Four Ways to Encourage Sensemaking in Math

by Annie Fetter

Many students (and adults!) believe that the goal of math is to get numerical answers really fast. This leads people to believe they aren't good at math and that they must not have a math brain. They don't know that doing math means noticing and wondering about situations, being curious, pondering different ideas, or even taking time to understand a story before trying to calculate anything. In fact, Standards for Mathematical Practice 1 states, in part:

> Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway *rather than simply jumping into a solution attempt.*

Yet when given a problem such as "The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?", many students will reflexively give an answer of 9 since 6 + 3 = 9. They've not done the work of making sense of the situation because they don't think that's part of "doing math."

ABOUT THIS PAPER

Annie Fetter worked on the project that developed the first version of The Geometer's Sketchpad and was a founding staff member of the Math Forum until it ended in 2017. Currently, she consults with schools, districts, states, and speaks at conferences—encouraging a focus on sensemaking and leveraging students' ideas. She is an author of McGraw Hill's new TK–12 textbook series, *California Reveal Math*, and works part-time for the 21st Century Partnership for STEM Education, continuing the Math Forum's work on two NSF grants. Her very first Ignite talk, "Ever Wonder What They'd Notice?", has been used in countless PD sessions around the world.

How might we better encourage students to make sense of this problem rather than grabbing numbers and picking a random operation? Consider this version: "The corner deli sells roses in bunches of 6. Dylan buys 3 bunches of roses. Draw a picture of the story." Students are going to engage their sensemaking skills because the problem is now a task that looks a lot more like literacy than math (and sensemaking tasks are a regular part of literacy instruction). Students are used to drawing pictures to illustrate stories, so they will do the work of making sense of the story to draw an accurate picture.

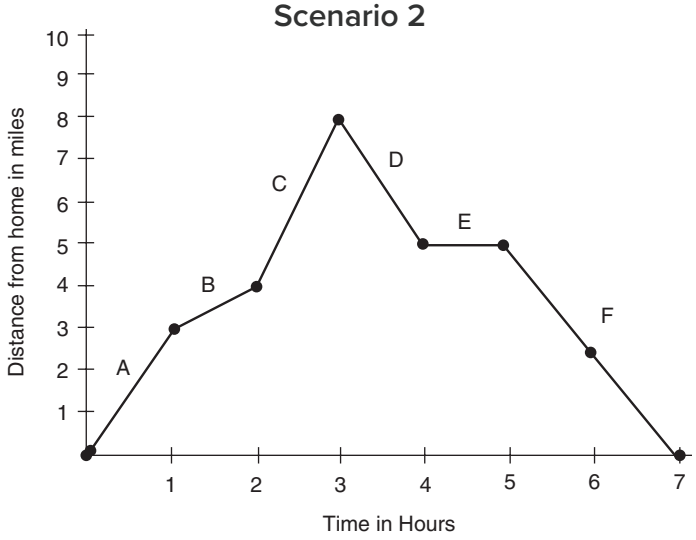
1. Get Rid of the Question

If you don't want students focused on finding answers, then don't include questions. Consider the following two "scenarios" (which is what we at The Math Forum called math situations without a stated question):

Scenario 1

Apple juice costs 50¢. The juice machine accepts quarters, dimes, and nickels.

Scenario 2



Source: Tina Cardone, *http://drawingonmath.blogspot.com/2014/11/distance-graph.html*

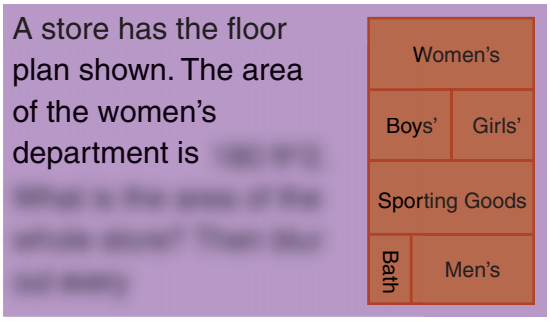
Present each of these scenarios to students and ask them what they Notice and Wonder © (a strategy we first developed at The Math Forum for use with teachers in a protocol for looking at student work in 2005 and formalized as an approach for problem-solving with our Problems of the Week in 2007). The work that students now do is focused on making sense of the situation, not answering a specific question. It's also a great formative assessment opportunity for the teacher to learn what knowledge students are bringing to the lesson.

In the first case, you might learn whether some students know that "quarter," "dime," and "nickel" are the names of coins, how many cents they are worth, and what that funny little "c" with the line through it means. Other students might wonder if the machine gives change or how many different ways there are to buy juice (or a number of other things!).

In the second scenario, which Tina Cardone (@TinaCardone) used as an introduction to slope and subsequently blogged about, students were asked to Notice and Wonder about the given graph before they were asked to answer any questions about it. They did such a thorough job of teasing out all of the details that when they did get around to answering the questions in the activity, they asked many fewer questions than classes had in previous years, and Tina was able to skip the wrap-up discussion since it was clear that the students really understood the mathematics.

2. Get Rid of the Question and the Numbers

Consider this picture:



Asking students to Notice and Wonder about this is going to bring out a lot of relationships since there aren't any numbers. In fact, some students will claim that you can't do anything since there aren't any numbers! But many students will notice the relative sizes of the different departments.

Another version of this is to take traditional word problems and remove the numbers so that you have a story that students can use to "make a movie in their minds" (a great sensemaking strategy often used in literacy). Here is an example from Brian Bushart (@bstockus). It comes in four different parts.

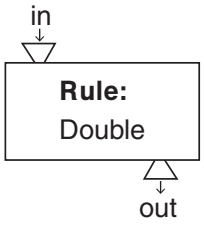
> Raul had some pet mice. Xavier gave him some more mice.
>
> Raul had some pet mice. Xavier gave him 3 more mice.
>
> Raul had some pet mice. Xavier gave him 3 more mice. Now he has 8 mice.
>
> Raul had some pet mice. Xavier gave him 3 more mice. Now he has 8 mice.
> How many mice did Raul have to start with?
>
> Source: *Brian Bushart, https://bstockus.wordpress.com/numberless-word-problems/*

This structure allows students to make sense of the story first—who has mice? Who gets more mice? From whom?—before they are given enough numbers to answer the question. Brian has an entire set of these problems on his website.

3. Give Students the Answer

If you really don't want students answer-getting, just give them the answer. Now the answer isn't so important anymore, but the process is!

I remember seeing a whole class activity in a Grade 3 math textbook years ago that instructed the teacher to display a function machine, an In-Out table, and then explain to the students how it works. The script for the teacher asked students to imagine that when you put a number into the machine, the machine changes the number according to a rule, then the new number comes out the other end. It stated the rule for teachers to tell the students and then made a list of all of the things teachers should point out to students. Each item in the script was important, and I would want the students to understand them all, but the teacher is doing all of the thinking and work!



In	Out
4	8
50	100
100	200
25	50
70	140

What if you just gave them the picture and table above and asked students to tell you everything they could about it? The point is to have the students do the work of making sense of the situation. The teacher has a great guide in the book and can use that to monitor whether all of the important ideas get mentioned by the students, but the teacher shouldn't be telling the students much of anything when they don't have to.

One of my favorite examples of this involves teaching someone the rules for playing KenKen. (Don't already know them? You and your students should learn!) I could show you an empty starting grid and drone on and on about how it works, but I think it's easier to just show you a finished grid and ask you what you notice about it and what you wonder. That's probably what you're going to do when I'm done talking anyway.

5+ 2	3	3+ 1
4+ 3	3+ 1	2
1	2	3 3

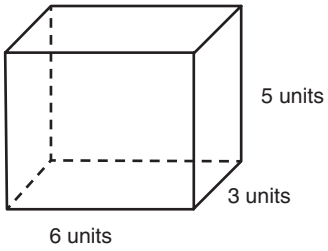
Another version of "Give Students the Answer" involves giving a scenario and then asking, "What question about this scenario would have an answer of 9?"

4. Ask About Ideas, Not Answers

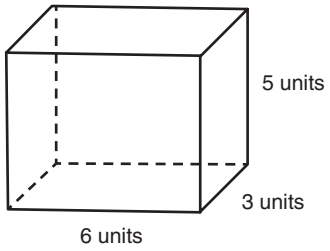
This might be really simple—instead of asking for the answer to question number 7, as we often do in the classroom, ask students, "Tell me something about question 7." They might tell you the answer, or they might tell you something else. In either case, ask other students to tell you something, too. After all, the answer is one thing about question 7, but it's not everything.

When we ask for the answer to question number 7, we already know that there are students in the room who can't tell us. We're almost telling some students they can't do math, which is not the message we want to be sending when we know everyone CAN do math! But in asking more broadly about question 7, we fully expect that every student could tell us something and are sending a message that everyone has math ideas to contribute to our learning community.

Joe Schwartz (@JSchwartz10a) recently blogged about a version of this in which a fifth-grade teacher at his school used the left picture instead of the right for an assessment:



Tell me everything you can about this figure.



Find the volume of the rectangular prism.

Source: *Joe Schwartz, http://exit10a.blogspot.com/2016/10/unknown-unknowns.html*

The teacher already had a pretty good idea about who could find volume and who couldn't, so he wouldn't have learned much by using the picture on the right, but he learned all sorts of stuff when he used the one on the left! On his blog, Joe writes about how they tackled some of the students' responses in subsequent activities.

Good problem-solvers employ their own sensemaking strategies automatically, but many students need support to develop those skills. By using explicit strategies such as these, you can slow down the "race to the answer" and help all students become sensemakers and mathematicians.

California Context: The 2023 Mathematics Framework and Sensemaking

The 2023 California Mathematics Framework encourages instruction designed to help students connect the why, how, and what of math. The framework identifies three Drivers of Investigation (DIs), which provide the “why” of learning mathematics, to pair with four categories of Content Connections (CCs), which provide the “how and what” of the mathematics that is to be learned in an activity. One of the key Drivers of Investigation for the California Math Framework is D1: Make Sense of the World (Understand and Explain). The others are D2: Predict What Could Happen (Predict) and D3: Impact the Future (Affect). The combination of investigation and content is what makes the learning meaningful and important, encouraging students to think deeply, develop understanding of mathematical relationships, make connections across mathematical ideas, and engage in authentic activities.

The framework explains that "students need to actually wonder. Active learning experiences enable students to engage in a full range of mathematical activity: exploring, noticing, questioning, solving, justifying, explaining—making clear that mathematics is far more than calculating."

In order for a situation or scenario to be "authentic" for a student, it doesn't have to be real-life for them (they may not have actually encountered it), but, it should cause them to actually wonder about the situation or the questions being asked of them.

From Chapter 1 of the framework, some examples of contexts that provoke student curiosity include:

- Environmental observations and issues on campus and in the local community (which concurrently help students develop their understanding of California’s Environmental Principles and Concepts)
- Puzzles
- Patterns—numerical or visual—in purely mathematical settings
- Real-world or fictional contexts in which something happens or changes over time

These types of activities invite students' questions and conjectures, supporting an equitable and engaging classroom community.

Source: *www.cde.ca.gov*