations with same rates and different rates (Lessons 10) • representations of ratio and rate situations (Lessons 16)

In addition, students are expected to describe and represent ratio associations, represent doubling and tripling of quantities in a ratio, represent equivalent ratios, justify whether ratios are or aren't equivalent and why information is needed to solve a ratio problem, generalize about equivalent ratios and about the usefulness of ratio representations, and critique representations of ratios.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

	New Terminology	
Lesson	**Receptive**	**Productive**
2-1	**ratio** __ to __ __ for every __	
2-2	diagram	
2-3	recipe batch same taste	**ratio** __ to __ __ for every __
2-4	mixture same color check (an answer)	batch
2-5	**equivalent ratios**	
2-6	**double number line diagram** tick marks representation	diagram
2-7	**per**	
2-8	**unit price** how much for 1 at this rate	**double number line**
2-9	**meters per second** constant speed	
2-10	**same rate**	**equivalent ratios**
2-11	**table** row column	
2-14	calculation	**per** **table**
2-15	**tape diagram** parts suppose	
2-16		**tape diagram**

Required Materials

- ☐ beakers (Lesson 4)
- ☐ colored pencils (Lesson 2)
- ☐ drink mix (Lesson 3)
- ☐ empty containers (Lesson 3)
- ☐ food coloring (Lesson 4)
- ☐ graduated cylinders (Lesson 4)
- ☐ graph paper (Lessons 15, 16)
- ☐ markers (Lesson 3, 4)
- ☐ masking tape (Lesson 9)
- ☐ meter sticks (Lesson 9)
- ☐ paper cups (Lesson 3, 4)
- ☐ rulers (Lessons 6-8, 16)
- ☐ snap cubes (Lesson 15)
- ☐ stopwatches (Lesson 9)
- ☐ string (Lessons 8-9)
- ☐ students' collections of objects (Lesson 1)
- ☐ teacher's collections of objects (Lesson 1)
- ☐ teaspoon (Lesson 3)
- ☐ tools for creating a visual display (Lessos 1-2, 5, 8, 15-17)
- ☐ water (Lesson 3)

Blackline Activity Masters

- ☐ 6.2.2.4, *Card Sort: Spaghetti Sauce*, one per group (Lesson 2, Activity 2.4)
- ☐ 6.2.13.3, *The International Space Station*, one per group (Lesson 13, Activity 13.3)
- ☐ 6.2.14.2, *Info Gap: Hot Chocolate and Potatoes* (Lesson 2, Activity 14.2)
- ☐ 6.2.16.3, *Info Gap: Salad Dressing and Moving Boxes* (Lesson 16, Activity 16.3)

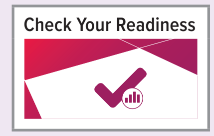
Unit Planner

Lessons		Days	Standards
Check Your Readiness Assessment		1	
Topic	**What Are Ratios?**		
	Lesson 2-1 Introducing Ratios and Ratio Language	1	6.RP.A.1
	Lesson 2-2 Representing Ratios with Diagrams	1	6.RP.A.1
Topic	**Equivalent Ratios**		
	Lesson 2-3 Recipes	1	6.RP.A.1
	Lesson 2-4 Color Mixtures	1	6.RP.A.1
	Lesson 2-5 Defining Equivalent Ratios	1	6.RP.A.1
Topic	**Representing Equivalent Ratios**		
	Lesson 2-6 Introducing Double Number Line Diagrams	1	6.RP.A.3
	Lesson 2-7 Creating Double Number Line Diagrams	1	6.RP.A.3
	Lesson 2-8 How Much for One?	1	6.RP.A.3.b
	Lesson 2-9 Constant Speed	1	6.RP.A.3.b
	Lesson 2-10 Comparing Situations by Examining Ratios	1	6.RP.A.2, 6.RP.A.3, 6.RP.A.3.b
Topic	**Solving Ratio and Rate Problems**		
	Lesson 2-11 Representing Ratios with Tables	1	6.RP.A.3.a
	Lesson 2-12 Navigating a Table of Equivalent Ratios	1	6.RP.A.3, 6.RP.A.3.a
	Lesson 2-13 Tables and Double Number Line Diagrams	1	6.RP.A.3, 6.RP.A.3.a
	Lesson 2-14 Solving Equivalent Ratio Problems	1	6.RP.A.3
Topic	**Part-Part-Whole Ratios**		
	Lesson 2-15 Part-Part-Whole Ratios	1	6.RP.A.3
	Lesson 2-16 Solving More Ratio Problems	1	6.RP.A.3
Topic	**Let's Put It to Work**		
	Lesson 2-17 A Fermi Problem	1	6.RP.A, 6.RP.A.3
End-of-Unit-Assessment		1	
	TOTAL	19	

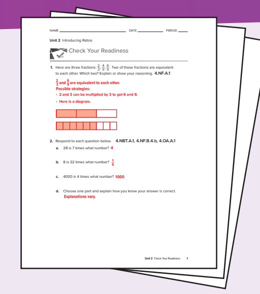
Pre-Unit Diagnostic Assessment

The pre-unit diagnostic assessment, *Check Your Readiness,* evaluates students' proficiency with prerequisite concepts and skills that students need to be successful in the unit. The Item Descriptions below offer guidance for students who may answer items incorrectly.

The assessment also may include problems that assess what students already know of the upcoming unit's key ideas, which you can use to pace or tune instruction. In rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.



Available as a digital assessment or a printable assessment.



1. Item Description

Students identify equivalent fractions and explain what it is that makes the fractions equivalent. This concept ties in with nearly all the work in this unit and the next: equivalent ratios, scaling, and units rates.

First Appearance of Skill or Concept: Lesson 6

If most students struggle with this item...

- Plan to use this item for some error analysis before beginning Lesson 6.

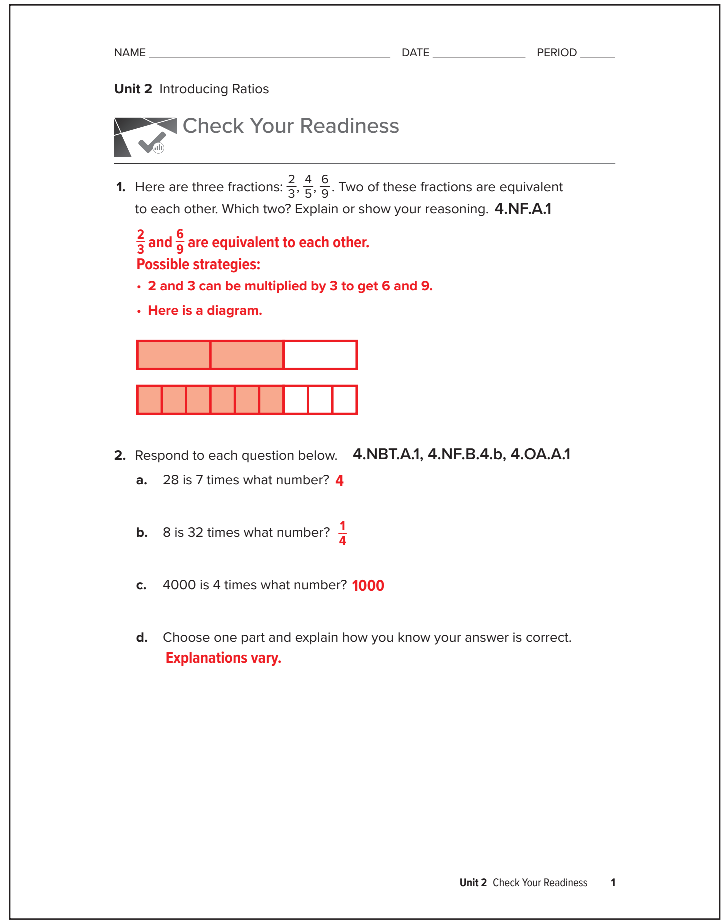
2. Item Description

In their work with equivalent ratios, students will often need to consider questions of the form, "B is A times what number?" The second part of this question involves reasoning with fractions; the third part requires reasoning with place value.

First Appearance of Skill or Concept: Lesson 2

If most students struggle with this item...

- Plan to expand the discussion during Activity 1 Number Talk.
- As you synthesize the Number Talk, consider including more related multiplication and division problems such as $24 \div 4 = 6$ and $4 \cdot 6 = 24$.



NAME ______________________ DATE ____________ PERIOD ______

Unit 2 Introducing Ratios

Check Your Readiness

1. Here are three fractions: $\frac{2}{3}, \frac{4}{5}, \frac{6}{9}$. Two of these fractions are equivalent to each other. Which two? Explain or show your reasoning. **4.NF.A.1**

$\frac{2}{3}$ and $\frac{6}{9}$ are equivalent to each other.
Possible strategies:
- **2 and 3 can be multiplied by 3 to get 6 and 9.**
- **Here is a diagram.**

2. Respond to each question below. **4.NBT.A.1, 4.NF.B.4.b, 4.OA.A.1**
   a. 28 is 7 times what number? **4**
   b. 8 is 32 times what number? $\frac{1}{4}$
   c. 4000 is 4 times what number? **1000**
   d. Choose one part and explain how you know your answer is correct. **Explanations vary.**

Unit 2 Check Your Readiness 1

(continued on the next page)

3. Item Description

Difficulty with placement of numbers and tick marks may be an indication that students need work with measurement conversions as discussed in the geometric measurement progression.

This issue may need more instructional attention than what is currently given. In this case, students need to subdivide the number line into fourths to figure out how to mark the three equally spaced tick marks between 0 and 24. Students will use this skill when they study double number lines in the upcoming unit.

First Appearance of Skill or Concept: Lesson 7

If most students struggle with this item....

- Plan to expand Lesson 7 Activity 1, *Ordering on a Number Line*, by including a few examples of partitioning a number line.

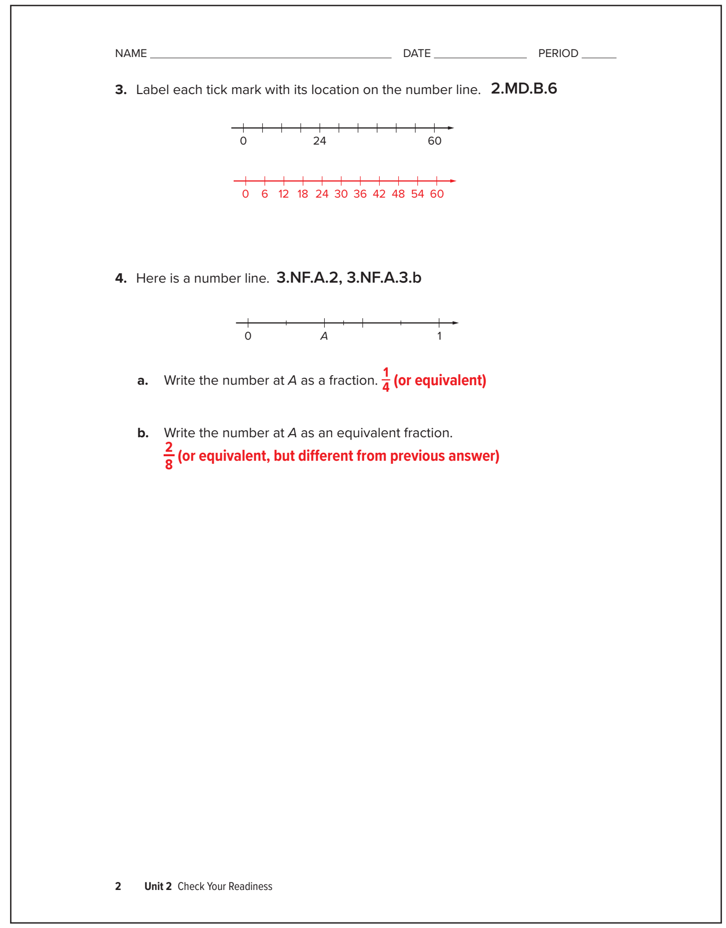
4. Item Description

This question is intended to assess whether the student understands that the interval from 0 to 1 must be partitioned into n parts of equal length in order for each subinterval to have length $\frac{1}{n}$.

First Appearance of Skill or Concept: Lesson 7

If most students struggle with this item....

- Plan to expand Lesson 7 Activity 1, *Ordering on a Number Line*, by including a few examples of equivalent fractions.



NAME ______________ DATE ________ PERIOD ____

3. Label each tick mark with its location on the number line. **2.MD.B.6**

4. Here is a number line. **3.NF.A.2, 3.NF.A.3.b**

a. Write the number at A as a fraction. $\frac{1}{4}$ **(or equivalent)**

b. Write the number at A as an equivalent fraction.
$\frac{2}{8}$ **(or equivalent, but different from previous answer)**

2 **Unit 2** Check Your Readiness

(continued on the next page)

5. Item Description

The two recipes in this problem contain the same ingredients, but in different amounts. Students will need to keep track of this information as they scale up the recipes. In addition to the conceptual work of scaling, this problem assesses students' comfort with multiplying a fraction by a whole number.

First Appearance of Skill or Concept: Lesson 3

If most students struggle with this item...

- Plan to do Activity 3, *Batches of Cookies*, to support their understanding of scaling using recipes.
- Then in Lesson 5 Activity 2, *Tuna Casserole*, students will practice multiplying a fraction times a whole number.

6. Item Description

This problem assesses students' conceptual understanding of speed. The car and the train both travel the same distance, but the car takes longer to get there. The fact that a longer time results in a slower speed may seem counterintuitive to students who do not have a solid understanding. Students having difficulty may need more real-life examples to help make this idea plausible to them.

First Appearance of Skill or Concept: Lesson 9

If most students struggle with this item...

- Plan to share a few examples similar to the one in the assessment before launching Activity 2, *Moving 10 Meters*.

NAME ______________________ DATE ____________ PERIOD ______

5. One batch of brownies calls for 1 box of brownie mix and the ingredients shown below. **4.NF.B.4.c**

Type of Brownie	Eggs	Water	Oil
fudge-like	2 eggs	$\frac{1}{4}$ cup	$\frac{1}{2}$ cup
cake-like	3 eggs	$\frac{1}{4}$ cup	$\frac{1}{2}$ cup

a. What amounts of eggs, water, and oil would you need for 2 batches of fudge-like brownies?

4 eggs, $\frac{1}{2}$ cup water; 1 cup oil (or equivalent)

b. What amounts of eggs, water, and oil would you need for 3 batches of cake-like brownies?

9 eggs, $\frac{3}{4}$ cup water; $1\frac{1}{2}$ cups oil (or equivalent)

6. Which travels faster: a car that travels 6 miles in 10 minutes at a constant speed, or a train that travels 6 miles in 8 minutes at a constant speed? Explain how you know. **6.RP.A.3**

The train travels faster because it covers the same distance as the car in less time.

Lesson 2-4

Color Mixtures

Goals

- Comprehend and respond (orally and in writing) to questions asking whether two ratios are equivalent, in the context of color mixtures.
- Draw and label a discrete diagram with circled groups to represent multiple batches of a color mixture.
- Explain equivalent ratios (orally and in writing) in terms of the amounts of each color in a mixture being multiplied by the same number to create another mixture that is the same shade.

Student Learning Goals

Let's see what color-mixing has to do with ratios.

Learning Targets

- I can explain the meaning of equivalent ratios using a color mixture as an example.
- I can use a diagram to represent a single batch, a double batch, and a triple batch of a color mixture.
- I know what it means to double or triple a color mixture.

Required Materials

- beakers
- food coloring
- graduated cylinders
- markers
- paper cups

Required Preparation

Mix blue water and yellow water; each group of 2 students will need 1 cup of each. To make colored water, add 1 teaspoon of food coloring to 1 cup of water. It would be best to give each mixture to students in a beaker or another container with a pour spout. If possible, conduct this lesson in a room with a sink.

Note that a digital version of this activity is available online. It is embedded in the digital version of the student materials, but if classrooms using the print version of materials have access to enough student devices, it could be used in place of mixing actual colored water.

Standards Alignment

Building On

4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

6.RP.A.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2 : 1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."

(continued on the next page)

Lesson Narrative

This is the second of two lessons that help students make sense of equivalent ratios through physical experiences. In this lesson, students mix different numbers of batches of a recipe for green water by combining blue and yellow water (created ahead of time with food coloring) to see if they produce the same shade of green. They also change the ratio of blue and yellow water to see if it changes the result. The activities here reinforce the idea that scaling a recipe up (or down) requires scaling the amount of each ingredient by the same factor. MP7

Students continue to use discrete diagrams as a tool to represent a situation.

For students who do not see color, the lesson can be adapted by having students make batches of dough with flour and water. 1 cup of flour to 5 tablespoons of water makes a very stiff dough, and 1 cup of flour to 6 tablespoons of water makes a soft (but not sticky) dough. In this case, doubling a recipe yields dough with the same tactile properties, just as doubling a colored-water recipe yields a mixture with the same color. The invariant property is stiffness rather than color. The principle that equivalent ratios yield products that are identical in some important way applies to both types of experiments.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3 Clarify, Critique, Correct
- MLR8 Discussion Supports
- Number Talk
- Think Pair Share

Lesson Pacing

	Pacing (min)
Warm Up 4.1 Number Talk: Adjusting a Factor	10
Activity 4.2 Turning Green	35
Activity 4.3 Perfect Purple Water*	10
Lesson Synthesis	5-10
Cool Down 4.4 Orange Water	5
optional activity* **TOTAL	**55-70**

Warm Up 4.1 Number Talk: Adjusting a Factor (10 minutes)

Standards Alignment

Building On: 4.NBT.B.5

This number talk encourages students to use the structure of base ten numbers and the properties of operations to find the product of two whole numbers. **MP7**

While many strategies may emerge, the focus of this string of problems is for students to see how adjusting a factor impacts the product, and how this insight can be used to reason about other problems. Four problems are given, however, it may not be possible to share every possible strategy. Consider gathering only two or three different strategies per problem. Each problem was chosen to elicit a slightly different reasoning, so as students explain their strategies, ask how the factors impacted how they approached the problem.

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- **Number Talk**
- **Mathematical Language Routines**
  - **MLR8 Discussion Supports**

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Support For Students with Disabilities

Representation: Internalize Comprehension To support working memory, provide students with sticky notes or mini whiteboards.

Supports Accessibility for: Memory, Organization

Student Task Statement

Find the value of each product mentally.

1. 6 • 15 90. Possible strategy: (6 • 10) + (6 • 5) = 90
2. 12 • 15 180. Possible strategy: Since the 6 from the first question doubled to 12, and the 15 stayed the same, the product doubles to 180. This is because there are twice as many groups of 15 than in the first question.
3. 6 • 45 270. Possible strategy: Since the 6 is the same as the 6 in the first question, and the 15 tripled to 45, the product triples to 270. This is because the number of groups stayed the same, but the amount in each group got three times as large.
4. 13 • 45 585. Possible strategy: Since the 45 is the same as the previous question, we can double the 6 and the product to get 540. We need one more group of 45, and 540 + 45 = 585.

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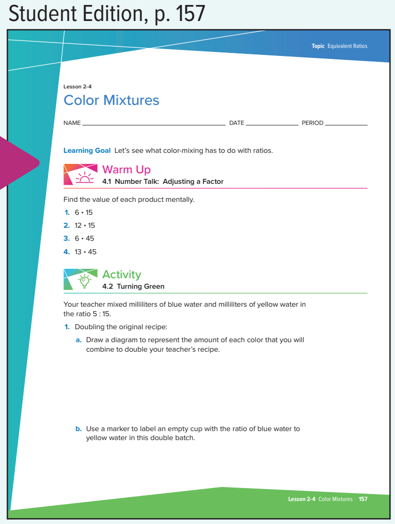

Topic Equivalent Ratios

Lesson 2-4

Color Mixtures

NAME ____________ DATE ________ PERIOD ________

Learning Goal Let's see what color-mixing has to do with ratios.

Warm Up
4.1 Number Talk: Adjusting a Factor

Find the value of each product mentally.

1. 6 • 15
2. 12 • 15
3. 6 • 45
4. 13 • 45

Activity
4.2 Turning Green

Your teacher mixed milliliters of blue water and milliliters of yellow water in the ratio 5 : 15.

1. Doubling the original recipe:
   a. Draw a diagram to represent the amount of each color that you will combine to double your teacher's recipe.
   b. Use a marker to label an empty cup with the ratio of blue water to yellow water in this double batch.

Lesson 2-4 Color Mixtures 157

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. Ask students if or how the factors in the problem impacted the strategy choice. To involve more students in the conversation, consider asking:

- Who can restate ___'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?
- Did anyone solve the problem in a different way?
- Does anyone want to add on to _____'s strategy?
- Do you agree or disagree? Why?

(continued on the next page)

Support For English Language Learners

Speaking

MLR8 Discussion Supports Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

Activity 4.2 Turning Green (35 minutes)

Standards Alignment

Addressing: 6.RP.A.1

In this activity, students mix different numbers of batches of a color recipe to obtain a certain shade of green. They observe how multiple batches of the same recipe produce the same shade of green as a single batch, which suggests that the ratios of blue to yellow for the two situations are equivalent. They also come up with a ratio that is not equivalent to produce a mixture that is a different shade of green.

As students make the mixtures, ensure that they measure accurately so they will get accurate outcomes. As students work, note the different diagrams students use to represent their recipes. Select a few examples that could be highlighted in discussion later.

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- **Mathematical Language Routines**
  - **MLR8 Discussion Supports**

Launch

Arrange students in groups of 2–4. (Smaller groups are better, but group size might depend on available equipment.) Each group needs a beaker of blue water and one of yellow water, one graduated cylinder, a permanent marker, a craft stick, and 3 opaque white cups (either styrofoam, white paper, or with a white plastic interior).

Print Activity: Show students the blue and yellow water. Demonstrate how to pour from the beakers to the graduated cylinder to measure and mix 5 ml of blue water with 15 ml of yellow water. Demonstrate how to get an accurate reading on the graduated cylinder by working on a level surface and by reading the measurement at eye level. Tell students they will experiment with different mixtures of green water and observe the resulting shades.

Digital Activity: Display the dynamic color mixing cylinders for all to see. Tell students, “The computer mixes colors when you add colored water to each cylinder. You can add increments of 1 or 5. You can’t remove water (once it’s mixed, it’s mixed), but you can start over. The computer mixes yellow and blue.” Ask students a few familiarization questions before they start working on the activity:

- What happens when you mix yellow and blue? A shade of green is formed.
- What happens if you add more blue than yellow? Darker green, blue green, etc.

If necessary, demonstrate how it works by adding some yellows and blues to both the left and the right cylinder. Show how the “Reset” button lets you start over.

Go online to find the digital version of this activity.

Support For Students with Disabilities

Representation: Internalize Comprehension Provide appropriate reading accommodations and supports to ensure students’ access to written directions, word problems and other text-based content.

Supports Accessibility for: Language, Conceptual Processing

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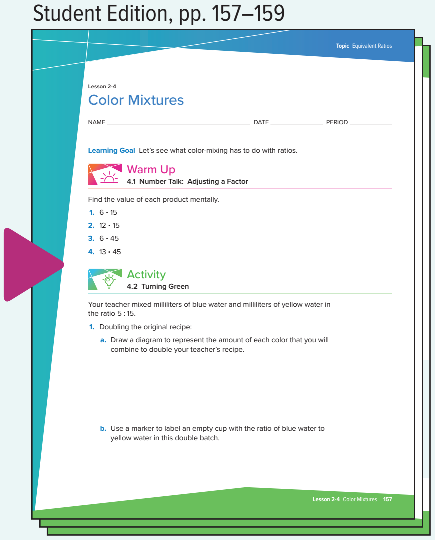


Topic Equivalent Ratios

Lesson 2-4

Color Mixtures

NAME ______ DATE ______ PERIOD ______

Learning Goal Let's see what color-mixing has to do with ratios.

Warm Up

4.1 Number Talk: Adjusting a Factor

Find the value of each product mentally.

1. 6 • 15
2. 12 • 15
3. 6 • 45
4. 13 • 45

Activity

4.2 Turning Green

Your teacher mixed milliliters of blue water and milliliters of yellow water in the ratio 5 : 15.

1. Doubling the original recipe:
   a. Draw a diagram to represent the amount of each color that you will combine to double your teacher's recipe.
   b. Use a marker to label an empty cup with the ratio of blue water to yellow water in this double batch.

Lesson 2-4 Color Mixtures 157

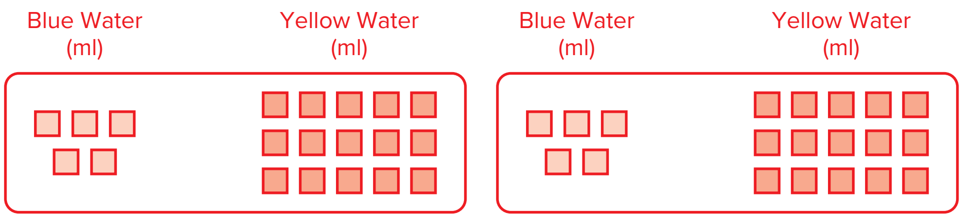
Anticipated Misconceptions

If any students come up with an incorrect recipe, consider letting this play out. A different shade of green shows that the ratio of blue to yellow in their mixture is not equivalent to the ratio of blue to yellow in the other mixtures. Rescuing the incorrect mixture to display during discussion may lead to meaningful conversations about what equivalent ratios mean.

Student Task Statement

Your teacher mixed milliliters of blue water and milliliters of yellow water in the ratio 5 : 15.

1. Doubling the original recipe:
   a. Draw a diagram to represent the amount of each color that you will combine to double your teacher's recipe. Here is one example of a diagram. Students may arrange the groups differently or use different symbols to represent 1 ml of water.

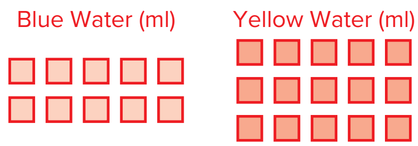


   b. Use a marker to label an empty cup with the ratio of blue water to yellow water in this double batch. A cup is labeled 10 : 30 or "10 to 30."
   c. Predict whether these amounts of blue and yellow will make the same shade of green as your teacher's mixture. Next, check your prediction by measuring those amounts and mixing them in the cup. If the recipe is correct, the shade of green is identical to the teacher's.
   d. Is the ratio in your mixture equivalent to the ratio in your teacher's mixture? Explain your reasoning. 10 : 30 is equivalent to 5 : 15 because it is 2 batches of the same recipe. It creates an identical shade of green.
2. Tripling the original recipe:
   a. Draw a diagram to represent triple your teacher's recipe. Like the previous diagram, except showing 3 batches.
   b. Label an empty cup with the ratio of blue water to yellow water. A cup is labeled 15 : 45 or "15 to 45."
   c. Predict whether these amounts will make the same shade of green. Next, check your prediction by mixing those amounts. If the recipe is correct, the shade of green is identical to the teacher's.
   d. Is the ratio in your new mixture equivalent to the ratio in your teacher's mixture? Explain your reasoning. 15 : 45 is equivalent to 5 : 15 because it is 3 batches of the same recipe. It creates an identical shade of green.

(continued on the next page)

3. Next, invent your own recipe for a *bluer* shade of green water.

a. Draw a diagram to represent the amount of each color you will combine. Answers vary. You might use more blue for the same amount of yellow, or less yellow for the same amount of blue. Sample response:



b. Label the final empty cup with the ratio of blue water to yellow water in this recipe. Answers vary. Sample responses: 10 : 15 (more blue for the same amount of yellow) or 5 : 10 (less yellow for the same amount of blue).

c. Test your recipe by mixing a batch in the cup. Does the mixture yield a bluer shade of green? If a correct ratio is used, the mixture should be a bluer shade of green than the other mixtures.

d. Is the ratio you used in this recipe equivalent to the ratio in your teacher's mixture? Explain your reasoning. No, it was not the same shade of green. The first and second parts were not, respectively, obtained by multiplying 5 and 15 by the same number.

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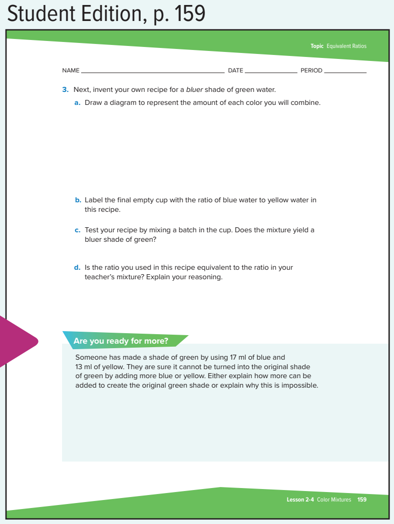
Topic: Equivalent Ratios

NAME ______ DATE ______ PERIOD ______

3. Next, invent your own recipe for a bluer shade of green water.

a. Draw a diagram to represent the amount of each color you will combine.

b. Label the final empty cup with the ratio of blue water to yellow water in this recipe.

c. Test your recipe by mixing a batch in the cup. Does the mixture yield a bluer shade of green?

d. Is the ratio you used in this recipe equivalent to the ratio in your teacher's mixture? Explain your reasoning.

Are you ready for more?

Someone has made a shade of green by using 17 ml of blue and 13 ml of yellow. They are sure it cannot be turned into the original shade of green by adding more blue or yellow. Either explain how more can be added to create the original green shade or explain why this is impossible.

Lesson 2-4 Color Mixtures 159

Are you ready for more?

Someone has made a shade of green by using 17 ml of blue and 13 ml of yellow. They are sure it cannot be turned into the original shade of green by adding more blue or yellow. Either explain how more can be added to create the original green shade, or explain why this is impossible. You could add 3 ml of blue to get 20 ml of blue, and 47 ml of yellow to get 60 ml of yellow. The blue to yellow ratio of 20 : 60 will make the same shade of green as 5 : 15. It's a quadruple batch.

Activity Synthesis

After each group has completed the task, have the students rotate through each group's workspace to observe the mixtures and diagrams. As they circulate, pose some guiding questions. (For students using the digital version, these questions refer to the mixtures on their computers.)

- Are each group's results for the first two mixtures the same shade of green?
- Are the ratios representing the double batch, the triple batch, and your new mixture all equivalent to each other? How do you know?
- What are some different ways groups drew diagrams to represent the ratios?

Highlight the idea that a ratio is equivalent to another if the two ratios describe different numbers of batches of the same recipe.

Support For English Language Learners

Conversing

MLR8 Discussion Supports Assign one member from each group to stay behind to answer questions from students visiting from other groups. Provide visitors with question prompts such as, "These look like the same shade, how can we be sure the ratios are equivalent?", "If you want a smaller amount with the same shade, what can you do?" or "What is the ratio for your new mixture?"

Design Principle(s): Cultivate conversation

Standards Alignment

Addressing: **6.RP.A.1**

Activity 4.3 Perfect Purple Water (optional, 10 minutes)

Students revisit color mixing—this time by producing purple-colored water—to further understand equivalent ratios. They recall that doubling, tripling, or halving a recipe for colored water yields the same resulting color, and that equivalent ratios can represent different numbers of batches of the same recipe.

As students work, monitor for students who use different representations to answer both questions, as well as students who come up with different ratios for the second question.

Instructional Routines

- **Anticipate, Monitor, Select, Sequence, Connect**
- **Mathematical Language Routines**
  - **MLR3** **Clarify, Critique, Correct**
- **Think Pair Share**

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

Launch

Arrange students in groups of 2. Remind students of the previous “Turning Green” activity. Ask students to discuss the following questions with a partner. Then, discuss responses together as a whole class:

- **Why did the different mixtures of blue and yellow water result in the same shade of green?** If mixed correctly, the amount of the ingredients were all doubled or all tripled. The ratio of blue water to yellow water was equivalent within each recipe.
- **How were you able to get a darker shade of green?** We changed the ratio of ingredients, so there was more blue for the same amount of yellow.

Explain to students that the task involves producing purple-colored water, but they won’t actually be mixing colored water. Ask students to use the ideas just discussed from the previous activity to predict the outcomes of mixing blue and red water.

Ensure students understand the abbreviation for milliliters is ml.

Support For Students with Disabilities

Representation: Develop Language and Symbols Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with snap cubes, blocks or printed representations.

Supports Accessibility for: Conceptual Processing

Anticipated Misconceptions

At a quick glance, students may think that since Andre is mixing a multiple of 8 with a multiple of 3, it will also result in Perfect Purple Water. If this happens, ask them to take a closer look at how the values are related or draw a diagram showing batches.

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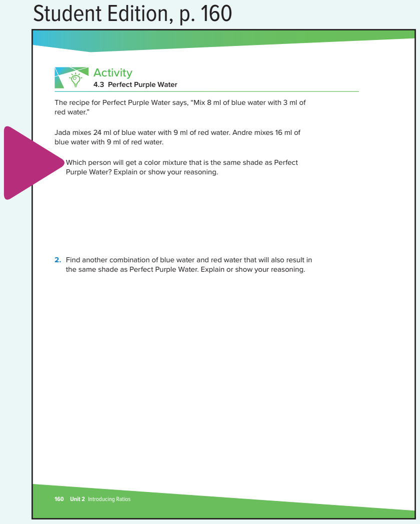
Activity
4.3 Perfect Purple Water
The recipe for Perfect Purple Water says, “Mix 8 ml of blue water with 3 ml of red water.”
Jada mixes 24 ml of blue water with 9 ml of red water. Andre mixes 16 ml of blue water with 9 ml of red water.
Which person will get a color mixture that is the same shade as Perfect Purple Water? Explain or show your reasoning.
2. Find another combination of blue water and red water that will also result in the same shade as Perfect Purple Water. Explain or show your reasoning.
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Student Task Statement

The recipe for Perfect Purple Water says, “Mix 8 ml of blue water with 3 ml of red water.” Jada mixes 24 ml of blue water with 9 ml of red water. Andre mixes 16 ml of blue water with 9 ml of red water.

1. Which person will get a color mixture that is the same shade as Perfect Purple Water? Explain or show your reasoning. Jada’s mixture will result in the same shade of purple, because both ingredients were tripled. $8 \cdot 3 = 24$ and $3 \cdot 3 = 9$. Andre’s mixture will not result in the same shade of purple, because the amount of red water is doubled, but the amount of blue water was tripled.

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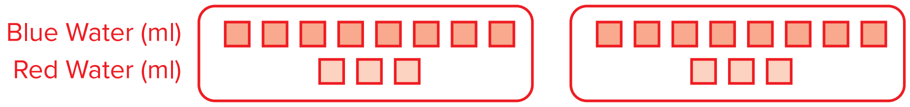
2. Find another combination of blue water and red water that will also result in the same shade as Perfect Purple Water. Explain or show your reasoning. Answers vary. One possible answer is 16 : 6 (each ingredient is doubled or multiplied by 2. 8 • 2 = 16, there are 16 ml blue. 3 • 2 = 6, there are 6 ml red.)



Activity Synthesis

Select students to share their answers to the questions.

- For the first question, emphasize that not only did Jada triple each amount of red and blue, but this means that amount of each color is being *multiplied by the same value*, in this case, 3.
- For the second question, list all the different ratios students brought up for all to see. Discuss how each ratio differed from that for the original mixture. Point out that as long as both terms are multiplied by the same quantity, the resulting ratio will be *equivalent* and will yield the same shade of purple.

Support For English Language Learners

Reading, Writing, Speaking

MLR3 **Clarify, Critique, Correct** Before students share their combination of blue water and red water that will make the same shade as Perfect Purple Water, present a flawed response. For example, "Mixing 18 ml of blue water with 13 ml of red water would result in the same shade of purple because I added 10 ml of each color." Ask students to identify the error, critique the reasoning, and write a correct explanation. Invite students to share their critiques and corrected combinations and explanations with the class. Listen for and amplify the language students use to justify the ratios are equivalent. This may include such language as multiply by the same quantity or represent the same shade. This helps students evaluate, and improve upon, the written mathematical arguments of others, as they clarify their understanding of equivalent ratios.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Lesson Synthesis Color Mixtures

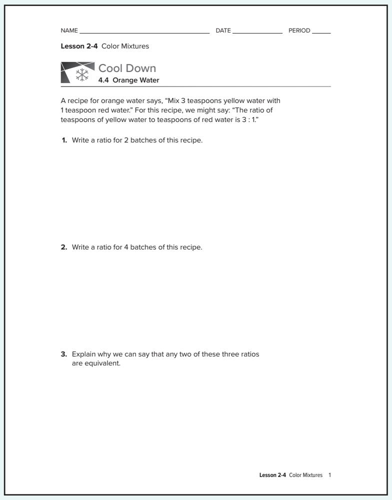
The important take-aways from this lesson are:

- To create more batches of a color recipe that will come out to be the same shade of the color, multiply each ingredient by the same number.
- We can think of equivalent ratios as representing different numbers of batches of the same recipe.

Remind students of the work done and observations made in this lesson. Some questions to guide the discussion might include:

- **How did you decide that 10 ml blue and 30 ml yellow would make 2 batches of 5 ml blue and 15 ml yellow?** Multiply each part by 2.
- **How did you decide that 15 ml blue and 45 ml yellow would make 3 batches?** Multiply each part by 3.
- **How did we know that 5:15, 10:30, and 15:45 were equivalent?** They created the same shade of green. Also, 10 : 30 has both parts of the original recipe multiplied by 2, and 15 : 45 has both parts of the original recipe multiplied by 3.

Printable Cool Down


NAME ____ DATE ____ PERIOD ____

Lesson 2-4 Color Mixtures

Cool Down
4.4 Orange Water

A recipe for orange water says, "Mix 3 teaspoons yellow water with 1 teaspoon red water." For this recipe, we might say: "The ratio of teaspoons of yellow water to teaspoons of red water is 3 : 1."

1. Write a ratio for 2 batches of this recipe.

2. Write a ratio for 4 batches of this recipe.

3. Explain why we can say that any two of these three ratios are equivalent.

Lesson 2-4 Color Mixtures 1

Standards Alignment

Addressing: 6.RP.A.1

Cool Down 4.4 Orange Water (5 minutes)

Student Task Statement

A recipe for orange water says, "Mix 3 teaspoons yellow water with 1 teaspoon red water." For this recipe, we might say: "The ratio of teaspoons of yellow water to teaspoons of red water is 3 : 1."

1. Write a ratio for 2 batches of this recipe. The ratio of teaspoons of yellow to teaspoons of red is 6 : 2 (or any sentence that accurately states this ratio). Note: a statement like "The ratio of yellow to red is 6 : 2" describes the association between the colors but does not indicate the amount of stuff in the mixture.
2. Write a ratio for 4 batches of this recipe. The ratio of teaspoons of yellow to teaspoons of red is 12 : 4 (or any sentence that accurately states this ratio).
3. Explain why we can say that any two of these three ratios are equivalent. These are equivalent ratios because they describe different numbers of batches of the same recipe. To make 2 batches, multiply the amount of each color by 2. To make 4 batches, multiply the amount of each color by 4. As long as you multiply the amounts for both colors by the same number, you will get a ratio that is equivalent to the ratio in the recipe.

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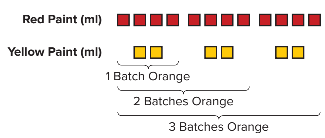
Summary
Color Mixtures

When mixing colors, doubling or tripling the amount of each color will create the same shade of the mixed color. In fact, you can always multiply the amount of *each* color by *the same number* to create a different amount of the same mixed color.

For example, a batch of dark orange paint uses 4 ml of red paint and 2 ml of yellow paint.

- To make two batches of dark orange paint, we can mix 8 ml of red paint with 4 ml of yellow paint.
- To make three batches of dark orange paint, we can mix 12 ml of red paint with 6 ml of yellow paint.

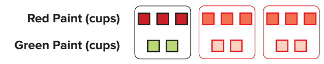
Here is a diagram that represents 1, 2, and 3 batches of this recipe.



We say that the ratios 4 : 2, 8 : 4, and 12 : 6 are *equivalent* because they describe the same color mixture in different numbers of batches, and they make the same shade of orange.

Practice
Color Mixtures

1. Here is a diagram showing a mixture of red paint and green paint needed for 1 batch of a particular shade of brown. Add to the diagram so that it shows 3 batches of the same shade of brown paint.



Answers vary. Sample response shown.

2. Diego makes green paint by mixing 10 tablespoons of yellow paint and 2 tablespoons of blue paint. Which of these mixtures produce the same shade of green paint as Diego's mixture? Select **all** that apply.

A. For every 5 tablespoons of blue paint, mix in 1 tablespoon of yellow paint.

B. Mix tablespoons of blue paint and yellow paint in the ratio 1 : 5.

C. Mix tablespoons of yellow paint and blue paint in the ratio 15 to 3.

D. Mix 11 tablespoons of yellow paint and 3 tablespoons of blue paint.

E. For every tablespoon of blue paint, mix in 5 tablespoons of yellow paint.

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3. To make 1 batch of sky blue paint, Clare mixes 2 cups of blue paint with 1 gallon of white paint.

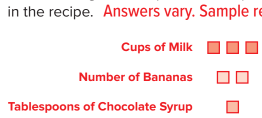
a. Explain how Clare can make 2 batches of sky blue paint.
Mix 4 cups of blue paint and 2 gallons of white paint.

b. Explain how to make a mixture that is a darker shade of blue than the sky blue.
Answers vary. Sample response: 3 cups of blue paint and 1 gallon of white paint. Mixing the same amount of white paint with *more* blue paint will make a darker shade of blue.

c. Explain how to make a mixture that is a lighter shade of blue than the sky blue.
Answers vary. Sample response: 2 cups of blue paint and 2 gallons of white paint. Mixing the same amount of blue paint with *more* white paint will make a lighter shade of blue.

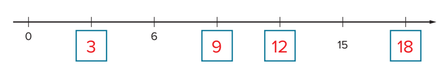
4. A smoothie recipe calls for 3 cups of milk, 2 frozen bananas, and 1 tablespoon of chocolate syrup. (Lesson 2-2)

a. Create a diagram to represent the quantities of each ingredient in the recipe. Answers vary. Sample response:

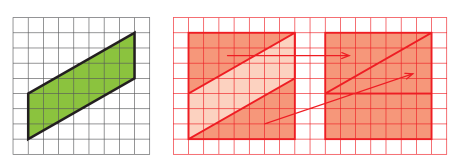


b. Write 3 different sentences that use a ratio to describe the recipe.
Answers vary. Sample response: The ratio of cups of milk to number of bananas is 3 : 2. The ratio of bananas to tablespoons of chocolate syrup is 2 to 1. For every tablespoon of chocolate syrup, there are 3 cups of milk.

5. Write the missing number under each tick mark on the number line. (Lesson 2-1)



6. Find the area of the parallelogram. Show your reasoning. (Lesson 1-4)



21 square units. Reasoning varies. Sample reasoning: Draw a square around the parallelogram; its area is 49 square units, because $7 \cdot 7 = 49$. Rearrange the triangles above and below the parallelogram to form a rectangle; the area of this rectangle is 28 square units, because $4 \cdot 7 = 28$. Subtracting the area of the triangles from the area of the square, we have 21 square units. $49 - 28 = 21$.

7. Complete each equation with a number that makes it true. (Lesson 2-1)

a. $11 \cdot \frac{1}{4} = \frac{11}{4}$ (or equivalent)

b. $7 \cdot \frac{1}{4} = \frac{7}{4}$ (or equivalent)

c. $13 \cdot \frac{1}{27} = \frac{13}{27}$ (or equivalent)

d. $13 \cdot \frac{1}{99} = \frac{13}{99}$ (or equivalent)

e. $x \cdot \frac{1}{y} = \frac{x}{y}$ (or equivalent)
(As long as y does not equal 0.)

Student Edition

Lesson 2-4

Color Mixtures

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Learning Goal Let's see what color-mixing has to do with ratios.

Warm Up

4.1 Number Talk: Adjusting a Factor

Find the value of each product mentally.

1. 6 • 15
2. 12 • 15
3. 6 • 45
4. 13 • 45

Activity

4.2 Turning Green

Your teacher mixed milliliters of blue water and milliliters of yellow water in the ratio 5 : 15.

1. Doubling the original recipe:
   a. Draw a diagram to represent the amount of each color that you will combine to double your teacher's recipe.

   b. Use a marker to label an empty cup with the ratio of blue water to yellow water in this double batch.

c. Predict whether these amounts of blue and yellow will make the same shade of green as your teacher's mixture. Next, check your prediction by measuring those amounts and mixing them in the cup.

d. Is the ratio in your mixture equivalent to the ratio in your teacher's mixture? Explain your reasoning.

2. Tripling the original recipe:

a. Draw a diagram to represent triple your teacher's recipe.

b. Label an empty cup with the ratio of blue water to yellow water.

c. Predict whether these amounts will make the same shade of green. Next, check your prediction by mixing those amounts.

d. Is the ratio in your new mixture equivalent to the ratio in your teacher's mixture? Explain your reasoning.

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3. Next, invent your own recipe for a *bluer* shade of green water.

a. Draw a diagram to represent the amount of each color you will combine.

b. Label the final empty cup with the ratio of blue water to yellow water in this recipe.

c. Test your recipe by mixing a batch in the cup. Does the mixture yield a bluer shade of green?

d. Is the ratio you used in this recipe equivalent to the ratio in your teacher's mixture? Explain your reasoning.

Are you ready for more?

Someone has made a shade of green by using 17 ml of blue and 13 ml of yellow. They are sure it cannot be turned into the original shade of green by adding more blue or yellow. Either explain how more can be added to create the original green shade or explain why this is impossible.

Activity

4.3 Perfect Purple Water

The recipe for Perfect Purple Water says, “Mix 8 ml of blue water with 3 ml of red water.”

Jada mixes 24 ml of blue water with 9 ml of red water. Andre mixes 16 ml of blue water with 9 ml of red water.

1. Which person will get a color mixture that is the same shade as Perfect Purple Water? Explain or show your reasoning.

2. Find another combination of blue water and red water that will also result in the same shade as Perfect Purple Water. Explain or show your reasoning.

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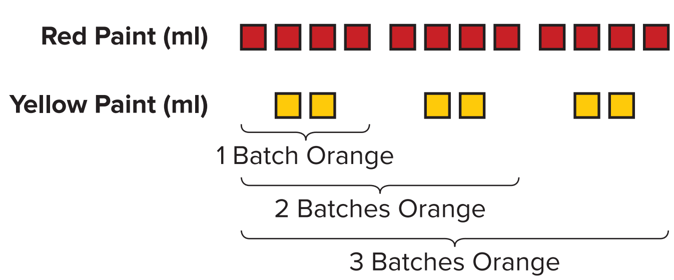

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Practice

Color Mixtures

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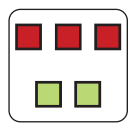
Red Paint (cups)

Green Paint (cups)



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   D. Mix 11 tablespoons of yellow paint and 3 tablespoons of blue paint.

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a. Explain how Clare can make 2 batches of sky blue paint.

b. Explain how to make a mixture that is a darker shade of blue than the sky blue.

c. Explain how to make a mixture that is a lighter shade of blue than the sky blue.

4. A smoothie recipe calls for 3 cups of milk, 2 frozen bananas, and 1 tablespoon of chocolate syrup. **(Lesson 2-2)**

a. Create a diagram to represent the quantities of each ingredient in the recipe.

b. Write 3 different sentences that use a ratio to describe the recipe.

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