8 Program Routines, Tools, and the Digital Environment

Contents

Program routines, such as Quick Looks and Name-Collection Boxes, and tools, such as calculators, rulers, pattern blocks, number lines, number grids, and Reference Books, are used in *Everyday Mathematics* to facilitate mathematical thinking and problem solving. *Everyday Mathematics* helps students learn why tools are used in mathematics, how to choose appropriate tools for a task, and how to use tools effectively to help solve problems.

Digital versions of program routines and tools have been especially designed and built for use in *Everyday Mathematics.* The eToolkit is a digital workspace containing electronic versions of physical manipulatives. Specific eTools are also built into teacher ePresentations and Student Learning Center activities, allowing teachers to use them within prepared activities.

Geometer's Sketchpad activities appear throughout the Reference Books and the Student Learning Center. These activities give students the ability to interact with and manipulate shapes, number grids, number lines, ten frames, and more.

Everyday Mathematics is designed to make doing mathematics in school resemble how mathematics is done in everyday life, which includes teaching students the power of tools and how to use them effectively.

8.1 Program Routines

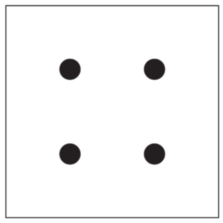
This section briefly summarizes the major *Everyday Mathematics* program routines that help students practice problem solving and develop fact fluency.

8.1.1 Quick Looks

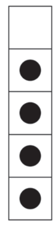
GRADE LEVEL K 1 2 3 4 5 6

Through *Quick Looks* activities, students develop the ability to **subitize,** to recognize a quantity without counting, and to *decompose* numbers in various ways. Showing numbers in different ways and asking students to describe how they see them elicits flexible thinking about numbers. As students encounter various combinations of numbers, they also develop strategies for basic facts. Research suggests developing the ability to subitize facilitates the development of fact fluency.

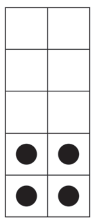
In Kindergarten through Grade 3, numerous activities use Quick Look images including dot patterns, five frames, ten frames, double ten frames, equal groups, and fractions. These images have been selected and sequenced to encourage concepts of number and strategy development. Students view images for 2–3 seconds and then share both *what* they saw and *how* they saw it. Quick Look Cards, including the following sample images, can be found in the Manipulatives Kit, as a printable PDF resource in the Teacher Center, in the eBooks, and in the teacher eToolkit.



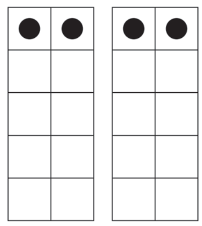
A dot-pattern representation of the number 4



A five-frame representation of the number 4

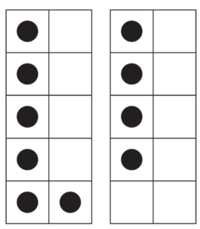


A ten-frame representation of the number 4



A double ten-frame representation of the number 4

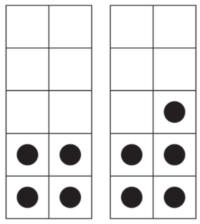
In this edition of *Everyday Mathematics,* five frames, ten frames, and double ten frames are oriented vertically to facilitate the addition and subtraction fact strategies that students explore in Grades 1 and 2. For example, the vertical orientation of the double ten frame in the image below encourages students to mentally manipulate the dots across ten frames to solve $6 + 4$ and record a number sentence in a left-to-right orientation.



Visualizing $6 + 4 = 10$

Section 8

Note that five, ten, and double ten frame images in this edition are not filled in a particular order (for example, from top to bottom, left to right). Since there are no restrictions on how the frames are filled, students encounter many opportunities to decompose numbers and develop helpful strategies, including addition doubles and the near doubles strategy. For example, the placement of the dots at the bottom of the double ten frame below encourages students to see 4 + 5 as a near doubles fact.



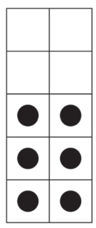
To solve 4 + 5, children mentally manipulate $4 + 4 + 1 = 8 + 1 = 9$

Quick Look Cards are available as an eTool. Teachers can choose the card or cards to display and the length of time each card will be shown.

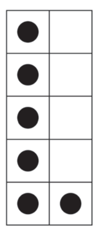
Quick Looks in Kindergarten

Kindergarten students begin Quick Looks in Section 1 by learning to recognize numbers as they are represented in simple dot patterns and five frames.

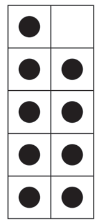
In Section 2 of Kindergarten, students begin practicing Quick Looks on ten frames. In later sections, students use ten frames to practice composing and decomposing the number 10. Teachers should emphasize discussing and sharing strategies such as:



Decomposing into "easy" combinations: "I saw 3 and 3, which makes 6;" or "I counted by 2s: 2, 4, 6."



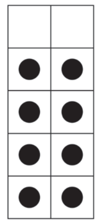
Relating to 5: "I saw one full line with 5 and then 1 more, and 5 and 1 more is 6."



Relating to 10: "I saw one empty space, and I know 9 is 1 less than 10."

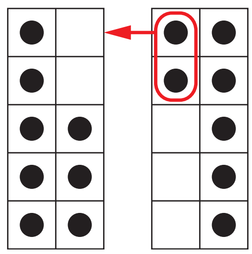
Quick Looks in Grades 1 and 2

In Grades 1 and 2, Quick Looks appear in the Focus activities of the addition fact strategy lessons to help students develop ideas such as doubles and combinations of 10 and strategies such as near doubles and making 10. Quick Looks are practiced throughout the year as Mental Math and Fluency exercises.

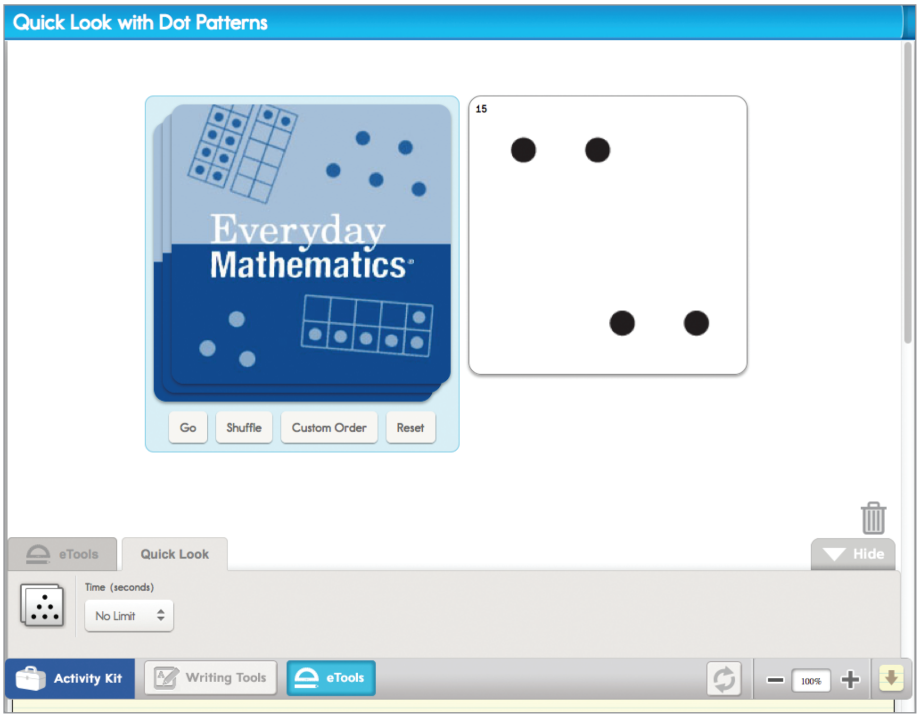


Quick Look Card representing the number 8 on a ten frame

First and second grade students may explain finding the total number of dots in the image above in different ways, such as "I saw 4 and 4 and that makes 8;" "I skip counted 2, 4, 6, 8;" and "There are 2 missing from the ten frame, so I know $10 - 2 = 8$." Quick Looks with more complex images are used to help students develop important fact strategies for solving more difficult addition facts as shown below.



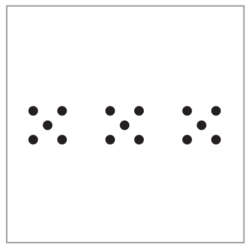
To solve $8 + 7$, children mentally manipulate the images to "make 10": $8 + 7 = 10 + 5 = 15$



Quick Look Cards in the Teacher Center ePresentations

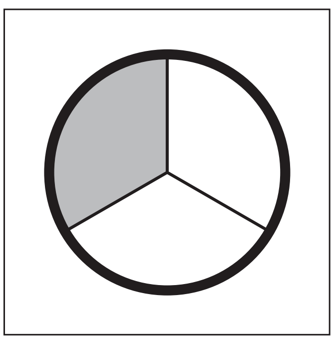
Quick Looks in Grade 3

Quick Looks activities in Grade 3 build on the foundations laid in Kindergarten through Grade 2. Third graders are challenged to use Equal Groups Quick Looks to make sense of arrays and equal-group images and to practice smaller multiplication facts. Many images encourage students to use strategies, such as skip counting, repeated addition, or instant recognition to build their understanding of multiplication. For example, for the Equal Groups Quick Look Card in the margin, third grade students might say, "I skip counted by 5s 3 times: 5, 10, 15;" or, "I see 3 groups of 5, so I think $5 + 5 = 10$ and 5 more is 15."



An Equal Groups Quick Look Card for 3 groups of 5 or 15

Fractions Quick Looks in Grade 3 support the development of visual images of fraction representations. A bold outline on each Fractions Quick Look indicates the whole for the image. These fraction images help students internalize representations of benchmark fractions and recognize the importance of identifying the whole. Third graders may describe the Fractions Quick Look card below as $\frac{1}{3}$, or as 1 of the 3 parts is shaded.



A Fractions Quick Look Card for showing 1 of 3 parts shaded or $\frac{1}{3}$

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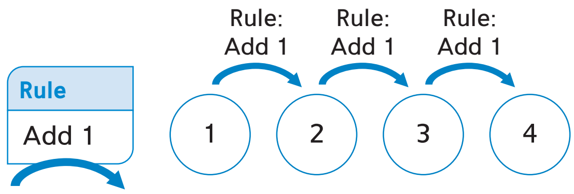
8.1.2 Frames-and-Arrows Diagrams

GRADE LEVEL K | 1 | 2 | 3 | 4 | 5 | 6

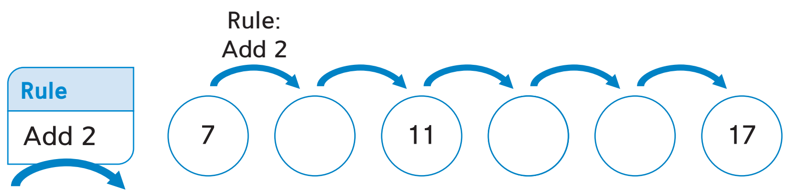
Frames-and-Arrows diagrams consist of a sequence of frames connected by arrows. Each frame contains a number; each arrow represents a rule that determines which number goes in the next frame, similar to a function machine.

The numbers in a Frames-and-Arrows diagram form a sequence; the arrow rule or rules represent the mathematical structure that generates the sequence. A simple example of a Frames-and-Arrows diagram for the rule "Add 1" is shown below.

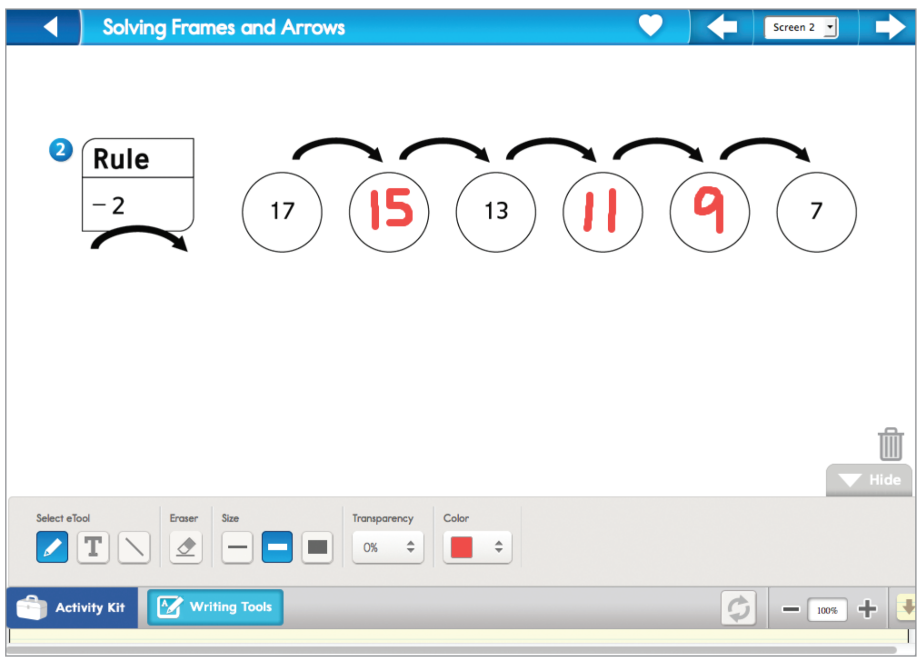


In Frames-and-Arrows problems, some information is missing. Several types of Frames-and-Arrows problems are described below.

- A rule is given but some of the frames are empty. Students use the rule to fill in the blank frames.

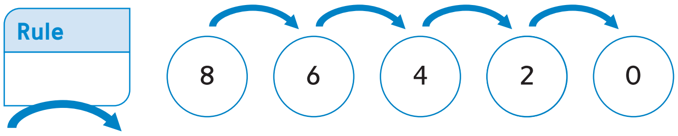


Solution: Write 9, 13, and 15 in the blank frames.



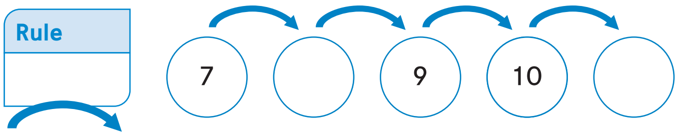
A Frames-and-Arrows problem completed in the Student Learning Center

- The frames are all filled in, but a rule is missing. Students find the rule. (Note that more than one rule is often possible.)



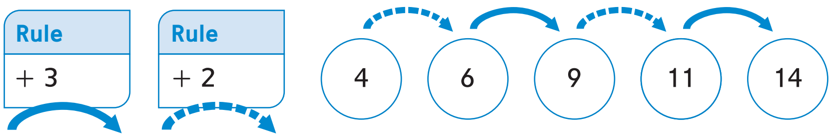
Solution: Possible rules include "Subtract 2," "Minus 2," or "– 2."

- There are some empty frames and a missing rule. Students find a rule and fill in the empty frames.



Solution: A rule is "Add 1" or "+ 1."

A Frames-and-Arrows problem can have more than one arrow rule; different rules are represented by different arrows. In the following problem, a solid arrow means + 3 and a dashed arrow means + 2.

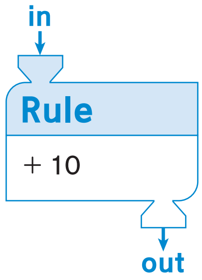


Sketchpad activities for Frames-and-Arrows are available in *My Reference Book* and *Student Reference Book*. Keypad entry is also available in the Student Learning Center.

8.1.3 "What's My Rule?" and Function Machines

GRADE LEVEL K 1 2 3 4 5 6

"What's My Rule?" is an activity in which students analyze a set of number pairs to determine a rule that relates the numbers in each pair. Students also use a rule to generate pairs of numbers. The data are often generated using a *function machine,* an imaginary device that receives inputs and produces outputs according to a rule. For example, the function machine below takes an input number and outputs 10 more than the input. The input/output pairs can then be displayed in a function table, also known as an in/out table or a "What's My Rule?" table.



A function machine

in	out
15	25
4	14
7	
	63

A function table, also known as an in/out table or a "What's My Rule?" table

Function machines and function tables help students visualize how a rule transforms each input value into an output value. In the ePresentations and Student Learning Center, a keypad is often available for entering missing *in* and *out* numbers. Interactive "What's My Rule?" and function machines are also available in all *Student Reference Books* and in *My Reference Book*.

Simple "What's My Rule?" games begin in *Kindergarten Everyday Mathematics*. The first games are attribute or rule activities that sort students into a specified group. For example, a teacher may sort students wearing red into one group and students not wearing red into a second group, without revealing the sorting rule. Students then determine what the rule is from their observations. In later sections, students play "What's My Rule?" with numbers, using a physical box that serves as the function machine.

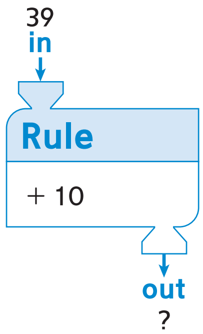
Late in Kindergarten and through fifth grade, "What's My Rule?" activities are extended to include problems in which pairs of numbers are given and the task is to find a rule that relates the numbers in each pair. The same rule has to work for every pair. Students record the input as the first number in the function table and the output as the second number, then work to identify the function machine's rule using the examples in the table.

In fifth grade, students use "What's My Rule?" tables to create ordered pairs, using the input as the first coordinate and the output as the second coordinate in the ordered pair. Then students graph the ordered pairs as another way to visualize the relationship between inputs and outputs.

Generally, in a "What's My Rule?" problem, two of the three parts of a function (input, output, and rule) are known. The goal is to find the unknown part.

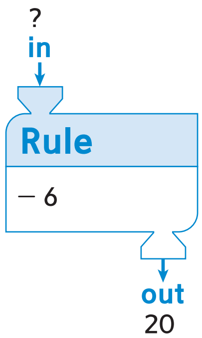
There are three basic types of problems:

- The rule and some input numbers are known. Students find the corresponding output numbers.



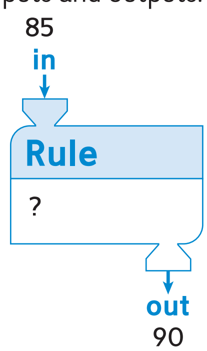
in	out
39	
54	
163	

- The rule and some output numbers are known. Students find the corresponding input numbers.



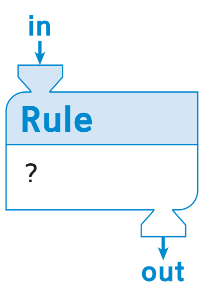
in	out
	6
	10
	20

- Some input and output numbers are known. Students find a rule that fits all of the available inputs and outputs.

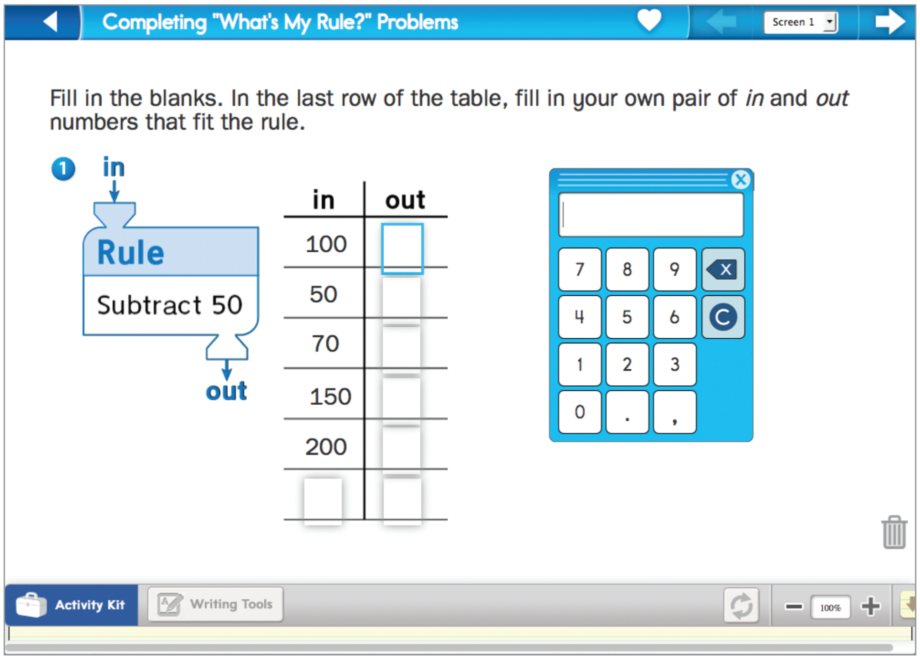


in	out
55	60
85	90
103	108

More than one of these problem types can be included in a single table. For example, students might be given a partially completed table with an unknown rule. If enough input and output clues are provided, students find the rule and fill in missing input and output numbers, as in the problem below.



in	out
15	25
4	14
7	
	63



"What's My Rule Problem" in Student Learning Center with keypad answer entry from Grade 3, Lesson 3-1

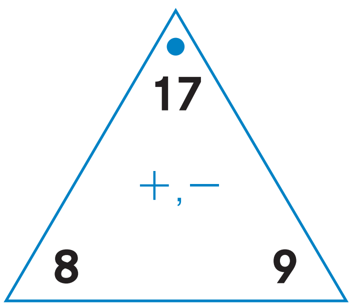
8.1.4 Fact Families and Fact Triangles

GRADE LEVEL K 1 2 3 4 5 6

A *fact family* is a collection of four related facts linking inverse operations, such as addition and subtraction or multiplication and division. Fact families improve fact fluency, and subsequently automaticity, by helping students connect facts they know to facts that are unknown. By recalling part of a fact family, students can reword or rewrite an unknown fact in ways that make sense to them. For example, when faced with 8 ÷ 4 = ? a student can think, "4 times what equals 8? 4 times 2 equals 8, so 8 divided by 4 equals 2."

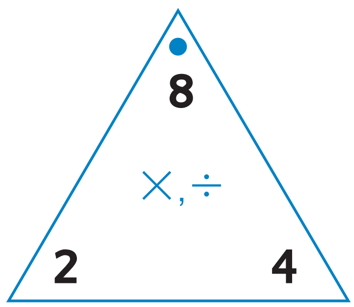
Fact Triangles are the *Everyday Mathematics* version of flash cards. Fact Triangles have two advantages over regular flash cards:

1. They connect fact families with fact practice, reinforcing the link between addition and subtraction and the link between multiplication and division.
2. They help improve fluency by linking four facts together. Knowing a single fact means you actually know four facts.



8 + 9 = 17 9 + 8 = 17
17 − 9 = 8 17 − 8 = 9

An addition/subtraction Fact Triangle



2 × 4 = 8 4 × 2 = 8
8 ÷ 2 = 4 8 ÷ 4 = 2

A multiplication/division Fact Triangle

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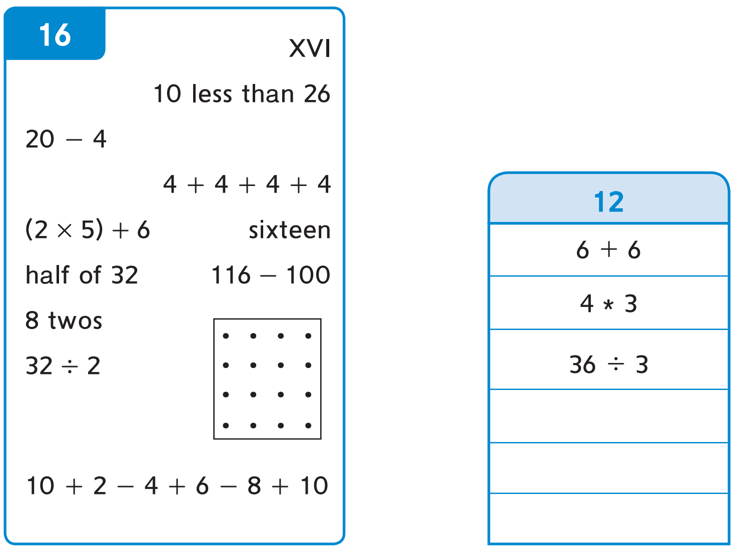
Fact Triangles are often employed in a cooperative-learning situation. One partner covers a corner with a finger and the other partner gives a fact that has the hidden number as an answer. For example, one partner might cover the 9 in the 8, 9, 17 addition/subtraction Fact Triangle. The other partner could say, "17 − 8 = 9" or "17 = 8 + 9." Students can also work independently with Fact Triangles to quiz themselves, sorting the Fact Triangles into known and unknown facts or by a strategy such as doubles, near doubles, times 3, or divided by 5, to make for efficient practice. Because Fact Triangles activities are easy to use at home, they are sent home as Home Links in Grades 1–3. Teachers can also customize Fact Triangles using the Fact Triangle eTool.

8.1.5 Name-Collection Boxes

GRADE LEVEL K 1 2 3 4 5 6

A *name-collection box* is used to collect many names for a given number. Beginning in *Kindergarten Everyday Mathematics*, name-collection boxes allow students to explore the notion that the same number can be expressed in many different ways. Names can include sets of objects, sums, differences, products, quotients, expressions, the results of combining several operations, words in English or other languages, tally marks, and arrays.



Name-collection boxes

For example, the box shown above is a name-collection box for the number 16. Beginning in fourth grade, students use a more compact name-collection box, such as the 12-box shown above.

8.1.6 Units and Unit Boxes

GRADE LEVEL K 1 2 3 4 5 6

Numbers and operations make the most sense to students when they relate to real-world contexts. Students must learn to connect the context of a problem to their results. Encourage students to attach appropriate unit labels, such as cents, lions, or feet, to the numbers they use when they measure, count, calculate, and record their work.

Unit
cents ¢

A unit box

Because labeling each number in a situation diagram or other problem can be tedious, *Everyday Mathematics* uses *unit boxes* beginning in first grade. A unit box can be displayed beside a problem or at the top of a page of problems. Unit boxes contain the labels or units of measure used in the problem(s). Unit boxes help students organize their mathematics while keeping a particular context in mind.

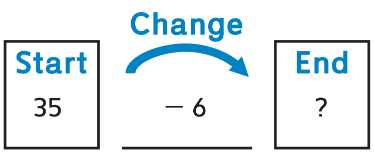
While unit boxes are not a focus in *Kindergarten Everyday Mathematics*, Kindergarten teachers should encourage students to identify and use the unit in their problems and answers. Teachers should model using units in the context of number stories, measurements, counts, and other activities and encourage students to develop this habit.

8.1.7 Situation Diagrams

GRADE LEVEL K **1** **2** **3** **4** 5 6

In *Everyday Mathematics*, students are encouraged to use *situation diagrams* to help them organize the information in a number story (or word problem) and make a plan for solving the problem. Addition, subtraction, multiplication, and division can each be applied in many different situations, but most of those situations can be sorted into a handful of categories.

The three basic categories for addition and subtraction situations are called change, parts-and-total, and comparison. Depending on what is known and what is unknown, each kind of situation can lead either to addition or subtraction. Multiplication and division situations are harder to sort out, but several basic situations can be distinguished: equal groups, arrays, and area. Each type of situation can lead either to multiplication or division problems, depending on what is unknown. *Everyday Mathematics* uses situation diagrams to help sort out these various kinds of problem situations. In Grades 1–3, students use change, parts-and-total, and comparison diagrams for addition or subtraction situations, and in Grades 3–4, students use multiplication/division diagrams. Sample diagrams are in the margin and below. Students fill out each section with a number or a question mark, depending on what is known and unknown, and determine which operation to use that makes sense to them.

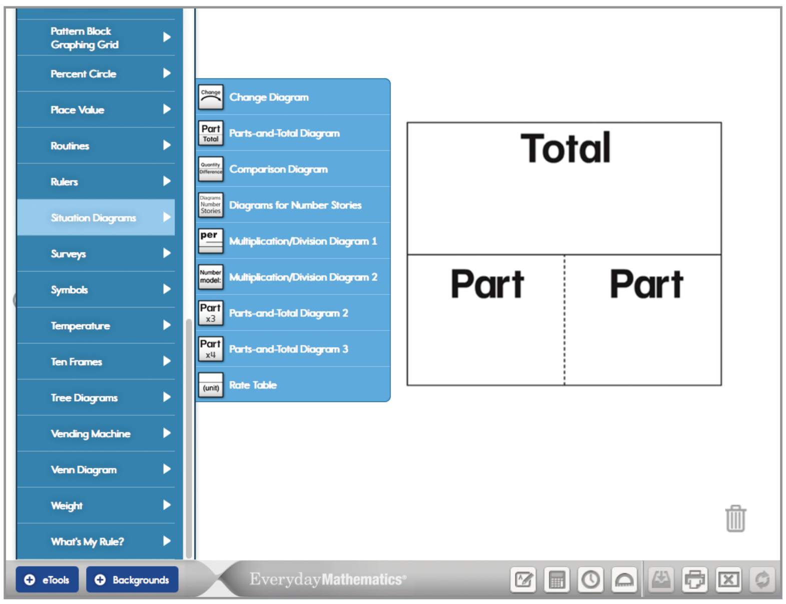


A change diagram

A comparison diagram

children	cards per child	cards
4	?	24

A multiplication/division diagram



Situation diagrams accessible in the Student Learning Center through the eToolkit

Addition and Subtraction Situation Diagrams

GRADE LEVEL K **1** **2** **3** 4 5 6

Most number stories that can be solved by addition or subtraction can be categorized as change, parts-and-total, or comparison situations.

Change Diagrams. In a *change* situation there is a starting quantity, a change in quantity, and an ending quantity. For example, a 15-cm tall plant might grow 5 cm in a week and end up 20 cm tall. Or, a student might have $5, spend $2, and then have only $3 left. Change situations can lead either to addition or subtraction problems, depending on the information that is known and unknown and how students interpret the problem. Note that some change situations can be interpreted as either addition or subtraction as shown in the examples below.

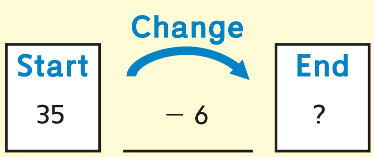
Example 1: A bus leaves school with 35 children. At the first stop, 6 children get off. How many children are left on the bus?

This is a change-to-less situation, or a decrease, with the ending quantity unknown.

Possible number models:

$35 - 6 = \_\_$

$6 + \_\_ = 35$



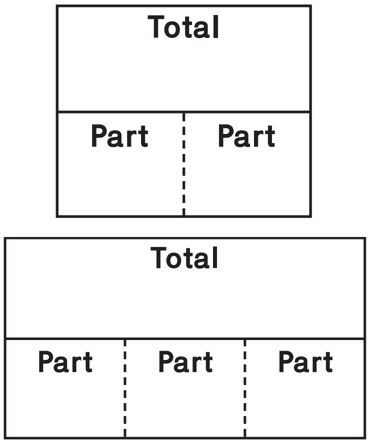
Example 2: Tom had some money. He earned $1.50 more. Then he had $6.50. How much money did Tom have to start with?

This is a change-to-more situation with the starting quantity unknown.

Possible number models:

$\$6.50 - \$1.50 = \_\_\_\_\_$

$\_\_\_\_\_ + \$1.50 = \$6.50$



Parts-and-total diagrams

Parts-and-Total Diagrams. In a *parts-and-total* situation, there is a total quantity that can be separated into two or more parts. For example, the total number of students in a class can be separated into the number of girls and the number of boys.

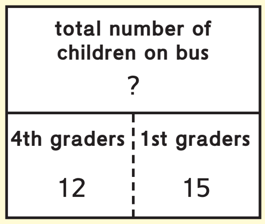
When drawing diagrams, quick and easy freehand drawings are best. The words "Part" and "Total" can be replaced with words that fit the problem situation better or omitted, as in Example 3.

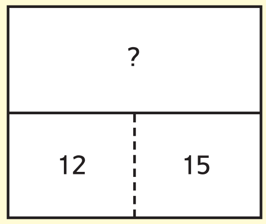
When all the parts are known but the total is unknown, the problem can be solved by adding the parts.

Example 3: Twelve fourth graders and 15 first graders are on a bus. How many children in all are on the bus?

The parts are known. You are looking for the total.

Possible number model: $12 + 15 =$ __





When the total is known but one of the parts is unknown, an addition or subtraction number model can be used to represent the situation.

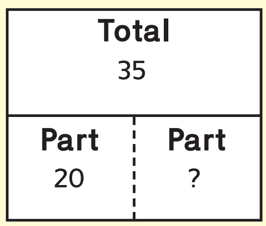
Example 4: Thirty-five children are riding on the bus. Twenty of them are boys. How many girls are riding on the bus?

One part and the total are known. You are looking for the other part.

Possible number models:

$20 +$ __ $= 35$

$35 - 20 =$ __



Comparison Diagrams. In a *comparison* situation, there are two separate quantities and the difference between them. For example, one person might be 60 inches tall and another 70 inches tall; the difference in heights is 10 inches. As with change and parts-and-total situations, comparison situations can lead to addition or subtraction, depending on what is known and what kind of comparison is being made. Note that the words "Quantity" and "Difference" can be replaced with words that fit the problem better or omitted.

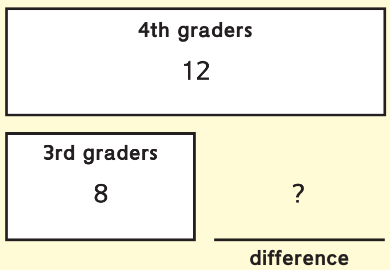
Example 5: There are 12 fourth graders and 8 third graders. How many more fourth graders are there than third graders?

Here both quantities being compared are known and the difference is unknown.

Possible number models:

12 − 8 = __

8 + __ = 12



Example 6: Vicky is 40 inches tall. Amelia is 4 inches shorter. How tall is Amelia?

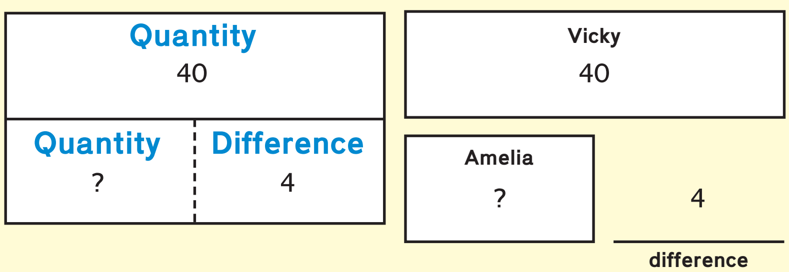
Here one of the quantities being compared and the difference are known. The other quantity being compared is unknown.

Possible number models:

40 − __ = 4

40 − 4 = __

__ + 4 = 40



Section 8

Multiplication and Division Situation Diagrams

GRADE LEVEL | K | 1 | 2 | **3** | **4** | 5 | 6

Multiplication and division arise in many different situations, but most can be sorted into the following categories: equal groups, arrays, and area.

Equal Groups. In an *equal-groups* situation, there are several groups of objects with the same number of objects in each group. Depending on what is unknown, equal-groups situations can lead either to multiplication or division problems.

In an equal-groups situation where the total is unknown but the number of groups and the number of objects in each group are known, the problem can be solved by multiplication.

Example 7: A vase holds 5 flowers with 6 petals on each flower. How many petals are there in all?

Possible number model: $5 \times 6 = \_\_$

flowers	petals per flower	total number of petals
5	6	?

In situations where the number of groups and the total number of objects are known, the problem is to find the number in each group. In *Everyday Mathematics*, these are called *equal-sharing* problems. Equal sharing is also known as *partitive division*. Many students solve equal-sharing problems by "dealing out" the objects to be shared.

Example 8: Twenty-eight baseball cards are to be shared equally by 4 children. How many cards does each child get?

Possible number models:

$4 \times \_\_ = 28$

$28/4 = \_\_$

$28 \div 4 = \_\_$

children	baseball cards per child	total number of cards
4	?	28

Section 8

In situations where the number in each group and the total number of objects are known, the problem is to find the number of groups. In *Everyday Mathematics*, these are called *equal-grouping* problems. Many students solve equal-grouping problems by making as many groups of the correct size as possible and then counting the number of groups.

Example 9: Twenty-four girl explorers are going on a canoe trip. Each canoe can hold 3 girls. How many canoes are needed?

Possible number models:

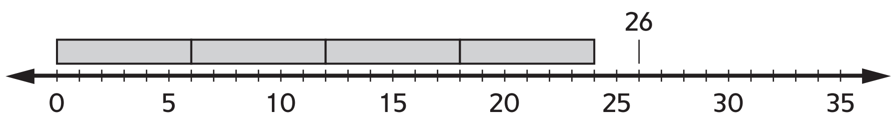
$3 \times \_\_ = 24$

$24/3 = \_\_$

$24 \div 3 = \_\_$

canoes	girls per canoe	total number of girls
?	3	24

Equal-grouping problems are also called *measurement division* or *quotitive division*. The term measurement division comes from thinking about using the divisor to "measure" the dividend. For example, consider 26 ÷ 6. Think: *How many 6s would it take to make 26?* Then imagine measuring off 6-unit lengths on a number line:



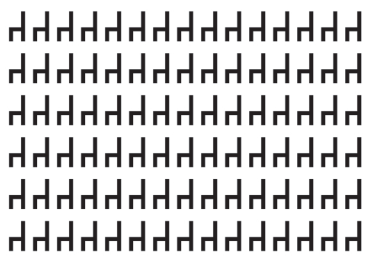
The figure above shows that there are four 6-unit lengths in 26, with 2 units left over. Thus, 26 ÷ 6 is 4 with a remainder of 2.

Arrays and Area. *Arrays* are closely related to equal-groups situations. If the equal groups are arranged in rows and columns, then a rectangular array is formed. As with equal-groups situations, arrays can lead either to multiplication or division problems.

Example 10: There are 6 rows with 15 chairs in each row. How many chairs are there in all?

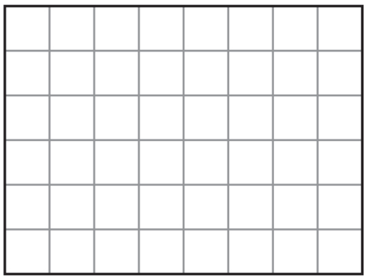
Possible number model: $6 \times 15 = \_\_$

rows	chairs per row	total chairs
6	15	?



A 6-by-15 array of chairs

Arrays are closely related to *area*. An array of square-centimeter tiles with no gaps between the tiles will have an area in square centimeters equal to the number of tiles (see Example 11 and the margin on next page). Unlike any of the previous multiplication models, an area model generalizes to multiplication with fractions, decimals, and mixed numbers.



An area model of 6×8

Example 11: The area of a rectangle is 48 cm^2. The rectangle's length is 8 cm. What is its width?

Possible number models:

$8 \times \_\_ = 48$

$48 \div 8 = \_\_$

length (cm)	width (cm)	area (cm^2)
8	?	48

Note that because of the Commutative Property of Multiplication, the factors in array and area situations can be interchanged without affecting the value of the product. The total number of chairs in Example 10 would not change if there were 15 rows with 6 chairs in each row. And if the rectangle in Example 11 had a width of 8 cm, its length would be 6 cm to give an area of 48 cm^2.

Even though the total number of chairs does not change based upon the order of the factors, there is a difference between 6 rows with 15 chairs in each row and 15 rows with 6 chairs in each row. Understanding the order of the factors in context is important for problem solving and in mathematics students may study later in school. A *matrix* is a mathematical term for an array, and to perform arithmetic with matrices requires keeping track of rows and columns. Therefore, it is best to talk about the dimensions of an array as meaning "rows" by "columns," in that order.

Teaching with Situation Diagrams

When situation diagrams are introduced in lessons, students are often asked to use the diagrams so that they become familiar with the process. Situation diagrams are available as Teaching Aid Masters in the *Math Masters* book and as backgrounds in the eToolkit. Once students are comfortable with the diagrams, *Everyday Mathematics* suggests that students follow these steps, although not necessarily in this order:

- Choose a diagram that fits the problem situation, but note that more than one diagram may fit. One person might think of a problem as a parts-and-total situation, while another person sees it as a change situation. Be flexible about which diagram is the most appropriate for a given situation, and remember that some students will prefer to use drawings, numbers, words, or other representations to make sense of and solve problems.
- Write the known quantities and a question mark for the unknown quantity (or quantities) in the appropriate parts of the diagram.
- Use the diagram to help decide how to solve the problem. Most problems can be solved in more than one way. A change-to-more problem, for example, might be solved by addition, by counting up mentally or on a number grid, by acting out with counters, or by drawing pictures.
- Calculate an answer.
- Write a number model that fits the problem.
- Write an answer. Be sure to include a measurement unit or other label.
- Check whether the answer makes sense.

8.2 Tools

The following sections highlight the purpose and use of some of the mathematical tools in *Everyday Mathematics*. It is not a comprehensive list of tools used in lessons, but the list discusses important points regarding the use of tools in the curriculum. Digital tools are available in the Teacher Center and the Student Learning Center through the eToolkit and Backgrounds.

8.2.1 Number and Counting Tools

Calculators

GRADE LEVEL K 1 2 3 4 5 6

Many researchers have studied the effects of calculators on how students learn. Evidence from these studies suggests that the proper use of calculators can enhance students' understanding and mastery of arithmetic, promote good number sense, improve problem-solving skills and attitudes toward mathematics, while not hindering the development of paper-and-pencil skills. As early as Kindergarten, students can use calculators to experiment with arithmetic concepts in problem-solving contexts.

Both teacher experience and educational research show that most students develop good judgment about when to use and when not to use calculators. *Everyday Mathematics* supports students in learning to decide when they should solve an arithmetic problem by estimating or calculating mentally, by using paper and pencil, or by using a calculator. Note that some student pages have been marked with a "no-calculator icon," which indicates that students should use a problem-solving strategy that does not involve a calculator.

Once students are introduced to algorithms, calculators can free students from spending time repeating a task they understand fully. Calculators also allow students to solve interesting everyday problems requiring computations that might otherwise be too difficult for them to perform, including problems that arise outside of mathematics class. They enable students to think about the problems themselves rather than focusing on carrying out algorithms without mistakes.

Calculator Basics. As with any tool, proper and effective use of a calculator requires instruction. Students must learn that calculators compute with programmed algorithms; they do not solve problems. The user must know which keys to press in what order and how to interpret the results. As with any technology or tool, it is important to check whether the answer obtained with the tool makes sense. Students might get a nonsensical calculator result for several reasons:

- They did not clear the calculator before starting a new operation.
- They entered a number or operation incorrectly.
- They analyzed the problem incorrectly and chose an incorrect algorithm or calculation.

Students must determine whether an answer is reasonable by asking whether it makes sense in terms of the original problem situation, just as they would after using paper and pencil or mental arithmetic. Making sense of results on a calculator is a fundamental skill necessary for effective use of calculators as a tool for problem solving. A four-function calculator is provided in the eToolkit.

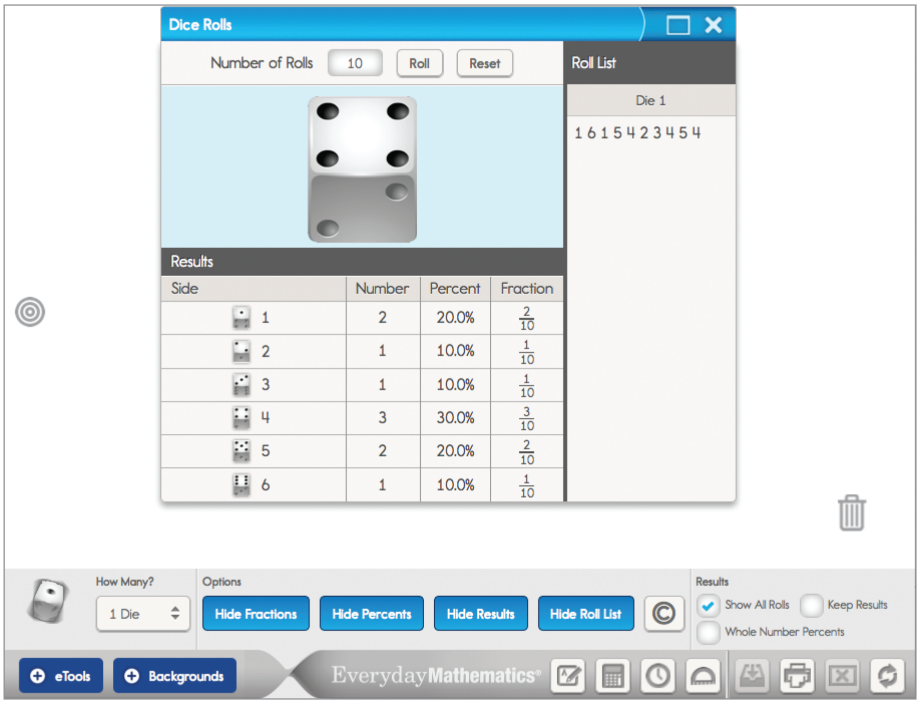
Random-Number Generators

GRADE LEVEL K 1 2 3 4 5 6

Everyday Mathematics uses a variety of devices to generate random numbers, primarily for use in games.

Dice. Various dice, such as 6-sided and polyhedral, are available in *Everyday Mathematics.* For example, use a regular 6-sided die to generate the numbers 1 through 6. Note that rolling more than one die and adding the resulting number of dots produces additional possible outcomes.

Everyday Mathematics eTools include digital dice for playing games and other activities and a Dice Roll tool that gathers large sets of data for analysis.



Dice rolls and data gathering in the eToolkit

Everything Math Deck. This deck of cards consists of four sets of number cards 0 through 10 and one set of number cards 11 through 20. You can limit the range of generated numbers simply by removing some of the cards from the deck. For a uniform distribution of the numbers 0 through 20, for example, use only one set of 0 through 10 cards and the set of 11 through 20 cards.

You can transform an ordinary deck of 54 playing cards to function as the whole-number side of an Everything Math Deck as follows:

- Change the four queens to 0s.
- Remove the four jacks, four kings, and two jokers. Label each of these ten cards with one of the numbers from 11 through 20.
- Change the four aces to 1s.
- Let all number cards represent their face value.

The Everything Math Deck and subsets of the deck are also available in the eToolkit. The deck automatically shuffles.

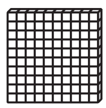
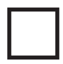
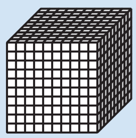
Other Card Decks. Numerous card decks exist in the print materials and online. These include fraction cards, shape cards, animal cards, elapsed time cards, and so on.

Base-10 Blocks

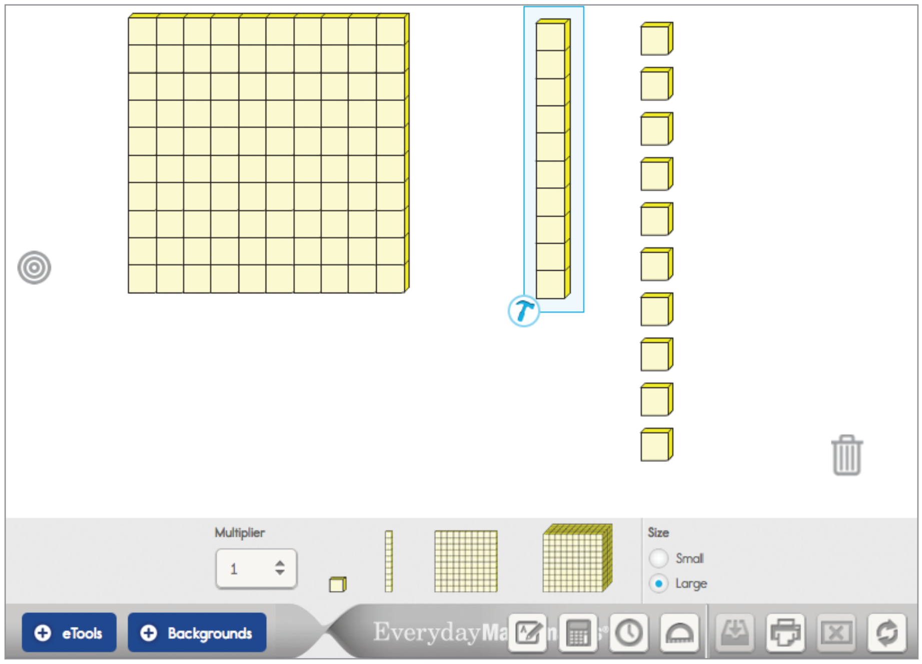
GRADE LEVEL K 1 2 3 4 5 6

In *Everyday Mathematics*, students use base-10 blocks to explore place value and operations. Although these blocks have a variety of names, it helps to have a common vocabulary for discussions and written work with the blocks. *Everyday Mathematics* uses the following names: *cube* for the 1-cm cube, *long* for the block consisting of ten 1-cm cubes (1 × 10), *flat* for the block consisting of one hundred 1-cm cubes (10 × 10), and *big cube* for the cube consisting of one thousand 1-cm cubes (10 × 10 × 10). Teachers may want students to make a written record of work with base-10 blocks. The shorthand shown below is handy for drawing quick pictures of base-10 blocks and can be used for explaining and representing solutions.

Base-10-Block Shorthand

Name	Block	Shorthand
cube		
long		
flat		
big cube		

As a supplement to physical base-10 blocks, *Everyday Mathematics* provides a digital version of the tool. The base-10 block eTool allows longs to break apart into ten cubes (as well as flats to ten longs, and so on). Groups of 10 or fewer blocks can be selected and the eTool combines them. The eTool prevents students from connecting more than 10 cubes into a single long or ten longs into a single flat. However, since the eTool allows combinations of fewer than 10 cubes, teachers should watch for students attempting to trade fewer than 10 cubes for a long, and so on. Note that because students can use the base-10 blocks eTool to show an answer on pages in the Student Learning Center, base-10 shorthand notation may not be necessary in those activities.



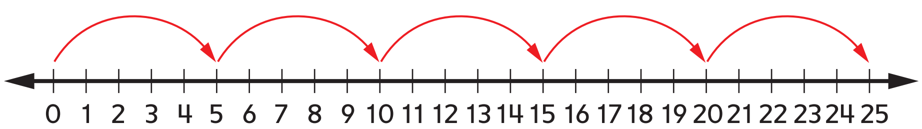
eTool base-10 blocks

Number Lines

GRADE LEVEL K 1 2 3 4 5 6

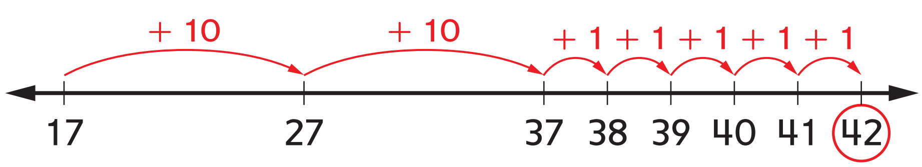
A **number line** is a line on which points are indicated by *tick marks*, usually at regularly spaced intervals from a point called the *origin*, the *zero point*, or simply 0. Numbers are associated with the tick marks, and the interval from 0 to 1 on the line is sometimes called the *unit interval*. Like any line, a number line extends without end in both directions. Any drawing of a number line is a representation of just part of the line.

Students use number lines for a variety of purposes in *Everyday Mathematics*. In the early grades, students use number lines with whole-number intervals when counting and skip counting, adding and subtracting, and making the connection between addition and subtraction. In Grade 1, students use number lines that are marked with tick marks at regular intervals.



Using a number line to skip count

In Grade 2, students begin using open number lines to add and subtract larger numbers as shown below.



Using an open number line to add 17 + 25

Beginning in Grade 3, students use number lines to represent fractions and open number lines to help calculate elapsed time. In Grade 4, students use number lines to represent decimals. This work is further developed in Grade 5, when students use number lines to create the axes for coordinate graphing systems.

The number line eTool allows students and teachers to create whole number, fraction, and decimal number lines. Teachers can automatically skip count and scroll. Students and teachers can add points, as well as segments. Teachers can add and hide labels and tick marks. Students and teachers can also use the line feature in the digital writing tools to easily display simple number lines.

Number Grids

GRADE LEVEL K 1 2 3 4 5 6

A *number grid* consists of rows of boxes, usually ten per row, with consecutive integers (positive and negative whole numbers). Number grids can be considered number lines that can fit nicely on a page or poster. However, number grids have other important features that serve other purposes, including pattern recognition and place value. As students move left and right on the number grid, they notice the numbers decrease or increase by 1. As students move up and down a column on the number grid, they notice that the numbers decrease or increase by 10. These patterns help students connect counting up and counting back with addition and subtraction.

–9	–8	–7	–6	–5	–4	–3	–2	–1	0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

A number grid

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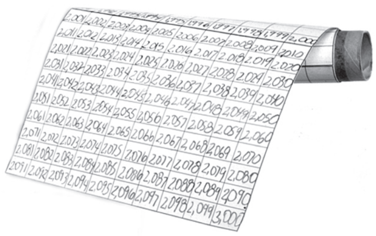
The *Everyday Mathematics* student digital number grid displays numbers −9 to 120. The teacher's digital number grid has all of the features of the student grid. Teachers can also show skip counts dynamically, highlight even and odd numbers, hide numbers, and create any contiguous subset of numbers (for example, from 23 to 47).



A number scroll

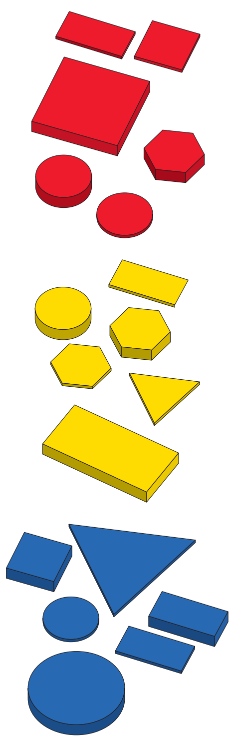
Number Scrolls

GRADE LEVEL K 1 2 3 4 5 6

Number scrolls are extended number grids used for number writing practice and place value and operations work in Kindergarten and Grade 1. Teachers can make number scrolls by adding single sheets of 100 numbers to existing sheets, either forward (adding positive integers) or backward (adding negative integers).

8.2.2 Geometry Tools

Many objects can be used to explore geometric figures, ranging from objects found around the house, such as shoe boxes or marbles, to commercially produced sets of shapes. The following are four readily available and relatively easily managed tools for studying geometric properties of objects.



Attribute blocks

Attribute Blocks

GRADE LEVEL K 1 2 3 4 5 6

Attribute blocks vary in color, size, shape, and thickness, so they are ideal manipulatives for sorting activities. In Kindergarten, students classify and sort attribute blocks and count and compare the number of blocks in each group. In first grade, students use attribute blocks for sorting and discussing defining and non-defining attributes of shapes.

Digital attribute blocks are available as an eTool that includes the same shapes, colors, and sizes.

Pattern Blocks and Geometric Solids

GRADE LEVEL K 1 2 3 4 5 6

Pattern blocks help students learn the names and attributes of geometric objects. In Kindergarten and first grade, students identify categories of shapes and colors of pattern blocks. Beginning in first grade, students are encouraged to find different ways of sorting blocks on their own, that is, to create multiple perspectives of a given set of blocks and to discuss defining and nondefining attributes. Pattern block eTools can cover each other. They come in two distinct sizes.

In the second half of Kindergarten, students use what they have learned about 2-dimensional shapes to begin exploring *geometric solids*. This work is continued in Grade 1 as students use 3-dimensional solids to determine defining and nondefining attributes and compose new shapes from other shapes. Grade 3 students use 3-dimensional solids to identify attributes of faces.

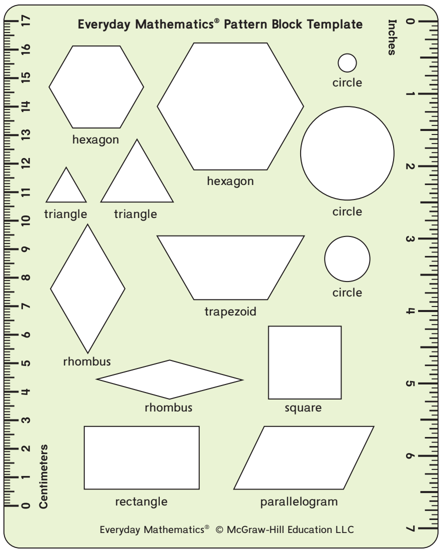
The 2-D Shape Library allows students and teachers to rotate, translate, and change the size of various figures. This library also provides teachers with examples of non-prototypical 2-dimensional shapes, which help students understand and clarify their understanding of shape categories, including defining and nondefining attributes of shapes.

Once students have had ample experience with 3-dimensional shapes in the real world, the 3-D Shape Library is a useful digital tool that allows students and teachers to rotate and change shape sizes. Users can also manipulate the size of shapes' bases, radii, and heights.

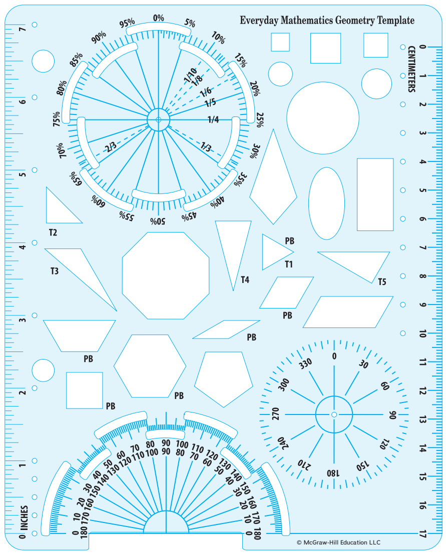
Pattern-Block Templates and Geometry Templates

GRADE LEVEL K 1 2 3 4 5 6

Students can use *Pattern-Block Templates* to explore plane figures. Teachers should encourage students to use the templates to draw shapes and to create composite shapes and designs.



Pattern-Block Template



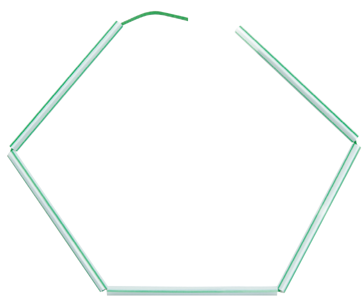
Geometry Template

Beginning in fourth grade, students use *Geometry Templates,* which have a greater variety and number of shapes than the Pattern-Block Templates. Students use the templates in their more detailed explorations of triangles and quadrangles. The measuring devices include inch and centimeter scales, a Percent Circle useful for making circle graphs, and both a full-circle and half-circle protractor.

Straws and Twist-Ties

GRADE LEVEL K 1 2 3 4 5 6

In Grades 1-4, students construct 2- and 3-dimensional objects using *straws and twist-ties.* Most students have little trouble constructing polygons with straws and connectors. The ends of the ties may need to be pinched a little to slide into the straws. If students use large-diameter straws, it may be necessary to fold back an inch or so of the end of the connector for a tighter joint. Students should use shorter straws when building polygons with more than five sides. Have students make polygons on flat surfaces and encourage them to try to keep the polygons flat when picking them up. For 3-dimensional figures, students can begin by putting two connectors, or one folded connector, into one end of a straw to connect it to two other straws.



Making a polygon with straws and twist ties

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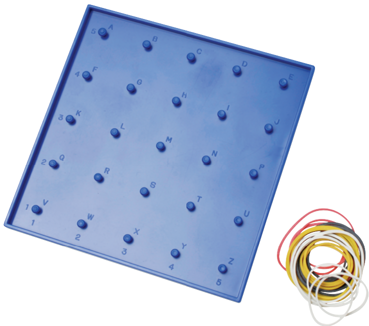


When more than three straws need to meet at a vertex, students may add additional connectors as needed, or first connect several pairs of straws and then make a bundle of one straw from each pair held together with an additional connector.

Geoboards

A *geoboard* is a small wooden or plastic board with nails or other posts, usually arranged at equally spaced intervals in a rectangular array. Students loop rubberbands around the posts to explore basic concepts in plane geometry.

8.2.3 Measurement Tools

A growing body of research shows the importance of early conceptual understanding of measurement, particularly of iterated measurement using same-size units placed end-to-end along a path with no gaps or overlaps. Students begin exploring these concepts in Kindergarten when they use non-standard units such as connecting cubes and stick-on notes to measure classroom objects. In first grade, students continue to measure with non-standard units in first grade, including paper clips, unsharpened pencils, and square pattern blocks. These rich experiences provide a foundation for understanding standard tools and units of measure in second grade and beyond.

Rulers and Tape Measures

GRADE LEVEL K 1 **2** **3** **4** **5** **6**

Rulers and tape measures are among the first tools used for practical everyday measurements. In the early grades, students learn to give ballpark estimates of heights and lengths. Over time, they get progressively more sophisticated, using measuring instruments to find more precise lengths. Teachers often find meter sticks and centimeter rulers instructive for exploring decimals.

If students use retractable tape measures, teachers should introduce and enforce the “2-inch, 5-centimeter no-zap rule.” That is, students should not “zap” the tape measure until no more than 2 inches or 5 centimeters show. This will extend the life of these tools, and make the classroom quieter.

A digital environment does not lend itself well to measuring with rulers and tape measures; the ease with which objects are resized conflicts with students’ developing understanding of standard units of measure. Therefore, teacher digital measurement tools exist for demonstration purposes only.

Pan Balances

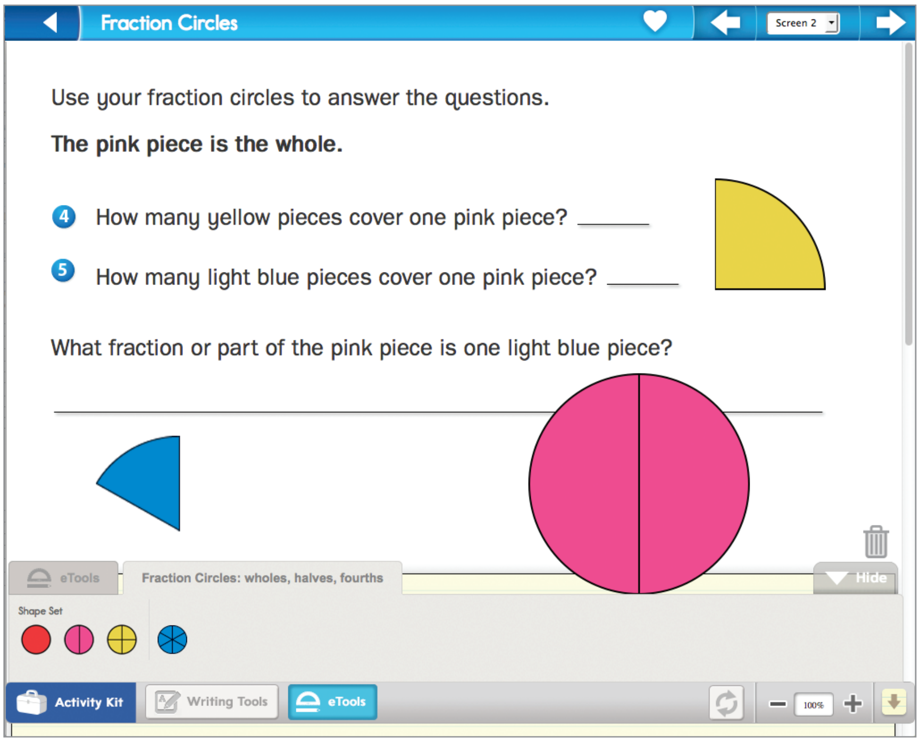
GRADE LEVEL **K** 1 2 **3** 4 5 **6**

A *pan balance* is used to weigh objects or to compare the weights of two objects. Students use pan balances in Kindergarten to compare the weights of classroom objects directly, or by using non-standard units of weight, such as paper clips, to quantify each object’s weight. In third grade, students use pan balances for measuring mass. In sixth grade, they are used for solving equations. A pan balance eTool can be used to solve problems involving standard masses.

8.2.4 Fraction Circles

Fraction circles are a new manipulative for *Everyday Mathematics 4*. Colored circles, cut into equal parts, represent unit fractions for one whole, halves, quarters, thirds, fifths, sixths, eighths, tenths, and twelfths. There are no fraction numerals printed on the fraction circle pieces, so that teachers can use the pieces to represent various wholes. While exploring the relative sizes of the fraction circle pieces, students learn that the name of a fractional part is linked with the size of the whole, that is, the same physical object can be both one-quarter of a circle and also one-half of a semicircle.

All of the fraction circles are available from the Activity Sheets in the journals, as downloads through the Teacher Center, or in the classroom manipulative kits. In addition, teachers can demonstrate with them through ePresentations in the eToolkit. Students also have complete access through their eToolkits in the Student Learning Center. Prior to introducing students to the fraction circle eTool, you may want to provide the physical fraction circle pieces so students can become familiar with the sizes, shapes, and colors of the pieces.



Fraction circles in the Student Learning Center

8.2.5 Toolkits

A *toolkit* gives students a sense of independence by giving them immediate access to basic tools. Every student in Kindergarten through Grade 6 can benefit by having a toolkit that includes a calculator, measuring tools, and any manipulatives used throughout the year. Tools may be stored in the *Everyday Mathematics* zippered bags or in some other container, such as a wooden box or a resealable plastic bag. You may want to use a permanent marker to write identification numbers on toolkits and on the non-consumable items in the toolkits.

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Early in the year, distribute toolkits with a few needed items. Hand out other items as they are first used in classroom activities. For example, eventually a Grade 3 toolkit may contain a tape measure, ruler, calculator, Pattern-Block Template, and clock face. Encourage students to take care of their toolkits and to store each tool after use so that it is there when needed.

8.3 The Digital Environment

The digital Teacher Center (TC) and Student Learning Center (SLC) allow teachers to implement *Everyday Mathematics* using available technology. These digital tools support a range of implementation options—from a fully digital delivery to a hybrid delivery. The fully digital classroom utilizes only the TC and SLC, while the hybrid classroom utilizes a mix of digital and print materials.

For complete information about the digital environment for *Everyday Mathematics 4,* see the Main Menu Help Center, or Professional Development.

In the digital environment, SLC screens take the place of paper journals and math masters, allowing students to respond in a variety of ways using keypad entry, writing tools, and eTools to show their thinking. Each SLC screen contains a yellow-lined scratchpad at the bottom as an extension of the student workspace. Students can flexibly favorite and copy screens, and use the "split screen" and "look back" features to track their work effectively. Immediate feedback such as "One of your answers is incorrect," or "Great job," is available on many student pages. Audio hints can help students solve challenging problems, and a Submit button sends students' work to the teacher. The grouping feature facilitates partner work, pushing work done on a single device to multiple student accounts.

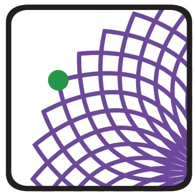
8.3.1 eTools

The digital environment provides a variety of eTools, including base-10 blocks, clocks, money, pattern blocks, card decks, dice, and so on. Having both physical and eTools available gives teachers and students a choice as to which mode of tool to use to complete a particular task. There are advantages to using both modes. For example, the base-10 blocks eTool may be more efficient for some tasks than the physical blocks. On the other hand, using a print number grid to calculate may be more effective than using the Number Grid eTool. It is important that teachers become familiar with both the physical manipulatives and their associated eTools so that they can make appropriate choices for student use of tools.

A link to the eToolkit is available on every home page and lesson screen of the Student Learning Center. Specific eTools that may be helpful for completing a problem are made available under the eTools tab on the students' screens. The positions of the etools on the screen are preserved so students can use them to show their work.

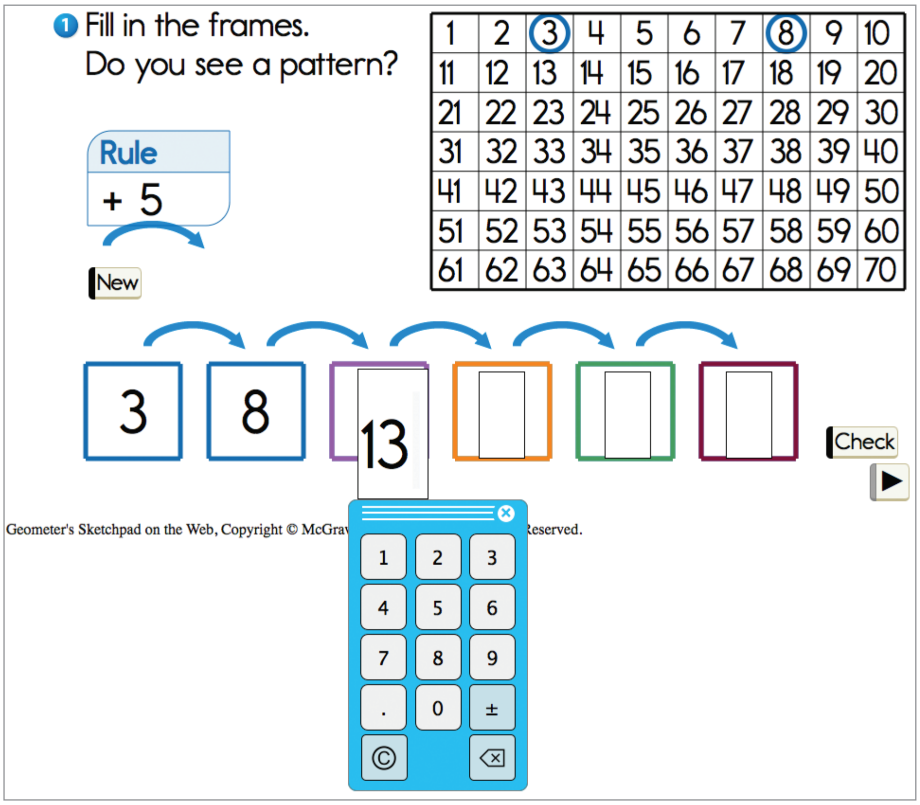
8.3.2 Geometer's Sketchpad

In the *Student Reference Book*, Geometer's Sketchpad activities are used to reinforce definitions and illustrate relationships between representations. Students can also complete interactive practice problems, modeled after worked examples, with the ability to check their work and receive feedback. The symbol shown in the margin guides students and teachers to the Geometer's Sketchpad activities.



Key Sketches

In the Student Learning Center, Sketchpad activities are available to take advantage of the dynamic digital environment. These customized activities are often significantly different from their print versions -- with dynamic feedback to encourage deep explorations and discussions.



Keypad answer entry for Geometer's Sketchpad activity in *My Reference Book*

The SLC also contains Sketchpad activities that allow students to create and explore mathematical models by constructing and composing pre-built Sketchpad objects. Students can test geometric properties by building and manipulating geometric figures -- using digital alternatives for straws and twist-ties, as well as geoboards and rubberbands. These unique digital tools also provide students with a space for unscripted exploration of many models and concepts, including fractions, arrays, functions, and area.

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