



**Program Overview**

Grades 6–8

# McGraw-Hill **Illustrative Mathematics**<sup>TM</sup>

The highest-rated curriculum. A partner you know and trust.





*“Students learn mathematics as a result of solving problems. Mathematical ideas are the outcomes of the problem-solving experience . . . ”<sup>1</sup>*

## Creating a World Where Learners Know, Use, and Enjoy Mathematics

Decades of research shows that if students are given a chance to notice and wonder while trying to solve a problem by themselves, they retain procedural skills, develop problem-solving skills, build conceptual understanding, and form a mental framework for how ideas fit together. It allows students to develop strategies for tackling non-routine problems while engaging in productive struggle.

*Illustrative Mathematics* is a problem-based curriculum designed to address content and practice standards to foster learning for all. Students are encouraged to take an active role to see what they can figure out before having things explained to them or being told what to do.

<sup>1</sup>Hiebert, J., et al. (1996). Problem solving as a basis for reform in curriculum and instruction: the case of mathematics. *Educational Researcher* 25(4), 12-21. [doi.org/10.3102/0013189X025004012](https://doi.org/10.3102/0013189X025004012)

# The Highest Rated Curriculum. A Partner You Know and Trust.

## McGraw-Hill

- Personalized service and support from a local McGraw-Hill sales representative
- A team of curriculum specialists to support your implementation
- On-demand customer service to get help when you need it
- Spend less time printing and more time teaching with reliable delivery of print resources

## Exclusive Features

- Interactive reports to drive instruction
- Student activities available digitally
- Autoscored practice problems for immediate feedback
- Engaging color print resources
- Improved layout of teacher materials support instruction more efficiently
- \*Options to bundle with *ALEKS*<sup>®</sup> personalized learning

\*ALEKS is not IM certified.

## Supporting the *Illustrative Mathematics* Mission

As an IM Certified™ Partner, McGraw-Hill is committed to providing the support needed to successfully implement *Illustrative Mathematics*. A portion of every purchase is earmarked toward supporting the continued development of high-quality math curriculum.

## Perfect Scores from EdReports

Grade 6	Grade 7	Grade 8
MEETS EXPECTATIONS	MEETS EXPECTATIONS	MEETS EXPECTATIONS
FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14	FOCUS & COHERENCE Score: 14/14
RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES Score: 18/18	RIGOR & MATHEMATICAL PRACTICES Score: 18/18

# The Illustrative Mathematics Classroom: Teacher Facilitated Learning



Teacher ensures students understand the question.



Students work individually or in groups. Teacher monitors, listens, and questions.



Teacher helps students synthesize their learning.

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## Mathematics is not a spectator sport

Decades of research shows that when students are given a chance to notice and wonder while trying to solve a problem by themselves, they retain procedural skills, develop problem-solving skills, build conceptual understanding, and form a mental framework for how ideas fit together. In order to learn mathematics, students need to spend class time doing mathematics.

## Each Lesson and Unit Tells a Story

Developing coherent learning progressions and connections among areas of study requires crafting lessons to tell a mathematical story. Lessons must coherently build across units and grade levels and attend to many things: the mathematics, representations, activity structures, and learning trajectories, to name a few. Each of these considerations impacts how students access mathematics and influences the belief that mathematics is a connected set of ideas that makes sense.

The story of each grade is told in nine units. Each unit has a narrative that describes the mathematical work that will unfold in that unit. Each lesson in the unit also has a narrative.





*“An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.”*

**Principles to Action by National Council of Teachers of Mathematics**

### **Lesson Narratives explain:**

- The mathematical content of the lesson and its place in the learning sequence.
- The meaning of any new terms introduced in the lesson.
- How the mathematical practices come into play, as appropriate.

### **Activities within lessons also have narratives, which explain:**

- The mathematical purpose of the activity and its place in the learning sequence.
- What students are doing during the activity.
- What the teacher needs to look for while students are working on an activity to orchestrate an effective synthesis.
- Connections to the mathematical practices, when appropriate.

# The Illustrative Mathematics Lesson



## Warm-Up

Help students get ready for the day's lesson or give students an opportunity to strengthen their number sense or procedural fluency.



## Instructional Activities

For each activity, the teacher helps students understand the problem, students work on the problem, and then the teacher makes sure that students synthesize what they have learned.




## Lesson Synthesis

Each lesson includes a lesson synthesis section that assists the teacher as they help students incorporate new insights into their big-picture understanding.



## Cool-Down

A brief formative assessment to determine whether students understood the lesson.

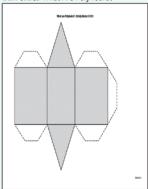


### Standards Alignment

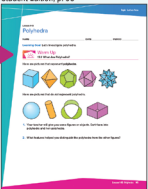
**Building Towards: 6.G.A.4**

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

**BLM 6.131 What Are Polyhedra?**



Student Edition, p. 95



### Warm Up 13.1 What Are Polyhedra? (10 minutes)

In this warm up, students analyze examples and counterexamples of polyhedra, observe their defining characteristics, and use their insights to sort objects into polyhedra and non-polyhedra. They then start developing a working definition of polyhedron.

Prepare physical examples of polyhedra and non-polyhedra for students to sort. These examples should be geometric figures rather than real-world objects such as shoe boxes or canisters. If such figures are not available, make some ahead of time using the nets in the blackline master.

As students work and discuss, notice those who can articulate defining features of a polyhedron. Invite them to share later.

#### Instructional Routines

- Notice and Wonder**

#### Launch


Arrange students in groups of 3–4. Give students 1 minute of quiet time to study the examples and non-examples in the task statement. Ask them to be ready to share at least one thing they notice and one thing they wonder. Give the class a minute to share some of their observations and questions.

Next, give each group a physical set of three-dimensional figures. The set should include some familiar polyhedra, some unfamiliar ones, and some non-polyhedra.


Ask groups to sort the figures into polyhedra and non-polyhedra (the first question). If groups members disagree about whether a figure is a polyhedron, discuss the disagreements. When the group has come to an agreement, give them 2–3 minutes of quiet time to complete the second question.

#### Student Task Statement

Here are pictures that represent **polyhedra**:



Here are pictures that do not represent polyhedra:



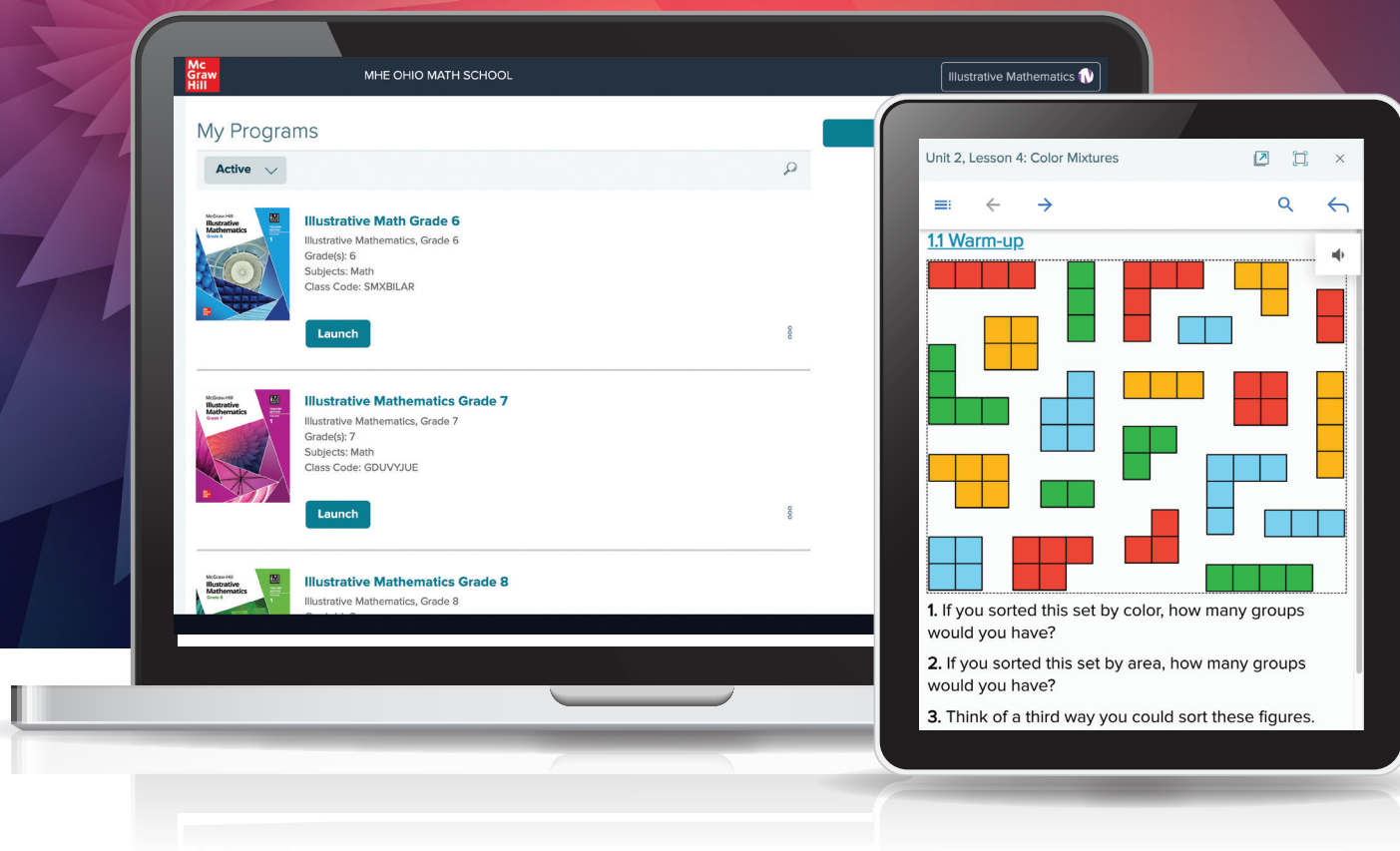
- Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra. **No response required.**
- What features helped you distinguish the polyhedra from the other figures?  
Answers vary. Sample responses:
  - Polyhedra are made from polygons.
  - Polyhedra don't have any unattached edges.
  - Non-polyhedra sometimes have curved or round surfaces.
  - Some non-polyhedra have a face that is not a polygon.

(continued on the next page)

150 Unit 1 Area and Surface Area

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## Digital Student and Teacher Editions

McGraw-Hill *Illustrative Mathematics* offers flexible implementations with both print and digital options that fit a variety of classrooms.

### Online resources offer:

- Customizable content.
- The ability to add resources.
- Auto-scoring of student practice work.
- Ongoing student assessments.
- Classroom performance reporting.

## Launch Presentations

Digital versions of lessons to present content.

## Reports

Review the performance of individual students, classrooms, and grade levels.

## Access Resources

Point-of-use access to resources such as assessments, eBooks, and course guides.

# Strengthen Skills With Instructional Routines

Instructional routines allow students and teachers to pay less attention to what they are supposed to do and more attention to the mathematics to be learned. Routines provide a structure that helps strengthen students' skills in listening and communicating their mathematical ideas.

There are 17 routines used in *Illustrative Mathematics*, including:

## Notice and Wonder

In this routine, students are shown a mathematical representation and are asked, “What do you notice? What do you wonder?” and the teacher steers the conversation to wondering about something mathematical that the class is about to focus on. This helps make a mathematical task accessible to all students.

## Number Talk

Number talks build computational fluency by encouraging students to think about the numbers in a computation problem and rely on what they know about structure, patterns, and properties of operations to mentally solve the problem.

Topic Let's Put It to Work

Lesson 1-19

### Designing a Tent

NAME \_\_\_\_\_ DATE \_\_\_\_\_ PERIOD \_\_\_\_\_

**Learning Goal** Let's design some tents.

**Activity**  
19.1 Tent Design—Part 1

Have you ever been camping?


You might know that sleeping bags are all about the same size, but tents come in a variety of shapes and sizes.

Your task is to design a tent to accommodate up to four people and estimate the amount of fabric needed to make your tent. Your design and estimate must be based on the information given and have mathematical justification.

First, look at these examples of tents, the average specifications of a camping tent, and standard sleeping bag measurements. Talk to a partner about:

- Similarities and differences among the tents
- Information that will be important in your designing process
- The pros and cons of the various designs

**Tent Styles**

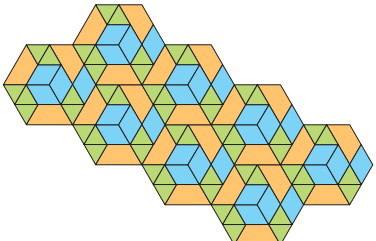


**Activity**  
1.2 More Orange, Green, or Blue?

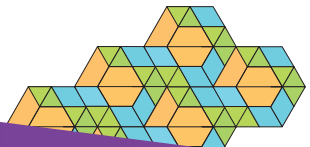
Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, orange trapezoids, or green triangles? Explain how you know.

**Pattern A**



**Pattern B**



## Note:

**Notice and Wonder** and **I Notice/I Wonder** are trademarks of NCTM and the Math Forum and are used in these materials with permission.





## Which one doesn't belong?

Students are presented with four figures, diagrams, graphs, or expressions with the prompt, “Which one doesn’t belong?” Typically, each of the four options “doesn’t belong” for a different reason, and the similarities and differences are mathematically significant. Students are prompted to explain their rationale for deciding that one option doesn’t belong and given opportunities to make their rationale more precise.

## Information Gap Cards

In an information gap, one partner gets a problem card with a math question that doesn’t have enough given information, and the other partner gets a data card with information relevant to the problem card. Students ask each other questions like “What information do you need?” and are expected to explain what they will do with the information. This routine provides opportunities for high-quality mathematical conversations.

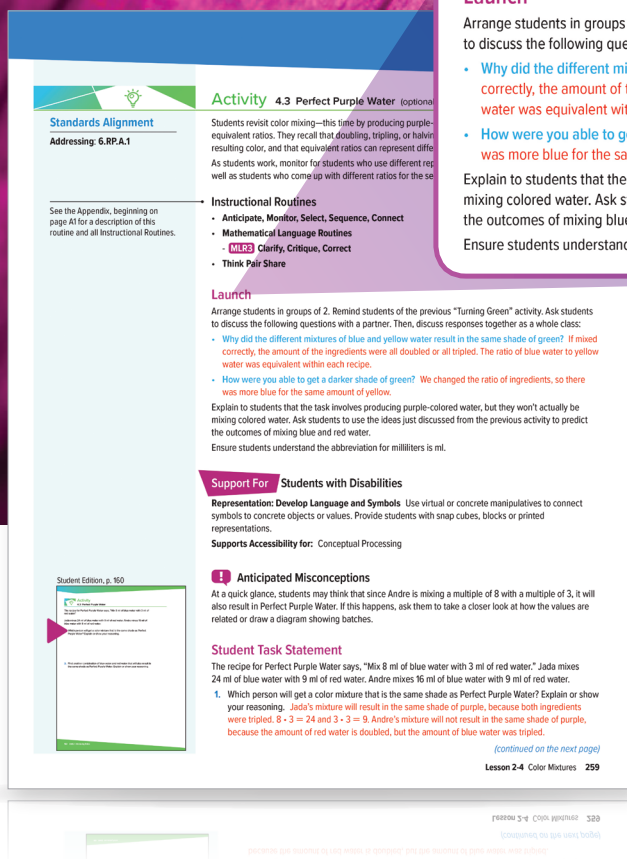




*“The 5 Practices provided the structure I needed as a teacher to put all of these good teaching strategies into a cohesive teaching style that was not only student-centered but also focused on the mathematical goal of the day.”*

**Alicia F. Grade 6–7 mathematics, Woodbury, Minnesota**





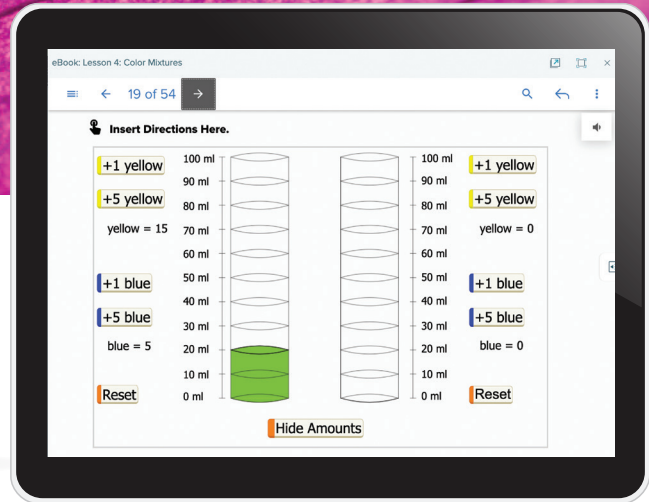
### Launch

Arrange students in groups of 2. Remind students of the previous "Turning Green" activity. Ask students to discuss the following questions with a partner. Then, discuss responses together as a whole class:

- **Why did the different mixtures of blue and yellow water result in the same shade of green?** If mixed correctly, the amount of the ingredients were all doubled or all tripled. The ratio of blue water to yellow water was equivalent within each recipe.
- **How were you able to get a darker shade of green?** We changed the ratio of ingredients, so there was more blue for the same amount of yellow.

Explain to students that the task involves producing purple-colored water, but they won't actually be mixing colored water. Ask students to use the ideas just discussed from the previous activity to predict the outcomes of mixing blue and red water.

Ensure students understand the abbreviation for milliliters is ml.



## Facilitating Productive Classroom Discussions

Activities are structured using the 5 Practices for Orchestrating Mathematical Discussions<sup>2</sup>.

### Anticipate

Consider how students might mathematically interpret the problem, the array of strategies that they might use to tackle it, and how those strategies and interpretations might relate to the mathematical concepts, representations, procedures, and practices that you would like your students to learn.

### Monitor

Pay close attention to students' mathematical thinking and solution strategies as they work on the task. Prompt students to make their thinking visible.

### Select

Select particular students to share their work with the rest of the class to get specific mathematics into the open for discussion.

### Sequence

Make purposeful choices about the order in which students' work is shared to maximize the chances of achieving the mathematical goals for the discussion.

### Connect

Help students draw connections between their solutions and other students' solutions as well as the key mathematical ideas in the lesson.

<sup>2</sup>(Smith & Stein, 2011), also described in Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014), and Intentional Talk: How to Structure and Lead Productive Mathematical Discussions (Kazemi & Hintz, 2014).

# Assessing Student Progress

*Illustrative Mathematics* contains formative and summative assessments in each unit to help gauge classroom and student progress.

## Pre-Unit Assessments

Each unit begins with a pre-unit diagnostic assessment titled Check Your Readiness. This assessment reviews prerequisite concepts and skills for the unit. Each assessment item identifies which lesson the skill or concept is needed for and provides guidance on what to do if students struggle or do well on the item. Teachers can use this knowledge to pace or tune instruction or move more quickly through a topic to optimize instructional time.

## Mid-Unit Assessments

In longer units, a mid-unit assessment is also available. This assessment has the same form and structure as an end-of-unit assessment.

## Summative Assessments

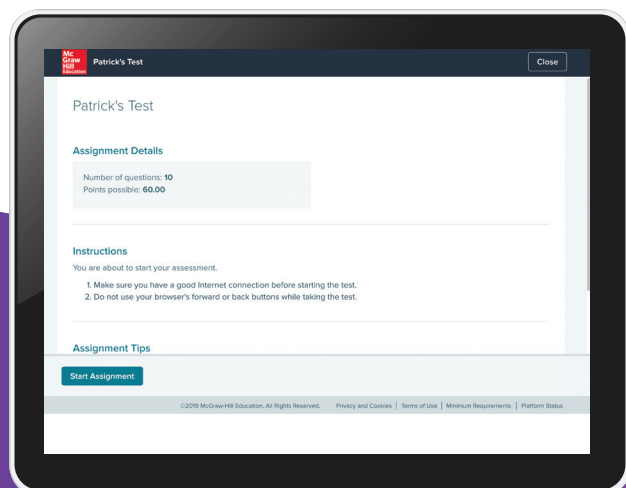
All summative assessment problems include a complete solution and standard alignment. Multiple-choice and multiple response problems often include a reason for each potential error a student might make. Restricted constructed response and extended response items include a rubric. Unlike formative assessments, problems on summative assessments generally do not prescribe a method of solution.

## End-of-Unit Assessments

End-of-unit assessments gauge students' understanding of the key concepts of the unit while also preparing students for new-generation standardized exams. Problem types include:

- Multiple-choice
- Multiple response
- Short answer
- Restricted constructed response
- Extended response

Problems vary in difficulty and depth of knowledge. In longer units, the end-of-unit assessment will include the breadth of all content for the full unit, with emphasis on the content from the second half of the unit.



**Are you ready for more?**

Rearrange the triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.

**Activity**  
3.3 Off the Grid

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.

Figure A

Figure B

Figure C

**Summary**  
Reasoning to Find Area

There are different strategies we can use to find the area of a region. We can:

1. Decompose it into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.

18 Unit 1 Area and Surface Area



# Engagement and Accessibility for All

## Consistent Lesson Structure

Each lesson has a warm-up activity, a synthesis, and a cool-down. By keeping the components in a similar form, the flow of work becomes predictable. This reduces cognitive demand, which enables students to focus on the mathematics rather than the lesson's mechanics.

## Logical Development of Concepts

Mathematical concepts are introduced simply, concretely, and repeatedly, with complexity and abstraction developing over time.

## Participation Progression

Students are allowed time to think through a situation or question independently before engaging with others. This allows them to carry the weight of learning, with just-in-time supports from a community of learners.

## Real-World Contexts

Opportunities to apply the mathematics they learn clarifies and deepens students' understanding of core math concepts and skills. Mathematical modeling is a powerful activity for all students, but especially students with disabilities.

For non-sighted or color-blind students, this activity can be adapted by giving them blocks of different shapes.

**Support For Students with Disabilities**

**Representation: Develop Language and Symbols** Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with snap cubes or blocks of different shapes.

**Supports accessibility for:** Conceptual Processing

See the Appendix, beginning on page A1 for a description of this routine and all Instructional Routines.

**Instructional Routines**

- Mathematical Language Routines
- MLR8 Discussion Supports

**Launch**

Orient students to the picture (if you have real cubes, use them). Review the meaning of "diagram." For example, to represent two green snap cubes, you might draw two green squares on the board, or two squares labeled "G" if colors are not available.

or

Arrange students in groups of 2. Provide access to colored pencils.

**Support For Students with Disabilities**

**Representation: Develop Language and Symbols** Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with snap cubes or blocks of different shapes.

**Supports accessibility for:** Conceptual Processing

**Anticipated Misconceptions**

Students might not draw discrete diagrams at first. They might be inclined to draw more detailed drawings. Emphasize that a diagram represents the number and type of objects, and does not need to represent details about the shapes of the snap cubes.

(continued on the next page)

234 Unit 2 Introducing Ratios

Topic: What Are Ratios?

Student Edition, p. 145

**Student Task Statement**

Here is a collection of snap cubes.

1. Choose two of the colors in the image, and draw a

**Support For English Language Learners**

**Representing**

**MLR8 Discussion Supports** Clarify mathematical use of the term "to" (as in 1 to 1 or 2 to 2) highlighting its use to compare the quantities. Listen for any misunderstanding about the use of the word "to".

**Design Principle:** Maximize meta-awareness

**Activity Synthesis**

Invite one or two pairs of students to share their sentences. Press for details as they explain, asking them to clarify, elaborate, or give examples. Revoice student ideas to demonstrate mathematical language. Discuss whether or not students were able to interpret one another's drawings accurately. If not, what may have led to confusion?

If no one wrote ratios in which all numbers are the same (e.g., 1 to 1 or 3 to 3), ask if the following sentence is acceptable and why or why not: "The ratio of green cubes to blue cubes is 2 to 2." If students suspect that ratios are only used to associate quantities with different values, clarify that this is not the case.

**Support For English Language Learners**

**Representing**

**MLR8 Discussion Supports** Clarify mathematical use of the term "to" (as in 1 to 1 or 2 to 2) highlighting its use to compare the quantities. Listen for any misunderstanding about the use of the word "to".

**Design Principle:** Maximize meta-awareness

**Standards Alignment**

Addressing: 6.RP.A.1

**Activity 2.3 Blue Paint and Art Paste (10 minutes)**

In this activity, students continue to draw connections between a diagram and the ratios it represents. Students work in pairs to discuss different ways to use ratio language to describe discrete diagrams. They first identify statements that would correctly describe a given diagram. Then, they create both a diagram and corresponding statements to represent a new situation involving ratio.

As students work, monitor for different ways in which students draw and discuss diagrams of the paste recipe. Identify a few pairs who draw different diagrams and use ratio language differently to share later. A few things to anticipate:

- Some students may draw very literal drawings of cups and pints. Encourage them to use simpler representations.

(continued on the next page)

Lesson 2.2 Representing Ratios with Diagrams 235



## A Personalized Pathway to Math Proficiency

\*ALEKS® is an online personalized learning solution for grades 6–12. ALEKS can be bundled with *Illustrative Mathematics* to provide targeted, supplemental assessment and instruction. It uses artificial intelligence to identify and provide instruction on the topics each student is most ready to learn. A continuous cycle of assessment, learning, and reinforcement adapts instruction to the individual needs of each student and customizes a unique learning pathway to help accelerate students to standard mastery. The program's three-phase cycle keeps students engaged by challenging them with concepts they are ready to learn, thus eliminating boredom and frustration.

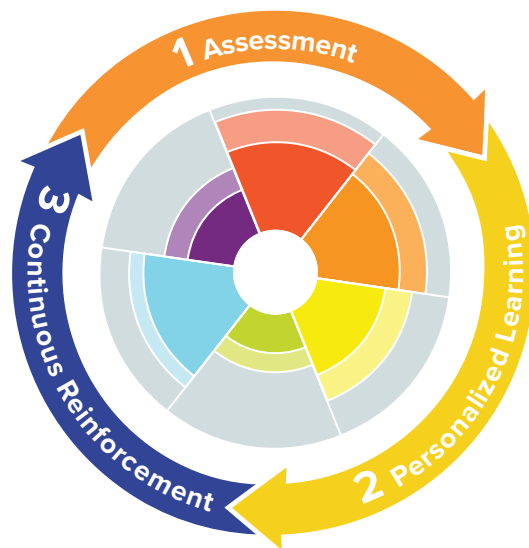
\*ALEKS is not IM certified.

### Features:

- An algorithm that generates a unique problem set for every student, every time.
- Detailed explanations for every problem—including dictionary and video resources.
- Learning Mode open-response problems and intuitive input tools provide an authentic measure of conceptual understanding.
- Pie reports provide in-depth analysis of student progress in multiple topics.
- Insights reports that identify students who may need intervention.
- Content in English and Spanish.
- Progress monitoring on student mastery of mathematical standards.
- Dynamic data at the student, class, school, and district level.

### \*2019 CODiE Award Winner

- Best Summative Assessment Solution
- Best College and Career-Readiness Solution



\*The only peer-recognized competition in education and business technology.





## IM Certified™ Professional Learning

McGraw-Hill is an IM Certified™ professional learning partner. Our facilitators are specially trained to deliver high-quality professional learning to teachers, coaches, and district leaders. McGraw-Hill partners with teachers and educational leaders to provide long-term, sustainable support for developing, refining, and reflecting on professional learning practices.



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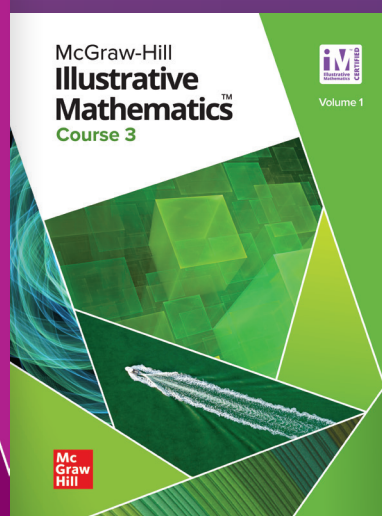
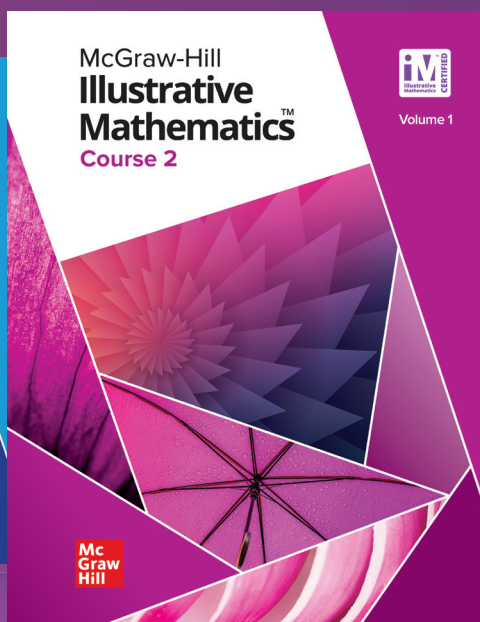
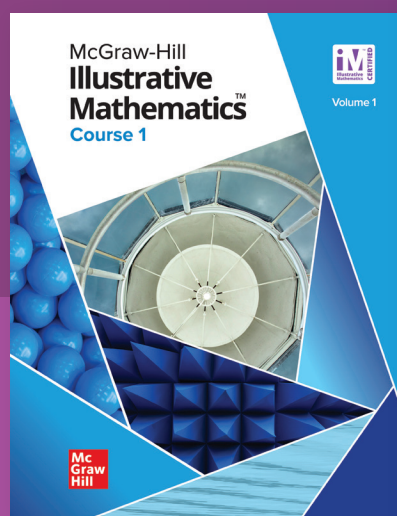
# Illustrative Mathematics™

Grades 6, 7, 8

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## Partnering with McGraw-Hill

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