Mc Graw Hill

**Program Overview**

Grades 9–12

# McGraw-Hill Illustrative Mathematics®

The highest-rated curriculum. A partner you know and trust.

Illustrative Mathematics CERTIFIED

*"Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience."*[1]

# Creating a World Where Learners Know, Use, and Enjoy Mathematics

Decades of research shows that if students are given a chance to notice and wonder while trying to solve a problem by themselves, they retain procedural skills, develop problem-solving skills, build conceptual understanding, and form a mental framework for how ideas fit together. It allows students to develop strategies for tackling non-routine problems while engaging in productive struggle.

*Illustrative Mathematics* is a problem-based curriculum designed to address content and practice standards to foster learning for all. Students are encouraged to take an active role to see what they can figure out before having things explained to them or being told what to do.

[1]Hiebert, J., et al. (1996). Problem solving as a basis for reform in curriculum and instruction: the case of mathematics. Educational Researcher 25(4), 12-21. doi.org/10.3102/0013189X025004012

# The Highest Rated Curriculum. A Partner You Know and Trust.

## McGraw-Hill

- Personalized service and support from a local McGraw-Hill sales representative
- A team of curriculum specialists to support your implementation
- On-demand customer service to provide help when you need it
- Spend less time printing and more time teaching with reliable delivery of print resources

## Exclusive Features

- Interactive reports to drive instruction
- Student activities available digitally
- Autoscored practice problems for immediate feedback
- Engaging color print resources
- Improved layout of teacher materials supports instruction more efficiently
- *Options to bundle with *ALEKS*® personalized learning

**ALEKS* is not IM certified.*

## Supporting the *Illustrative Mathematics* Mission

As an IM Certified™ Partner, McGraw-Hill is committed to providing the support needed to successfully implement *Illustrative Mathematics*. A portion of every purchase is earmarked toward supporting the continued development of high-quality math curricula.

# The Illustrative Mathematics Classroom: Teacher Facilitated Learning

The teacher ensures students understand the question.

Students work individually or in groups, while the teacher monitors, listens, and questions.

The teacher helps students synthesize their learning.

## Each Lesson and Unit Tells a Story

Developing coherent learning progressions and connections among areas of study requires crafting lessons that tell a mathematical story. Lessons must coherently build across units and grade levels and attend to many things: the mathematics, representations, activity structures, and learning trajectories, to name a few. Each of these considerations impacts how students access mathematics and influences the belief that mathematics is a connected set of ideas that makes sense.

Each unit has a narrative that describes the mathematical work that will unfold in that unit. Each lesson in the unit also has a narrative.

*"An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world."*

**Principles to Action by National Council of Teachers of Mathematics**

## Lesson Narratives Explain:

- The mathematical content of the lesson and its place in the learning sequence.
- The meaning of any new terms introduced in the lesson.
- How the mathematical practices come into play, as appropriate.

## Activities within lessons also have narratives, which explain:

- The mathematical purpose of the activity and its place in the learning sequence.
- What students are doing during the activity.
- What the teacher needs to look for while students are working on an activity to orchestrate an effective synthesis.
- Connections to the mathematical practices, when appropriate.

# The *Illustrative Mathematics* Lesson

## Warm-Up

Help students get ready for the day's lesson or give students an opportunity to strengthen their number sense or procedural fluency.

## Instructional Activities

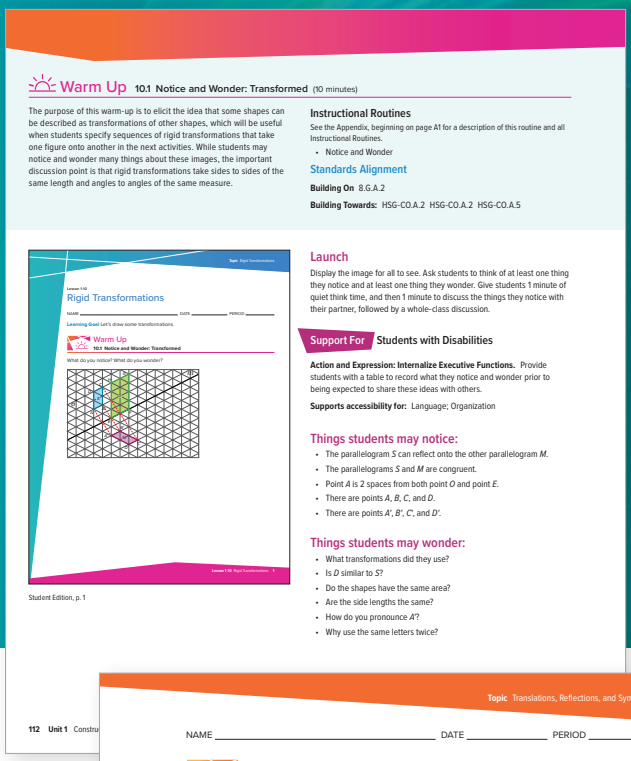
For each activity, the teacher helps students understand the problem, students work on the problem, and then the teacher makes sure that students synthesize what they have learned.

## Lesson Synthesis

Each lesson includes a lesson synthesis section that assists the teacher as they help students incorporate new insights into their big-picture understanding.

## Cool-Down

A brief formative assessment to determine whether students understood the lesson.



Warm Up 10.1 Notice and Wonder: Transformed (10 minutes)

The purpose of this warm-up is to elicit the idea that some shapes can be described as transformations of other shapes, which will be useful when students specify sequences of rigid transformations that take one figure onto another in the next activities. While students may notice and wonder many things about these images, the important discussion point is that rigid transformations take sides to sides of the same length and angles to angles of the same measure.

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Notice and Wonder

Standards Alignment

Building On 8.G.A.2

Building Towards: HSG-CO.A.2 HSG-CO.A.2 HSG-CO.A.5

Rigid Transformations

Warm Up 10.1 Notice and Wonder: Transformed

What do you notice? What do you wonder?

Student Edition, p. 1

Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Support For Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a table to record what they notice and wonder prior to being expected to share these ideas with others.

Supports accessibility for: Language; Organization

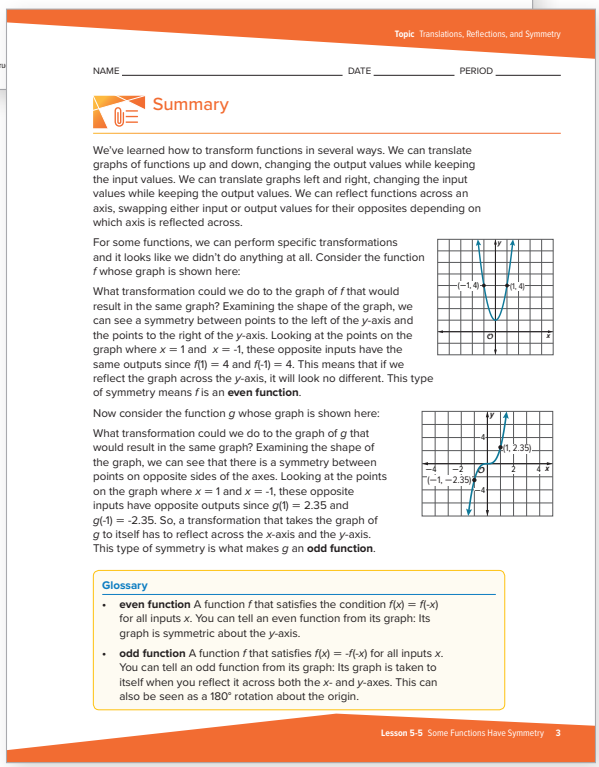
Things students may notice:

- The parallelogram *S* can reflect onto the other parallelogram *M*.
- The parallelograms *S* and *M* are congruent.
- Point *A* is 2 spaces from both point *O* and point *E*.
- There are points *A*, *B*, *C*, and *D*.
- There are points *A'*, *B'*, *C'*, and *D'*.

Things students may wonder:

- What transformations did they use?
- Is *D* similar to *S*?
- Do the shapes have the same area?
- Are the side lengths the same?
- How do you pronounce *A'*?
- Why use the same letters twice?

112 Unit 1 Construc



Topic Translations, Reflections, and Symmetry

NAME _____ DATE _____ PERIOD _____

Summary

We've learned how to transform functions in several ways. We can translate graphs of functions up and down, changing the output values while keeping the input values. We can translate graphs left and right, changing the input values while keeping the output values. We can reflect functions across an axis, swapping either input or output values for their opposites depending on which axis is reflected across.

For some functions, we can perform specific transformations and it looks like we didn't do anything at all. Consider the function *f* whose graph is shown here:

What transformation could we do to the graph of *f* that would result in the same graph? Examining the shape of the graph, we can see a symmetry between points to the left of the *y*-axis and the points to the right of the *y*-axis. Looking at the points on the graph where $x = 1$ and $x = -1$, these opposite inputs have the same outputs since $f(1) = 4$ and $f(-1) = 4$. This means that if we reflect the graph across the *y*-axis, it will look no different. This type of symmetry means *f* is an **even function**.

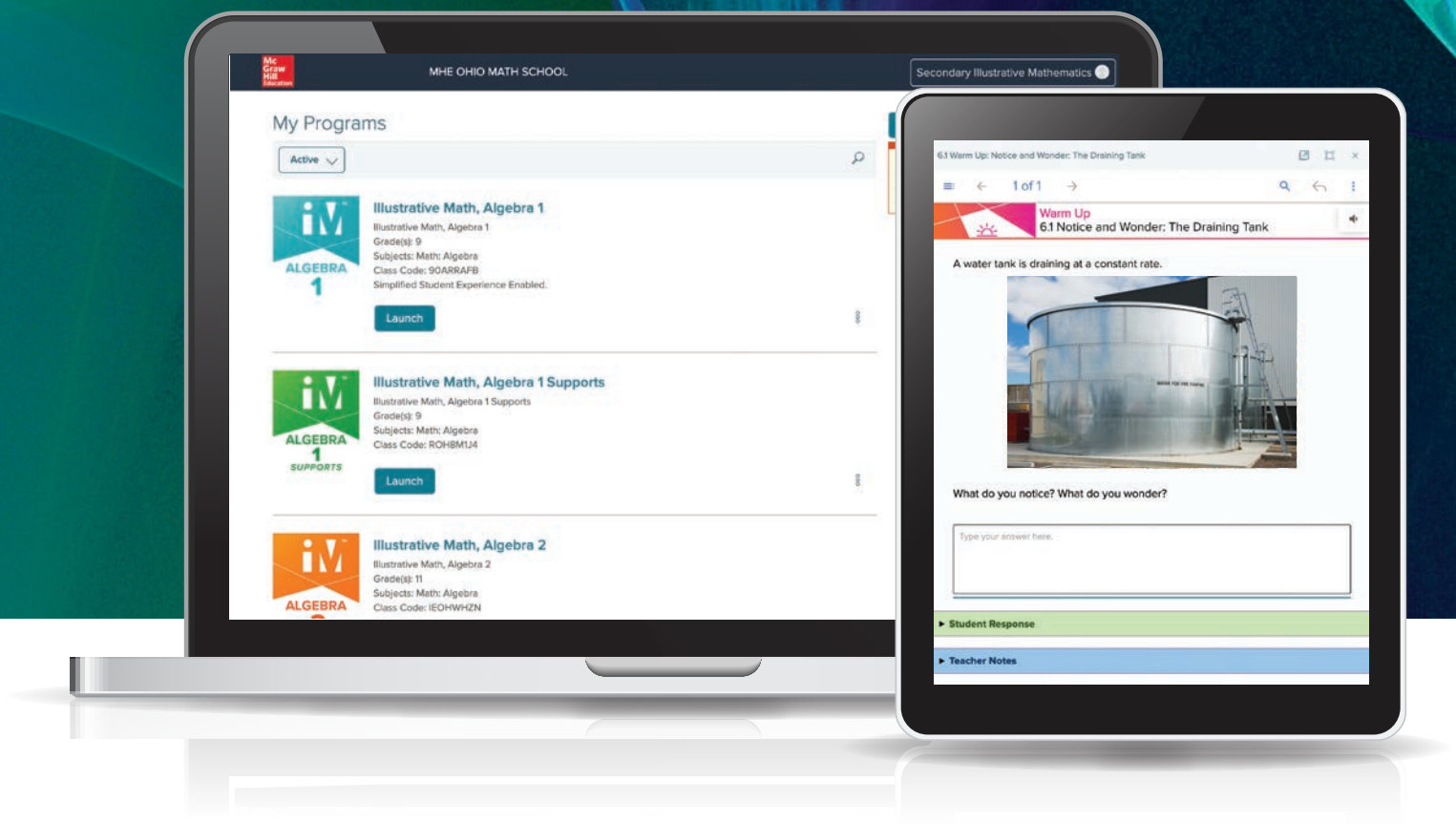
Now consider the function *g* whose graph is shown here:

What transformation could we do to the graph of *g* that would result in the same graph? Examining the shape of the graph, we can see that there is a symmetry between points on opposite sides of the axes. Looking at the points on the graph where $x = 1$ and $x = -1$, these opposite inputs have opposite outputs since $g(1) = 2.35$ and $g(-1) = -2.35$. So, a transformation that takes the graph of *g* to itself has to reflect across the *x*-axis and the *y*-axis. This type of symmetry is what makes *g* an **odd function**.

Glossary

- **even function** A function *f* that satisfies the condition $f(x) = f(-x)$ for all inputs *x*. You can tell an even function from its graph: Its graph is symmetric about the *y*-axis.
- **odd function** A function *f* that satisfies $f(x) = -f(-x)$ for all inputs *x*. You can tell an odd function from its graph: Its graph is taken to itself when you reflect it across both the *x*- and *y*-axes. This can also be seen as a 180° rotation about the origin.

Lesson 5-5 Some Functions Have Symmetry 3




## Digital Student and Teacher Editions

McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

**Online resources offer:**

- Customizable content.
- The ability to add resources.
- Auto-scoring of student practice work.
- Ongoing student assessments.
- Classroom performance reporting.

## Launch Presentations

Access and present digital versions of lessons.

## View Actionable Reports

Review the performance of individual students, classrooms, and grade levels.

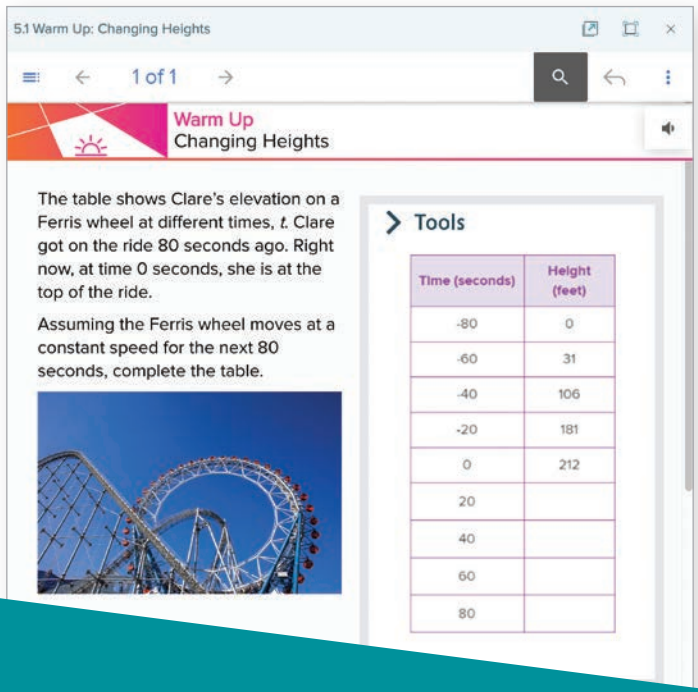
## Access Resources

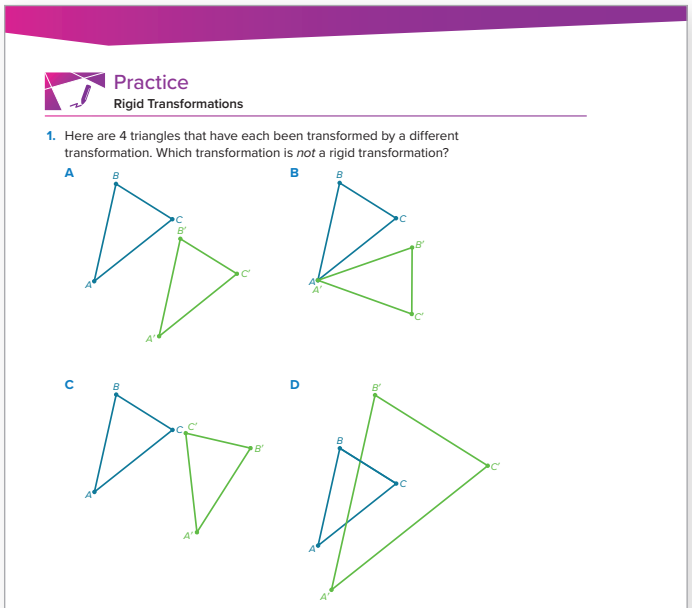
Point-of-use access to resources such as assessments, eBooks, and course guides.

# Mathematical Modeling Prompts

Mathematics is a tool for understanding the world better and making decisions. School mathematics instruction often neglects giving students opportunities to understand this, and reduces mathematics to disconnected rules for moving symbols around on paper. Mathematical modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (NGA 2010). This mathematics will remain important beyond high school in students' lives and education after high school (NCEE 2013).

- Modeling Prompts can be thought of like a project or assignment. They are meant to be launched in class by a teacher but can be worked on independently or in small groups by students in or out of class. We built in maximum flexibility for a teacher to implement these in a way that will work for them.
- The purpose of mathematical modeling is for students to understand that they can use math to better understand things they are interested in in the world.
- Mathematical modeling is different from solving word problems. There should be room to interpret the problem and a range of acceptable assumptions and answers. Modeling requires genuine choices to be made by the student.
- Modeling with mathematics is not a solitary activity. Students should have support from their teacher and classmates, while assessments focus on providing feedback that helps students improve their modeling skills.




National Governors Association Center for Best Practices (2010). Common Core State Standards for Mathematics.

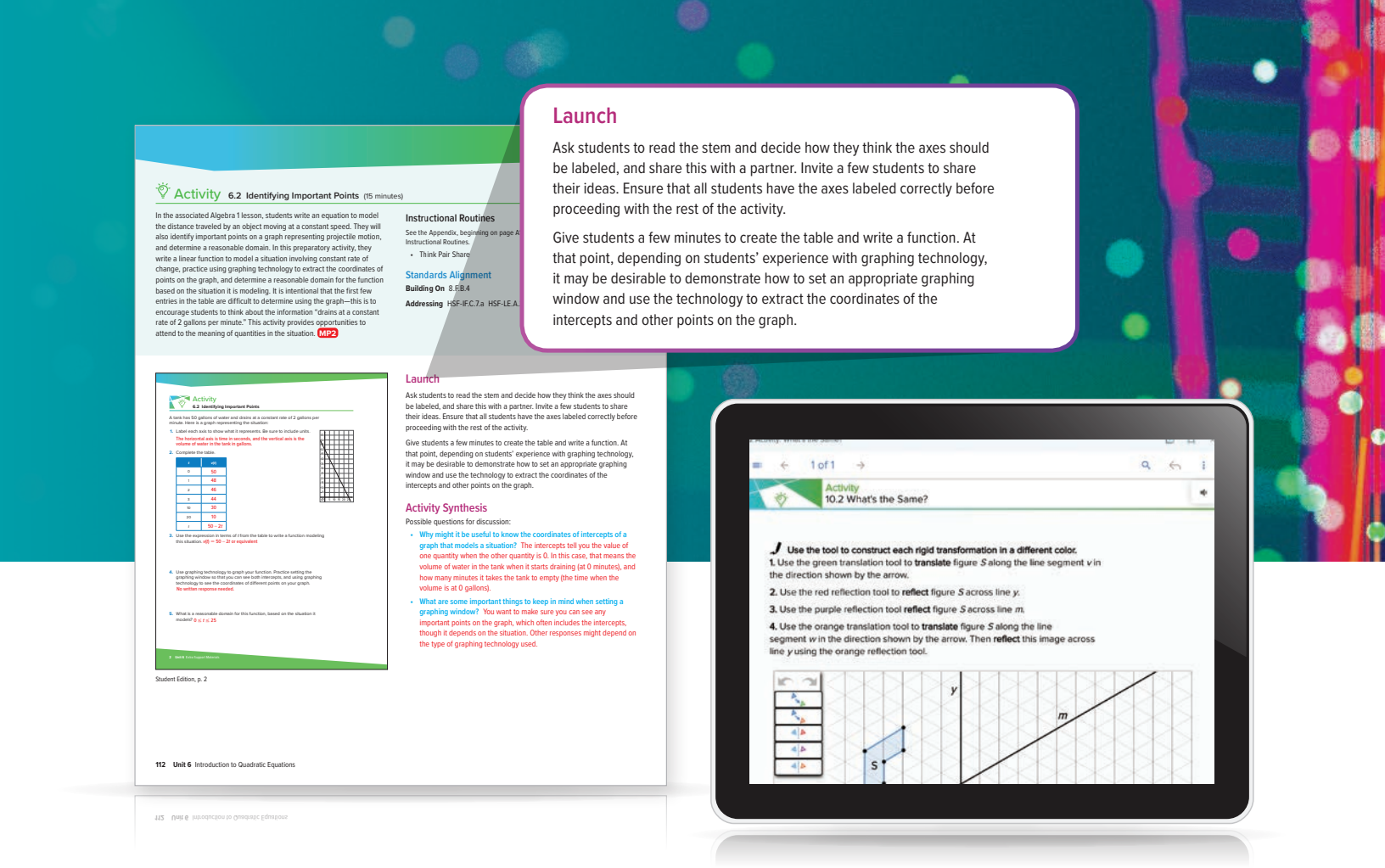
NCEE (2013). What Does It Really Mean to Be College and Work Ready? Retrieved November 20, 2017 from http://ncee.org/college-and-work-ready/

# Built-in Instructional Routines

Instructional routines allow students and teachers to pay less attention to what they are supposed to do and more attention to the mathematics being learned. Routines provide a structure that helps strengthen students' skills in listening and communicating their mathematical ideas.

*"The 5 Practices provided the structure I needed as a teacher to put all of these good teaching strategies into a cohesive teaching style that was not only student-centered but also focused on the mathematical goal of the day."*

**Alicia F. Woodbury, Minnesota**




# Facilitating Productive Classroom Discussions

Activities are structured using the 5 Practices for Orchestrating Mathematical Discussions[2].

## Anticipate

Consider how students might mathematically interpret the problem, the array of strategies that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that you would like your students to learn.

## Monitor

Pay close attention to students' mathematical thinking and solution strategies as they work on the task. Prompt students to make their thinking visible.

## Select

Select particular students to share their work with the rest of the class to get specific mathematics into the open for discussion.

## Sequence

Make purposeful choices about the order in which students' work is shared to maximize the chances of achieving the mathematical goals for the discussion.

## Connect

Help students draw connections between their solutions and other students' solutions as well as the key mathematical ideas in the lesson.

[2](Smith & Stein, 2011), also described in Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014), and Intentional Talk: How to Structure and Lead Productive Mathematical Discussions (Kazemi & Hintz, 2014).

# Assessing Student Progress

*Illustrative Mathematics* contains formative and summative assessments in each unit to help gauge classroom and student progress.

## Pre-Unit Assessments

Each unit begins with a pre-unit diagnostic assessment titled Check Your Readiness. This assessment reviews prerequisite concepts and skills for the unit. Each assessment item identifies which lesson the skill or concept is needed for and provides guidance on what to do if students struggle or do well on the item. Teachers can use this knowledge to pace or tune instruction or move more quickly through a topic to optimize instructional time.

## Mid-Unit Assessments

In longer units, a mid-unit assessment is also available. This assessment has the same form and structure as an end-of-unit assessment.

## Summative Assessments

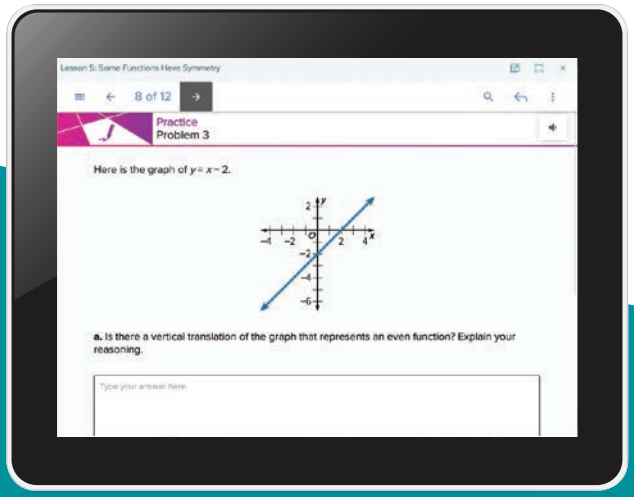
All summative assessment problems include a complete solution and standard alignment. Multiple-choice and multiple response problems often include a reason for each potential error a student might make. Restricted constructed response and extended response items include a rubric. Unlike formative assessments, problems on summative assessments generally do not prescribe a method of solution.

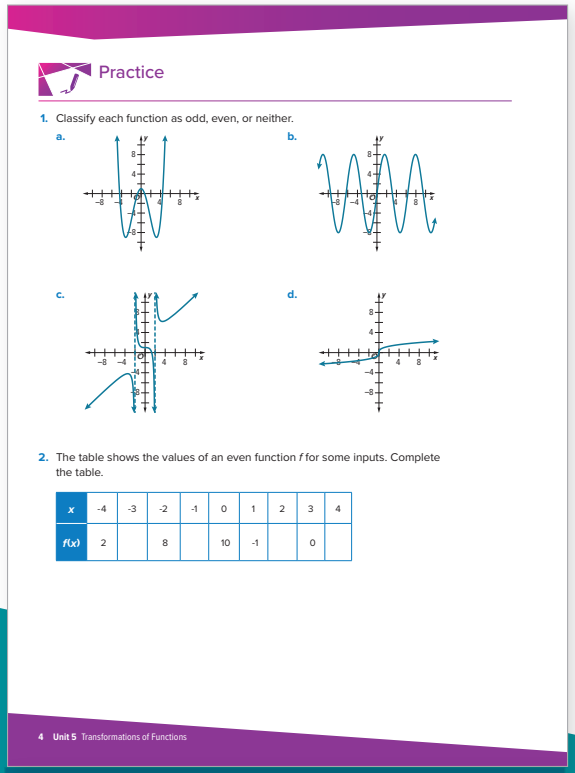
## End-of-Unit Assessments

End-of-unit assessments gauge students' understanding of the key concepts of the unit while also preparing students for new-generation standardized exams. Problem types include:

- Multiple-choice.
- Multiple response.
- Short answer.
- Restricted constructed response.
- Extended response.

Problems vary in difficulty and depth of knowledge. In longer units, the end-of-unit assessment will include the breadth of all content for the full unit, with emphasis on the content from the second half of the unit.



# Algebra 1 Supports

Each Algebra 1 Supports lesson is associated with a lesson in the Algebra 1 course. The Algebra 1 Support lesson helps students learn or remember a skill or concept that is needed to access and find success with the associated Algebra 1 lesson.

# Engagement and Accessibility for All

## Consistent Lesson Structure

By keeping lesson components in a similar form, the flow of work becomes predictable. This reduces cognitive demand, which enables students to focus on the mathematics rather than the lesson's mechanics.

## Participation Progression

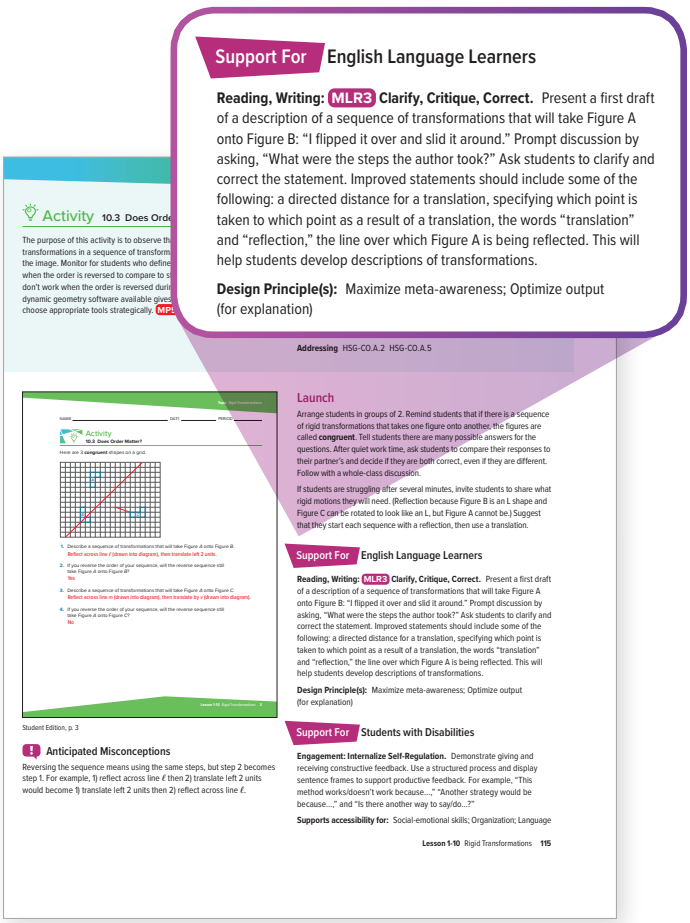
Students are allowed time to think through a situation or question independently before engaging with others. This allows them to carry the weight of learning, with just-in-time supports from a community of learners.

## Logical Development of Concepts

Mathematical concepts are introduced simply, concretely, and repeatedly, with complexity and abstraction developing over time.

## Real-World Contexts

Opportunities to apply the mathematics they learn clarifies and deepens students' understanding of core math concepts and skills. Mathematical modeling is a powerful activity for all students, especially students with disabilities.



**Support For** English Language Learners

**Reading, Writing:** MLR3 **Clarify, Critique, Correct.** Present a first draft of a description of a sequence of transformations that will take Figure A onto Figure B: "I flipped it over and slid it around." Prompt discussion by asking, "What were the steps the author took?" Ask students to clarify and correct the statement. Improved statements should include some of the following: a directed distance for a translation, specifying which point is taken to which point as a result of a translation, the words "translation" and "reflection," the line over which Figure A is being reflected. This will help students develop descriptions of transformations.

**Design Principle(s):** Maximize meta-awareness; Optimize output (for explanation)

Activity 10.3 Does Orde

The purpose of this activity is to observe th transformations in a sequence of transform the image. Monitor for students who define when the order is reversed to compare to s don't work when the order is reversed duri dynamic geometry software available given choose appropriate tools strategically. MP5

Addressing HSG-CO.A.2 HSG-CO.A.5

Launch

Arrange students in groups of 2. Remind students that if there is a sequence of rigid transformations that takes one figure onto another, the figures are called **congruent**. Tell students there are many possible answers for the questions. After quiet work time, ask students to compare their responses to their partner's and decide if they are both correct, even if they are different. Follow with a whole-class discussion.

If students are struggling after several minutes, invite students to share what rigid motions they will need. (Reflection because Figure B is an L shape and Figure C can be rotated to look like an L, but Figure A cannot be.) Suggest that they start each sequence with a reflection, then use a translation.

Support For English Language Learners

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**Design Principle(s):** Maximize meta-awareness; Optimize output (for explanation)

Activity 10.3 Does Order Matter?

Here are 3 **congruent** shapes on a grid.

1. Describe a sequence of transformations that will take Figure A onto Figure B. Reflect across line ℓ (drawn into diagram), then translate left 2 units.
2. If you reverse the order of your sequence, will the reverse sequence still take Figure A onto Figure B? Yes
3. Describe a sequence of transformations that will take Figure A onto Figure C. Reflect across line m (drawn into diagram), then translate by v (drawn into diagram).
4. If you reverse the order of your sequence, will the reverse sequence still take Figure A onto Figure C? No

Student Edition, p. 3

Support For Students with Disabilities

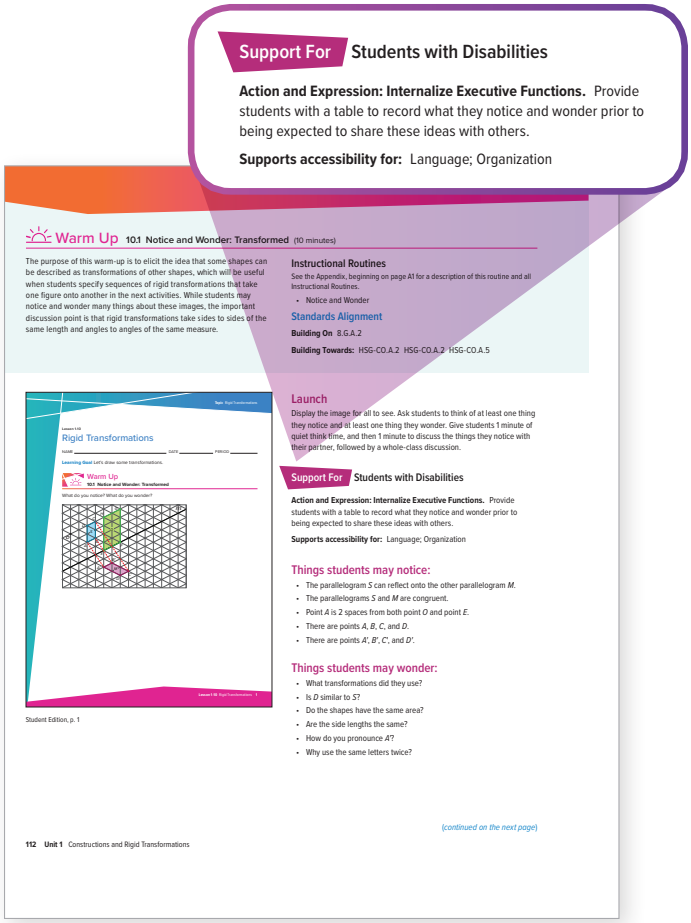
**Engagement: Internalize Self-Regulation.** Demonstrate giving and receiving constructive feedback. Use a structured process and display sentence frames to support productive feedback. For example, "This method works/doesn't work because…." "Another strategy would be because…," and "Is there another way to say/do…?"

**Supports accessibility for:** Social-emotional skills; Organization; Language

Anticipated Misconceptions

Reversing the sequence means using the same steps, but step 2 becomes step 1. For example, 1) reflect across line ℓ then 2) translate left 2 units would become 1) translate left 2 units then 2) reflect across line ℓ.

Lesson 1-10 Rigid Transformations 115



**Support For** Students with Disabilities

**Action and Expression: Internalize Executive Functions.** Provide students with a table to record what they notice and wonder prior to being expected to share these ideas with others.

**Supports accessibility for:** Language; Organization

Warm Up 10.1 Notice and Wonder: Transformed (10 minutes)

The purpose of this warm-up is to elicit the idea that some shapes can be described as transformations of other shapes, which will be useful when students specify sequences of rigid transformations that take one figure onto another in the next activities. While students may notice and wonder many things about these images, the important discussion point is that rigid transformations take sides to sides of the same length and angles to angles of the same measure.

Instructional Routines

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

- Notice and Wonder

Standards Alignment

**Building On** 8.G.A.2

**Building Towards:** HSG-CO.A.2 HSG-CO.A.5

Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Support For Students with Disabilities

**Action and Expression: Internalize Executive Functions.** Provide students with a table to record what they notice and wonder prior to being expected to share these ideas with others.

**Supports accessibility for:** Language; Organization

Rigid Transformations

Learning Goal Let's draw some transformations.

Warm Up 10.1 Notice and Wonder: Transformed

What do you notice? What do you wonder?

Student Edition, p. 1

Things students may notice:

- The parallelogram *S* can reflect onto the other parallelogram *M*.
- The parallelograms *S* and *M* are congruent.
- Point *A* is 2 spaces from both point *O* and point *E*.
- There are points *A*, *B*, *C*, and *D*.
- There are points *A'*, *B'*, *C'*, and *D'*.

Things students may wonder:

- What transformations did they use?
- Is *D* similar to *S*?
- Do the shapes have the same area?
- Are the side lengths the same?
- How do you pronounce *A*?
- Why use the same letters twice?

(continued on the next page)

112 Unit 1 Constructions and Rigid Transformations

# ALEKS®

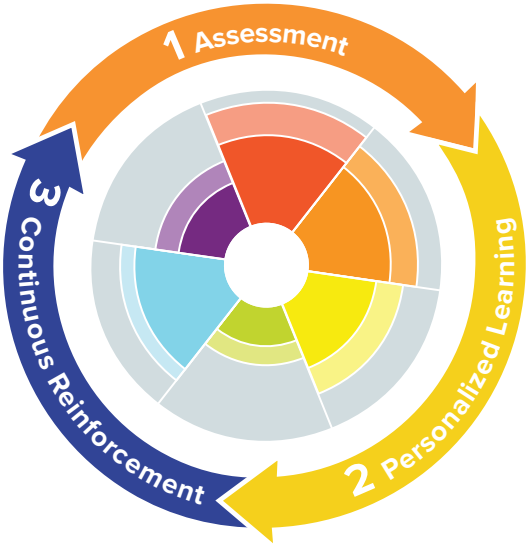
## A Personalized Pathway to Math Proficiency

**ALEKS*® is an online personalized learning solution for grades 6–12. *ALEKS* can be bundled with *Illustrative Mathematics* to provide targeted, supplemental assessment and instruction. It uses artificial intelligence to identify and provide instruction on the topics each student is most ready to learn. A continuous cycle of assessment, learning, and reinforcement adapts instruction to the individual needs of each student and customizes a unique learning pathway to help accelerate students to standard mastery. The program's three-phase cycle keeps students engaged by challenging them with concepts they are ready to learn, thus eliminating boredom and frustration.

**ALEKS* is not IM certified.

### Features:

- An algorithm that generates a unique problem set for every student, every time.
- Detailed explanations for every problem—including dictionary and video resources.
- Learning Mode open-response problems and intuitive input tools provide an authentic measure of conceptual understanding.
- Pie reports provide an in-depth analysis of student progress in multiple topics.
- Insights reports that identify students who may need intervention.
- Content in English and Spanish.
- Progress monitoring of students' mastery of the mathematical standards.
- Dynamic data at the student, class, school, and district level.

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# Illustrative Mathematics®

Algebra 1 | Algebra 1 Supports | Geometry | Algebra 2

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- Engaging color print resources for students and teachers
- Enhanced teacher materials support instruction more efficiently
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