Using the Mathematical Practices Effectively in the Classroom

By John SanGiovanni, M.Ed.

What Are the Standards for Mathematical Practice?

Mathematics is more than the pursuit of answers; it is a tool and a way of thinking. The mathematics content that we teach in elementary schools can be summarized into ideas about place value, number relationships, operations, fractions, measurement, and data, but there are also habits of mind and habits of interaction that are embedded in the practices that we intend to develop in our students and form the basis for math learning.

The Standards for Mathematical Practice (Council of Chief State School Officers [CCSSO], 2010) identify these behaviors as eight "practices" and reflect a deep conceptual and procedural understanding. They are referred to as practices or MPs. These practices have had long-standing importance in mathematics (CCSCO, 2012), and they are not new. The predecessor of today's practices appeared in the National Council of Teachers of Mathematics (NCTM, 2000) landmark publication, "Principles and Standards for School Mathematics." That work organized elementary mathematics learning into content and process.

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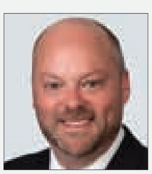

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The National Research Council's (NRC, 2001) report, "Adding It Up," described the strands of mathematical proficiency. These strands outlined behaviors of mathematically proficient students including adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

Content Standards	Process Standards
Number and operations	Problem-solving
Algebra	Connections
Geometry	Communication
Measurement	Reasoning
Data and probability	Representation

The NCTM process standards and strands of mathematical proficiency were used to establish the eight MPs (CCSCO, 2012). They are:

- Practice 1: Make sense of problems and persevere in solving them (MP 1).
- Practice 2: Reason abstractly and quantitatively (MP 2).
- Practice 3: Construct viable arguments and critique the reasoning of others (MP 3).
- Practice 4: Model with mathematics (MP 4).
- Practice 5: Use appropriate tools strategically (MP 5).
- Practice 6: Attend to precision (MP 6).
- Practice 7: Look for and make sense of structure (MP 7).
- Practice 8: Look for and make use of repeated reasoning (MP 8).

Mathematics skills and concepts change from grade to grade but these practices do not. The practices are not grade-specific. They are nurtured and develop in relation to progressions in mathematics content. Ultimately, these standards "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise" (CCSSO, 2010). Students attend to the precision (MP 6) of their calculations whether they are finding sums of addends less than five (Kindergarten), finding sums of mixed numbers with unlike denominators (Grade 5), or solving ratio and percent problems (Grade 7). Tools are used (MP 5) to represent combinations of 10 (Grade 1) and for comparison of fractions (Grade 4). Students use the structure (MP 7) of the distributive property for leveraging partial products in fourth and fifth grades just as students in first grade use the structure of place value for understanding quantities greater than 10. Of course, students construct arguments (MP 3) to justify their reasoning and solutions regardless of the content or problems they are working with.

Summaries and Implications of Mathematical Practices

Practice 1: Make sense of problems and persevere in solving them (MP 1)

This first practice is at the heart of mathematics. It seems self-explanatory, but there are important ideas just below the surface. MP 1 calls for problem-solving to be a daily experience. Problem-solving is to be a thinking and reasoning experience that emphasizes process and sensemaking as much as, if not more than, solutions. Students must be armed with ideas about how to persevere through problems. Opportunities to discuss problems, strategies, and solutions are necessary for students to become proficient with this practice.

Practice 2: Reason Abstractly and Quantitatively (MP 2)

Symbols and equations are efficiencies for solving problems and making sense of the world mathematically. Students exhibit this practice by using equations to represent and solve problems and then contextualizing their solutions. As students do this, they determine whether their solutions make sense and whether further interpretation is needed. For example, six cars holding four people each are needed to transport 22 people rather than 5.5 cars. This practice also calls for students to think flexibly about numbers and quantities.

Practice 3: Construct Viable Arguments and Critique the Reasoning of Others (MP 3)

Like MP 1, MP 3 seems rather straightforward. Students should construct arguments to justify their reasoning and solutions. However, their arguments do not have to be clad in written form. Instead, students should be charged with creating arguments scaffolded by representations, diagrams, and computations. This MP also calls for students to actively listen to the arguments of others and to ask questions and counter ideas as appropriate.

Practice 4: Model With Mathematics (MP 4)

Using mathematics to model situations, make predictions, and draw conclusions is the intent of MP 4. This practice is applied frequently to medicine, engineering, and weather forecasts. Modeling with mathematics is a bit different in elementary grades. Here, students can model mathematics concepts and use models to solve problems. In middle grades, students use equations and functions to make predictions about problems and investigations or to make conjectures about patterns in the world around them.

Practice 5: Use Appropriate Tools Strategically (MP 5)

Tools help complete tasks and solve problems. Clearly, rulers, protractors, and calculators are tools. Paper and pencil can be a tool as well. Students realize this practice when they show how tools work as well as make good decisions about when to use them. For example, 9 + 14 can be solved with a number chart, base ten blocks, or paper and pencil; however, fifth-grade students are able to find the sum without the use of any of these tools.

Similarly, students in sixth grade are able to decide that 750 ÷ 250 doesn't necessarily need an algorithm if they understand benchmark numbers or recognize the relationship to 75 ÷ 25. Moreover, this practice requires students to understand tools and how they work. This means that students understand that rulers measure the distance between two points and measuring with them does not have to begin with zero.

Practice 6: Attend to Precision (MP 6)

Precision with computation and communication is at the core of MP 6. Precision with calculation increases as students develop and refine strategies for efficient computation as well as a sense of reasonableness. Communicating precisely means that students use mathematics terms correctly. It also means that they communicate precise solutions noting correct units of measure. As with the other practices, greater precision comes with experience and opportunity on a frequent and consistent basis. Therefore, students need daily experiences to acquire and use mathematics vocabulary and work with operations and number relationships.

Practice 7: Look For and Make Sense of Structure (MP 7)

Patterns and structure in mathematics help us understand quantity and concept. MP 7 calls for students to look for and make sense of structures in mathematics. A foundational example of structure is inherent in base ten place value. Another example is within the properties of operations. For example, the distributive property of multiplication is a structure that enables students to leverage partial products when multiplying multidigit numbers. Exploration and discussion are critical for exposing and understanding the structure of mathematics. Recording patterns so that students clearly see relationships and make generalizations also plays an important role in this practice. This practice also calls for students to see complicated things as a collection of things. "For example, they can see 5 − 3(x − y)2 as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y" (CCSCO, 8).

Practice 8: Look For and Make Use of Repeated Reasoning (MP 8)

Finding and using patterns is an effective problem-solving strategy. MP 8 comes to life as students see and leverage relationships between problems and calculations. For example, students who exhibit proficiency with MP 8 conclude that 5 × 8 = 40 because they know that 5 × 7 = 35 and that 5 × 8 is one more group of 5 (Illustrative Mathematics, 2014). In other situations, students might transfer understanding of combinations of 10 (e.g., 4 + 6, 5 + 5, 7 + 3) to quickly find the sum of 40 and 60 or 400 and 600. In eighth grade, students might use repeated reasoning with right triangles to discover and use ideas about 3:4:5 triangles. Repeated reasoning enables students to take shortcuts grounded in understanding. Implications for instruction are similar to other practices in that intentional selection of problems and computations, exploration, and discussion must be in place.

Why Do the Mathematical Practices Matter?

The math practices and their predecessors have always mattered. Advances in technology influence the changing needs of employers and society in general (Boaler, 2015). The need for critical-thinking, problem-solving, and communication skills, as well as abilities to use tools effectively, seek evidence, and make sense of patterns, has never been greater. Evidence of this need is reflected in other disciplines. The Next Generation Science Standards Lead States (2013) outline practices that embody similar behaviors. Language arts also calls for practices or habits of mind outlined in "student capacities" (CCSSO, 2010). Today, learning to do mathematics is more than a set of procedures that one learns and uses without deep thinking. Instead, students learn mathematics content and mathematics practices so that they are armed with skills to prepare them for their future.

Myths and Facts About the Mathematical Practices

There is a cloud of myths about teaching and learning mathematics. There are notions that men are better at mathematics than women. There is the notion that there is a "best" way to do mathematics or that it's bad to count on one's fingers (University of Fairbanks, 2018). These beliefs are false. Naturally, there are myths about the MPs (Mateas, 2016). Some of those myths include:

1. **The Mathematical Practices don't really matter.**
   We must prepare students for their future, yet there are many jobs that exist today that won't by the time our elementary students enter the workforce (Forbes, 2018). It is difficult to imagine that people once thought that elevator operators and telephone operators were irreplaceable. Changes are greatly influenced by technology, and in today's world of ever-advancing technology, thinking may be the most important skill we can develop in our students. MPs are mathematical habits of mind. They are the behaviors of mathematicians and thinkers. They may matter even more than the mathematics content that can be learned by watching a video or reading a blog.

2. **You must include every practice in every lesson.**
   A year's worth of topics cannot be shoehorned into a single lesson. The same is true for the MPs. A high-quality lesson will naturally illustrate multiple practices. However, it is virtually impossible for every lesson to feature every practice. Instead, teachers should take careful note of the frequency of student exposure and experience with the practices so that there is some level of balance. Certain practices, such as MP 1 and MP 3, are central to the experience of mathematics and likely appear everyday.

3. **You can only cover one practice per lesson.**
   Converse to the myth that every lesson includes every practice is the myth that a lesson only features one practice. Behaviors and thinking are not that simplistic or linear. Any high-quality or worthwhile task naturally weaves in multiple practices. As mentioned above, MP 1 and MP 3 should come alive in every lesson. MP 5 and MP 6 are other practices that are likely to appear

in many lessons. Some MPs require a more intentional evocation. What is most important is that teachers call attention to these practices and evidence them as students engage in mathematics so that the practices are better understood and ultimately used.

4. **Practices can be taught in one lesson.**
   These practices are complex. As with any complex skill or concept, they cannot be taught and mastered in a single class period. Instead, they are nurtured over long periods of time. Instances of those practices should be highlighted and reflected upon. Revisiting practices throughout a week or unit helps students better understand them. It helps students recognize that careful selection of tools extends beyond a study of measurement and that reasoning about symbols and quantities is a daily experience.

5. **Only special, complicated tasks elicit the practices.**
   The math practices should come about through rich problems and quality tasks. Clearly, a page of 40 problems requiring nothing more than rote procedure is unlikely to elicit these practices. However, an eccentric prompt and complicated tasks aren't required either. There is no one special task that causes students to master constructing viable arguments. Rather, it happens over time through a collection of authentic tasks that consistently ask students to explain how they represented problems, how they solved problems, why their solutions are accurate, or how they made connections between numbers, representations, and operations.

6. **Practices are separate from content.**
   The MPs are unique yet interconnected to one another. They are also closely connected to the mathematics content that is taught. Content such as place value, two-dimensional figures, or fractions is the what of mathematics instruction. These practices embody the how. Students learn about the structure of place value (MP 8). They communicate precisely (MP 6) about the attributes of two-dimensional figures and use that information to solve problems (MP 1). Students reason about a fraction's relationship to zero, one-half, or one (MP 2) and use that relationship to argue why one fraction is greater than another (MP 3).

7. **Practices are a poster for students to read.**
   Practices must be experienced. Teachers and students must understand what they are and how they "appear" in a mathematics class. Posters and anchor charts are quality references. Yet, hanging these in a classroom do not ensure that the mathematics practices actually occur.

8. **Practices aren't tested.**
   There are misconceptions that math practices are soft skills that aren't as important as mathematics content. These ideas lead to false conclusions that practices aren't tested. Thinking and reasoning are on almost every test. Many state and district assessments now call for students to explain their reasoning, represent their ideas, and use tools to solve problems on the assessment. Some assessments report how students perform with some of the practices

(PARCC, 2018; Smarter Balanced, 2018). Other tests do not report performance with practices. Even so, the skills and behaviors of these practices are incorporated into assessment items and design.

9. **The practices aren't expected in primary grades.**
"The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students" (CCSSO, 2010). The practices are curriculum expectations in all grades. All elementary students are to gain experience with and develop behaviors inherent in these practices. Certain practices such as solving problems, constructing arguments, and attending to precision are obviously present in these grades; the others must be as well. Students can make sense of symbols and equations when connected to their representations of problems. Students can make decisions about how tools help them and when tools are unnecessary. Students can see patterns within numbers and results to find efficiencies. Simply, the content may be different from grade to grade, but these practices remain a constant.

How Do We Bring the Practices to Life?

The practices won't magically appear in a mathematics lesson. They have to be understood and planned for intentionally. They must be discussed and reflected upon. They come to life when we:

- Understand what they mean.
- Recognize what they look like.
- Plan for them intentionally and daily, though they are not a list to be checked off.
- Select authentic tasks that elicit behaviors that are evidence of the practices and spark curiosity.
- Discuss how the practices were evident in a lesson or investigation by reflecting and discussing them.

What Is Evidence of the Practices in a Mathematics Lesson?

There will be evidence that the practices are alive and well in a lesson or classroom, but is important to keep in mind that these classrooms can look different in respect to how students are engaging in the different processes. Teachers can find evidence in artifacts of a lesson plan or in the writing and representations of student work. Teacher and student behaviors are strong indicators of the practices as well because student attitudes, perspectives, and behaviors are related to what teachers do and say (Northern, 2017). In essence, student behaviors are not random.

The charts below can be useful for teachers and students as they describe and define the practices. They can help students reflect on how they behaved like a mathematician during the lesson. Administrators might use the descriptors as indicators on a classroom walk-through or to buoy feedback to teachers. The charts are not exhaustive.

Mathematical Practice 1: Make sense of problems and persevere in solving them.

Students	Teachers
▪ Make sense of a problem ▪ Represent problems with physical tools, drawings, and equations ▪ Use a process for solving problems ▪ Apply strategies for solving problems ▪ Adjust strategies as needed when solving problems ▪ Determining whether solutions are reasonable	▪ Pose problems daily ▪ Develop student strategies for solving problems ▪ Provide strategies for students when they struggle ▪ Discuss student strategies ▪ Emphasize process rather than solution ▪ Have students estimate and compare to develop a sense of reasonableness

Mathematical Practice 2: Reason abstractly and quantitatively.

Students	Teachers
▪ Represent problems with equations ▪ Connect equations to problems and contexts ▪ Put solutions into the context of the problem ▪ Think flexibly about numbers ▪ Compute flexibly and efficiently	▪ Connect equations to problems and contexts ▪ Provide opportunities to manipulate numbers ▪ Model flexible strategies for computation ▪ Discuss student strategies

Mathematical Practice 3: Construct viable arguments and critique the reasoning of others.

Students	Teachers
▪ Justify their solutions and processes ▪ Communicate their thinking accurately with numbers, words, and/or pictures ▪ Listen actively to the reasoning of others ▪ Question the ideas of others	▪ Facilitate discussion rather than supply solutions and procedures ▪ Question students so that arguments are clear ▪ Establish an environment of respect and rapport for effective conversations ▪ Avoid giving too much assistance

Mathematical Practice 4: Model with mathematics.

Students	Teachers
▪ Use physical tools to make predictions and solve problems ▪ Use drawings and diagrams to make predictions and solve problems ▪ Use tables and graphs to predict and solve problems	▪ Make tools available for students to use ▪ Highlight how tools, drawings, diagrams, tables, and equations support reasoning and solutions ▪ Use intentional and appropriate representations

Mathematical Practice 5: Use appropriate tools strategically.

Students	Teachers
▪ Understand how tools work ▪ Determine when tools are helpful ▪ Can describe why they used a tool for a given situation ▪ Evaluate whether the results generated with a tool are reasonable	▪ Make tools available to students ▪ Use tools during instruction ▪ Focus on how tools work as well as how to use them ▪ Discuss whether tools are necessary for a given situation ▪ Discuss whether results are reasonable

Mathematical Practice 6: Attend to precision.

Students	Teachers
▪ Use vocabulary correctly ▪ Calculate precisely ▪ Specify units of measure ▪ Use appropriate symbols	▪ Use vocabulary correctly ▪ Model efficient strategies for computation, noting accurate solutions ▪ Highlight units of measure and symbols ▪ Correct inaccuracies

Mathematical Practice 7: Look for and make sense of structure.

Students	Teachers
▪ Look for patterns in numbers and operations ▪ Use structure to compute and solve problems ▪ Use structure and patterns to determine whether results are reasonable	▪ Arrange expressions and equations so that patterns are easily observed ▪ Provide time for students to discover and discuss patterns ▪ Point out patterns and structures, noting generalizations and usefulness

Mathematical Practice 8: Look for and make use of repeated reasoning.

Students	Teachers
▪ Look for relationships between problems and calculations ▪ Make generalizations between problems and calculations ▪ Use generalizations to develop and use shortcuts	▪ Focus on relationships between problems ▪ Present expressions and equations that are related ▪ Discuss how reasoning can be transferred from one problem to a related problem ▪ Develop understanding of patterns and relationships that allow for shortcuts

Practical Tips for Realizing the Mathematical Practices

1. Avoid key words and similar strategies for solving problems. Making sense of problems and solving them is not a procedure to be mastered (MP 1).
2. Make explicit connections between problems, drawings and diagrams, and equations (MP 2 and 4).
3. Explore numbers, number relationships, and operations daily (MP 2).
4. Challenge students to determine whether a tool is needed for a task or computation (MP 5).
5. Reflect on the frequency of tools used, the type of tools used, and discussion about the tools used during mathematics class (MP 5).
6. Avoid shortcuts for tools. For example, when measuring with rulers, one doesn't have to line up the edge of an object with zero on the ruler, instead, focus on understanding of the concept and the tool.

7. Record related equations in ways so that patterns are easily observed (MP 7).

8. Representations used must align to the mathematics. Equal group multiplication problems should be represented with equal groups rather than area models.

Questions to Ask

- MP 1: Do my students engage in problems daily? Do my students have diverse strategies for solving problems?
- MP 2: Do my students have opportunities to reason about numbers?
- MP 3: Do my students justify their thinking? Do they communicate their ideas clearly? Do they passively agree with classmates?
- MP 4: Do my students construct models of problems and situations with physical tools, drawings, and equations?
- MP 5: Do my students know how to use tools?
- MP 6: Do my students use correct vocabulary, attend to unit labels, and recognize precision?
- MP 7: Do my students have opportunities to observe, discuss, and make sense of structures in mathematics?
- MP 8: Do my students explain why shortcuts work?

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