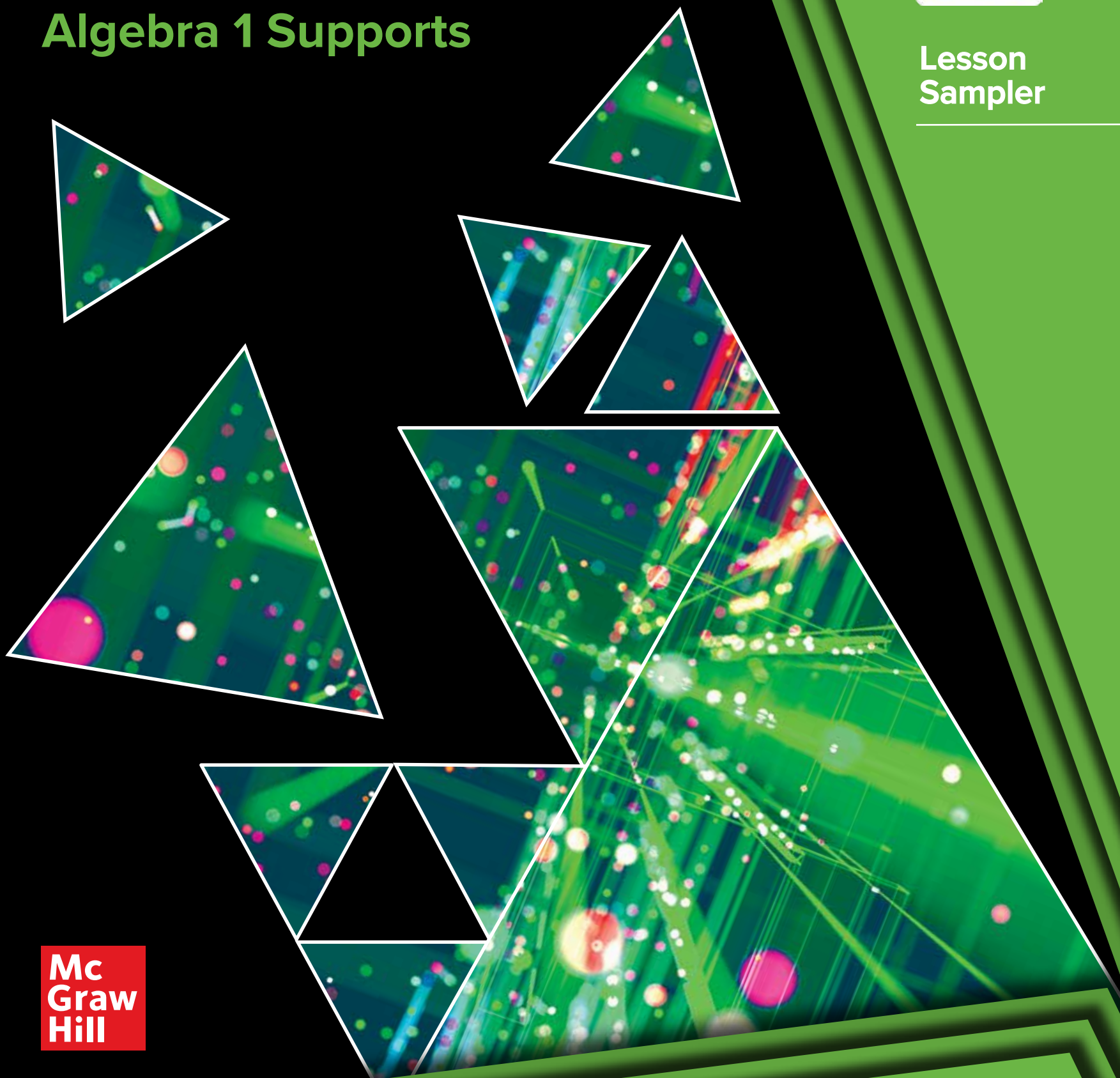


# McGraw-Hill Illustrative Mathematics®

## Algebra 1 Supports



Lesson  
Sampler



Mc  
Graw  
Hill



# McGraw-Hill **Illustrative Mathematics<sup>®</sup>**

## **Algebra 1 Supports**

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*Illustrative Mathematics* is a problem-based core curriculum designed to address content and practice standards to foster learning for all. Students learn by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Teachers can shift their instruction and facilitate student learning with high-leverage routines to guide learners to understand and make connections between concepts and procedures.

## What is a Problem-based Curriculum?

In a problem-based curriculum, students work on carefully crafted and sequenced mathematics problems during most of the instructional time. Teachers help students understand the problems and guide discussions to be sure that the mathematical takeaways are clear to all. In the process, students explain their ideas and reasoning and learn to communicate mathematical ideas. The goal is to give students just enough background and tools to solve initial problems successfully, and then set them to increasingly sophisticated problems as their expertise increases.

The value of a problem-based approach is that students spend most of their time in math class doing mathematics: making sense of problems, estimating, trying different approaches, selecting and using appropriate tools, and evaluating the reasonableness of their answers. They go on to interpret the significance of their answers, noticing patterns and making generalizations, explaining their reasoning verbally and in writing, listening to the reasoning of others, and building their understanding.





# Design Principles

## **Balancing Conceptual Understanding, Procedural Fluency, and Applications**

These three aspects of mathematical proficiency are interconnected: procedural fluency is supported by understanding, and deep understanding often requires procedural fluency. In order to be successful in applying mathematics, students must both understand, and be able to do, the mathematics.

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## **Mathematical Practices are the Verbs of Math Class**

In a mathematics class, students should not just learn about mathematics, they should do mathematics. This can be defined as engaging in the mathematical practices: making sense of problems, reasoning abstractly and quantitatively, making arguments and critiquing the reasoning of others, modeling with mathematics, making appropriate use of tools, attending to precision in their use of language, looking for and making use of structure, and expressing regularity in repeated reasoning.

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## **Build on What Students Know**

New mathematical ideas are built on what students already know about mathematics and the world, and as they learn new ideas, students need to make connections between them (NRC 2001). In order to do this, teachers need to understand what knowledge students bring to the classroom and monitor what they do and do not understand as they are learning. Teachers must themselves know how the mathematical ideas connect in order to mediate students' learning.

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## **Good Instruction Starts with Explicit Learning Goals**

Learning goals must be clear not only to teachers, but also to students, and they must influence the activities in which students participate. Without a clear understanding of what students should be learning, activities in the classroom, implemented haphazardly, have little impact on advancing students' understanding. Strategic negotiation of whole-class discussion on the part of the teacher during an activity synthesis is crucial to making the intended learning goals explicit. Teachers need to have a clear idea of the destination for the day, week, month, and year, and select and sequence instructional activities (or use well-sequenced materials) that will get the class to their destinations. If you are going to a party, you need to know the address and also plan a route to get there; driving around aimlessly will not get you where you need to go.

## Different Learning Goals Require a Variety of Types of Tasks and Instructional Moves

The kind of instruction that is appropriate at any given time depends on the learning goals of a particular lesson. Lessons and activities can:

- provide experience with a new context
- introduce a new concept and associated language
- introduce a new representation
- formalize the definition of a term for an idea previously encountered informally
- identify and resolve common mistakes and misconceptions
- practice using mathematical language
- work toward mastery of a concept or procedure
- provide an opportunity to apply mathematics to a modeling or other application problem

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## Each and Every Student Should Have Access to the Mathematical Work

With proper structures, accommodations, and supports, all students can learn mathematics. Teachers' instructional tool boxes should include knowledge of and skill in implementing supports for different learners. This curriculum incorporates extensive tools for specifically supporting English Language Learners and Students with Disabilities



# Instructional Model

## Learning Goals and Targets

### Learning Goals

Teacher-facing learning goals appear at the top of lesson plans. They describe, for a teacher audience, the mathematical and pedagogical goals of the lesson. Student-facing learning goals appear in student materials at the beginning of each lesson and start with the word “Let’s.” They are intended to invite students into the work of that day without giving away too much and spoiling the problem-based instruction. They are suitable for writing on the board before class begins.

### Learning Targets

These appear in student materials at the end of each unit. They describe, for a student audience, the mathematical goals of each lesson. Teachers and students might use learning targets in a number of ways. Some examples include:

- targets for standards-based grading
- prompts for a written reflection as part of a lesson synthesis
- a study aid for self-assessment, review, or catching up after an absence from school

## Lesson Structure

1. INTRODUCE	2. EXPLORE AND DEVELOP
<p><b>Warm Up</b></p> <p>Warm Up activities either:</p> <ul style="list-style-type: none"><li>■ give students an opportunity to strengthen their number sense and procedural fluency.</li><li>■ make deeper connections.</li><li>■ encourage flexible thinking.</li></ul> <p>or:</p> <ul style="list-style-type: none"><li>■ remind students of a context they have seen before.</li><li>■ get them thinking about where the previous lesson left off.</li><li>■ preview a calculation that will happen in the lesson.</li></ul>	<p><b>Classroom Activities</b></p> <p>A sequence of one to three classroom activities. The activities are the heart of the mathematical experience and make up the majority of the time spent in class.</p> <p><b>Each classroom activity has three phases.</b></p> <p><b>The Launch</b></p> <p>The teacher makes sure that students understand the context and what the problem is asking them to do.</p>

## Practice Problems

Each lesson includes an associated set of practice problems that may be assigned as homework or for extra practice in class. They can be collected and scored or used for self-assessment. It is up to teachers to decide which problems to assign (including assigning none at all).

The design of practice problem sets looks different from many other curricula, but every choice was intentional, based on learning research, and meant to efficiently facilitate learning. The practice problem set associated with each lesson includes a few questions about the contents of that lesson, plus additional problems that review material from earlier in the unit and previous units. Our approach emphasizes distributed practice rather than massed practice

## Mathematical Modeling Prompts

Mathematics is a tool for understanding the world better and making decisions. School mathematics instruction often neglects giving students opportunities to understand this, and reduces mathematics to disconnected rules for moving symbols around on paper. Mathematical modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (NGA 2010). This mathematics will remain important beyond high school in students’ lives and education after high school (NCEE 2013).

- Modeling Prompts can be thought of as a project or assignment. They are meant to be launched in class by a teacher, but but can be worked on independently or in small groups by students in or out of class.. We built in maximum flexibility for a teacher to implement these in a way that will work for them.
- The purpose of mathematical modeling is for students to understand that they can use math to better understand things they are interested in in the world.
- Mathematical modeling is different from solving word problems. There should be room to interpret the problem and a range of acceptable assumptions and answers. Modeling requires genuine choices to be made by the modeler.
- Modeling with mathematics is not a solitary activity and students should have support from their teacher and classmate while assessments focus on providing feedback that helps students improve their modeling skills.

### 3. SYNTHESIZE

#### Student Work Time

Students work individually, with a partner, or in small groups.

#### Activity Synthesis

The teacher orchestrates some time for students to synthesize what they have learned and situate the new learning within previous understanding.

#### Lesson Synthesis

Students incorporate new insights gained during the activities into their big-picture understanding.

#### Cool Down

A task to be given to students at the end of the lesson. Students are meant to work on the Cool Down for about 5 minutes independently and turn it in.

National Governors Association Center for Best Practices (2010). Common Core State Standards for Mathematics. NCEE (2013). What Does It Really Mean to Be College and Work Ready? Retrieved November 20, 2017 from <http://ncee.org/college-and-work-ready/>



## Instructional Routines

The kind of instruction appropriate in any particular lesson depends on the learning goals of that lesson. Some lessons may be devoted to developing a concept, others to mastering a procedural skill, yet others to applying mathematics to a real-world problem. These aspects of mathematical proficiency are interwoven. These materials include a small set of activity structures and reference a small, high-leverage set of teacher moves that become more and more familiar to teachers and students as the year progresses.

Like any routine in life, these routines give structure to time and interactions. They are a good idea for the same reason all routines are a good idea: they let people know what to expect, and they make people comfortable.

Why are routines in general good for learning academic content? One reason is that students and the teacher have done these interactions before, in a particular order, and so they don't have to spend much mental energy on classroom choreography. They know what to do when, who is expected to talk when, and when they are expected to write something down. The structure of the routine frees them up to focus on the academic task at hand. Furthermore, a well-designed routine opens up conversations and thinking about mathematics that might not happen by themselves.

- Analyze It
- Anticipate, Monitor, Select, Sequence, Connect
- Aspects of Mathematical Modeling
- Card Sort
- Construct It
- Draw It
- Estimation
- Extend It
- Fit It
- Graph It
- Math Talk
- Notice and Wonder
- Poll the Class
- Take Turns
- Think Pair Share
- Which One Doesn't Belong?

## Warm Up 10.1 Notice and Wonder: Transformed (10 minutes)

The purpose of this warm-up is to elicit the idea that some shapes can be described as transformations of other shapes, which will be useful when students specify sequences of rigid transformations that take one figure onto another in the next activities. While students may notice and wonder many things about these images, the important discussion point is that rigid transformations take sides to sides of the same length and angles to angles of the same measure.

### Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Notice and Wonder

### Standards Alignment

**Building On:** 8.G.A.2 HSG-CO.A.2 HSG-CO.A.5

### Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

### Support For Students with Disabilities

**Action and Expression: Internalize Executive Functions.** Provide students with a table to record what they notice and wonder prior to being expected to share these things with others.

**Supports accessibility for:** Language; Organization

### Things students may notice:

- The parallelogram  $S$  can reflect onto the other parallelogram  $M$ .
- The parallelograms  $S$  and  $M$  are congruent.
- Point  $A$  is 2 spaces from both point  $O$  and point  $E$ .
- There are points  $A$ ,  $B$ ,  $C$ , and  $D$ .
- There are points  $A'$ ,  $B'$ ,  $C'$ , and  $D'$ .

### Things students may wonder:

- What transformations did they use?
- Is  $D$  similar to  $S$ ?
- Do the shapes have the same area?
- Are the side lengths the same?
- How do you pronounce  $A'$ ?
- Why use the same letters twice?

Student Edition, p. 1

(continued on the next page)

112 Unit 1 Constructions and Rigid Transformations

Topic Translations, Reflections, and Symmetry

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

## Summary

We've learned how to transform functions in several ways. We can translate graphs of functions up and down, changing the output values while keeping the input values. We can translate graphs left and right, changing the input values while keeping the output values. We can reflect functions across an axis, swapping either input or output values for their opposites depending on which axis is reflected across.

For some functions, we can perform specific transformations and it looks like we didn't do anything at all. Consider the function  $f$  whose graph is shown here:

What transformation could we do to the graph of  $f$  that would result in the same graph? Examining the shape of the graph, we can see a symmetry between points to the left of the  $y$ -axis and the points to the right of the  $y$ -axis. Looking at the points on the graph where  $x = 1$  and  $x = -1$ , these opposite inputs have the same outputs since  $f(1) = 4$  and  $f(-1) = 4$ . This means that if we reflect the graph across the  $y$ -axis, it will look no different. This type of symmetry means  $f$  is an **even function**.

Now consider the function  $g$  whose graph is shown here:

What transformation could we do to the graph of  $g$  that would result in the same graph? Examining the shape of the graph, we can see that there is a symmetry between points on opposite sides of the axes. Looking at the points on the graph where  $x = 1$  and  $x = -1$ , these opposite inputs have opposite outputs since  $g(1) = 2.35$  and  $g(-1) = -2.35$ . So, a transformation that takes the graph of  $g$  to itself has to reflect across the  $x$ -axis and the  $y$ -axis. This type of symmetry is what makes  $g$  an **odd function**.

### Glossary

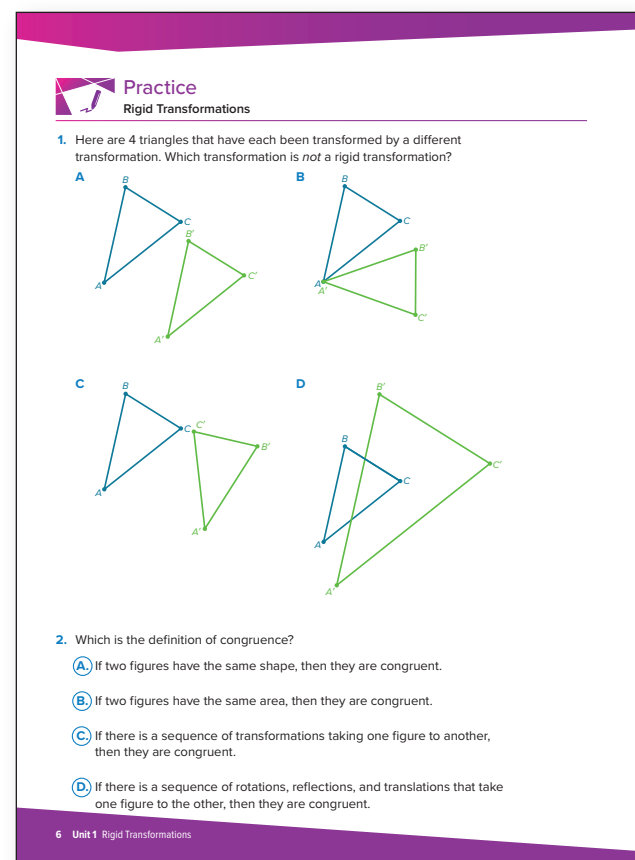
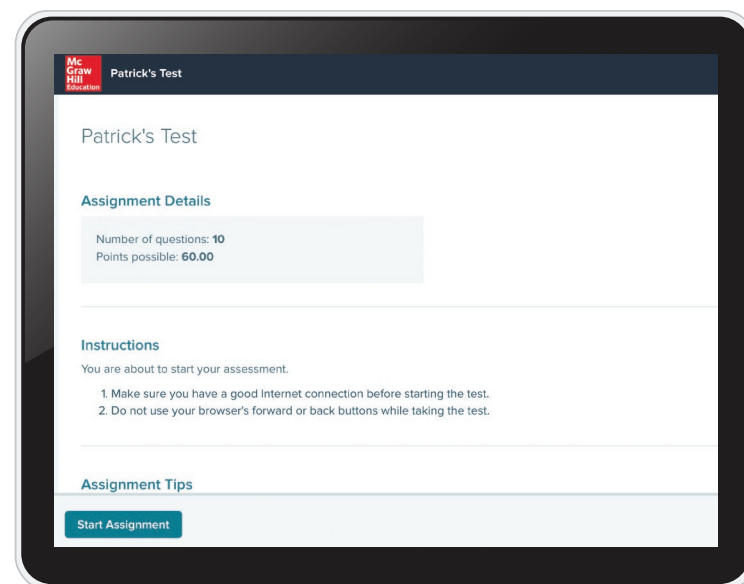
- **even function** A function  $f$  that satisfies the condition  $f(x) = f(-x)$  for all inputs  $x$ . You can tell an even function from its graph: Its graph is symmetric about the  $y$ -axis.
- **odd function** A function  $f$  that satisfies  $f(x) = -f(-x)$  for all inputs  $x$ . You can tell an odd function from its graph: Its graph is taken to itself when you reflect it across both the  $x$ - and  $y$ -axes. This can also be seen as a  $180^\circ$  rotation about the origin.

Lesson 5.5 Some Functions Have Symmetry 3

## How to Assess Progress

*Illustrative Mathematics* contains many opportunities and tools for both formative and summative assessment. Some things are purely formative, but the tools that can be used for summative assessment can also be used formatively.

- Each unit begins with a diagnostic assessment (“Check Your Readiness”) of concepts and skills that are prerequisite to the unit as well as a few items that assess what students already know of the key contexts and concepts that will be addressed by the unit.
- Each instructional task is accompanied by commentary about expected student responses and potential misconceptions so that teachers can adjust their instruction depending on what students are doing in response to the task. Often there are suggested questions to help teachers better understand students’ thinking.
- Each lesson includes a cool-down (analogous to an exit slip or exit ticket) to assess whether students understood the work of that day’s lesson. Teachers may use this as a formative assessment to provide feedback or to plan further instruction.
- A set of cumulative practice problems is provided for each lesson that can be used for homework or in-class practice. The teacher can choose to collect and grade these or simply provide feedback to students.
- Each unit includes an end-of-unit written assessment that is intended for students to complete individually to assess what they have learned at the conclusion of the unit. Longer units also include a mid-unit assessment. The mid-unit assessment states which lesson in the middle of the unit it is designed to follow.





## Supporting Students with Disabilities

All students are individuals who can know, use, and enjoy mathematics. *Illustrative Mathematics* empowers students with activities that capitalize on their existing strengths and abilities to ensure that all learners can participate meaningfully in rigorous mathematical content. Lessons support a flexible approach to instruction and provide teachers with options for additional support to address the needs of a diverse group of students.

## Supporting English-language Learners

*Illustrative Mathematics* builds on foundational principles for supporting language development for all students. Embedded within the curriculum are instructional supports and practices to help teachers address the specialized academic language demands in math when planning and delivering lessons, including the demands of reading, writing, speaking, listening, conversing, and representing in math (Aguirre & Bunch, 2012). Therefore, while these instructional supports and practices can and should be used to support all students learning mathematics, they are particularly well-suited to meet the needs of linguistically and culturally diverse students who are learning mathematics while simultaneously acquiring English.

Aguirre, J.M. & Bunch, G. C. (2012). What's language got to do with it?: Identifying language demands in mathematics instruction for English Language Learners. In S. Celedón-Pattichis & N.

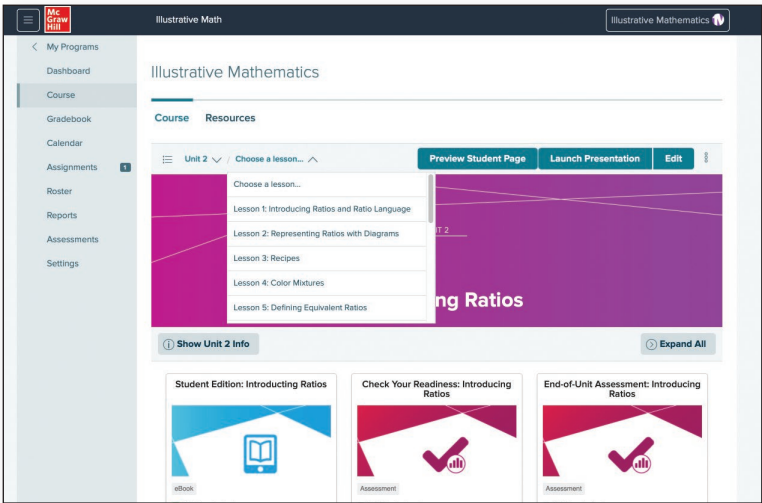


# Digital

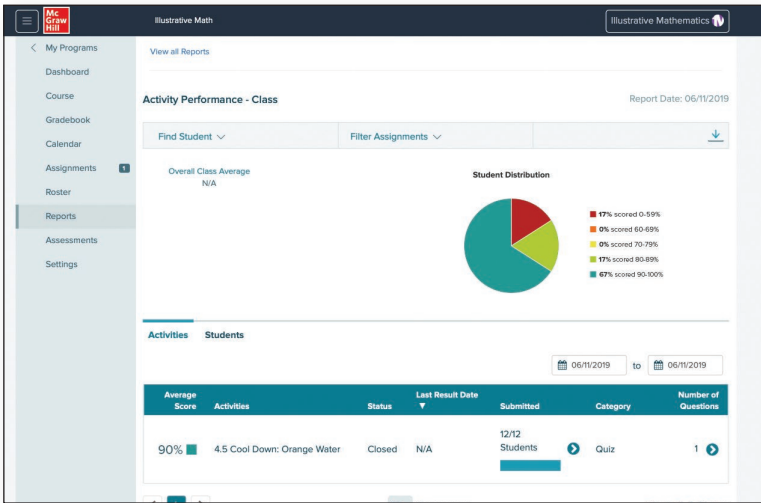
McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

Online resources offer:

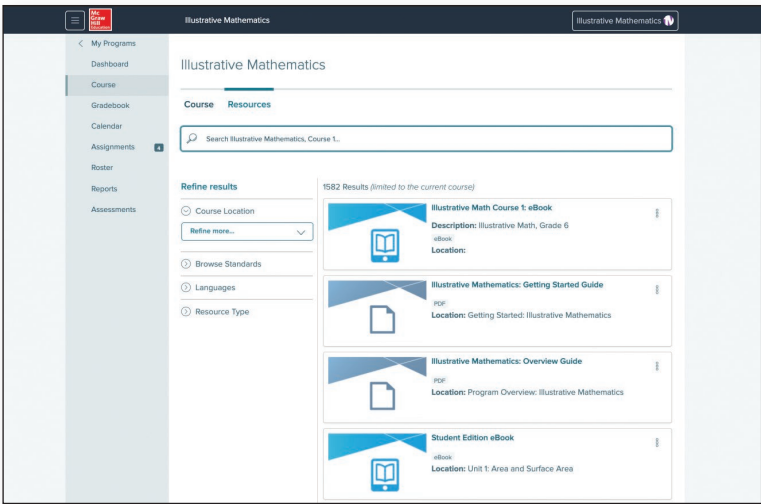
- customizable content
- the ability to add resources
- auto-scoring of student practice work
- on-going student assessments
- classroom performance reporting



**Launch** Presentations Digital versions of lessons to present content.



**Reports** Review the performance of individual students, classrooms, and grade levels.



**Access Resources** Point-of-use access to resources such as assessments, eBooks, and course guides.



## Unit 1

# One-variable Statistics



### Getting to Know You

- Lesson 1-1 Human Box Plot
- 1-2 Lesson 2: Human Dot plot
- 1-3 Which One?

### Distribution Shapes

- 1-4 The Shape of Data Distributions
- 1-5 Watch Your Calculations

### Manipulating Data

- 1-9 Using Technology for Statistics
- 1-10 Measures of Center
- 1-11 Decisions, Decisions
- 1-12 Variability
- 1-13 Standard Deviation in Real-World Contexts
- 1-14 Outliers & Means
- 1-15 Where Are We Eating?

### Analyzing Data

- 1-16 Compare & Contrast

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## Unit 2

# Linear Equations, Inequalities and Systems



### Writing and Modeling with Equations

- Lesson 2-1** Expressing Mathematics
- 2-2** Words and Symbols
- 2-3** Setting the Table
- 2-4** Solutions in Context
- 2-5** Graphs, Tables, and Equations

### Manipulating Equations and Understanding Their Structure

- 2-6** Equality Diagrams
- 2-7** Why Is That Okay?
- 2-8** Reasoning About Equations
- 2-9** Same Situation, Different Symbols
- 2-10** Equations and Relationships
- 2-11** Slopes and Intercepts

### Systems of Linear Equations in Two Variables

- 2-12** Connecting Situations and Graphs
- 2-13** Making New, True Equations
- 2-14** Making More New, True Equations
- 2-15** Off the Line
- 2-16** Elimination
- 2-17** Number of Solutions in One-Variable Equations

### Linear Inequalities in One Variable

- 2-18** Inequalities in Context
- 2-19** Queuing on the Number Line
- 2-20** Interpreting Inequalities

### Linear Inequalities in Two Variables

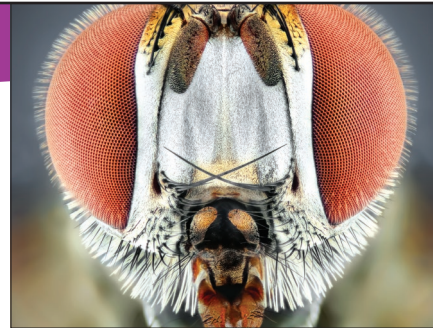
- 2-21** From One- to Two-Variable Inequalities
- 2-22** Situations with Constraints
- 2-23** Modeling Constraints

### Systems of Linear Inequalities in Two Variables

- 2-24** Reasoning with Graphs of Inequalities
- 2-25** Representing Systems of Inequalities
- 2-26** Testing Points to Solve Inequalities

## Unit 3

# Two-Variable Statistics



## Two-way Tables

- Lesson 3-1 Human Frequency Table
- 3-2 Talking Percents
- 3-3 Associations and Relative Frequency Tables

## Scatterplots

- 3-4 Interpret This, Interpret That
- 3-5 Goodness of Fit
- 3-6 Actual Data vs. Predicted Data

## Correlation Coefficients

- 3-7 Confident Models
- 3-8 Correlations
- 3-9 What's the Correlation?

## Estimating Lengths

- 3-10 Putting It All Together

source7/123RF

## Unit 4

# Functions



## Functions and Their Representations

- Lesson 4-1 Describing Graphs
- 4-2 Understanding Points in Situations
- 4-3 Using Function Notation
- 4-4 Interpreting Functions
- 4-5 Function Representations

## Analyzing and Creating Graphs of Functions

- 4-6 Finding Interesting Points on a Graph
- 4-7 Slopes of Segments
- 4-8 Interpreting and Drawing Graphs for Situations
- 4-9 Increasing and Decreasing Functions

## A Closer Look at Inputs and Outputs

- 4-10 Interpreting Inputs and Outputs
- 4-11 Examining Domains and Ranges
- 4-12 Functions with Multiple Parts
- 4-13 Number Line Distances
- 4-14 Absolute Value Meaning

## Inverse Functions

- 4-15 Finding Input Values and Function Values
- 4-16 Rewriting Equations for Perspectives
- 4-17 Interpreting Function Parts in Situations

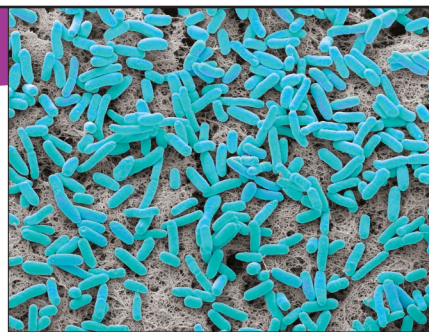
## Putting it All Together

- 4-18 Modeling Price Information



## Unit 5

# Introduction to Exponential Functions



## Looking at Growth

- Lesson 5-1 Reviewing Exponents
- 5-2 Growth Patterns

## A New Kind of Relationship

- 5-3 Properties of Exponents
- 5-4 Working with Fractions
- 5-5 Connections between Representations
- 5-6 Find That Factor
- 5-7 Negative Exponents

## Exponential Functions

- 5-8 Representing Functions
- 5-9 Interpreting Functions
- 5-10 Rate of Change
- 5-11 Skills for Modeling with Mathematics
- 5-12 Connections between Graphs and Equations
- 5-13 Representations of Exponential Functions

## Percent Growth and Decay

- 5-14 Percent Increase and Decrease
- 5-15 Changing the Score
- 5-16 Over and Over
- 5-17 Annually, Quarterly, or Monthly?
- 5-18 Bases and Exponents

## Comparing Linear and Exponential Functions

- 5-19 Adjusting Windows
- 5-20 Evaluating Functions over Equal Intervals

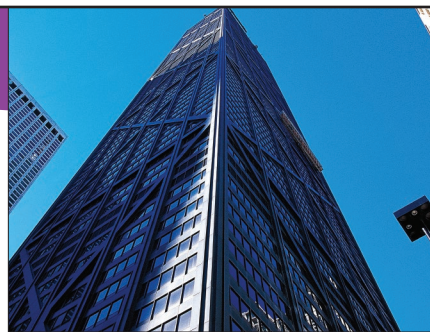
## Putting It All Together

- 5-21 Skills for Mathematical Modeling

STEVE GSCHMEISSNER/SCIENCE PHOTO LIBRARY/Getty Images

## Unit 6

# Introduction to Quadratic Functions



## A Different Kind of Change

**Lesson 6-1** Accessing Areas and Pondering Perimeters

**6-2** Describing Patterns

## Quadratic Functions

**6-3** Lots of Rectangles

**6-4** Evaluating Quadratic and Exponential Functions

**6-5** Distance To and Distance From

**6-6** Graphs of Situations that Change

**6-7** Accurate Representations

## Working with Quadratic Expressions

**6-8** Areas and Equivalent Expressions

**6-9** Working with Signed Numbers

**6-10** Relating Linear Equations and their Graphs

## Features of Graphs of Quadratic Functions

**6-11** Zeros of Functions and Intercepts of Graphs

**6-12** Changing the Equation

**6-14** Interpreting Representations

**6-15** Preparing for Vertex Form

**6-16** Graphing from the Vertex Form

**6-17** Parameters and Graphs

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## Unit 7

# Quadratic Functions



### Finding Unknown Inputs

- Lesson 7-1 Areas Around Areas
- 7-2 Equations and Graphs

### Solving Quadratic Equations

- 7-3 Squares and Equations
- 7-4 Equations and Their Zeros
- 7-5 Steps in Solving Equations
- 7-6 Sums and Products
- 7-7 Integers of Quadratics
- 7-8 Multiplying Expressions
- 7-9 Equivalent Equations and Functions
- 7-10 Quadratic Zeros

### Completing the Square

- 7-11 Finding Perfect Squares
- 7-12 Forms of Quadratic Equations
- 7-13 Constants in Quadratic Equations
- 7-14 Rewriting Quadratic Expressions
- 7-15 Irrational Numbers

### The Quadratic Formula

- 7-16 Working with Quadratics
- 7-17 Quadratic Meanings
- 7-18 Solving Quadratics
- 7-19 Quadratic Steps
- 7-20 Quadratics and Irrationals
- 7-21 Odd and Even Numbers

### Vertex Form Revisited

- 7-22 Features of Parabolas
- 7-23 Comparing Functions

### Putting It All Together

- 7-24 Quadratic Situations

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	Lessons	Days	Standards
Topic	<b>A Different Kind of Change</b>		
	<a href="#">Lesson 6-1</a> Accessing Areas and Pondering Perimeters	1	3.MD.D, 4.MD.A.3, 6.EE.C.9, 8.F.B.5, HSF-BF.A.1.a, HSF-LE.A
	<a href="#">Lesson 6-2</a> Describing Patterns	1	HSF-LE.A.1, HSF-BF.A.1.a, HSF-LE.A.1, HSF-BF.A.1.a
Topic	<b>Quadratic Functions</b>		
	<a href="#">Lesson 6-3</a> Lots of Rectangles	1	5.NF.B.4.b, 6.EE.A.3, 6.EE.A.4, HSA-SSE.A.2, HSF-BF.A.1.a
	<a href="#">Lesson 6-4</a> Evaluating Quadratic and Exponential Functions	1	6.EE.A.2.c, 6.EE.A.1, HSF-IF.A.2, HSF-LE.A.3
	<a href="#">Lesson 6-5</a> Distance To and Distance From	1	8.F.B.4, HSF-BF.A.1.a, HSF-IF.A.2, HSF-IF.C.7.a, HSF-IF.C.7.a
	<a href="#">Lesson 6-6</a> Graphs of Situations that Change	1	8.F.B.4, HSF-IF.C.7.a, HSF-LE.A.2, HSF-IF.C.7.a
	<a href="#">Lesson 6-7</a> Accurate Representations	1	HSF-IF.B.5, HSF-IF.C.7.a, HSF-IF.B.5, HSF-IF.C.7.a
Topic	<b>Working with Quadratic Expressions</b>		
	<a href="#">Lesson 6-8</a> Areas and Equivalent Expressions	1	4.NBT.B.5, 6.EE.A.3, HSA-SSE.B.3
	<a href="#">Lesson 6-9</a> Working with Signed Numbers	1	7.EE.A.1, 7.NS.A.1, 7.NS.A.2, HSA-SSE.A.2, HSA-SSE.B.3
	<a href="#">Lesson 6-10</a> Relating Linear Equations and their Graphs	1	HSA-REI.D.10, 8.F.A.3, 8.F.B.4, HSF-IF.C.7.a, HSA-SSE.B.3, HSF-IF.B.4
Topic	<b>Features of Graphs of Quadratic Functions</b>		
	<a href="#">Lesson 6-11</a> Zeros of Functions and Intercepts of Graphs	1	8.F.A.1, HSF-IF.A.2, HSF-IF.B.4, HSF-IF.C, HSF-IF.C.7, HSF-IF.C.8
	<a href="#">Lesson 6-12</a> Changing the Equation	1	6.EE.A.2, 7.NS.A.2.c, HSF-IF.C.7.a, HSF-BF.B.3, HSF-IF.A.2
	<a href="#">Lesson 6-14</a> Interpreting Representations	1	HSF-IF.C.9, HSF-IF.C.9
	<a href="#">Lesson 6-15</a> Preparing for Vertex Form	1	7.EE.A.1, HSF-BF.B.3, HSF-IF.C.7.a, HSF-BF.B.3, HSF-IF.C.7.a, HSF-IF.C.8.a
	<a href="#">Lesson 6-16</a> Graphing from the Vertex Form	1	6.EE.A.2, 7.NS.A.2.c, HSF-IF.C.7.a, HSF-IF.C.7.a
	<a href="#">Lesson 6-17</a> Parameters and Graphs	1	8.G.A, HSF-BF.B.3, HSF-BF.B.3
	TOTAL	16-17	

## Required Materials

- ☐ Four-function calculators ([Lesson 1](#))
- ☐ Graphing technology ([Lessons 6, 12, 15, 17](#))

- ☐ Graph paper ([Lesson 1](#))
- ☐ Tools for creating a visual display ([Lesson 14](#))



# Graphs of Situations that Change

These materials, when encountered before Algebra 1 Lesson 6-6: *Building Quadratic Functions to Describe Situations (Part 2)*, support success in that lesson.

## Goals (Teacher-Facing)

- Create functions that model situations with a constant rate of change or constant growth factor.
- Identify the coordinates of the intercepts of a graph using technology or other methods.

## Student Learning Goals

Let’s identify intercepts on a graph.

## Required Materials

- Graphing technology

## Lesson Narrative

In the associated Algebra 1 lesson, students make sense of the vertex of a graph and zeros of a quadratic function that represents a situation. This supporting lesson gives students a chance to practice identifying intercepts of graphs that represent linear and exponential functions. This lesson is a good opportunity to help students become more comfortable extracting the coordinates of important points of graphs using the graphing technology that is available to them. The first activity explicitly asks students to use graphing technology to determine the coordinates of some points on a graph and draw connections between the various representations and the contexts they represent. The practice activity offers several opportunities to practice these skills. In the warm up, students have an opportunity to start to make sense of a situation so that in the next activity they are in a better position to make sense of what is being asked and make sense of the quantities in the situation. **MP1 MP2**

## Instructional Routines

- Notice and Wonder
- Think Pair Share

## Lesson Pacing

	Pacing (min)
<b>Warm Up 6.1</b> Notice and Wonder: The Draining Tank	5
<b>Activity 6.2</b> Identifying Important Points	15
<b>Activity 6.3</b> Three Situations	25
<b>TOTAL</b>	<b>45</b>

## Standards Alignment

### Building On

**8.F.B.4** Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two  $(x, y)$  values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

### Addressing

**HSF-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.

**HSF-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

### Building Towards

**HSF-IF.C.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.

## Warm Up 6.1 Notice and Wonder: The Draining Tank (5 minutes)

This warm up prompts students to make sense of a problem before solving it by familiarizing themselves with a context and the mathematics that might be involved. **MP1**

In the next activity, they will hear more details about the situation and be asked specific questions about it. When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use. **MP6**

Students might first propose less formal or imprecise language and then restate their observation with more precise language in order to communicate more clearly. For example, for this prompt, they might use words like full, empty, volume, time, minutes or seconds, and rate of change.

### Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Notice and Wonder

### Standards Alignment

**Building Towards** HSF-IF.C.7.a

Topic Quadratic Functions

Lesson 6-6

## Graphs of Situations that Change


NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**Learning Goal** Let's identify intercepts on a graph.

**Warm Up**

**6.1 Notice and Wonder: The Draining Tank**

A water tank is draining at a constant rate.



What do you notice? What do you wonder?

**Students may notice:**

- There is some water in a tank.
- The water is draining out of the tank.
- It says the water is draining at a constant rate.

**Students may wonder:**

- What is the capacity of the tank?
- How much water is in the tank?

- How long has it been draining for?
- When will the tank be empty?
- How quickly is it draining?
- How would you measure the rate at which it is draining?
- Where is the tank located?
- Is the tank supposed to be draining, or is this a leak?

Lesson 6-6 Graphs of Situations that Change
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Student Edition, p. 105

### Launch

Display the prompt for all to see. Give students 1 minute of quiet think time and ask them to be prepared to share at least one thing they notice and one thing they wonder. Give students another minute to discuss their observations and questions.

### Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the task statement. After all responses have been recorded without commentary or editing, ask students, *Is there anything on this list that you are wondering about now?* Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

## Activity 6.2 Identifying Important Points (15 minutes)

In the associated Algebra 1 lesson, students write an equation to model the distance traveled by an object moving at a constant speed. They will also identify important points on a graph representing projectile motion and determine a reasonable domain. In this preparatory activity, they write a linear function to model a situation involving constant rate of change, practice using graphing technology to extract the coordinates of points on the graph, and determine a reasonable domain for the function based on the situation it is modeling. It is intentional that the first few entries in the table are difficult to determine by using the graph—this is to encourage students to think about the information “drains at a constant rate of 2 gallons per minute.” This activity provides opportunities to attend to the meaning of quantities in the situation. **MP2**

### Instructional Routines


See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Think Pair Share

### Standards Alignment

**Building On** 8.F.B.4

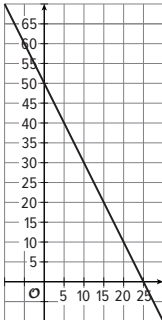
**Addressing** HSF-IF.C.7.a, HSF-LE.A.2

**Activity**  
6.2 Identifying Important Points

A tank has 50 gallons of water and drains at a constant rate of 2 gallons per minute. Here is a graph representing the situation:

1. Label each axis to show what it represents. Be sure to include units.  
**The horizontal axis is time in seconds, and the vertical axis is the volume of water in the tank in gallons.**
2. Complete the table.

$t$	0	1	2	3	10	20	$t$
$v(t)$	50	48	46	44	30	10	$50 - 2t$
3. Use the expression in terms of  $t$  from the table to write a function modeling this situation.  
 **$v(t) = 50 - 2t$  or equivalent**
4. Use graphing technology to graph your function. Practice setting the graphing window so that you can see both intercepts, and using graphing technology to see the coordinates of different points on your graph.  
**No written response needed.**
5. What is a reasonable domain for this function, based on the situation it models?  **$0 \leq t \leq 25$**



106 Unit 6 Introduction to Quadratic Functions

### Launch

Ask students to read the stem and decide how they think the axes should be labeled, and share this with a partner. Invite a few students to share their ideas. Ensure that all students have the axes labeled correctly before proceeding with the rest of the activity.

Give students a few minutes to create the table and write a function. At that point, depending on students’ experience with graphing technology, it may be desirable to demonstrate how to set an appropriate graphing window and use the technology to extract the coordinates of the intercepts and other points on the graph.

### Activity Synthesis

Possible questions for discussion:

- **Why might it be useful to know the coordinates of intercepts of a graph that models a situation?** The intercepts tell you the value of one quantity when the other quantity is 0. In this case, that means the volume of water in the tank when it starts draining (at 0 minutes), and how many minutes it takes the tank to empty (the time when the volume is at 0 gallons).
- **What are some important things to keep in mind when setting a graphing window?** You want to make sure you can see any important points on the graph, which often includes the intercepts, though it depends on the situation. Other responses might depend on the type of graphing technology used.

## Activity 6.3 Three Situations (25 minutes)

This activity is an opportunity to practice using graphing technology to determine important points on a graph and to practice writing a function to represent a situation described verbally. Students can choose to find the coordinates of the intercepts either by using the technological tool or by reasoning about the definition of the function. For example, on the graph of function  $d$ , they can either use technology to find the  $y$ -coordinate when  $x$  is 0, or they can evaluate  $81 \cdot 3^0$ .

Note that in function  $b$ , the  $x$ -intercept is a very small negative number. In a few cases, students will encounter a small, negative  $x$ -intercept in the projectile motion lessons. Although intercepts like this aren't generally meaningful in the context, they are mentioned a bit in the associated Algebra 1 lessons. So that's the reason why such a function was included here.

### Standards Alignment

**Addressing** HSF-IF.C.7.a, HSF-LE.A.2

Topic Quadratic Functions

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**Activity**  
**6.3 Three Situations**

1. Create a graph of each function using graphing technology. Make a rough sketch of each graph. On each graph, label the coordinates of any intercepts.
  - $a(x) = 4 + -3x + 50$   
**A sketch of  $54 - 3x$  with the points (0,54) and (18,0) labeled.**
  - $b(x) = 10(x - 0.5) + 17$   
**A sketch of  $10x + 12$  with (0,12) and (-1.2,0) labeled.**
  - $c(x) = 81 - \frac{1}{3}x$   
**A sketch of  $81 - \frac{1}{3}x$  with (0,81) and (243,0) labeled.**
  - $d(x) = 8x - x^2$   
**A sketch of  $8x - x^2$  with (0,0) and (8,0) labeled.**
2. Function  $d$  has a maximum point. Can you find the coordinates of this point?  
**(4,16)**

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### Launch

The second question asks students to find the coordinates of the vertex of the graph of the quadratic function  $d$ . Depending on the specific graphing technology used, they may be able to figure this out on their own, or they may need explicit instruction on how to use the technology to find the coordinates of this point.

### Activity Synthesis

Display one or more students' work on the last question for all to see. Point out examples where students made sure to name the variables they used in their equations. If needed, provide others time to add that to their work.

- **How are the graphs of the three situations alike? How are they different?** They all have the same  $y$ -intercept. Two of the graphs are lines and the third is the graph of an exponential function.
- **How are the equations you wrote alike? How are they different?** The equations all have a 128. The two linear equations either add  $4t$  or subtract  $4t$  from 128. The exponential equation has a growth factor of  $\frac{1}{2}$  and the variable is the exponent.
- **Which graphs have  $x$ -intercepts? Which graphs have a  $y$ -intercept?**  
 The graph of  $y = 128\left(\frac{1}{2}\right)^x$  does not have an  $x$ -intercept. All the graphs have the same  $y$ -intercept, (0, 128).



3. Here are some situations. For each situation:

a. Write an equation representing the situation. If you get stuck, consider making a table of values, thinking about what type of function it is, or thinking about the initial value and rate of change or growth factor. Be sure to explain the meaning of any variables you use.

b. Sketch a graph representing each situation. Label the coordinates of any intercepts or other important points.

- A person has \$128 saved, and adds \$4 to their savings per week.

A person has \$128 dollars saved, and adds \$4 to their savings per week:  $y = 128 + 4x$ , where  $x$  represents time in weeks and  $y$  represents amount saved in dollars.

- A tank has 128 gallons of water, and drains at a constant rate of 4 gallons per minute.

A tank has 128 gallons of water, and drains at a constant rate of 4 gallons per minute:  $y = 128 - 4x$ , where  $x$  represents time in minutes and  $y$  represents volume of water in the tank in gallons.

- A patient is given 128 milligrams of a medication, and half of the medication leaves the patient's bloodstream every hour.

A patient is given 128 mg of a medication, and half of the medication leaves the patient's bloodstream every hour:  $128 \cdot \left(\frac{1}{2}\right)^x$ , where  $x$  represents time in hours and  $y$  represents amount of medication in the patient's bloodstream in milligrams.

- What does the  $y$ -intercept of each graph tell you about the situation? It is the initial amount.
- What does the  $x$ -intercept (if there is one) tell you about the situation? For the graph of  $y = 128 - 4x$ , the  $x$ -intercept tells when the tank is empty. For the savings account, the  $x$ -intercept is not meaningful because it would have a negative  $x$ -coordinate and negative values for time don't make sense in this situation.



**Student Edition**

Lesson 6-6

# Graphs of Situations that Change

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**Learning Goal** Let’s identify intercepts on a graph.



## Warm Up

### 6.1 Notice and Wonder: The Draining Tank

A water tank is draining at a constant rate.



What do you notice? What do you wonder?



## Activity

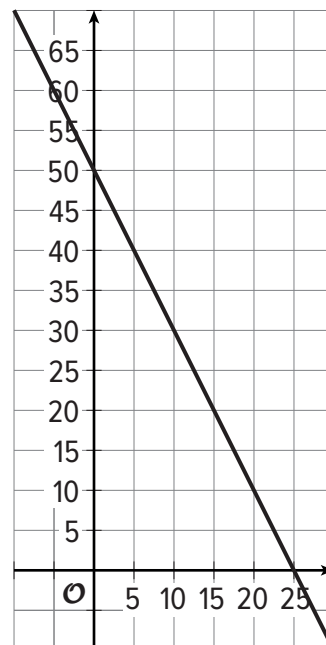
### 6.2 Identifying Important Points

A tank has 50 gallons of water and drains at a constant rate of 2 gallons per minute. Here is a graph representing the situation:

1. Label each axis to show what it represents. Be sure to include units.

2. Complete the table.

$t$	0	1	2	3	10	20	$t$
$v(t)$							



3. Use the expression in terms of  $t$  from the table to write a function modeling this situation.
4. Use graphing technology to graph your function. Practice setting the graphing window so that you can see both intercepts, and using graphing technology to see the coordinates of different points on your graph.
5. What is a reasonable domain for this function, based on the situation it models?



NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_



## Activity

### 6.3 Three Situations

1. Create a graph of each function using graphing technology. Make a rough sketch of each graph. On each graph, label the coordinates of any intercepts.

- $a(x) = 4 + -3x + 50$

- $b(x) = 10(x - 0.5) + 17$

- $c(x) = 81 - \frac{1}{3}x$

- $d(x) = 8x - x^2$

2. Function  $d$  has a maximum point. Can you find the coordinates of this point?

**3.** Here are some situations. For each situation:

- a.** Write an equation representing the situation. If you get stuck, consider making a table of values, thinking about what type of function it is, or thinking about the initial value and rate of change or growth factor. Be sure to explain the meaning of any variables you use.
- b.** Sketch a graph representing each situation. Label the coordinates of any intercepts or other important points.

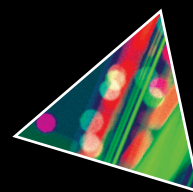
- A person has \$128 saved, and adds \$4 to their savings per week.
- A tank has 128 gallons of water, and drains at a constant rate of 4 gallons per minute.
- A patient is given 128 milligrams of a medication, and half of the medication leaves the patient's bloodstream every hour.

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