McGraw-Hill Illustrative Mathematics Algebra 2



Lesson Sampler



McGraw-Hill Illustrative Mathematics

Algebra 2

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Illustrative Mathematics is a problem-based core curriculum designed to address content and practice standards to foster learning for all. Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Teachers can shift their instruction and facilitate student learning with high-leverage routines to guide learners to understand and make connections between concepts and procedures.



What is a Problem-based Curriculum?

In a problem-based curriculum, students work on carefully crafted and sequenced mathematics problems during most of the instructional time. Teachers help students understand the problems and guide discussions to be sure that the mathematical takeaways are clear to all. In the process, students explain their ideas and reasoning and learn to communicate mathematical ideas. The goal is to give students just enough background and tools to solve initial problems successfully, and then set them to increasingly sophisticated problems as their expertise increases.

The value of a problem-based approach is that students spend most of their time in math class doing mathematics: making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers. They go on to interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding.

Design Principles

Balancing Conceptual Understanding, Procedural Fluency, and Applications

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand, and be able to do, the mathematics.

Mathematical Practices are the Verbs of Math Class

In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

Build on What Students Know

New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them (NRC 2001). In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students' learning.

Good Instruction Starts with Explicit Learning Goals

Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students' understanding. Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.

Different Learning Goals Require a Variety of Types of Tasks and Instructional Moves

The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- provide experience with a new context
- introduce a new concept and associated language
- introduce a new representation
- formalize the definition of a term for an idea previously encountered informally
- identify and resolve common mistakes and misconceptions
- practice using mathematical language
- work toward mastery of a concept or procedure
- provide an opportunity to apply mathematics to a modeling or other application problem

Each and Every Student Should Have Access to the Mathematical Work

With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional tool boxes should include knowledge of and skill in implementing supports for different learners. This curriculum incorporates extensive tools for specifically supporting English Language Learners and Students with Disabilities



Instructional Model

Learning Goals and Targets

Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson. Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word "Let's." They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson. Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

Lesson Structure

1. INTRODUCE

Warm Up

Warm Up activities either:

- give students an opportunity to strengthen their number sense and procedural fluency.
- make deeper connections.
- encourage flexible thinking.

or:

- remind students of a context they have seen before.
- get them thinking about where the previous lesson left off.
- preview a calculation that will happen in the lesson.

2. EXPLORE AND DEVELOP

Classroom Activities

A sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class.

Each classroom activity has three phases.

The Launch

The teacher makes sure that students understand the context and what the problem is asking them to do.

Practice Problems

Each lesson includes an associated set of practice problems that may be assigned as homework or for extra practice in class. They can be collected and scored or used for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The design of practice problem sets looks different from many other curricula, but every choice was intentional, based on learning research, and meant to efficiently facilitate learning. The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Our approach emphasizes distributed practice rather than massed practice

Mathematical Modeling Prompts

Mathematics is a tool for understanding the world better and making decisions. School mathematics instruction often neglects giving students opportunities to understand this, and reduces mathematics to disconnected rules for moving symbols around on paper. Mathematical modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (NGA 2010). This mathematics will remain important beyond high school in students' lives and education after high school (NCEE 2013).

- Modeling Prompts can be thought of as a project or assignment. They are meant to be launched in class by a teacher, but but can be worked on independently or in small groups by students in or out of class.. We built in maximum flexibility for a teacher to implement these in a way that will work for them.
- The purpose of mathematical modeling is for students to understand that they can use math to better understand things they are interested in in the world.
- Mathematical modeling is different from solving word problems. There should be room to interpret the problem and a range of acceptable assumptions and answers. Modeling requires genuine choices to be made by the modeler.
- Modeling with mathematics is not a solitary activity and students should have support from their teacher and classmate while assessments focus on providing feedback that helps students improve their modeling skills.

3. SYNTHESIZE

Student Work Time

Students work individually, with a partner, or in small groups.

Activity Synthesis

The teacher orchestrates some time for students to synthesize what they have learned and situate the new learning within previous understanding.

National Governors Association Center for Best Practices (2010). Common Core State Standards for Mathematics. NCEE (2013). What Does It Really Mean to Be College and Work Ready? Retrieved November 20, 2017 from http://ncee.org/college-and-work-ready/

Lesson Synthesis

Students incorporate new insights gained during the activities into their big-picture understanding.

Cool Down

A task to be given to students at the end of the lesson. Students are meant to work on the Cool Down for about 5 minutes independently and turn it in.

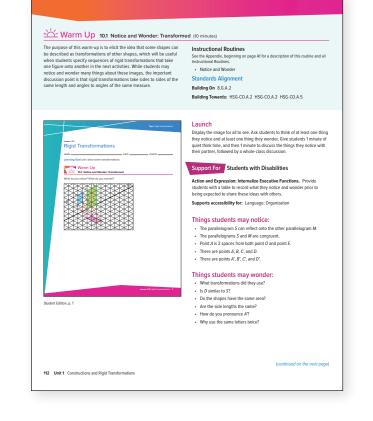
Instructional Routines

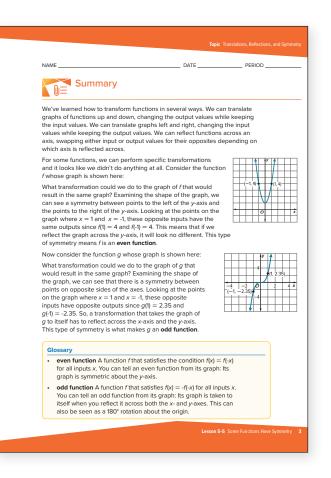
The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, yet others to applying mathematics to a real-world problem. These aspects of mathematical proficiency are interwoven. These materials include a small set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

Like any routine in life, these routines give structure to time and interactions. They are a good idea for the same reason all routines are a good idea: they let people know what to expect, and they make people comfortable.

Why are routines in general good for learning academic content? One reason is that students and the teacher have done these interactions before, in a particular order, and so they don't have to spend much mental energy on classroom choreography. They know what to do when, who is expected to talk when, and when they are expected to write something down. The structure of the routine frees them up to focus on the academic task at hand. Furthermore, a welldesigned routine opens up conversations and thinking about mathematics that might not happen by themselves.

- Analyze It
- Anticipate, Monitor, Select, Sequence, Connect
- Aspects of Mathematical Modeling
- Card Sort
- Construct It
- Draw It
- Estimation
- Extend It
- Fit It
- Graph It
- Math Talk
- Notice and Wonder
- Poll the Class
- Take Turns
- Think Pair Share
- Which One Doesn't Belong?



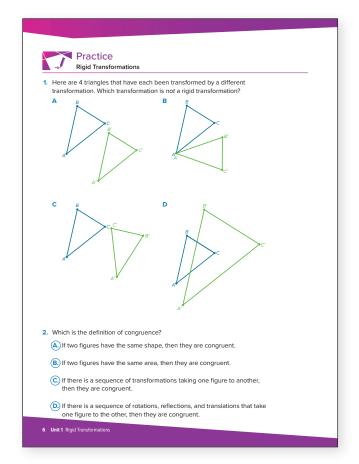


How to Assess Progress

Illustrative Mathematics contains many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

- Each unit begins with a diagnostic assessment ("Check Your Readiness") of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.
- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students' thinking.
- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day's lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.
- A set of cumulative practice problems is provided for each lesson that can be used for homework or inclass practice. The teacher can choose to collect and grade these or simply provide feedback to students.
- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.

Patrick's Test		
Assignment Details		
Number of questions: 10 Points possible: 60.00		
Instructions		
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Assignment Tips		
Start Assignment		





Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. *Illustrative Mathematics* empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

Supporting English-language Learners

Illustrative Mathematics builds on foundational principles for supporting language development for all students. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

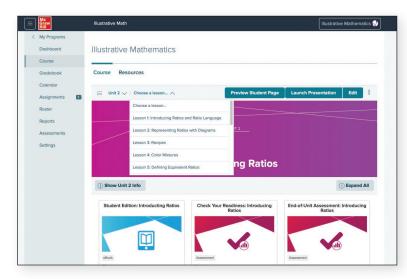
Aguirre, J.M. & Bunch, G. C. (2012). What's language got to do with it?: Identifying language demands in mathematics instruction for English Language Learners. In S. Celedón-Pattichis & N.

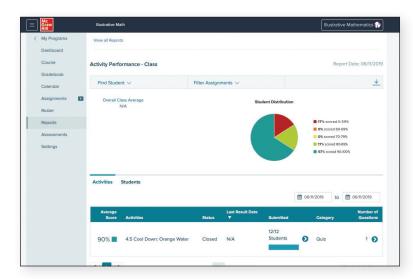
Digital

McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

Online resources offer:

- customizable content
- the ability to add resources
- auto-scoring of student practice work
- on-going student assessments
- classroom performance reporting





Launch Presentations Digital versions of lessons to present content.

Reports Review the performance of individual students, classrooms, and grade levels.

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	Gradebook Calendar Assignments	Course Resources	urse 1			
	Reports Assessments	Refine results Course Location Refine more Course Standards	1582 Results (limited to the co	Intent course) Illustrative Math Course 1: eBook Description: Illustrative Math, Grade 6 eBook Location:	I	
		Languages Resource Type		Illustrative Mathematics: Getting Starte PDF Location: Getting Started: Illustrative M	8	
				Illustrative Mathematics: Overview Gui	1	
			I	Student Edition eBook eBook Location: Unit 1: Area and Surface Area	8	

Access Resources Point-of-use access to resources such as assessments, eBooks, and course guides.



Sequences and Functions

A Towering Sequence

Lesson 1-1 A Towering Sequence

Sequences

- 1-2 Introducing Geometric Sequences
- **1-3** Different Types of Sequences
- 1-4 Using Technology to Work with Sequences
- **1-5** Sequences are Functions
- **1-6** Representing Sequences
- 1-7 Representing More Sequences

What's the Equation?

- 1-8 The Term
- **1-9** What's the Equation?
- 1-10 Situations and Sequence Types
- 1-11 Adding Up



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Polynomials and Rational Functions

What Is a Polynomial?

- Lesson 2-1 Let's Make a Box
 - 2-2 Funding the Future
 - 2-3 Introducing Polynomials
 - 2-4 Combining Polynomials

Working with Polynomials

- 2-5 Connecting Factors and Zeros
- 2-6 Different Forms
- 2-7 Using Factors and Zeros
- 2-8 End Behavior (Part 1)
- 2-9 End Behavior (Part 2)
- 2-10 Multiplicity
- 2-11 Finding Intersections
- 2-12 Polynomial Division (Part 1)
- 2-13 Polynomial Division (Part 2)
- 2-14 What Do You Know About Polynomials?
- 2-15 The Remainder Theorem

Rational Functions

- 2-16 Minimizing Surface Area
- 2-17 Graphs of Rational Functions (Part 1)
- 2-18 Graphs of Rational Functions (Part 2)
- 2-19 End Behavior of Rational Functions

Rational Equations

- 2-20 Rational Equations (Part 1)
- 2-21 Rational Equations (Part 2)
- 2-22 Solving Rational Equations

Polynomial Identities

- 2-23 Polynomial Identities (Part 1)
- 2-24 Polynomial Identities (Part 2)
- 2-25 Summing Up
- 2-26 Using the Sum



Complex Numbers and Rational Exponents

Exponent Properties

Lesson 3-1 Properties of Exponents

- 3-2 Square Roots and Cube Roots
- **3-3** Exponents That Are Unit Fractions
- 3-4 Positive Rational Exponents
- 3-5 Negative Rational Exponents

Solving Equations with Square and Cube Roots

- 3-6 Squares and Square Roots
- 3-7 Inequivalent Equations
- 3-8 Cubes and Cube Roots
- 3-9 Solving Radical Equations

A New Kind of Number

- 3-10 A New Kind of Number
- 3-11 Introducing the Number i
- **3-12** Arithmetic with Complex Numbers
- 3-13 Multiplying Complex Numbers
- 3-14 More Arithmetic with Complex Numbers
- 3-15 Working Backwards

Solving Quadratics with Complex Numbers

- **3-16** Solving Quadratics
- 3-17 Completing the Square and Complex Solutions
- 3-18 The Quadratic Formula and Complex Solutions
- 3-19 Real and Non-Real Solutions



Exponential Functions and Equations

Growing and Shrinking

- Lesson 4-1 Growing and Shrinking
 - 4-2 Representations of Growth and Decay

Understanding Non-Integer Inputs

- 4-3 Understanding Rational Inputs
- 4-4 Representing Functions at Rational Inputs
- 4-5 Changes Over Rational Intervals
- **4-6** Writing Equations for Exponential Functions
- 4-7 Interpreting and Using Exponential Functions

Missing Exponents

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Dure

- 4-8 Unknown Exponents
- 4-9 What is a Logarithm?
- 4-10 Interpreting and Writing Logarithmic Equations
- 4-11 Evaluating Logarithmic Expressions

The Constant e

- 4-12 The Number e
- 4-13 Exponential Functions with Base e
- 4-14 Solving Exponential Equations

Logarithmic Functions and Graphs

- 4-15 Lesson 15: Using Graphs and Logarithms to Solve Problems (Part 1)
- 4-16 Using Graphs and Logarithms to Solve Problems (Part 2)
- **4-17** Logarithmic Functions
- 4-18 Applications of Logarithmic Functions



Transformations of Functions

Warming Up to Decimals

Lesson 5-1 Matching up to Data

- **5-2** Moving Functions
- 5-3 More Movement
- 5-4 Reflecting Functions
- 5-5 Some Functions Have Symmetry
- 5-6 Symmetry in Equations
- 5-7 Expressing Transformations of Functions Algebraically

Scaling Outputs and Inputs

- 5-8 Scaling the Outputs
- **5-9** Scaling the Inputs

Putting It All Together

- **5-10** Combining Functions
- 5-11 Making a Model for Data

baona/Getty Images



Trigonometric Functions

The Unit Circle

Lesson 6-1 Moving in Circles

- 6-2 Revisiting Right Triangles
- 6-3 The Unit Circle (Part 1)
- 6-4 The Unit Circle (Part 2)
- 6-5 The Pythagorean Identity (Part 1)
- 6-6 The Pythagorean Identity (Part 2)
- 6-7 Finding Unknown Coordinates on a Circle

Periodic Functions

angMan Photography/Getty Image

- 6-8 Rising and Falling
- 6-9 Introduction to Trigonometric Functions
- 6-10 Beyond 2π
- 6-11 Extending the Domain of Trigonometric Functions
- 6-12 Tangent

Trigonometry Transformations

- 6-13 Amplitude and Midline
- 6-14 Transforming Trigonometric Functions
- 6-15 Features of Trigonometric Graphs (Part 1)
- 6-16 Features of Trigonometric Graphs (Part 2)
- 6-17 Comparing Transformations
- 6-18 Modeling Circular Motion

Putting It All Together

6-19 Beyond Circles



Statistical Inferences

Study Types

Lesson 7-1 Being Skeptical

- 7-2 Study Types
- 7-3 Randomness in Groups

Distributions

- 7-4 Describing Distributions
- 7-5 Normal Distributions
- 7-6 Areas in Histograms
- 7-7 Areas under a Normal Curve

Not All Samples Are the Same

- 7-8 Not Always Ideal
- 7-9 Variability in Samples
- 7-10 Estimating Proportions from Samples
- 7-11 Reducing Margin of Error
- 7-12 Estimating a Population Mean

Analyzing Experimental Data

- 7-13 Experimenting
- 7-14 Using Normal Distributions for Experiment Analysis
- 7-15 Questioning Experimenting
- 7-16 Heart Rates

Prior Work

Polynomial, Radical, and Exponential Functions

Prior to this unit, students have worked with a variety of function types, such as polynomial, radical, and exponential. The purpose of this unit is for students to consider functions as a whole and understand how they can be transformed to fit the needs of a situation, which is an aspect of modeling with mathematics. MP4

An important takeaway of the unit is that we can transform functions in a predictable manner using translations, reflections, scale factors, and by combining multiple functions. Throughout the unit students analyze graphs, tables, equations, and contexts as they work to connect representations and understand the structure of different transformations. MP7

Work in This Unit

Transform Functions

The unit begins with students informally describing transformations of graphs, eliciting their prior knowledge and establishing language that will be refined throughout the unit. Students consider the graphs of two possible functions as fits for a data set and make an argument about why one is a better fit. **MP3**

Students return to this data set in future lessons as they learn more ways to transform a given equation to fit data.

The first types of transformations students consider are vertical and horizontal translations. While these types of transformations have been studied briefly for specific function types, such as quadratics, here they are studied for all function types. In parallel with their study of the effect of translations on graphs and tables, students learn to write equations for functions that are defined in terms of another to describe transformations using function notation.

Next, students investigate how transformations such as reflections across the horizontal and vertical axes are defined using function notation and make connections to the same topic from geometry. These ideas are expanded to consider the properties of even functions, odd functions, and functions that are neither even nor odd from both a graphical and algebraic perspective. From translations and reflections, students move on to explore the effect of multiplying the output or input of a function by a scale factor. They fit quadratic functions to parabolic arches in photos in order to better understand how to "squash" or "stretch" outputs. Students consider the change in height over time of a rider on different Ferris wheels as another application of scale factors as they contrast multiplying outputs against multiplying inputs of functions. The use of clear and precise language is emphasized as students make sense of the effects of different scale factors. MP6

In a future unit, students use their knowledge of transformations to transform trigonometric functions to model a variety of periodic situations. By saving the introduction of trigonometric functions until after a study of transformations, students have the opportunity to revisit transformations from a new perspective which reinforces the idea that all functions, even periodic ones, behave the same way with respect to translations, reflections, and scale factors.

	Lessons	Days	Standards
	Check Your Readiness Assessment	1	HSG-CO.A.3, 7.G.A.1, HSS-ID.B.6.c, HSG-CO.A.3, HSF-IF.A.2, HSF-IF.B.4
Торіс	Translations, Reflections, and Symmetry		
	Lesson 1 Matching up to Data	1	HSF-IF.B.4, HSF-BF.B.3, HSF-BF.B.3, HSS-ID.B.6.a
	Lesson 2 Moving Functions	1	HSF-IF.A, HSF-BF.B.3
	Lesson 3 More Movement	1	HSF-BF.B.3, HSF-BF.B.3, HSS-ID.B.6.a
	Lesson 4 Reflecting Functions	1	HSF-BF.B.3, HSF-BF.B.3
	Lesson 5 Some Functions Have Symmetry	1	HSF-BF.B.3, HSF-IF.C, HSF-BF.B.3
	Lesson 6 Symmetry in Equations	1	HSF-BF.B.3, HSF-IF.C.8
	Lesson 7 Expressing Transformations of Functions Algebraically	1	HSF-BF.B.3, HSS-ID.B.6.a
Торіс	Scaling Outputs and Inputs		
	Lesson 8 Scaling the Outputs	1	HSA-APR.B, HSF-BF.B.3, HSS-ID.B.6.a
	Lesson 9 Scaling the Inputs	1	HSF-IF.B.4, HSF-BF.B.3
Торіс	Putting It All Together		
	Lesson 10 Combining Functions	1	HSF-BF.B, HSF-BF.A.1.b, HSF-BF.A.1.b
	Lesson 11 Making a Model for Data	1	HSF-BF.A.1.b, HSF-BF.B.3, HSF-IF.B.4, HSF-LE.B, HSS-ID.B.6.a
	End-of-Unit Assessment	1	HSF-BF.B.3, HSF-BF.B.3, HSF-BF.A.1.b, HSF-BF.B, HSF-BF.B.3, HSF-BF.B.3, HSF-BF.B.3
	TOTAL	13	

Required Materials		Blackline Activity Masters
 Colored pencils (Lessons 4, 9) Graphing technology (Lessons 2, 3, 8, 10, 11) Graph paper (Lesson 2) 	 Pre-printed slips, cut from copies of the blackline master (Lessons 1, 5) Tracing paper (Lessons 4, 6, 7) 	 Card Sort: Two Types of Graphs (Lesson 5, Activity 2) Card Sort: Two Types of Coordinates (Lesson 5, Activity 3)

Pre-Unit Diagnostic Assessment

The pre-unit diagnostic assessment, *Check Your Readiness,* evaluates students' proficiency with prerequisite concepts and skills that they need to be successful in the unit. The item descriptions below offer guidance for students who may answer items incorrectly.

The assessment also may include problems that assess what students already know of the upcoming unit's key ideas, which you can use to pace or tune instruction. In rare cases, this may signal the opportunity to move more quickly through a topic to optimize instructional time.

Materials: Graphing calculators should not be used because some of the problems involve students considering how expressions for transformed functions alter their graphs. Use of a four-function or scientific calculator is acceptable.

1. Item Description

Students selecting A or B may not understand reflections. Students who select C may recognize that a reflection across the line y = -x aligns one vertex and may think aligning one of the vertices must align the others as well.

First Appearance of Skill or Concept: Lesson 4

If most students struggle with this item....

- Plan to make tracing paper available in Lessons 4, 5, and 6 to assist students with visualizing reflections.
- If needed, invite students to share how they could use tracing paper to solve this problem as a refresher on tracing paper techniques.

2. Item Description

Ideas closely related to this skill or concept first appear in Lesson 1 when students examine vertical and horizontal stretches. Vertical and horizontal stretches are not dilations because the stretch is only applied in a single direction while a dilation applies the same scale factor in all directions. Nonetheless, the idea of scaling is common to both situations.

First Appearance of Skill or Concept: Lesson 8

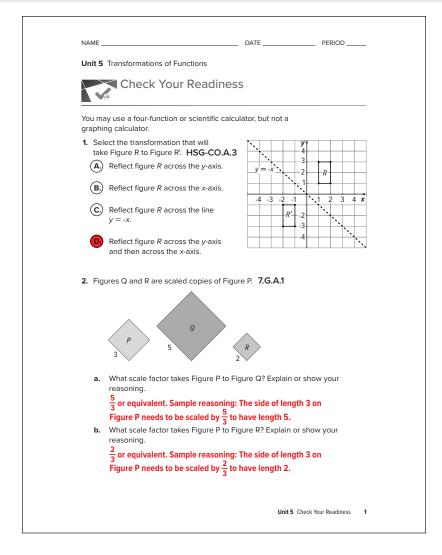
If most students struggle with this item....

- Plan to briefly review the concept of scale factor prior to Lesson 8.
- Geometry Unit 3 Lesson 1 Activity 4, Match the Scale Factors, is an example of a brief activity that could be added to review scale factors.



Available as a digital assessment or a printable assessment.





3. Item Description

First Appearance of Skill or Concept: Lesson 1

If most students do well with this items....

• Plan to connect what students know about transforming linear functions to the work they do with non-linear functions in Lesson 1.

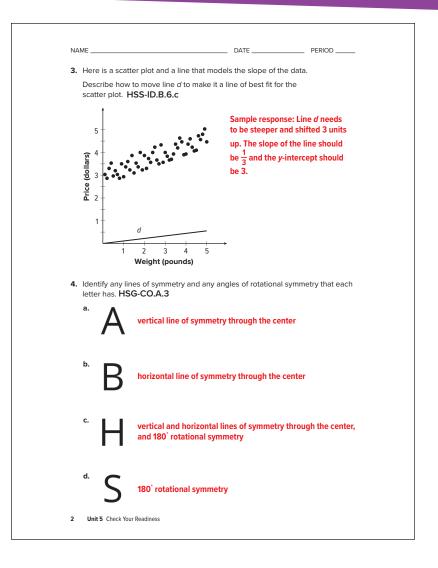
If most students struggle with this item....

- Plan to make tracing paper available in Lesson 1 to assist students with visualizing transformations described by their partner.
- Invite a student to demonstrate how they would propose shifting the graph of *g*(*x*) in Activity 2 using tracing paper to show their thinking.

4. Item Description

First Appearance of Skill or Concept: Lesson 5 If most students struggle with this item....

- Plan to make tracing paper available in Lesson 5 to assist students with visualizing rotations.
- If needed, invite students to share how they could use tracing paper to solve this problem as a refresher on tracing paper techniques.



5. Item Description

First Appearance of Skill or Concept: Lesson 2

If most students struggle with this item....

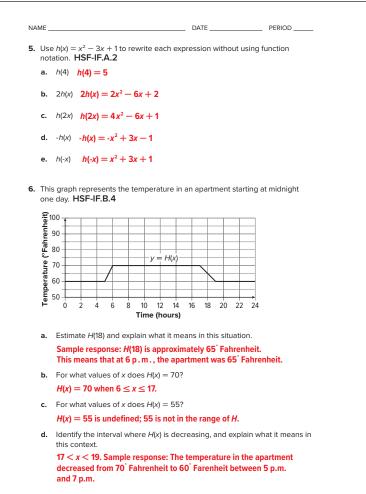
- Plan to incorporate a review of function notation during Lesson 1 Activity 2.
- Students will have opportunities throughout the unit to both practice and make sense of scaling, adding to, and subtracting from the inputs and outputs of a function.
- If students make errors involving squaring negative inputs, distributing, or negative signs, use this prompt to do error analysis throughout the unit.
- Focus on operations with signed inputs, such as *h*(-*x*), prior to Lesson 5, and on distributing scale factors on inputs and outputs, such as *h*(2*x*), prior to Lesson 8.

6. Item Description

First Appearance of Skill or Concept: Lesson 2

If most students struggle with this item....

- Plan to incorporate a review of function notation and interpreting functions during Lesson 1 Activity 2.
- Students will have opportunities throughout the unit to practice interpreting functions and using function notation to describe points on the graphs of functions.



Unit 5 Check Your Readiness 3

Goals (Teacher-Facing)

- Generalize (orally and in writing) what is true for coordinate pairs of even and odd functions.
- Identify (orally) features that graphs of even functions have in common and features that graphs of odd functions have in common.

Student Learning Goals

Let's look at symmetry in graphs of functions.

Learning Targets

• I can identify even and odd functions by their graphs.

Required Materials

· Pre-printed slips, cut from copies of the blackline master

Lesson Narrative

In the previous lesson, students learned to graph the reflection of a function across an axis. They paid close attention to the signs of inputs and outputs when reflecting over the *x*- and *y*-axes. Building on that experience, this is the first of two lessons where students learn to identify **even functions** and **odd functions**. A function *f* is even if the outputs for *x* and *-x* are the same. Visually, the graph of *f* appears symmetric across the vertical axis. Algebraically, we say that f(x) = f(-x) for any input *x*. A function *g* is odd if the output for *x* is the opposite of the output for *-x*. Visually, the graph of *g* has a type of symmetry defined by successive reflections across both the *x*- and *y*-axes taking the graph of *g* to itself. Algebraically, we say that g(x) = -g(-x) for any input *x*.

In this lesson, students first identify key features of each type of function by sorting graphs into two groups. They then match tables of values to the graphs and refine their ideas about what makes a function odd or even, giving students the opportunity to use repeated reasoning as they establish definitions for even and odd functions. MP8

In the next lesson, students formalize these ideas using function notation established in the Lesson Synthesis and learn to identify a function as even, odd, or neither from an equation.

Instructional Routines

- Card Sort
- Mathematical Language Routines
 - MLR2 Collect and Display
 - MLR8 Discussion Supports

Lesson Pacing

			Pacing (min)
Warm Up 5.1 C	hanging Heights		5
Activity 5.2 C	ard Sort: Two Types of Graphs		15
Activity 5.3 C	ard Sort: Two Types of Coordinates		15
Lesson Synthes	sis		5
Cool Down 5.4	Even or Odd?		5
	то	TAL	45

Standards Alignment

Addressing

HSF-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

HSF-IF.C Analyze functions using different representations.

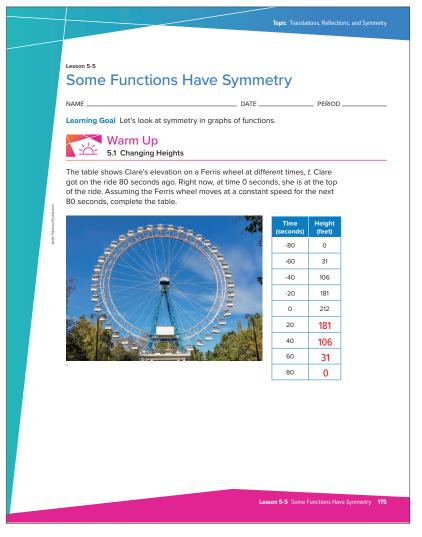
Building Towards

HSF-BF.B.3 Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Warm Up 5.1 Changing Heights (5 minutes)

This warm up invites students to use their understanding of the motion of a Ferris wheel to complete a table of values where the data has distinct symmetry.

Standards Alignment Building Towards HSF-BF.B.3



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Activity Synthesis

Select students to explain how they completed the table. If not brought up by students, ask, What does the graph of this function look like? A curve that is symmetric about the vertical axis. Invite students to share their descriptions or any graphs they make of the data. If no students sketched a graph that can be shared, display one for all to see to help highlight the symmetry.

S.2 Card Sort: Two Types of Graphs (15 minutes)

The purpose of this activity is for students to identify common features in graphs of functions that are the same when reflected across the vertical axis and those that look the same when reflected across both axes. This leads to defining these types of functions as even or odd, respectively. In the following activity, students match tables to the graphs and refine their definitions, so groups should keep their copies of the blackline master from this activity to use in the next activity.

Monitor for different ways groups choose different categories, but especially for categories that distinguish between graphs of even functions and graphs of odd functions. As students work, encourage them to refine their descriptions of the graphs using more precise language and mathematical terms.

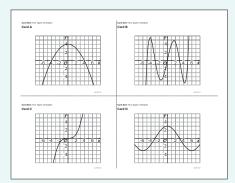
Instructional Routines

See the Appendix, beginning on page A1 for a description of these routines and all Instructional Routines.

- Card Sort
- Mathematical Language Routines
 - MLR2 Collect and Display

Standards Alignment

Building Towards HSF-BF.B.3



5.5.2 Card Sort: Two Types of Graphs



ACTIVITY 5.2 Card Sort: Two Types of Graphs

Your teacher will give you a set of cards that show graphs. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.

Sample response: Reflecting graphs A, D, F, and G across the *y*-axis results in the same graph. Reflecting graphs B, C, E, and H across both axes results in the same graph.



Your teacher will give you a set of cards to go with the cards you already have.

- Match each table of coordinate pairs with one of the graphs from earlier. Card A - Card L, Card B - Card K. Card C - Card O, Card D - Card J, Card E - Card M, Card F - Card Q, Card G - Card N, Card H - Card P.
- Describe something you notice about the coordinate pairs of even functions.

Sample response: Even functions have the same *y*-values for inputs +x and -x. For example, if the graph of an even function has the point (-2, 3), then it also has the point (2, 3).

Describe something you notice about the coordinate pairs of odd functions.
 Sample response: Odd functions have the opposite *y*-values for opposite inputs *x*. For example, if the graph of an odd function has the point (-2, 3), then it also has the point (2, -3).

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Launch

Arrange students in groups of 2. Tell them that in this activity, they will sort some cards into categories of their choosing. When they sort the graphs, they should work with their partner to come up with categories. Distribute pre-cut slips to each group.

Support For English Language Learners

Conversing: MLR2 Collect and Display. As students discuss their matchings with a partner, listen for and collect vocabulary, gestures, and diagrams students use to identify and describe what is the same and different. Capture student language that reflects a variety of ways to describe the differences between even and odd functions. Write the students' words on a visual display and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their partner and whole-group discussions.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Support For Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Provide students with six cards to sort, ensure that the set includes three even functions and three odd functions. This will allow students additional processing time.

Supports accessibility for: Organization; Attention; Social-emotional skills

Anticipated Misconceptions

Some students may focus too closely on identifying specific points on the graph to use to make their categories. Encourage these students to look at the graph as a whole while they sort.

Activity Synthesis

Select groups to share their categories and how they sorted their graphs. Choose as many different types of categories as time allows, but ensure that one set of categories distinguishes between graphs of even functions and graphs of odd functions. Attend to the language that students use to describe their categories, giving them opportunities to describe the types of graphs more precisely.

It is possible students will think of graphs of odd functions as ones where a 180° rotation using the origin as the center of rotation results in the same graph. While it is true that this type of rotation appears the same as successive reflections of the graph across both axes, focus the conversation on thinking in terms of reflections since the function notation students will use to describe odd functions, g(x) = -g(-x), algebraically describes reflections.

At the conclusion of the sharing, display the graphs of the even functions next to the odd functions. Tell students that functions whose graphs look the same when reflected across the *y*-axis are called **even functions**. Functions whose graphs that look the same when reflected across both axes are called **odd functions**. In the next activity, students refine their understanding of even and odd functions by pairing each of the graphs with a table of values and writing their own description for these two types of functions based on inputs and outputs.

Activity 5.3 Card Sort: Two Types of Coordinates (15 minutes)

The purpose of this activity is for students to deepen their understanding of even and odd functions. Using the graphs from the previous activity, students first match each graph to a table of coordinate pairs and then use both representations to identify defining characteristics of even functions and odd functions.

In the next lesson, students will learn how to use an equation to prove if a function is even or odd, so an important result of this activity is describing even and odd functions using function notation.

Monitor for students making connections between the transformations described in the previous activity (a reflection across the *y*-axis versus successive reflections across both axes) and the coordinates in the tables to share during the whole-class discussion.

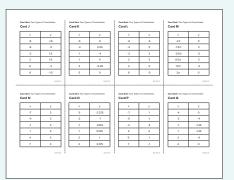
Instructional Routines

See the Appendix, beginning on page A1 for a description of these routines and all Instructional Routines.

- Card Sort
- Mathematical Language Routines
 - MLR8 Discussion Supports

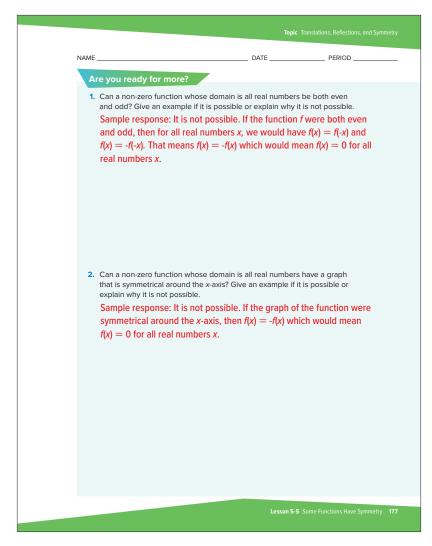
Standards Alignment

Addressing HSF-BF.B.3, HSF-IF.C



5.5.3 Card Sort: Two Types of Coordinates

(continued on the next page)



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Launch

Keep students in the same groups. If students do not already have their slips from the previous activity arranged into two groups, one for graphs of even functions and one for graphs of odd functions, ask them to do so now. Distribute pre-cut slips.

Support For English Language Learners

Conversing: MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: "_____ and _____ are alike because...", and "I noticed _____, so I matched...." Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about even and odd functions.

Design Principle(s): Support sense-making; Maximize meta-awareness

Support For Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Provide students with six cards to match, ensure that the set includes the table of coordinate pairs that match the six graphs used in the previous activity. This will allow students additional processing time.

Supports accessibility for: Organization; Attention; Social-emotional skills

Activity Synthesis

The goal of this discussion is for students to move from observations about specific even and odd functions to generalizing things that are true for all even functions and for all odd functions.

Select previously identified students to share the connections they see between the transformations associated with graphs of even and odd functions and features of the coordinate pairs belonging to even and odd functions. Record these observations for all to see in two lists: one for even, one for odd.

If time allows, assign groups to write a single sentence describing even or odd functions that summarizes one of the lists.

Support For Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, invite students to highlight positive values in one color and negative values in a different color within the table of coordinate pairs.

Supports accessibility for: Visual-spatial processing

Example 2 Lesson Synthesis (5 minutes)

The goal of this discussion is for students to use function notation to summarize their understanding of what makes a function even and what makes a function odd.

Here are some questions for discussion to help students transition to using function notation:

- Using the the language of inputs and outputs, what is true about even functions? Opposite inputs have the same output.
- Using the the language of inputs and outputs, what is true about odd functions? Opposite inputs have opposite outputs.
- If a function *f* is even and *f*(3) = 7, what is something else you know about *f*? Since *f* is even, if an input of 3 has an output of 7, then an input of -3 also has an output of 7, so *f*(-3) = 7.
- If a function g is odd and g(5) = -1, what is something else you know about g? Since g is odd, if an input of 5 has an output of -1, then an input of -5 has an output of 1, so g(-5) = 1.

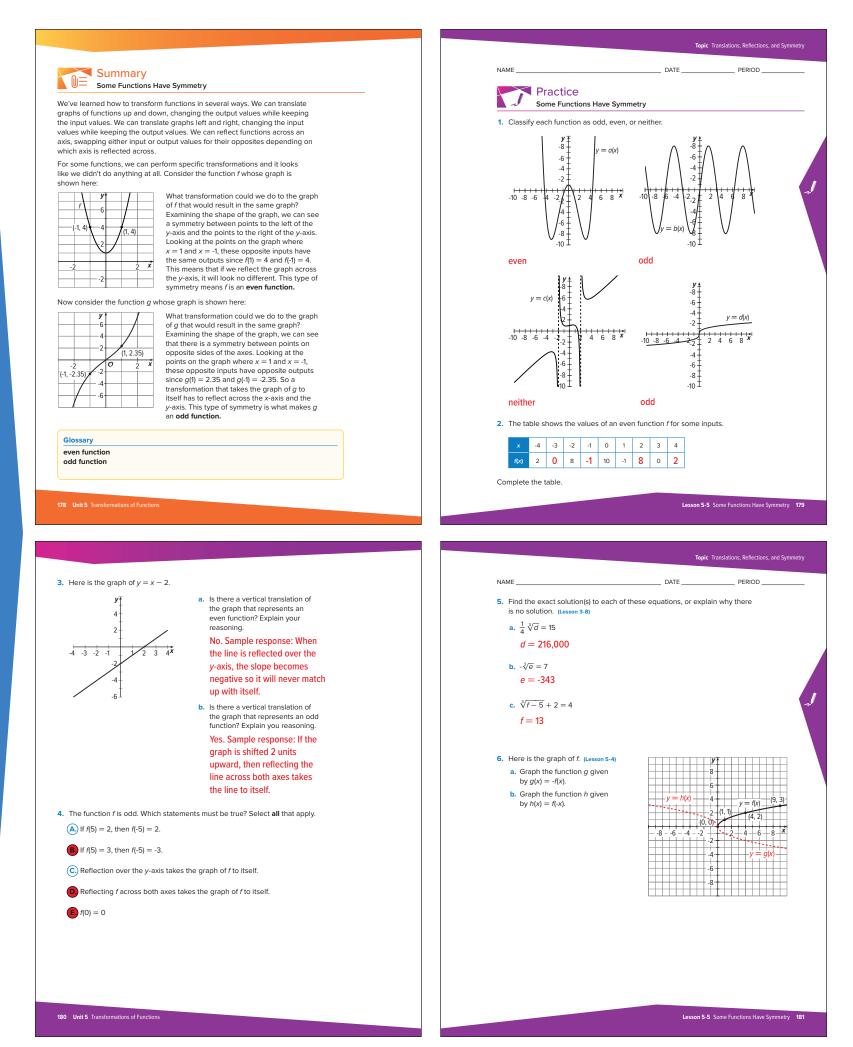
Tell students that these observations can be generalized for all even and odd functions. If a function *f* is even, then f(x) = f(-x) is true. If a function *g* is odd, then g(x) = -g(-x). Students will focus on these definitions in the next lesson.

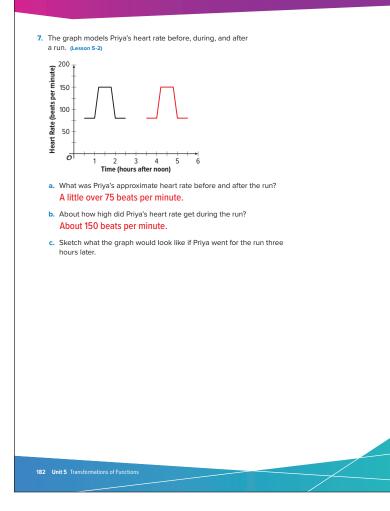
Standards Alignment Addressing HSF-BF.B.3

Lesson 5-5 Some Functions Have Symmetry COOL DOWN 5.4 Even or Odd? Let <i>h</i> be a function where $y = h(x)$. 1. What is the value of <i>a</i> if <i>h</i> is an even function? 2. What is the value of <i>a</i> if <i>h</i> is an odd function? $\frac{1}{2}$ 1. What is the value of <i>a</i> if <i>h</i> is an odd function? $\frac{1}{2}$	NAME	DATE		PERIOD	
Let <i>h</i> be a function where $y = h(x)$. 1. What is the value of <i>a</i> if <i>h</i> is an even function? $ \begin{array}{c c} x & y \\ -4 & a \\ 0 & 0 \\ \hline 2. What is the value of a if h is an odd function? 4 \frac{1}{2}$	Lesson 5-5 Some Functions Have Symmetry				
1. What is the value of a if h is an even function?xy $\frac{1}{2}$ -4a002. What is the value of a if h is an odd function?4 $\frac{1}{2}$	Cool Down 5.4 Even or Odd?				
$\frac{1}{2}$ 2. What is the value of <i>a</i> if <i>h</i> is an odd function? $\frac{x}{-4}$ $\frac{y}{-4}$ $\frac{a}{0}$ 0 $\frac{1}{2}$	Let <i>h</i> be a function where $y = h(x)$.				
2	$\frac{1}{2}$ 2. What is the value of <i>a</i> if <i>h</i> is an odd function		-4 0	a 0	
2		n?			
	2				

Cool Down 5.4 Even or Odd? (5 minutes)

Printable Cool Down





Student Edition

Topic Translations, Reflections, and Symmetry

Lesson 5-5

Some Functions Have Symmetry

NAME ____

_____ DATE _____ PERIOD ____

Learning Goal Let's look at symmetry in graphs of functions.

Warm Up

5.1 Changing Heights

The table shows Clare's elevation on a Ferris wheel at different times, t. Clare got on the ride 80 seconds ago. Right now, at time 0 seconds, she is at the top of the ride. Assuming the Ferris wheel moves at a constant speed for the next 80 seconds, complete the table.

adim Petrakov/Shutterstock



Time (seconds)	Height (feet)
-80	0
-60	31
-40	106
-20	181
0	212
20	
40	
60	
80	

Activity 5.2 Card Sort: Two Types of Graphs

Your teacher will give you a set of cards that show graphs. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.



Your teacher will give you a set of cards to go with the cards you already have.

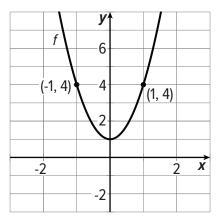
- 1. Match each table of coordinate pairs with one of the graphs from earlier.
- **2.** Describe something you notice about the coordinate pairs of even functions.
- 3. Describe something you notice about the coordinate pairs of odd functions.

AME			
 Can a non-zero function whose domain is all real numbers be both even and odd? Give an example if it is possible or explain why it is not possible. Can a non-zero function whose domain is all real numbers have a graph that is symmetrical around the <i>x</i>-axis? Give an example if it is possible or 	JAME	DATE	PERIOD
 and odd? Give an example if it is possible or explain why it is not possible. 2. Can a non-zero function whose domain is all real numbers have a graph that is symmetrical around the <i>x</i>-axis? Give an example if it is possible or 	Are you ready for more?		
that is symmetrical around the <i>x</i> -axis? Give an example if it is possible or			
	that is symmetrical around the <i>x</i> -		

Some Functions Have Symmetry

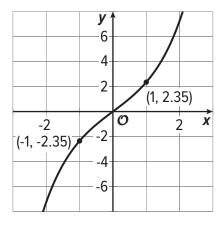
We've learned how to transform functions in several ways. We can translate graphs of functions up and down, changing the output values while keeping the input values. We can translate graphs left and right, changing the input values while keeping the output values. We can reflect functions across an axis, swapping either input or output values for their opposites depending on which axis is reflected across.

For some functions, we can perform specific transformations and it looks like we didn't do anything at all. Consider the function *f* whose graph is shown here:



What transformation could we do to the graph of *f* that would result in the same graph? Examining the shape of the graph, we can see a symmetry between points to the left of the *y*-axis and the points to the right of the *y*-axis. Looking at the points on the graph where x = 1 and x = -1, these opposite inputs have the same outputs since f(1) = 4 and f(-1) = 4. This means that if we reflect the graph across the *y*-axis, it will look no different. This type of symmetry means *f* is an **even function**.

Now consider the function g whose graph is shown here:



What transformation could we do to the graph of *g* that would result in the same graph? Examining the shape of the graph, we can see that there is a symmetry between points on opposite sides of the axes. Looking at the points on the graph where x = 1 and x = -1, these opposite inputs have opposite outputs since g(1) = 2.35 and g(-1) = -2.35. So a transformation that takes the graph of *g* to itself has to reflect across the *x*-axis and the *y*-axis. This type of symmetry is what makes *g* an **odd function.**

Glossary

even function odd function

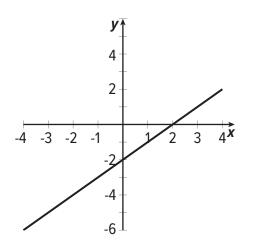
Topic Translations, Reflections, and Symmetry _____ DATE _____ PERIOD _____ NAME _____ **Practice** Some Functions Have Symmetry 1. Classify each function as odd, even, or neither. У -8 v = a(x)-6 -6 -4 -4 -2 -2 -²-2 6 8 **x** -10 -8 -10 -8 -6 -4 8 2 2 4 4 -2 -16 = b(x)2 -10 -10 8 y = c(x)-6 -4 y = d(x)-2 4 6 8 **x** 6 8 X -10 -8 -6 -4 -10 -8 -6 -4 22 2 4 -4 -6 -8 -10

2. The table shows the values of an even function *f* for some inputs.

x	-4	-3	-2	-1	0	1	2	3	4
f(x)	2		8		10	-1		0	

Complete the table.

3. Here is the graph of y = x - 2.



 a. Is there a vertical translation of the graph that represents an even function? Explain your reasoning.

 b. Is there a vertical translation of the graph that represents an odd function? Explain you reasoning.

- **4.** The function *f* is odd. Which statements must be true? Select **all** that apply.
 - (A) If f(5) = 2, then f(-5) = 2.
 - **(B.)** If f(5) = 3, then f(-5) = -3.
 - C. Reflection over the *y*-axis takes the graph of *f* to itself.
 - **(D.)** Reflecting *f* across both axes takes the graph of *f* to itself.
 - $(\mathbf{E}, f(\mathbf{0}) = \mathbf{0}$

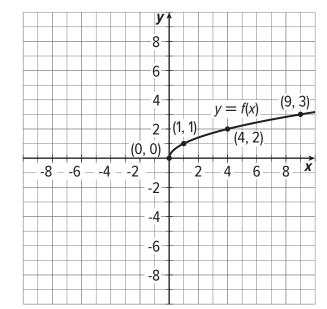
	Ic	ppic Translations, Reflections, and Symmetry	
NAME	DATE	PERIOD	
5. Find the exact solution(s) to each of these e is no solution. (Lesson 3-8)	quations, or ex	plain why there	

a.
$$\frac{1}{4}\sqrt[3]{d} = 15$$

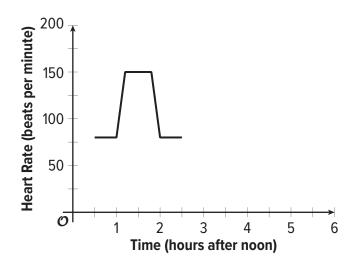
b. $-\sqrt[3]{e} = 7$

c.
$$\sqrt[3]{f-5} + 2 = 4$$

- 6. Here is the graph of f. (Lesson 5-4)
 - **a.** Graph the function g given by g(x) = -f(x).
 - **b.** Graph the function *h* given by h(x) = f(-x).



7. The graph models Priya's heart rate before, during, and after a run. (Lesson 5-2)



- a. What was Priya's approximate heart rate before and after the run?
- **b.** About how high did Priya's heart rate get during the run?
- **c.** Sketch what the graph would look like if Priya went for the run three hours later.

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