A Key to Deep Understanding: The Importance of Rich Tasks in K-12 Mathematics

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Introduction: The Evolution of Problems Into Rich Tasks

Think back to your experiences learning mathematics in elementary, middle or high school. What comes to mind? What did a page in your textbook look like? How would you describe a typical mathematics class?

For many of us, our recollections are of a teacher standing at the board showing the entire class how to do a mathematical procedure and perhaps explaining why it worked. This was followed by a series of practice exercises in a book or with a worksheet, usually ending with a few word problems (story problems) that used that procedure.

The use of rich tasks to develop mathematical concepts and deepen student understanding is not new. Content and process standards over the past 25 years (NCTM 1989, 2001) have called for problem solving to be a focus of mathematical instruction and the platform for learning mathematics. That call is reflected in today's college and career ready standards and effective teaching practices at both the national and state level. (CCSSO 2011, NCTM 2014).

The evolution of problems into rich tasks continues. In this paper, I will provide some examples of rich tasks, take a closer look at the characteristics of a rich task, and explain what effectively using rich tasks in mathematics instruction entails.

"An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically."

- National Council of Teachers of Mathematics, Principles to Actions (NCTM 2014)

"Standard for Mathematical Practice: Make sense of problems and persevere in solving them."

- The Common Core Standards for Mathematics. (CCSSO, 2011)

Helpful Definitions

Often the words we use to describe instructional tasks are confusing. What follows is a brief explanation of some terms related to rich tasks.

Task: A *task* is any job or assignment we set out to complete. Completing 20 multiplication exercises on a worksheet or solving a word problem are examples of mathematical tasks.

Exercise: An *exercise* is a task that is routine, and the path to a solution is known. We use exercises to give students opportunities to practice a skill. *Exercises* include calculating with given numbers or solving word problems that provide practice applying a previously learned concept.

Story problem: A *story problem* is a worded situation that might be an exercise (for example, the student knows how to multiply to find the solution) or it might be used to introduce a new concept or to apply a known concept in a new situation (for example, the student doesn't know what to do, so he or she explores and uses a variety of strategies to reach a solution to the problem).

Rich task: A *rich task* or problem can be presented in words or expressed using only numbers. It can be a game or a puzzle. It is a task to which there is no immediate known solution-path on the part of the learner. A rich task presents a high level of cognitive demand and requires students to think abstractly in order to make connections to and among mathematical concepts. As students work to reach a solution, new mathematics concepts unfold, and deeper understanding occurs.

Note that what is considered a rich task for one student may not be a rich task for another student. Rich tasks are often referred to as open-ended or non-routine problems.

Examples of Rich Tasks

Rich Task 1: Prince Peter's Palace Garden (Grade 3)

Prince Peter is planning a garden for the palace. He has 24 petunia plants that must be in straight rows with the same number of plants in each row. How can he arrange the plants in his garden?

Follow-up: Which arrangement do you recommend for the prince to use? Explain why you suggest that arrangement.

This example represents a rich task for third graders with no previous experience using array models for multiplication. The students will explore using various representations, such as modeling with concrete materials or drawing pictures, to represent the situation.

On the other hand, this exercise would not be a rich task for fourth graders who already understand the array model for multiplication. For them, it is more of an exercise; an opportunity to practice what they already know how to do.

Rich Task 2: The Summer Swings Baseball League (multiple grade levels)

You have joined the Summer Swings baseball league. There are 10 teams in the league. If each team plays each of the other teams exactly once, what is the total number of games that will be played in the league?

Extension: Next summer the league will increase to include 5 additional teams. How many games will need to be scheduled?

Students at a variety of grade levels can solve this task. They can tackle it with a range of strategies such as *solve a simpler problem and generalize, find a pattern, draw a diagram, or act it out*. Most students will use a combination of several strategies.

"A rich task has a range of characteristics that together offer different opportunities to meet the different needs of learners at different times.... much of what it takes to make a rich task "rich" is the environment in which it is presented, which includes the support and questions that are used by the teacher and the roles that learners are encouraged to adopt."

- Jennifer Piggott, Rich Tasks and Contexts (http://nrich.maths.org/5662)

Characteristics of a Rich Task

Teachers must recognize rich tasks and build a library of resources, including tasks from math programs that provide students the opportunity to explore and make sense of mathematics concepts. What follows is a list of the characteristics of rich tasks and reflection questions to ask when examining a task.

Not all tasks will have all of these characteristics, but the richer the task, the more it will fit these qualities. This list is not meant to be all-inclusive. Your description of a rich task will expand as you begin to recognize and use rich tasks with your students.

Notice that many of these characteristics align with the effective teaching practices from NCTM's *Principles to Actions* (2014) and the Standards for Mathematical Practice (NGA/CCSSO 2011). Most likely, they also align with your own state's process or practice standards.

1. A rich task is engaging.

The context of a rich task is interesting to students. It inherently draws in the student and causes him/her to be curious. The task sets the stage for exploration and discovery.

Tasks can certainly provide interesting and helpful contexts, yet they can also take the forms of puzzles, fantasy or fairy tales. With a rich task, students wonder about the situation and the solution.

Question: *Will my students find this task interesting enough to explore even if appears to be too difficult at first?*

2. A rich task requires substantive mathematics to reach a solution.

The mathematical content necessary to solve the problem should be clear to you and developmentally appropriate for students. It should be accessible and aligned to the content and learning intention of the lesson.

While using rich tasks for enrichment or extensions is admirable, they are more impactful when they build deeper understanding of concepts on which the students are working. Rich tasks often provide the opportunity for students to connect mathematical concepts. For example, in Prince Peter's Palace Garden, students are exploring the use of arrays as a model for multiplication and will connect that model to the area model to find the area of a rectangle as they progress.

Although the Summer Swings Baseball League task can be solved on many levels, connecting it to finding a rule (algebraic expression) for any number of teams presents students with the important algebra concept of generalizing.

Question: *Does this task align with the mathematics I want my students to learn? How will I check to ensure they have learned it?*

3. A rich task is accessible for all students in the class.

Every mathematics class includes students with a broad range of interests and abilities. A rich task should have a variety of possible entry points, so each student can begin to work on the task at different levels.

In the Prince Peter's Palace task, most students will be able to model a garden that has 24 petunias using concrete materials such as counters or tiles. The task can be scaled down or scaled up to meet the needs of each student. For instance, Prince Peter's garden can have more petunias for students who are able to explore with a greater number or fewer petunias for struggling students.

You can increase or decrease the number of teams in The Summer Swings task to give all students a reachable starting point. A rich task should be designed so that it inherently supports differentiation in the classroom. Some educators refer to these as "low floor/high ceiling tasks."

Question: *How can I adjust this task to meet the needs of different learning levels in my class?*

4. A rich task has multiple solution paths.

When faced with a rich task, a student might lament by saying, "I don't get what they want me to do!" The more experience students have with learning and using a variety of problem-solving strategies, the more confident they become in their approach to solving a rich task.

Students need formal experience with problem-solving strategies and opportunities in order to collaborate in groups using a particular strategy. Once a repertoire of strategies has been developed, a student can determine which strategies make the most sense in a given situation and how to adjust thinking if the attempt is not leading to a reasonable solution.

A rich task allows students to be creative in their approach, to use multiple representations (physical, visual, symbolic, verbal and contextual), and to change paths when one approach to finding a solution is not successful.

Question: *What strategy/strategies might students use to complete this task?*

Problem Solving Strategies	
Make a Table	Account for All Possibilities
Look for a Pattern	Solve A Simpler Problem
Make a Drawing or Diagram	Work Backwards
Make a Model	Write an Expression or Equation
Guess and Check	Identify a Subgoal
Act It Out	Change Your Point of View
Make an Organized List	Generalize
Make or Use a Graph	

5. A rich task has multiple solutions.

Prince Peter's Garden is an example of this. There are several arrays that can fit the conditions of the garden (24 plants). These solutions should lead to interesting classroom discussions, including having students explain how they know they have all of the possible solutions, or whether a garden with 6 row and 4 petunias in each row is the same as a garden with 4 rows with 6 petunias in each.

Some rich tasks have one acceptable, accurate solution. In the Summer Swings task, there is one accurate solution even though there are several strategies that can lead students to find it. The important aspect of this task is the mathematical thinking that leads to the response. Classroom discussion should focus around:

- Comparing solution methods and finding similarities and differences among those methods
- Describing patterns found while solving and extending the task, which can be done by making generalizations for a league with a greater number of teams

Question: *What solution(s) would I expect my students to find for this task? What types of questions can I ask that will support a rich classroom discussion?*

6. A rich task encourages student discourse and additional questions among students as they work to reach a solution.

The value of metacognition (thinking about one's thinking) becomes apparent when students are expected to explain their reasoning in classroom discussions through writing and/or showing their work.

The importance of student reflection has been recognized in all curriculum areas. In mathematics, metacognition helps students build conceptual understanding and link that understanding to procedural skills. As such, rich tasks provide a context for deep learning. They also give teachers the opportunity to change the conditions of the problem to extend understanding and make generalizations, and they encourage students to ask, "What if?"

Question: *What questions will I ask students who are struggling with the task without telling them what to do? How will I support students who struggle to clearly explain their thinking?*

7. A rich task sets the stage for a solution process that is as important as the solution.

Think about the two rich task examples shown earlier: Prince Peter's Palace Garden and The Summer Swings Baseball League. While both are interesting tasks in and of themselves, once an acceptable solution is reached, little attention will be paid to the situation. The importance and richness of each of these tasks is the mathematical reasoning that takes place during the solution process. Prince Peter provides students with a new model for multiplication. That model can easily be extended to explore other numbers, so that once students recognize and understand the process, they can use it again in other situations.

The Summer Swings task can be solved on a variety of levels. It allows students to recognize a particular pattern (triangular numbers), describe a pattern for finding the number of games to play by any number of teams, and (for middle school students) find a way to express that pattern algebraically.

Within this rich task, there are many ways to engage students with triangular numbers (or square numbers or other number patterns). The solution process that students use will help them recognize number patterns and use similar strategies with similar tasks. There is a great deal of learning and understanding happening while the task is being tackled!

Question: *What mathematical ideas will students take away from the solution process? How does it align with the content and mathematical practice learning intentions for the lesson?*

Instructional Strategies for Using Rich Tasks

Among the many benefits of using rich mathematical tasks in the classroom are 1) a deeper understanding of mathematical concepts and 2) the transfer of conceptual understanding to procedural skills.

That said, it is not enough to simply hand out a task and put students in groups to collaboratively work on a solution. Research has shown that it is crucial for the teacher to orchestrate the implementation of a rich task to ensure students are using the task to promote understanding through mathematical thinking, reasoning, and problem-solving. (Stein, Lane 1996).

During the process of selecting and using rich tasks, a teacher should consider:

- What mathematics will my students learn from completing this task?
- What prerequisite understandings are essential for students to have entry into the task?
- How will I introduce the task?
- What are the potentially correct and incorrect solution paths students may take?
- What questions will support student work during the solution process?
- What questions will support connecting mathematical ideas during whole class discussion?
- What evidence will inform me of a student's understanding as he/she completes the task?

In addition to these questions, teachers should be aware that instruction using rich tasks differs from traditional instruction in the following ways:

- The role of the teacher is not to show and tell students about a procedure. Rather, the teacher is there to support students by asking purposeful questions. The student takes charge of his or her own learning.
- It may take more than a class "period" to fully complete a rich task.
- Students work on a task collaboratively to make sense of the mathematics. They will use a variety of strategies, such as discussing their ideas and using multiple representations both to reach a solution and to show their thinking.
- Students experience productive struggle when working on a rich task. They may head down an unproductive path while seeking a solution—perhaps more than once! However, part of the solution process is recognizing false starts and rethinking the approach. Through experience, students learn to recognize when and how to adjust their thinking.
- Student work looks different. The focus is not only on the answer. The process used to reach a solution is an essential part of the learning, as it helps students to make sense of mathematical ideas.
- The teacher assesses students throughout the solution process by asking carefully constructed questions. These questions are designed to check student understanding and to help students connect concepts.

Online Resources for Rich Tasks

The Internet offers some good sources of rich mathematical tasks. Beware: some sites offer tasks labeled "rich tasks" that are actually low cognitive demand tasks.

Browse through these sites to get your library of rich tasks started. Be on the lookout for additional websites and books that offer more examples.

- http://www.insidemathematics.org/
- http://www.nrich.maths.org/
- http://illuminations.nctm.org
- http://www.illustrativemathematics.org
- http://www.nctm.org/ARCs/
- https://nzmaths.co.nz/

Summary

While the benefits of using rich tasks far outweigh the limiting nature of drill and practice, teaching and learning with rich tasks requires much more from both the teacher and the learner. Both teachers and students must understand that this type of learning takes more effort and deeper thinking. The solution is never obvious, and simply following a set of steps is not an approach that will lead to success or to understanding.

If you are thinking of using rich tasks, start slowly and prepare thoroughly. Choose tasks that align with content you are teaching. Consider how a task gives students opportunities to use, and more deeply understand, the Standards for Mathematical Practice.

Teachers should also solve the problem in as many ways as they can before giving it to students. This gives them a better understanding of how students may approach the task while also surfacing potential connections to previously learned or future content.

A best practice is to have all materials that students might need on hand. If working with rich tasks is new for your students, understand that they may become frustrated by the new kind of thinking that is expected. Be prepared to support your students without reverting to showing them how to solve the problem.

Lastly, be willing to learn with your students. Hold fast to your expectations that students are responsible for their mathematical thinking. You do not need to show students "how" to do the mathematics. Your efforts will be rewarded as you see your students become more engaged and excited about learning mathematics!

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