10 Differentiating Instruction with *Everyday Mathematics*

Contents

> "Differentiation is a philosophy that enables teachers to plan strategically in order to reach the needs of the diverse learners in classrooms today." (Gregory, 2003)

This section provides background, ideas, and strategies for differentiating instruction with *Everyday Mathematics.* It highlights differentiation that is integrated into the program and also points to features that can be readily adapted to meet the needs of students. This section will help teachers support the needs of exceptional learners—learners who need support in developing concepts, learners who are ready to extend or expand their understanding of mathematical concepts and skills, and learners who speak a language other than English at home and are in the process of learning English.

Creating a classroom where rigorous mathematics content is accessible and engaging to all students is among a teacher's most challenging tasks. An important objective of this edition of *Everyday Mathematics* is to make differentiation achievable for all teachers who use the program. To that end, the authors examined every new and revised lesson through the lens of the diverse learning needs of students in a typical classroom. Each lesson includes considerable guidance along with a range of tools to assist teachers with differentiation.

A differentiated classroom is a rich learning environment that provides students with multiple avenues for acquiring content, making sense of ideas, developing skills, and demonstrating what they know. In this sense, differentiated instruction is synonymous with good teaching. The fundamental design of *Everyday Mathematics* creates many natural opportunities for such teaching. The *Everyday Mathematics* program:

- begins with an appreciation of the mathematical sensibilities that students bring with them to the classroom and connects to students' prior interests and experiences;
- offers multiple opportunities for teachers to assess and monitor students' progress over time, and to analyze their mathematical strengths and misconceptions;
- incorporates predictable routines that help engage students in mathematics and regular practice in a variety of contexts;
- provides many opportunities throughout the year for students to acquire, process, and express mathematical concepts in concrete, pictorial, and symbolic ways;
- extends student thinking about mathematical ideas through questioning and discussion with peers that leads to deepened understandings of concepts;
- incorporates and validates a variety of learning strategies;
- emphasizes the process of problem solving as well as finding solutions;
- provides suggestions for enhancing or supporting students' learning in each lesson;
- encourages collaborative and cooperative groupings in addition to individual and whole-class work;
- facilitates the development and use of mathematical language and promotes academic discourse;
- provides teachers with information about the learning trajectories or paths to achieving learning goals;
- suggests how students can demonstrate what they know in multiple ways;
- encourages students to reflect on their own strengths and weaknesses.

For more information, see Section 2 *Guiding Principles for the Design and Development of* Everyday Mathematics.

Providing good differentiated instruction, however, goes beyond "good teaching". Differentiated instruction requires careful analysis and planning with target populations in mind so that those populations will be more likely to benefit from good teaching. The following information will help you use *Everyday Mathematics* to meet the needs of all learners.

10.1 Differentiating Instruction in *Everyday Mathematics*: For Whom?

Good teaching requires some degree of differentiation for every student, every day. However, most discussions about differentiation focus on groups of students who typically require more differentiation than normally expected. *Everyday Mathematics* lessons offer specific differentiation advice for four groups of learners.

- Students[1] Who Need More Scaffolding
- Advanced Learners
- Beginning English Language Learners
- Intermediate and Advanced English Language Learners

Each of these groups is briefly described in the following sections to help teachers understand the intended audience for different differentiation strategies. Note, however, that each of the four categories named above include highly heterogeneous groupings of students with mixed abilities and needs, creating additional challenges for differentiating instruction.

Although differentiation strategies are targeted for these four groups, teachers will find that many of the suggested strategies will benefit all students. Thus a suggestion for Students Who Need More Scaffolding is likely a good strategy to use with the whole class. Similarly, most of the language-related strategies suggested for Intermediate and Advanced English Language Learners will have broad applicability, especially for English-dominant students who have challenges involving language. Reviewing the differentiation suggestions for each lesson will provide useful tools for teaching all students.

For information on support in the teacher materials for these groups of students, see Section 10.2 *Overview of Differentiation Features in* Everyday Mathematics.

10.1.1 Students Who Need More Scaffolding

Students in this group include those who are struggling generally with mathematics as well as those who may struggle with specific concepts at different points in the curriculum. Instructional "scaffolding" refers to the supports needed to assist students in developing their understanding of particular concepts and skills. All students need some degree of scaffolding that builds upon prior understandings of a concept. Sometimes the scaffolds target the mathematical content, such as suggesting the use of fraction circle pieces to help students develop fraction concepts. Sometimes the scaffolds address habits of mind, such as using graphic organizers to help students organize their thinking and see connections between related concepts.

This is a fluid group. Some students will routinely need additional scaffolding and others will need support less frequently, often for specific concepts and skills. Different cognitive competencies are required for acquiring skills such as counting; recalling basic facts quickly; or developing effective arithmetic procedures with whole numbers, fractions, and decimals than for learning mathematics that requires visual or spatial understandings, such as geometry or data representation; making connections among concepts; or solving complex problems. Students may struggle in one domain but function well in

[1] In Grades K–3, this group is referred to as Children Who Need More Scaffolding.

another. As such, determining the scaffolding needed for individual students often requires teacher judgment.

Some students who need extra scaffolding likely have been identified for special education services. With regard to special education disability classifications, *Everyday Mathematics*' focus is on "high-incidence disabilities," such as speech and language impairments, specific learning disabilities, emotional disturbances, mild intellectual disabilities, and other health impairments, such as attention deficit disorder.

Researchers caution, however, that diagnosing these disabilities is complex and that many students are misdiagnosed. They suggest that school efforts are better spent strengthening the instructional repertoires of teachers and their knowledge of good assessment practices. Teachers can then develop the appropriate tools for addressing individual student needs. *Everyday Mathematics* has adopted this approach, rather than attempting to provide suggestions for addressing specific disabilities. That said, in developing a student's Individualized Educational Plan, school staff and families must always make decisions about instructional scaffolds based upon the specific needs of the individual student. Understanding the many tools in the curriculum that support student understanding can assist with the process of making those determinations.

The supports suggested in *Everyday Mathematics* lessons may also be useful for many students who have "low-incidence disabilities," such as more severe cognitive impairments, severe autism, or physical disabilities. However, such students often require additional accommodations or modifications that are beyond the scope of this discussion.

For more information on differentiation strategies to support this group of students, see Section 10.3 *Differentiation Strategies in* Everyday Mathematics.

Increasingly, most students with identified disabilities receive their mathematics instruction in general education (inclusion) settings. Depending upon the school and the specific disabilities of the students involved, a special education co-teacher or a teacher's aide may be present in the general education classroom or there may be no additional support for classroom teachers. Differentiation supports in *Everyday Mathematics* can work in any of these environments, as well as in more restrictive settings.

10.1.2 Advanced Learners

For more information on tools and features that address this group of students, see Section 10.4 *Working with Advanced Learners.*

All students need an appropriate academic challenge, and every student should be challenged in mathematics class daily. Advanced learners—sometimes called "gifted" or "talented" students—routinely grasp concepts quickly, finish assignments early, and often can benefit from a "deeper dive" into the mathematics of a lesson. These students need more "stretch" or extensions in the work they are doing so that they develop the habit of persisting in problem solving and learn the value of working to understand mathematical concepts.

10.1.3 English Language Learners

Teachers in every area of the United States now find students in their classrooms who speak a language other than English at home and are in the process of developing English language proficiency. These teachers, many of whom have little or no training or experience teaching English as a second

language, are expected to make cognitively challenging content accessible to students who are developing their English language proficiency.

Frameworks developed by professional organizations have identified five English language proficiency levels: Entering (Level 1), Emerging (Level 2), Developing (Level 3), Expanding (Level 4), and Bridging (Level 5).[2] Differentiating mathematics lessons for students who are at various levels on this continuum of language development means that teachers have to use different language accessibility strategies for different groups of English language learners. Effective differentiation strategies address four language domains—listening, speaking, reading, and writing—as well as vocabulary usage, language forms and conventions, and discourse complexity.

Beginning English Language Learners

Everyday Mathematics addresses the needs of three groups of English language learners based upon their English language proficiency: beginning, intermediate, and advanced. Beginning English language learners fall into the Entering (Level 1) and Emerging (Level 2) proficiencies. This group is typically within the first year of learning English; students' basic communication skills with everyday language are in their early development. These students require the most intensive language-related accommodations in order to access the mathematics in most lessons.

Intermediate and Advanced English Language Learners

Intermediate and Advanced English language learners represent Levels 3, 4, and 5 (Developing, Expanding, and Bridging) in the English language proficiencies identified above. Students in this category are typically in their second to fourth year of learning English. They may be proficient with basic communication skills in English and able to carry on everyday conversations, but they are still developing proficiencies with the more cognitively demanding academic language of the mathematics class. Their still-emerging English language proficiency with listening, speaking, reading, and writing will impact the quality of their communication in class. However, because these students have reasonable English language skills with basic, everyday communication, teachers often misinterpret their language difficulties as problems with their mathematical understanding.

For information on support for this group of students, see Section 10.5 *Working with English Language Learners in* Everyday Mathematics.

[2] The language proficiencies are outlined in the *PreK-12 English Language Proficiency Standards,* developed by Teachers of English to Speakers of Other Languages (TESOL). (http://www.tesol.org/docs/books/bk_prek-12elpstandards_framework_318.pdf) They are based upon the *English Language Development Standards* developed by the World-Class Instructional Design and Assessments (WIDA) consortium, a consortium comprised of representatives of nine states. (http://www.wida.us/standards/eld.aspx)

10.2 Overview of Differentiation Features in *Everyday Mathematics*

This section provides an overview of support for differentiation that is included at point of use in the lessons and online in the Differentiation Support pages.

10.2.1 Differentiation Features in Lessons

Everyday Mathematics lessons are designed to accommodate students with a wide range of academic abilities and experience levels. This overview highlights some of the strategies and opportunities for differentiating instruction that are incorporated into the lessons.

Common Core State Standards Focus Clusters help teachers identify and focus on the most important mathematics in each lesson.

Spiral Snapshot provides a quick way to see how a particular content goal develops over the course of the year, helping teachers understand when to expect fluency with a concept or skill. It makes the learning trajectory for a concept more visible to teachers and helps them determine when differentiation approaches are needed. A more complete **Spiral Tracker** is available online.

Each lesson includes multiple opportunities for engaging **practice** of skills and concepts.

Lesson 6-8

Partial-Sums Addition, Part 2

Overview Children are introduced to partial-sums addition.

Vocabulary
partial-sums addition

Common Core State Standards

Focus Clusters
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

1 Warm Up 15–20 min

	Materials	
Mental Math and Fluency Children compare 3-digit numbers.	slate	2.NBT.4
Daily Routines Children complete daily routines.	See pages 4–43.	See pages xiv–xvii.

2 Focus 30–40 min

Math Message Children represent 2-digit numbers with base-10 blocks and expanded form.	base-10 blocks	2.NBT.1, 2.NBT.3 SMP2
Using Expanded Form to Find Partial Sums Children use expanded form to find partial sums and are introduced to partial-sums addition.	base-10 blocks	2.NBT.3, 2.NBT.5, 2.NBT.7, 2.NBT.9 SMP1, SMP2, SMP6
Estimating and Adding with Partial Sums Children practice using partial-sums addition and use ballpark estimates to check that their answers make sense.	*Math Journal 2*, p. 158; base-10 blocks (optional)	2.NBT.3, 2.NBT.5, 2.NBT.7, 2.NBT.9 SMP1
Assessment Check-In See page 579.	*Math Journal 2*, p. 158	2.NBT.5, 2.NBT.7

Spiral Snapshot 2.NBT.7
GMC Add multidigit numbers using models or strategies.

5-10 Focus Practice | 6-1 Practice | 6-2 through 6-7 Warm Up Focus Practice | 6-8 Focus Practice | 7-1 Warm Up Practice | 7-3 Warm Up | 7-9 Practice

Spiral Tracker — Go Online to see how mastery develops for all standards within the grade.

3 Practice 15–20 min

Playing *Salute!* Game Children find missing addends.	*My Reference Book*, pp. 162–163; per group: 4 each of number cards 0–10	2.OA.2 SMP6
Math Boxes 6-8 Children practice and maintain skills.	*Math Journal 2*, p. 159	See page 579.
Home Link 6-8 Homework Children practice partial-sums addition.	*Math Masters*, p. 176	2.NBT.5, 2.NBT.7

connectED.mheducation.com
Plan your lessons online with these tools.

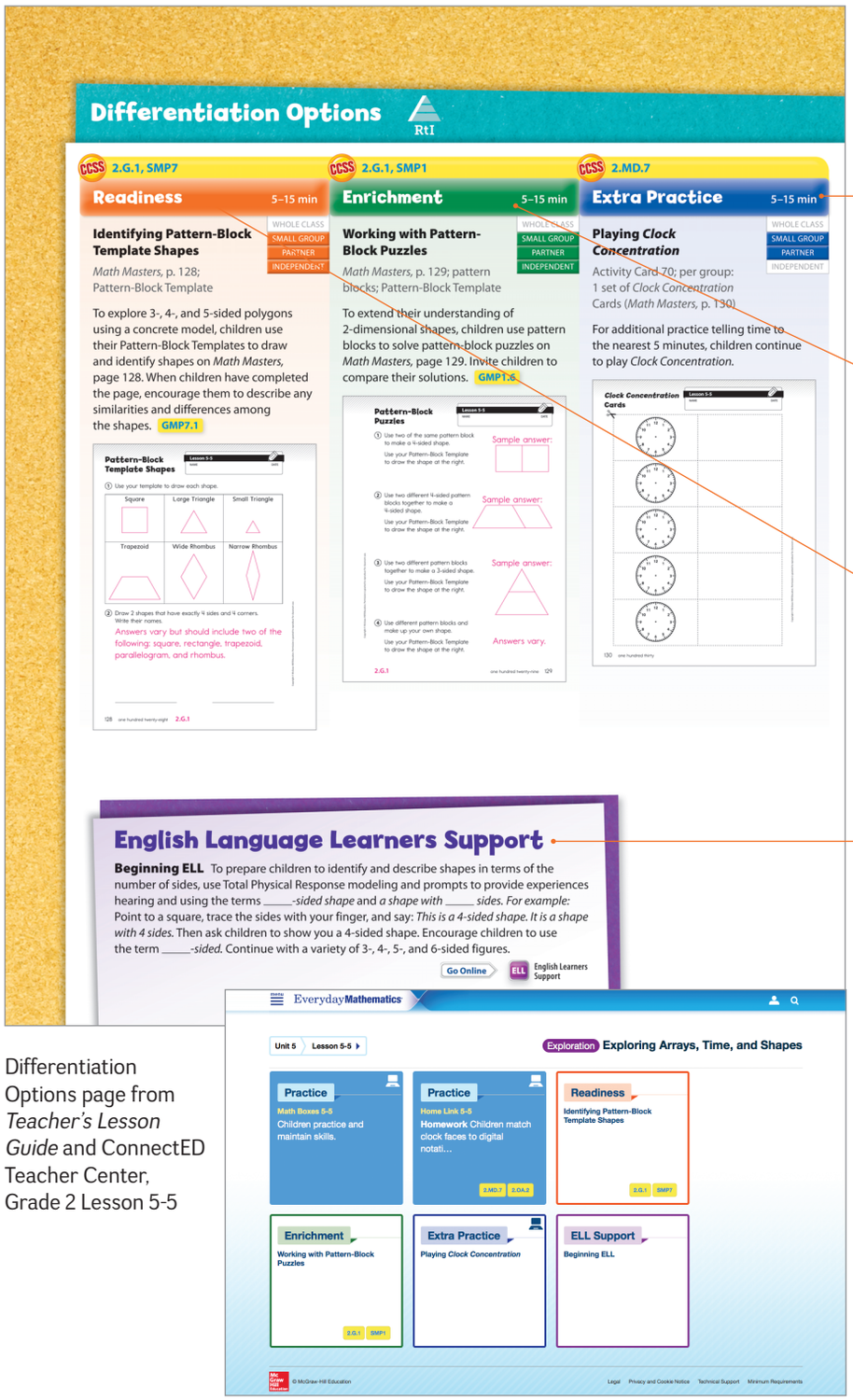
ePresentations | Student Learning Center | Facts Workshop Game | eToolkit | Professional Development | Home Connections | Spiral Tracker | Assessment and Reporting | English Learners Support | Differentiation Support

574 Unit 6 | Whole Number Operations and Number Stories

Lesson Opener from *Teacher's Lesson Guide*, Grade 2 Lesson 6-8

Differentiation Options are included in all regular lessons. They include optional Readiness, Enrichment, Extra Practice, and English Language Learners Support activities that can be used with individual students, small groups, or the whole class. The activities build on the standards highlighted in each lesson opener. In Grades 1–6 extended, lesson-specific differentiation tools and strategies are available in online **Differentiation Support** pages.

For more information, see Section 10.2.2 *Online Differentiation Support in Grades 1–6.*



Differentiation Options page from *Teacher's Lesson Guide* and ConnectED Teacher Center, Grade 2 Lesson 5-5

Extra Practice Activities provide students with additional opportunities to practice mathematical content embedded in the lesson.

Enrichment Activities apply or deepen students' understanding of lesson content.

Readiness Activities preview lesson content or provide students with alternative strategies for learning concepts and skills.

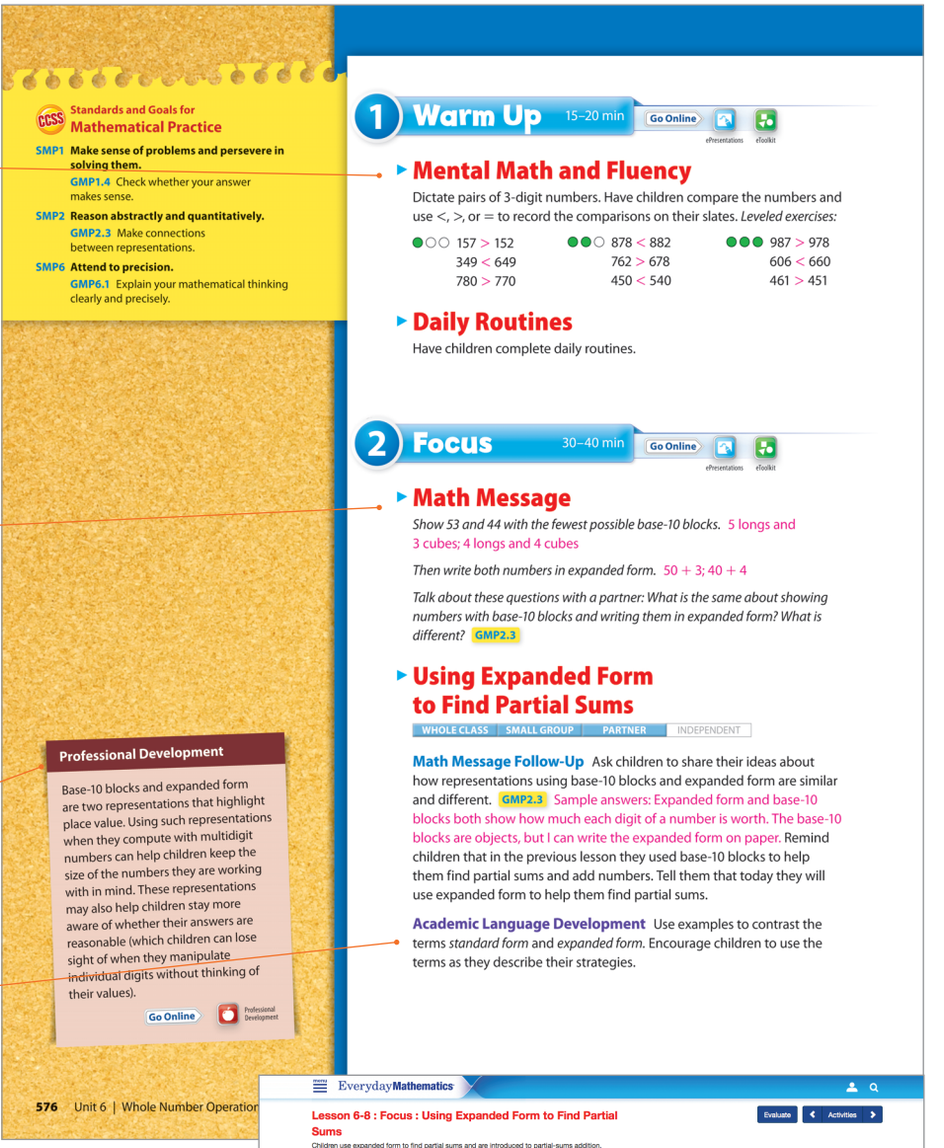
English Language Learners Support activities help **beginning English language learners** acquire or practice vocabulary or other language skills used in the lesson.

Mental Math and Fluency problems begin lessons in Grades 1–6. They range in difficulty, beginning with easier exercises and progressing to more difficult ones. Levels are designated by the symbols ●○○ ●●○ ●●●. Many of these activities are presented in a "slate" format that engages all students in answering questions and allows teachers to quickly assess students' understanding.

Math Message activates and builds on students' prior knowledge and creates a context for the material to be learned. Follow-up discussion provides an opportunity to assess and anticipate students' readiness for the lesson.

Professional Development notes provide content background that help teachers focus their instruction on important mathematical ideas.

Academic Language Development notes use etymology, graphic organizers, and sentence frames, among other methods, to suggest ways to encourage academic language use by all students.



Standards and Goals for Mathematical Practice

SMP1 Make sense of problems and persevere in solving them.
GMP1.4 Check whether your answer makes sense.
SMP2 Reason abstractly and quantitatively.
GMP2.3 Make connections between representations.
SMP6 Attend to precision.
GMP6.1 Explain your mathematical thinking clearly and precisely.

1 Warm Up 15–20 min Go Online ePresentations eToolkit

Mental Math and Fluency

Dictate pairs of 3-digit numbers. Have children compare the numbers and use <, >, or = to record the comparisons on their slates. *Leveled exercises:*

●○○ 157 > 152; 349 < 649; 780 > 770
●●○ 878 < 882; 762 > 678; 450 < 540
●●● 987 > 978; 606 < 660; 461 > 451

Daily Routines

Have children complete daily routines.

2 Focus 30–40 min Go Online ePresentations eToolkit

Math Message

Show 53 and 44 with the fewest possible base-10 blocks. 5 longs and 3 cubes; 4 longs and 4 cubes

Then write both numbers in expanded form. 50 + 3; 40 + 4

Talk about these questions with a partner: What is the same about showing numbers with base-10 blocks and writing them in expanded form? What is different? GMP2.3

Using Expanded Form to Find Partial Sums

WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

Math Message Follow-Up Ask children to share their ideas about how representations using base-10 blocks and expanded form are similar and different. GMP2.3 Sample answers: Expanded form and base-10 blocks both show how much each digit of a number is worth. The base-10 blocks are objects, but I can write the expanded form on paper. Remind children that in the previous lesson they used base-10 blocks to help them find partial sums and add numbers. Tell them that today they will use expanded form to help them find partial sums.

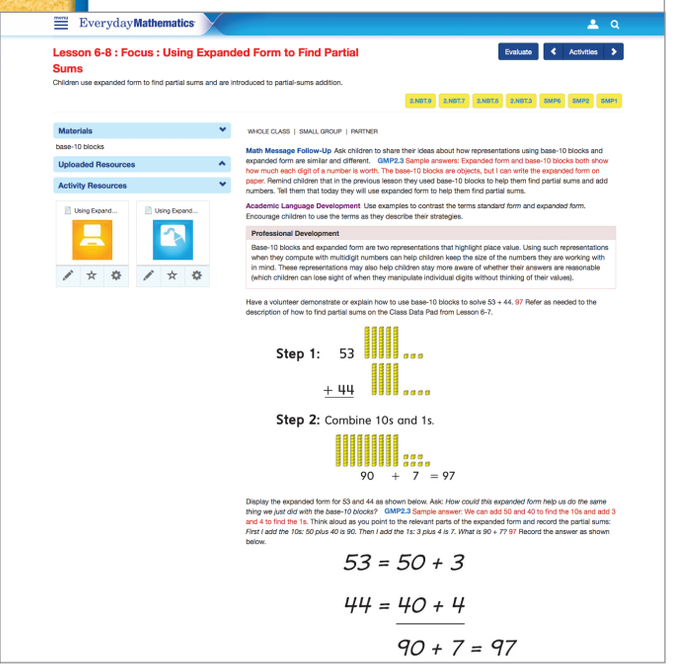
Academic Language Development Use examples to contrast the terms *standard form* and *expanded form*. Encourage children to use the terms as they describe their strategies.

Professional Development

Base-10 blocks and expanded form are two representations that highlight place value. Using such representations when they compute with multidigit numbers can help children keep the size of the numbers they are working with in mind. These representations may also help children stay more aware of whether their answers are reasonable (which children can lose sight of when they manipulate individual digits without thinking of their values).

Go Online Professional Development

576 Unit 6 | Whole Number Operation



EverydayMathematics

Lesson 6-8 : Focus : Using Expanded Form to Find Partial Sums

Children use expanded form to find partial sums and are introduced to partial-sums addition.

Evaluate Activities

2.NBT.9 2.NBT.7 2.NBT.5 2.NBT.3 SMP6 SMP2 SMP1

Materials: base-10 blocks
Uploaded Resources
Activity Resources

WHOLE CLASS | SMALL GROUP | PARTNER

Math Message Follow-Up Ask children to share their ideas about how representations using base-10 blocks and expanded form are similar and different. GMP2.3 Sample answers: Expanded form and base-10 blocks both show how much each digit of a number is worth. The base-10 blocks are objects, but I can write the expanded form on paper. Remind children that in the previous lesson they used base-10 blocks to help them find partial sums and add numbers. Tell them that today they will use expanded form to help them find partial sums.

Academic Language Development Use examples to contrast the terms *standard form* and *expanded form*. Encourage children to use the terms as they describe their strategies.

Professional Development

Base-10 blocks and expanded form are two representations that highlight place value. Using such representations when they compute with multidigit numbers can help children keep the size of the numbers they are working with in mind. These representations may also help children stay more aware of whether their answers are reasonable (which children can lose sight of when they manipulate individual digits without thinking of their values).

Have a volunteer demonstrate or explain how to use base-10 blocks to solve 53 + 44. 97 Refer as needed to the description of how to find partial sums on the Class Data Pad from Lesson 6-7.

Step 1: 53 + 44

Step 2: Combine 10s and 1s.

90 + 7 = 97

Display the expanded form for 53 and 44 as shown below. Ask: *How could this expanded form help us do the same thing we just did with the base-10 blocks?* GMP2.3 Sample answer: We can add 50 and 40 to find the 10s and add 3 and 4 to find the 1s. Think aloud as you point to the relevant parts of the expanded form and record the partial sums: *First I add the 10s: 50 plus 40 is 90. Then I add the 1s: 3 plus 4 is 7. What is 90 + 7? 97* Record the answer as shown below.

53 = 50 + 3
44 = 40 + 4
90 + 7 = 97

Page from *Teacher's Lesson Guide* and ConnectED Teacher Center, Grade 2 Lesson 6-8

Have a volunteer demonstrate or explain how to use base-10 blocks to solve 53 + 44. 97 Refer as needed to the description of how to find partial sums on the Class Data Pad from Lesson 6-7. (*See margin.*)

Display the expanded form for 53 and 44 as shown below. Ask: *How could this expanded form help us do the same thing we just did with the base-10 blocks?* GMP2.3 Sample answer: We can add 50 and 40 to find the 10s and add 3 and 4 to find the 1s. Think aloud as you point to the relevant parts of the expanded form and record the partial sums: *First I add the 10s: 50 plus 40 is 90. Then I add the 1s: 3 plus 4 is 7. What is 90 + 7?* 97 Record the answer as shown below.

$$53 = 50 + 3$$
$$44 = 40 + 4$$
$$90 + 7 = 97$$

Pose another 2-digit addition problem, such as 36 + 75. 111 Write the problem vertically. Ask children to tell you how to write each number in expanded form, and record the expanded form off to the side. You may want to use a second color. (*See margin.*) Challenge children to work with a partner to use expanded form to help them solve the problem without base-10 blocks. Encourage them to think about adding the 10s, adding the 1s, and then adding the partial sums.

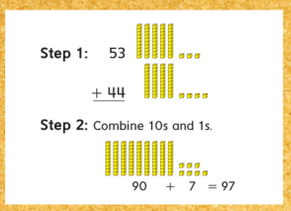
Differentiate **Adjusting the Activity**

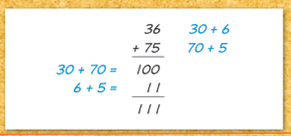
If children struggle writing the expanded form for each number, have them model the numbers with base-10 blocks or sketch base-10 shorthand and record their work on paper.

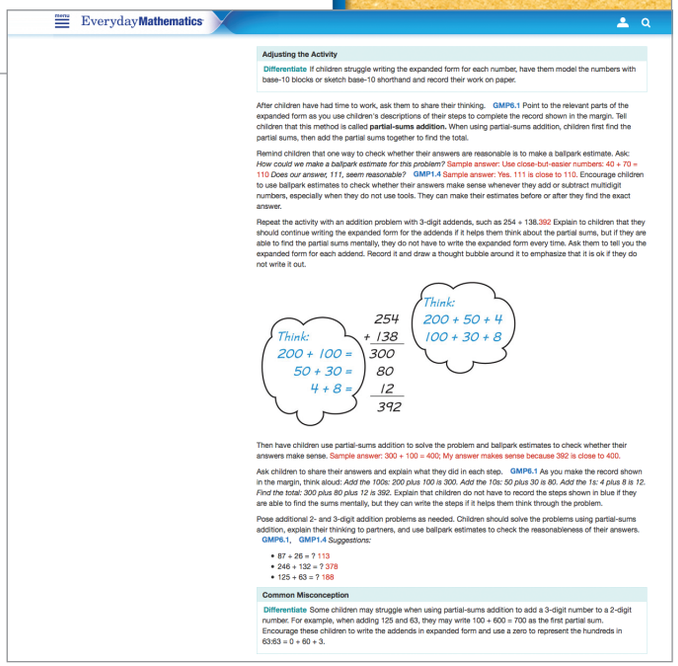
Go Online | Differentiation Support

After children have had time to work, ask them to share their thinking. GMP6.1 Point to the relevant parts of the expanded form as you use children's descriptions of their steps to complete the record shown in the margin. Tell children that this method is called **partial-sums addition.** When using partial-sums addition, children first find the partial sums, then add the partial sums together to find the total.

Remind children that one way to check whether their answers are reasonable is to make a ballpark estimate. Ask: *How could we make a ballpark estimate for this problem?* Sample answer: Use close-but-easier numbers: 40 + 70 = 110 *Does our answer, 111, seem reasonable?* GMP1.4 Sample answer: Yes. 111 is close to 110. Encourage children to use ballpark estimates to check whether their answers make sense whenever they add or subtract multidigit numbers, especially when they do not use tools. They can make their estimates before or after they find the exact answer.



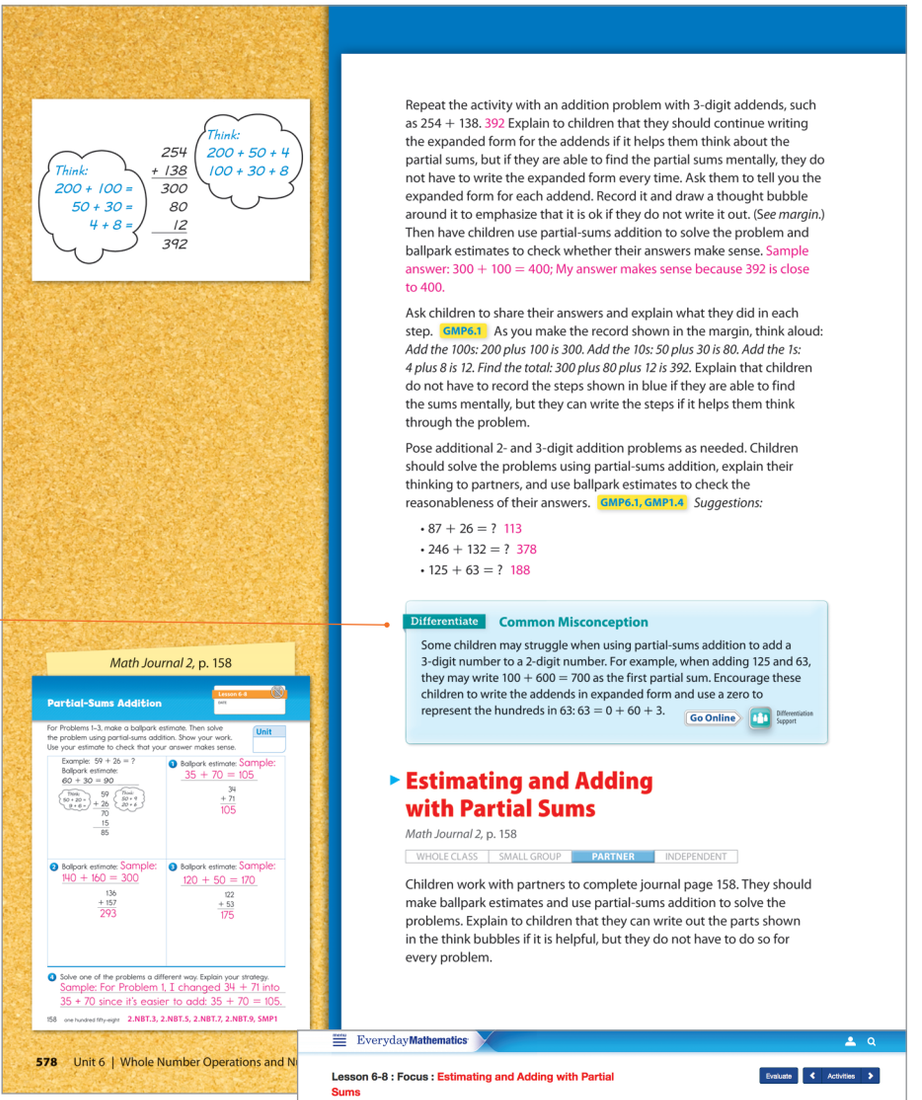




Page from *Teacher's Lesson Guide* and ConnectED Teacher Center, Grade 2 Lesson 6-8

Adjusting the Activity notes include recommendations for tools, visual aids, and other instructional strategies that provide immediate support for exceptional learners. Sometimes these notes also provide suggestions for questions to extend students' thinking.

Common Misconception notes suggest how to use observations of students' work to adapt instruction. These notes help teachers anticipate and recognize common errors and misconceptions in students' thinking and alert teachers to multiple solution strategies or unique insights students may offer.



Repeat the activity with an addition problem with 3-digit addends, such as 254 + 138. 392 Explain to children that they should continue writing the expanded form for the addends if it helps them think about the partial sums, but if they are able to find the partial sums mentally, they do not have to write the expanded form every time. Ask them to tell you the expanded form for each addend. Record it and draw a thought bubble around it to emphasize that it is ok if they do not write it out. (*See margin.*) Then have children use partial-sums addition to solve the problem and ballpark estimates to check whether their answers make sense. Sample answer: 300 + 100 = 400; My answer makes sense because 392 is close to 400.

Ask children to share their answers and explain what they did in each step. GMP6.1 As you make the record shown in the margin, think aloud: *Add the 100s: 200 plus 100 is 300. Add the 10s: 50 plus 30 is 80. Add the 1s: 4 plus 8 is 12. Find the total: 300 plus 80 plus 12 is 392.* Explain that children do not have to record the steps shown in blue if they are able to find the sums mentally, but they can write the steps if it helps them think through the problem.

Pose additional 2- and 3-digit addition problems as needed. Children should solve the problems using partial-sums addition, explain their thinking to partners, and use ballpark estimates to check the reasonableness of their answers. GMP6.1, GMP1.4 *Suggestions:*

- 87 + 26 = ? 113
- 246 + 132 = ? 378
- 125 + 63 = ? 188

Differentiate **Common Misconception**

Some children may struggle when using partial-sums addition to add a 3-digit number to a 2-digit number. For example, when adding 125 and 63, they may write 100 + 600 = 700 as the first partial sum. Encourage these children to write the addends in expanded form and use a zero to represent the hundreds in 63: 63 = 0 + 60 + 3.

Go Online | Differentiation Support

Estimating and Adding with Partial Sums

Math Journal 2, p. 158

WHOLE CLASS | SMALL GROUP | **PARTNER** | INDEPENDENT

Children work with partners to complete journal page 158. They should make ballpark estimates and use partial-sums addition to solve the problems. Explain to children that they can write out the parts shown in the think bubbles if it is helpful, but they do not have to do so for every problem.

Math Journal 2, p. 158

Partial-Sums Addition

Lesson 6-8

For Problems 1–3, make a ballpark estimate. Then solve the problem using partial-sums addition. Show your work. Use your estimate to check that your answer makes sense.

Example: 59 + 26 = ?
Ballpark estimate: 60 + 30 = 90
Think: 50 + 20 = 70; 9 + 6 = 15
59 + 26 = 70 + 15 = 85

1. Ballpark estimate: Sample: 35 + 70 = 105
34 + 71 = 105

2. Ballpark estimate: Sample: 140 + 160 = 300
136 + 157 = 293

3. Ballpark estimate: Sample: 120 + 50 = 170
122 + 53 = 175

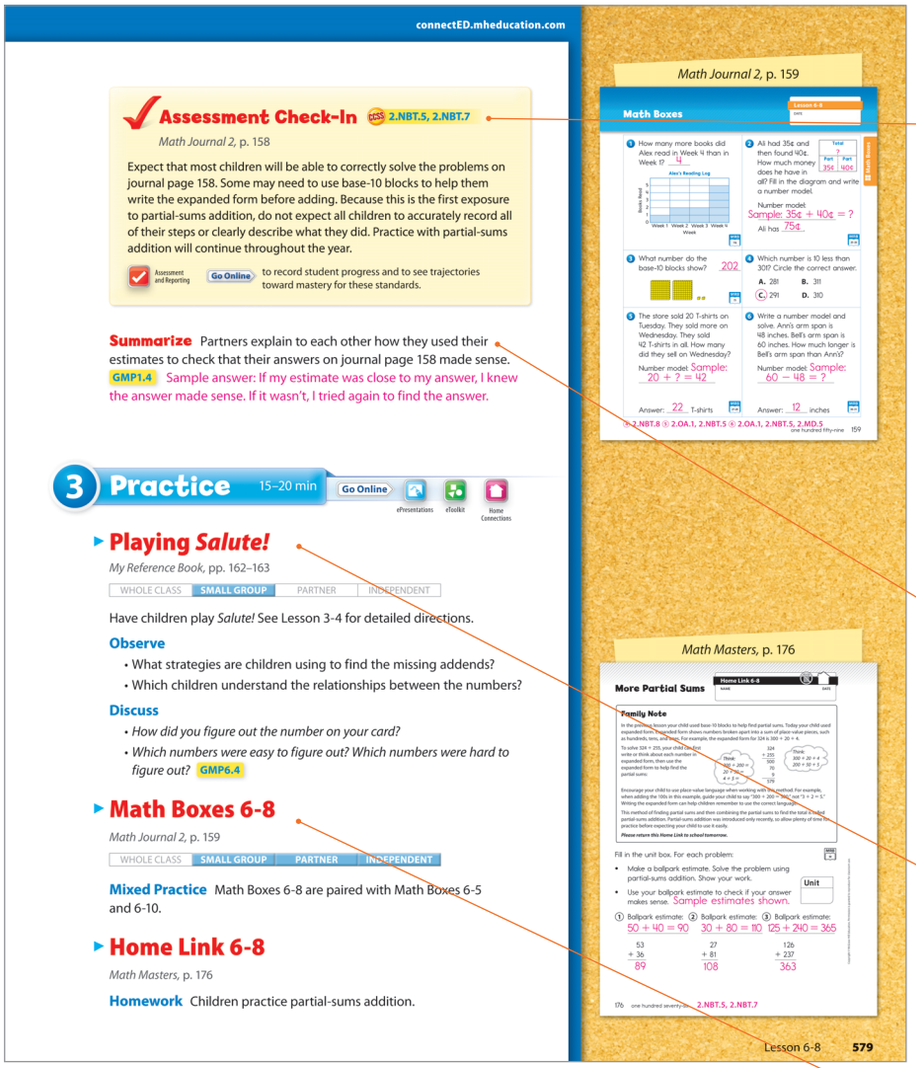
4. Solve one of the problems a different way. Explain your strategy.
Sample: For Problem 1, I changed 34 + 71 into 35 + 70 since it's easier to add: 35 + 70 = 105.

158 one hundred fifty-eight 2.NBT.3, 2.NBT.5, 2.NBT.7, 2.NBT.9, SMP1

578 Unit 6 | Whole Number Operations and N

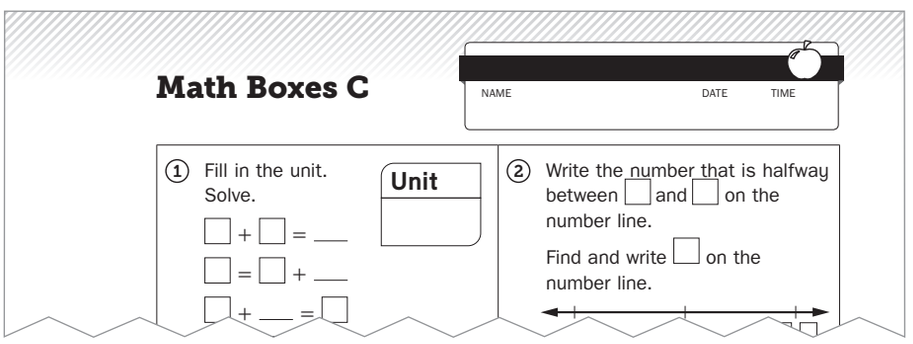
Think: 200 + 100 = 300; 50 + 30 = 80; 4 + 8 = 12
254 + 138 = 392
Think: 200 + 50 + 4; 100 + 30 + 8

Page from *Teacher's Lesson Guide* and ConnectED Teacher Center, Grade 2 Lesson 6-8



Page from *Teacher's Lesson Guide*, Grade 2 Lesson 6-8

In addition to the Math Boxes provided for each lesson, there are Math Boxes Teaching Aid Masters that can be used for differentiation purposes. In general, each Math Boxes Teaching Aid Master is aligned with the concepts of 2 to 3 units. For Grade 1, Math Boxes A is aligned with Units 2 and 3, Math Box B with 4 and 5, and so on. There is also a blank Math Boxes master. For Grade 2 Math Box C is aligned with Units 1, 2, 3, Math Box D is aligned with Units 4 and 5, and so on. For Grades 3–6, Math Box C is aligned with Units 1 and 2, Math Box D with Units 3 and 4, and so on. For Grades 2–6, there are 2 blank Math Boxes masters (A and B).



Grade 3, Math Boxes C

An **Assessment Check-In** in the focus activity highlights specific tasks that can be used for assessment to monitor students' progress. Assessment Check-Ins identify the expectations for students who are making adequate progress and point to skills or strategies that some students may be able to demonstrate. Most Assessment Check-Ins include information on how to address the needs of students who struggle to meet expectations at the given point in the year.

Summarize activities bring closure to a lesson and help students make connections among the concepts studied, which is particularly important for many struggling learners.

Games provide engaging opportunities to apply, practice, and extend mathematical skills and concepts. Each game includes specific suggestions for modifications, making them powerful differentiation tools. Many game modifications can be found online on the Differentiation Support pages.

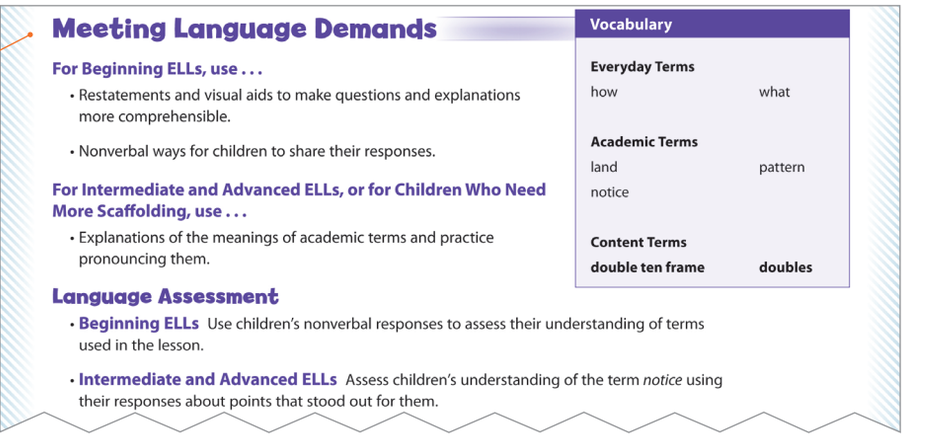
Math Boxes provide distributed practice, revisiting content to help students build and maintain important concepts and skills. Two sets of special Preview Math Boxes appear toward the end of each unit. The Preview Math Boxes can be used to gauge students' readiness for the upcoming unit so that teachers can better plan instruction and choose appropriate differentiation activities.

10.2.2 Online Differentiation Support in Grades K–6

In Grades K–6 extended, lesson-specific differentiation tools and strategies are available in online **Differentiation Support** pages.

Differentiation Support pages for all lessons (except Open Response and Reengagement lessons and Progress Check lessons) provide expanded, lesson-specific suggestions for working with diverse learners, including English language learners and students who need more scaffolding. Each section lists the target population the suggestions address. These pages are available online and can be printed as a two-page document. They can be accessed directly from each lesson within the digital Teacher Center, providing easy, point-of-use access to differentiation supports. Go Online to access the **Differentiation Support** ebook.

Meeting Language Demands is a collation of suggestions for addressing the language demands of the lesson. It also includes vocabulary for the lesson and suggested strategies for assessing English language learners' understanding of particularly important words needed for accessing the lesson.



Meeting Language Demands

For Beginning ELLs, use . . .

- Restatements and visual aids to make questions and explanations more comprehensible.
- Nonverbal ways for children to share their responses.

For Intermediate and Advanced ELLs, or for Children Who Need More Scaffolding, use . . .

- Explanations of the meanings of academic terms and practice pronouncing them.

Vocabulary

Everyday Terms	
how	what
Academic Terms	
land	pattern
notice	
Content Terms	
double ten frame	**doubles**

Language Assessment

- **Beginning ELLs** Use children's nonverbal responses to assess their understanding of terms used in the lesson.
- **Intermediate and Advanced ELLs** Assess children's understanding of the term *notice* using their responses about points that stood out for them.

Page 1 from Online Differentiation Support, Grade 1 Lesson 4-7

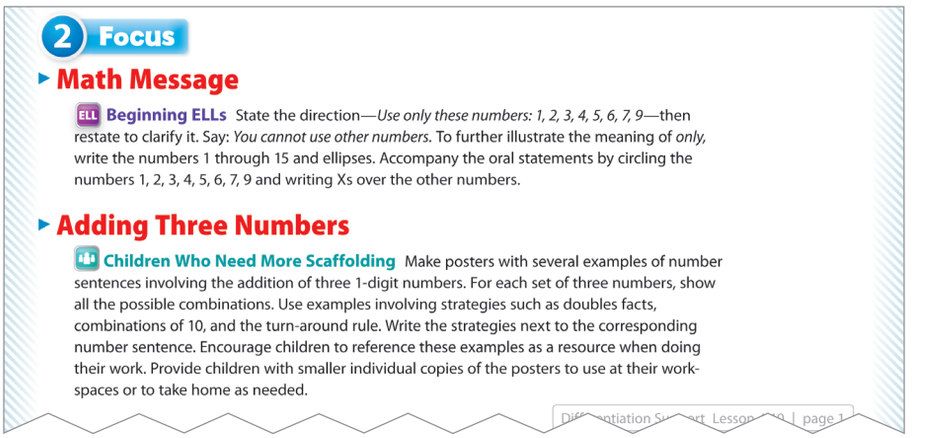
For designated parts of each lesson, specific tips are provided to help address the learning needs of English language learners and students who need more scaffolding.

1 Warm Up

Mental Math and Fluency

Children Who Need More Scaffolding Provide ten frames (*Math Masters,* page TA19) and counters to help children replicate what they saw on the Quick Look Cards; and to help them visualize the quantities in the frame as they add or take away one.

Support for *Children Who Need More Scaffolding* for the Warm Up, Grade 1 Lesson 4-8 from Differentiation Support



2 Focus

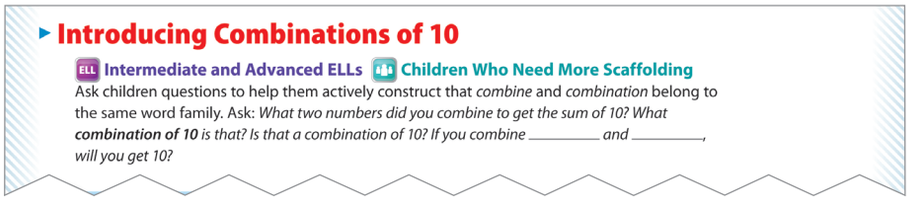
Math Message

Beginning ELLs State the direction—*Use only these numbers: 1, 2, 3, 4, 5, 6, 7, 9*—then restate to clarify it. Say: *You cannot use other numbers.* To further illustrate the meaning of *only,* write the numbers 1 through 15 and ellipses. Accompany the oral statements by circling the numbers 1, 2, 3, 4, 5, 6, 7, 9 and writing Xs over the other numbers.

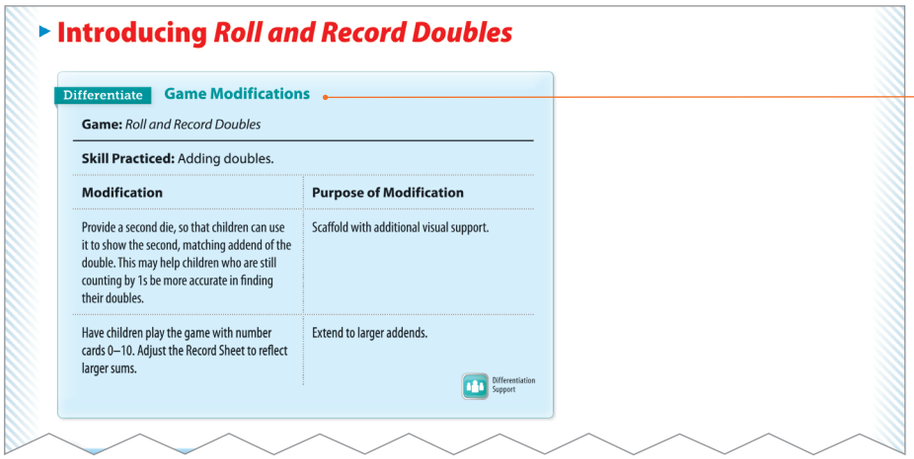
Adding Three Numbers

Children Who Need More Scaffolding Make posters with several examples of number sentences involving the addition of three 1-digit numbers. For each set of three numbers, show all the possible combinations. Use examples involving strategies such as doubles facts, combinations of 10, and the turn-around rule. Write the strategies next to the corresponding number sentence. Encourage children to reference these examples as a resource when doing their work. Provide children with smaller individual copies of the posters to use at their work-spaces or to take home as needed.

Support for the *Focus* part of Grade 1 Lesson 4-10 from Differentiation Support

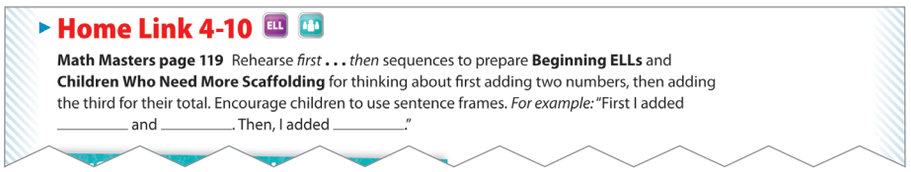


Support for an activity in Grade 1 Lesson 4-8 from Differentiation Support

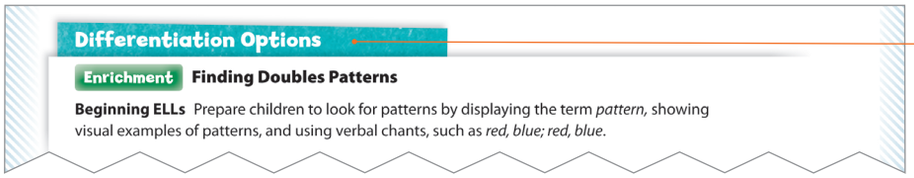


Game Modifications for Grade 1 Lesson 4-7 from the Differentiation Support

Game Modifications are provided for every game that is introduced in a lesson. Game modifications provide scaffolds for making the games accessible to all learners, as well as extensions to challenge students who would benefit more from a stretch.



Support for the *Home Link* in Grade 1 Lesson 4-10 from Differentiation Support



Additional *Differentiation Option* for an Enrichment activity in Grade 1 Lesson 4-7 from Differentiation Suppport

Differentiation Options include additional suggestions, as needed, for specific populations for the Readiness, Enrichment, or Extra Practice options in the lesson.

10.3 Differentiation Strategies in *Everyday Mathematics*

Differentiation strategies in *Everyday Mathematics* can be considered "accessibility strategies" or instructional approaches and tools that help students access the mathematics while maintaining the rigor and expectations of the concepts and skills being covered. *Everyday Mathematics* lends itself naturally to use as part of a Response to Intervention (RTI) plan or a Multi-Tiered System of Supports. Students receive Tier 1, high-quality instruction in the general education classroom when *Everyday Mathematics* is being used. Going beyond Tier 1, there are numerous ways that students receive Tier 2 support when being taught with *Everyday Mathematics.* The Readiness, Extra Practice, Adjusting the Activity features, as well as the Differentiation Support pages provide a variety of suggestions for providing small-group Tier 2 support. Such differentiation suggestions may also be appropriately adapted for Tier 3 intensive interventions. An RTI system also requires that students' progress to be monitored, and appropriate interventions used when students do not make adequate progress. *Everyday Mathematics* has many ways to monitor progress for planning targeted instructional needs.

This section outlines different strategies of this type that are included in the curriculum. In considering accessibility strategies, a guiding principle is that while the embedded and called-out differentiation strategies in lessons may target students who need more scaffolding, they typically will benefit many other students as well.

10.3.1 Looking within *Everyday Mathematics* for Differentiation Tools and Strategies

One of the benefits of a rich, comprehensive mathematics curriculum is that the program itself provides a wide range of differentiation opportunities in each lesson. This edition of *Everyday Mathematics* has an expanded toolbox of differentiation strategies beyond those provided in previous editions such as new Math Boxes Teaching Aid Masters. While each student has different educational needs, teachers will likely find sufficient differentiation strategies to meet the needs of nearly all their students within the program.

Embedded Differentiation Strategies

Many differentiation strategies and tools are woven into the core teacher and student materials and may not be immediately apparent as differentiation strategies. They build on the features of the curriculum that are inherently well suited for use with diverse learners. Examples of embedded differentiation strategies in *Everyday Mathematics* include the use of a particular manipulative, diagram, or other representation of a mathematical concept; the use of multiple strategies and algorithms for solving problems; the careful tailoring of language in the student materials to increase accessibility to all learners; the inclusion of many everyday contexts for problems; the regular routines and lesson structures; and specific discussion prompts to elicit students' thinking about a problem situation.

Explicitly Called-Out Differentiation Strategies

Everyday Mathematics includes features that are explicitly labeled to highlight their potential role in supporting students who need additional scaffolding. In the *Teacher's Lesson Guide*, each regular and Explorations lesson includes a **Differentiation Options** page that provides optional activities that can enhance readiness for the lesson, offer enrichment, provide extra practice, and address the needs of beginning English language learners. Each lesson includes Differentiation Notes at point of use. These include notes for Adjusting the Activity, Common Misconceptions, and English Language Learners.

Each regular lesson includes online **Differentiation Support** pages that provide additional suggestions and resources to assist with differentiation.

For more information, see Section 10.2.1 *Differentiation Features in Lessons*.

For more information, see Section 10.2.2 *Online Differentiation Support in Grades 1–6*.

10.3.2 Effective Differentiation Begins with Good Assessment Practices

The bedrock of effective differentiation is a solid understanding of a student's strengths, misconceptions, and weaknesses. The key to that understanding is a rich body of assessment data gathered formally and informally as students engage in the mathematics. *Everyday Mathematics* provides assessment opportunities to help teachers identify strengths and areas of need and consider which alternative approaches may be most effective for students. Specific differentiation supports are included with the assessment opportunities.

For more information on assessment opportunities, see Section 9 *Assessment in Everyday Mathematics*.

Assessment Check-Ins

Lessons include Assessment Check-Ins in the focus activity that highlight specific tasks that can be used for assessment to monitor students' progress. Each Assessment Check-In provides information on expectations for particular standards at that point in the curriculum. In regular lessons, the expectations statement is accompanied with specific recommendations for supporting students who struggle to meet expectations and often for students who excel.

Assessment Check-In CCSS 1.OA.1, 1.OA.6

Math Masters, p. TA4

Tell another simple number story using up to 10 pennies. Ask children to record their solutions and show their work on an Exit Slip (*Math Masters*, page TA4). Expect most children to solve the number story by finding the correct sum or difference. Many will also illustrate their solutions. Have children who struggle to solve the number story model solutions using counters. For children who excel, consider providing a more challenging follow-up problem in which the start or middle quantity is unknown. *For example: I had 6 pennies, and my mom gave me some more pennies. Now I have 10 pennies. How many pennies did she give me?*

Assessment and Reporting Go Online to record student progress and to see trajectories toward mastery for these standards.

Assessment Check-In with suggestions for supporting children who struggle and who exceed expectations, Grade 1 Lesson 1-10

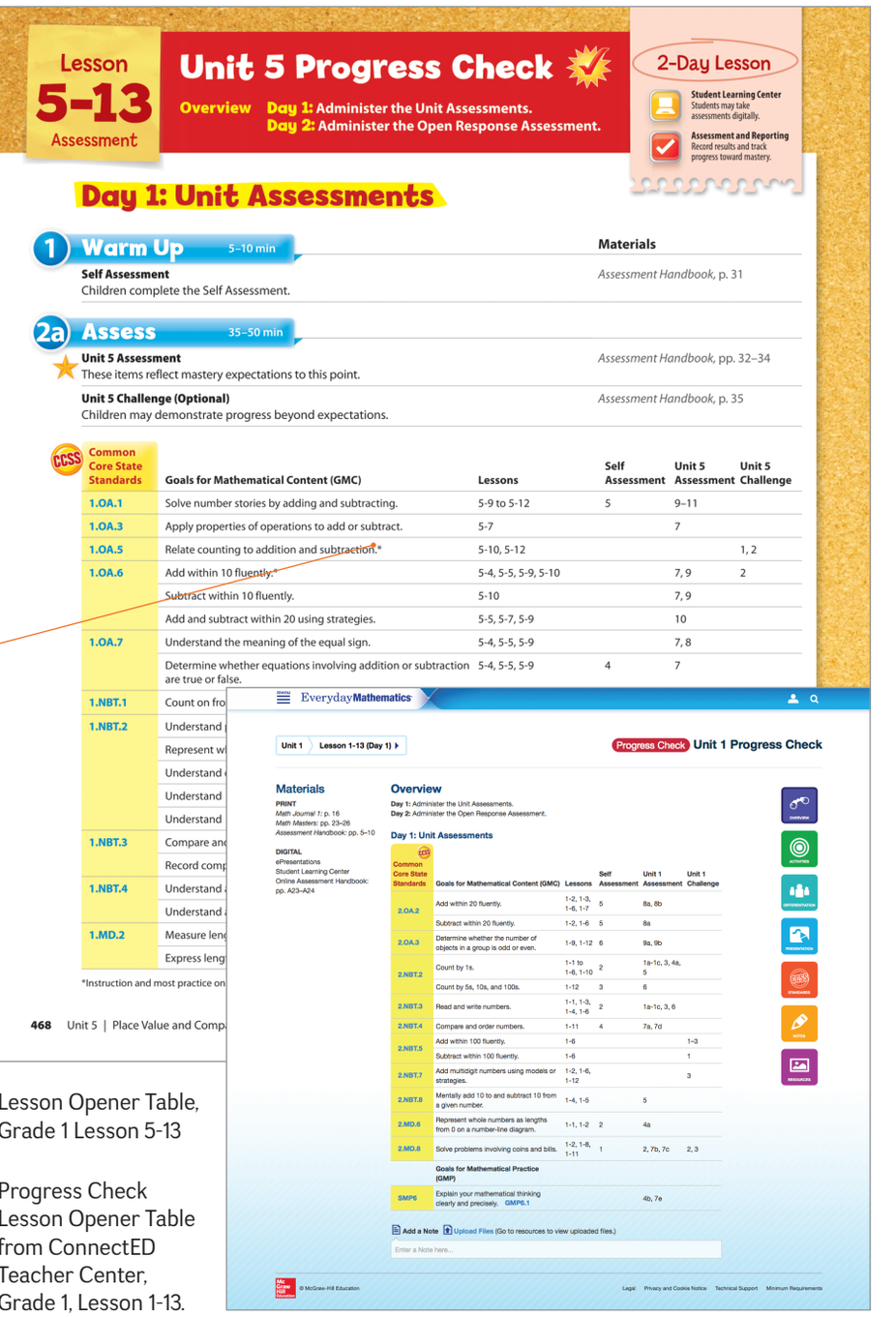
For information about the Digital Evaluation Tool, see Section 9.4.

Progress Check Lessons

Progress Check Lessons include the following features to help make differentiation decisions:

- **Lesson Opener Table** Progress Check lessons include tables listing the Common Core State Standards and Goals for Mathematical Content addressed by the assessment along with the items that address each standard and goal. The openers provide information that help teachers know when to intervene and when "watchful waiting" is appropriate.

Asterisks on specific content goals in the table indicate when instruction and most practice on that content is complete and differentiation strategies or intervention may be appropriate for students who have not met those goals at this point in the curriculum.



Lesson 5-13 Assessment

Unit 5 Progress Check

Overview **Day 1:** Administer the Unit Assessments. **Day 2:** Administer the Open Response Assessment.

2-Day Lesson

Student Learning Center: Students may take assessments digitally.

Assessment and Reporting: Record results and track progress toward mastery.

Day 1: Unit Assessments

1 Warm Up 5–10 min

Materials

Self Assessment
Children complete the Self Assessment.
Assessment Handbook, p. 31

2a Assess 35–50 min

Unit 5 Assessment
These items reflect mastery expectations to this point.
Assessment Handbook, pp. 32–34

Unit 5 Challenge (Optional)
Children may demonstrate progress beyond expectations.
Assessment Handbook, p. 35

Common Core State Standards	Goals for Mathematical Content (GMC)	Lessons	Self Assessment	Unit 5 Assessment	Unit 5 Challenge
1.OA.1	Solve number stories by adding and subtracting.	5-9 to 5-12	5	9–11	
1.OA.3	Apply properties of operations to add or subtract.	5-7		7	
1.OA.5	Relate counting to addition and subtraction.*	5-10, 5-12			1, 2
1.OA.6	Add within 10 fluently.*	5-4, 5-5, 5-9, 5-10		7, 9	2
	Subtract within 10 fluently.	5-10		7, 9	
	Add and subtract within 20 using strategies.	5-5, 5-7, 5-9		10	
1.OA.7	Understand the meaning of the equal sign.	5-4, 5-5, 5-9		7, 8	
	Determine whether equations involving addition or subtraction are true or false.	5-4, 5-5, 5-9	4	7	
1.NBT.1	Count on fro				
1.NBT.2	Understand				
	Represent w				
	Understand				
	Understand				
	Understand				
1.NBT.3	Compare an				
	Record comp				
1.NBT.4	Understand				
	Understand				
1.MD.2	Measure leng				
	Express leng				

*Instruction and most practice on

468 Unit 5 | Place Value and Comp

Lesson Opener Table, Grade 1 Lesson 5-13

EverydayMathematics

Unit 1 Lesson 1-13 (Day 1)

Progress Check Unit 1 Progress Check

Materials

PRINT
Math Journal 1: p. 16
Math Masters: pp. 23–26
Assessment Handbook: pp. 5–10

DIGITAL
ePresentations
Student Learning Center
Online Assessment Handbook: pp. A23–A24

Overview

Day 1: Administer the Unit Assessments.
Day 2: Administer the Open Response Assessment.

Day 1: Unit Assessments

Common Core State Standards	Goals for Mathematical Content (GMC)	Lessons	Self Assessment	Unit 1 Assessment	Unit 1 Challenge
2.OA.2	Add within 20 fluently.	1-2, 1-3, 1-6, 1-7	5	8a, 8b	
	Subtract within 20 fluently.	1-2, 1-6	5	8a	
2.OA.3	Determine whether the number of objects in a group is odd or even.	1-9, 1-12	6	9a, 9b	
2.NBT.2	Count by 1s.	1-1 to 1-6, 1-10	2	1a-1c, 3, 4a, 5	
	Count by 5s, 10s, and 100s.	1-12	3	6	
2.NBT.3	Read and write numbers.	1-1, 1-3, 1-4, 1-6	2	1a-1c, 3, 6	
2.NBT.4	Compare and order numbers.	1-11	4	7a, 7d	
2.NBT.5	Add within 100 fluently.	1-6			1–3
	Subtract within 100 fluently.	1-6			1
2.NBT.7	Add multidigit numbers using models or strategies.	1-2, 1-6, 1-12			3
2.NBT.8	Mentally add 10 to and subtract 10 from a given number.	1-4, 1-5		5	
2.MD.6	Represent whole numbers as lengths from 0 on a number-line diagram.	1-1, 1-2	2	4a	
2.MD.8	Solve problems involving coins and bills.	1-2, 1-8, 1-11	1	2, 7b, 7c	2, 3
	Goals for Mathematical Practice (GMP)				
SMP6	Explain your mathematical thinking clearly and precisely. GMP6.1			4b, 7e	

Add a Note Upload Files (Go to resources to view uploaded files.)

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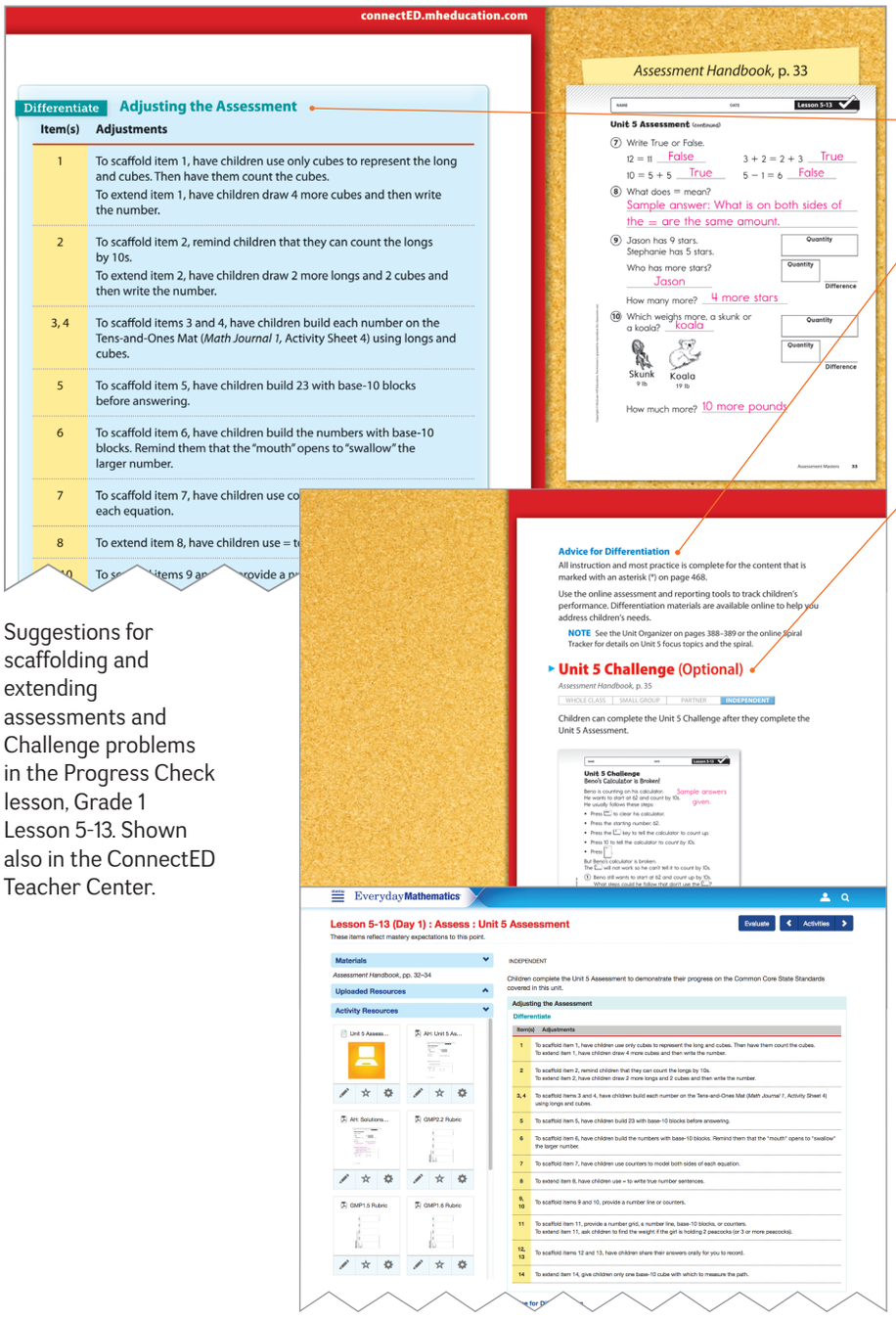
Progress Check Lesson Opener Table from ConnectED Teacher Center, Grade 1, Lesson 1-13.

- **Adjusting the Assessment** Progress Check lessons include tables withsuggestions to scaffold or extend assessed items.
- **Advice for Differentiation** Progress Check lessons provide notes alerting the teacher when the assessed content was introduced and if it will be revisited in subsequent units.



Suggestions for scaffolding and extending assessments and Challenge problems in the Progress Check lesson, Grade 1 Lesson 5-13. Shown also in the ConnectED Teacher Center.

Suggestions for scaffolding and extending assessment items Progress Check assessments are accompanied by a table with specific suggestions for scaffolding and extending selected items on the assessments.

Challenge Each Progress Check lesson includes one or more challenge problems related to important ideas from the unit.

10.3.3 Effective Differentiation Maintains the Cognitive Demand of the Mathematics

Increasingly mathematics education researchers have recognized how the quality of mathematical tasks impacts what students learn. Researchers broadly categorize mathematical tasks into two categories: low cognitive demand tasks and high cognitive demand tasks.[3] Low cognitive demand tasks, such as memorizing facts and procedures or solving routine arithmetic problems (for example, 23 x 16) are an essential component of every mathematics program, including *Everyday Mathematics*. However, a curriculum composed largely of such tasks will not provide adequate opportunities for students to develop the full range of mathematical competencies that are required by the Common Core State Standards for Mathematics. Researchers recommend that mathematics curricula also include ongoing and substantive experiences with high cognitive demand tasks that require more thinking, reasoning, communication, and application. *Everyday Mathematics* has a deep foundation of such high cognitive demand tasks.

In an effort to assist students with high cognitive demand tasks, teachers often attempt to break down a complex task into a series of easy-to-follow steps. This helps students get through the problems but in the process, it does much of the thinking for them and thereby reduces the cognitive demand of the task. Thus, what begins as a high cognitive demand task gets *implemented* as a low cognitive demand task, depriving students of the necessary experience of doing the more demanding work.

An important skill in providing effective scaffolding for students is increasing accessibility to the problems but *maintaining the cognitive demand*. This requires that teachers diagnose the stumbling blocks that are impeding access to the essential mathematics of the problem, and then adopt a strategy that removes the obstacles while keeping the student in the driver's seat of engaging with the mathematics. The differentiation strategies suggested in *Everyday Mathematics* help teachers with this work.

For more information, see Section 6 *Open Response and Reengagement*.

Open Response and Reengagement Lessons

One new feature of this edition of *Everyday Mathematics* is the expanded use of open response problems with an emphasis on providing all students with opportunities to solve problems with high cognitive demand and reengagement in the problem-solving process tailored to students' needs. Every unit includes a two-day Open Response and Reengagement lesson. In these lessons, students first complete an open response problem, where they apply mathematical concepts and skills. Teachers then analyze students' responses, carefully selecting student work that highlights important aspects of the problem or common misconceptions. Following a consistent process, teachers develop a reengagement discussion based around the selected work. Finally, students revise their work based on the discussion and their new understandings. For students who struggled with the open response task, the reengagement discussion provides an opportunity to revisit the problem and consider where they may have gotten off track. For students who succeeded with the task, the reengagement lessons involve them in analyzing the thinking

[3] While the discussion of cognitive demand in mathematics lessons is discussed widely, see Stein, M.K., Grover, B.W. & Henningsen, M. (1996) for an introduction to the concept of high and low cognitive demand tasks.

of others, an important and difficult mathematical process. As such, the Open Response and Reengagement lessons provide powerful new ways to create naturally differentiated lessons and to learn about students' mathematical thinking and proficiencies.

10.3.4 Effective Differentiation Requires Daily Reflection about Student Learning

This edition of *Everyday Mathematics* was written and engineered to enable *all*[4] students to access and learn the mathematics outlined in the Common Core State Standards for Mathematics. That emphasis on equity issues has been a fundamental principle of every edition of *Everyday Mathematics*. It requires that all students have the opportunity to learn the full range of mathematics in the program. Using the many tools provided by *Everyday Mathematics* can help teachers increase accessibility to the mathematics in the program and ensure that all students are challenged appropriately in each lesson.

10.3.5 General Differentiation Strategies

> "All tasks should respect each learner. Every student deserves work that is focused on the essential knowledge, understanding, and skills targeted for the lesson. Every student should be required to think at a high level and should find his or her work interesting and powerful." (Tomlinson, 2003)

Each *Everyday Mathematics* lesson addresses a variety of mathematical concepts and skills. The most prominent content goals and the focus goals for mathematical practice are highlighted in the *Teacher's Lesson Guide*. Planning for differentiated instruction involves considering individual students' current readiness in relation to those goals, then selecting accessibility strategies that support students' learning of the core concepts, skills, and practices in the lesson. Examples of some of the instructional strategies incorporated into lessons are described here.

Intentional Student Groupings for Activities

Everyday Mathematics lessons involve whole-class and small-group instruction, as well as time for students to work individually. This allows teachers many opportunities to regroup students for differentiation purposes. For many activities, teachers will want to group students who need more scaffolding with those who have a better understanding of a concept. For other activities teachers may want to create groups of students with similar levels of understanding of a concept. From a differentiation perspective, what is most important is to be intentional about creating groups that will most effectively help students access and learn the lesson content. Groupings should be fluid; they should change regularly, based upon student needs and the nature of the content to be learned. To ensure all students have equal access to the full range of mathematics content in the curriculum, avoid static groupings based on student ability (grouping all of the struggling learners or most advanced learners together all or most of the time).

For more information about creating student groupings, see Section 5.2 *Organizing Students in Small Groups*.

[4] It is understood that there is a small group of students with severe cognitive disabilities or other problems that will require alternative approaches to learning mathematics.

Framing the Lesson

Explicitly framing the goals of each lesson for students sets the stage and supports learning by mentally preparing students for the content of the lesson or by activating prior knowledge and making connections to earlier learning. Framing statements are generally included as part of the Math Message Follow-Up in each lesson, as illustrated here.

▸ Math Message

Show 53 and 44 with the fewest possible base-10 blocks. 5 longs and 3 cubes; 4 longs and 4 cubes

Then write both numbers in expanded form. 50 + 3; 40 + 4

Talk about these questions with a partner: What is the same about showing numbers with base-10 blocks and writing them in expanded form? What is different? GMP2.3

▸ Using Expanded Form to Find Partial Sums

WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

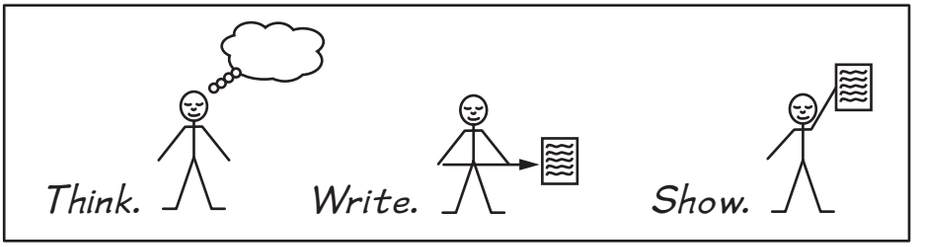
Math Message Follow-Up Ask children to share their ideas about how representations using base-10 blocks and expanded form are similar and different. GMP2.3 Sample answers: Expanded form and base-10 blocks both show how much each digit of a number is worth. The base-10 blocks are objects, but I can write the expanded form on paper. Remind children that in the previous lesson they used base-10 blocks to help them find partial sums and add numbers. Tell them that today they will use expanded form to help them find partial sums.

Example of suggestions for framing the lesson in the Math Message Follow-Up, Grade 2 Lesson 6-8

Providing Wait Time

During the whole-class portion of a lesson and as you work with students in other groupings, allow time for students to think and process information before eliciting answers to questions posed. Waiting even a few seconds for an answer will help many students process information and, in turn, participate more fully in class discussions.

Wait time is also beneficial when you pose Mental Math and Fluency problems at the beginning of each lesson. Encourage students to stop and think before they write on their slates and show their answers. Consider displaying these three steps on a poster. Establish a routine by pointing to the steps in sequence, pausing at each for several seconds.



Establish a routine for Mental Math and Fluency in which students *Think*, *Write*, and *Show*

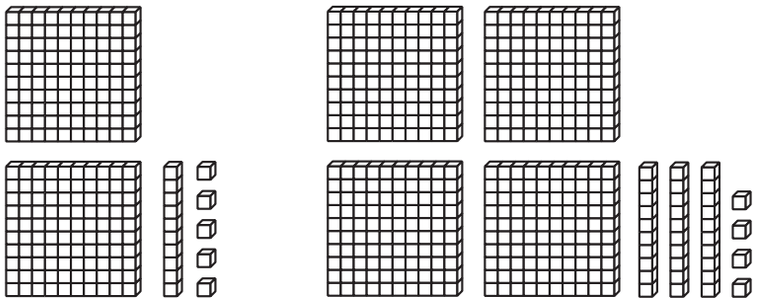
Making Connections to Everyday Life

Everyday Mathematics was founded on the principle that students will learn mathematics better by using the mathematics found in everyday situations. Lessons offer regular opportunities to make connections between students' everyday experiences and new mathematics concepts and skills. This provides a context for learners that increases the accessibility of the mathematics.

Modeling Concretely

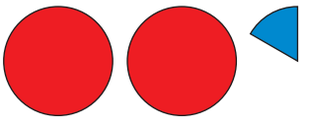
Lessons frequently include the use of manipulatives. They should be available at most times and for all students. Modeling concretely not only makes lesson content more accessible for some students, it deepens all students' understanding of concepts and skills.

Effective use of manipulatives to model mathematical situations includes connecting the concrete representation with other representations of the situation, such as number models, graphs and diagrams, and written descriptions. Making explicit connections between the different representations of a problem situation helps develop students' understanding of the mathematical concepts involved and promotes their abilities to apply the concepts to new contexts.

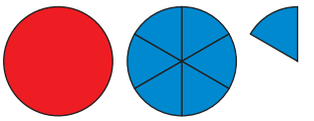


215 + 434 = _____

Using base-10 blocks to model place value and addition



Show $2\frac{1}{6}$.



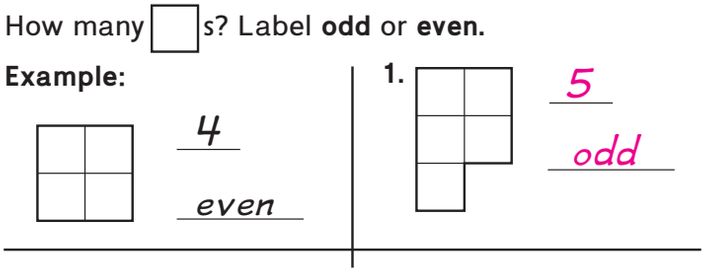
Trade 1 whole for 6 sixths.

Take away $1\frac{4}{6}$. $\frac{3}{6}$ is left.

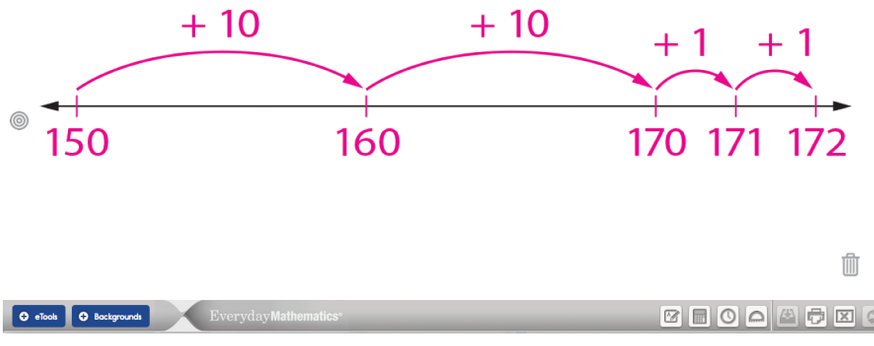
Using fraction circles to model subtracting fractions

Modeling Visually

Classrooms tend to be highly verbal places, and this can be overwhelming for some students. Simple chalkboard drawings, anchor charts, diagrams, and other visual representations can help students make sense of the flow of words around them and also help them connect words to the actual items. As stated above, making connections between different representations of a mathematical situation is an important strategy for promoting understanding of the mathematical concept and adaptability in its use.



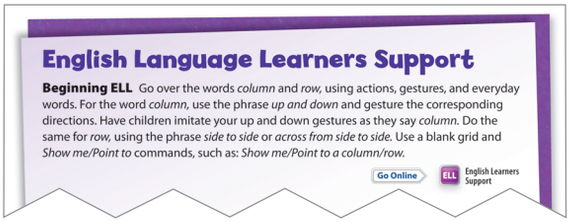
A visual representation of odd and even numbers



Number line from eToolkit Backgrounds in Student Learning Center

Modeling Physically

Lessons suggest ways to have students demonstrate concepts and skills with gestures or movements. This strategy helps many students better understand and retain the concept or skill.



English Language Learners Support

Beginning ELL Go over the words *column* and *row,* using actions, gestures, and everyday words. For the word *column,* use the phrase *up and down* and gesture the corresponding directions. Have children imitate your up and down gestures as they say *column.* Do the same for *row,* using the phrase *side to side* or *across from side to side.* Use a blank grid and *Show me/Point to* commands, such as: *Show me/Point to a column/row.*

Go Online | ELL English Learners Support

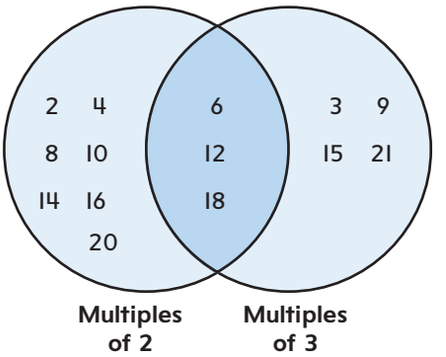
ELL note suggesting gestures to use with a number grid in Grade 1

−9	−8	−7	−6	−5	−4	−3	−2	−1	0
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110

Students can move their fingers on number lines or number grids to model operations

Providing Organizational Tools

Everyday Mathematics lessons often include tools to help students organize their thinking. Have students use diagrams, tables, charts, and graphs when these materials are included in lessons and as appropriate. This is another way to make the lesson content more accessible for some students while at the same time deepening other students' understanding of concepts and skills.



Students can use Venn diagrams to compare, contrast, and sort properties of numbers and shapes

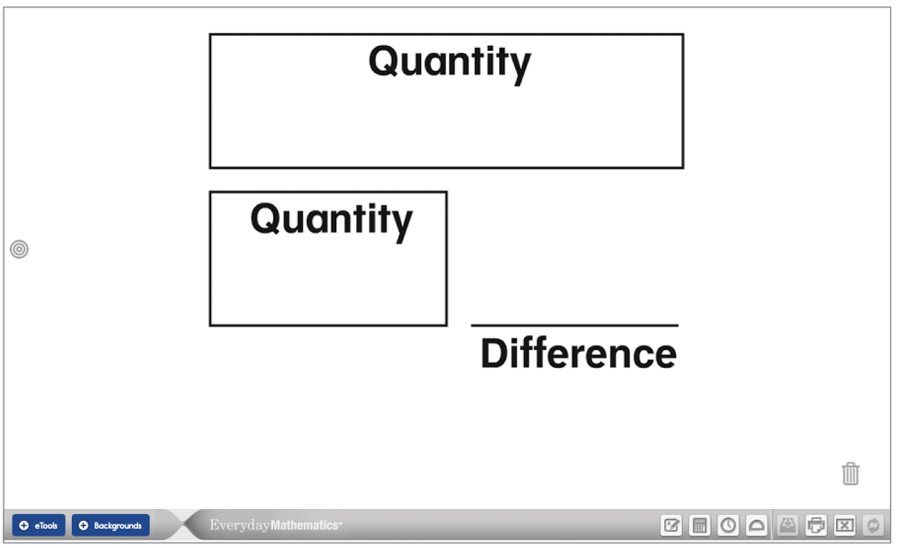
rows	chairs per row	chairs in all
3	?	15

Total
?
Part 47 | Part 15

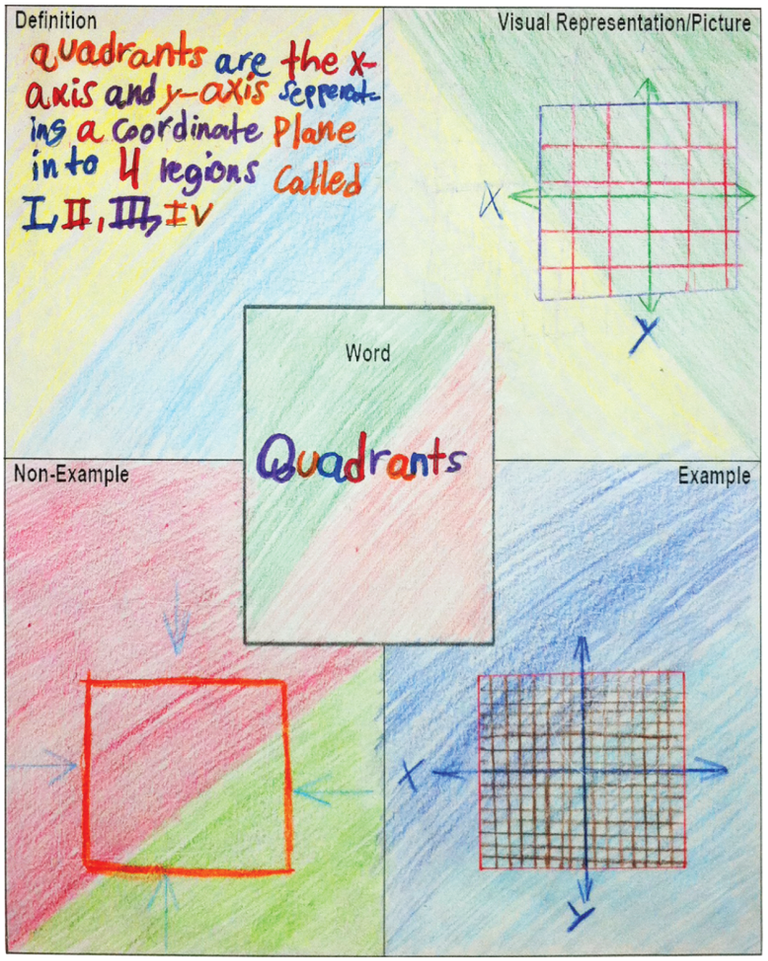
Students use situation diagrams to model operations

Ten Thousands	Thousands	Hundreds	Tens	Ones
4	3	0	0	2

Students use place-value charts to help represent place-value concepts



Comparison diagram from the eToolkit Backgrounds in the Student Learning Center.



Students use 4-Square Graphic Organizers to construct understanding of academic terms.

Promoting "Math Talk" in Class

A key to effective differentiation is making students' thinking "visible". Student thinking is readily visible within a classroom environment that fosters discussion about the mathematics of each lesson. Lessons often suggest discussion prompts or questions that support the development of good communication skills in the context of mathematics. Although finding the correct solution is one important goal, *Everyday Mathematics* lessons also emphasize sharing and comparing solution strategies. This type of "math talk" involves not only explaining what is done (explanation), but also why it is done (reasoning), and why it is correct or incorrect to do it a particular way (justification). These discussions help students deepen their understanding of mathematical concepts and processes. Teachers can model and explicitly discuss how to conduct academic discourse to help students understand what is expected of them. For example, encouraging students to look at other students when they are speaking, to build on each other's ideas, and to ask for clarification when they don't understand a statement. Academic Language Development notes provide lesson-specific suggestions for facilitating such academic discourse.

For more information, see Section 5.1 *Facilitating Discussions.*

Engaging Students in Writing about Math

Writing about the mathematics in a lesson is another important strategy for making students' thinking visible. Journal pages and assessment problems frequently prompt students to explain their thinking and strategies in words, pictures, and diagrams. Writing/reasoning prompts are included in Math Boxes in about half the lessons. Writing offers students opportunities to reflect on their thinking and can help teachers assess students' mathematical understandings and communication skills.

Sharing a Cracker

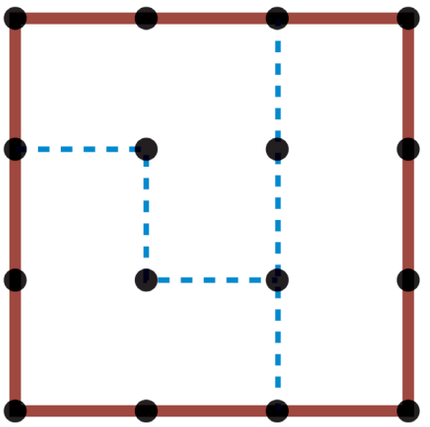
Lesson 9-2

DATE

Math Message

Juan has a cracker he wants to share equally with 2 friends.

He divided the cracker in an unusual way:



Do you think Juan divided the cracker into 3 equal parts? _____

Explain your answer.

_____

_____

_____

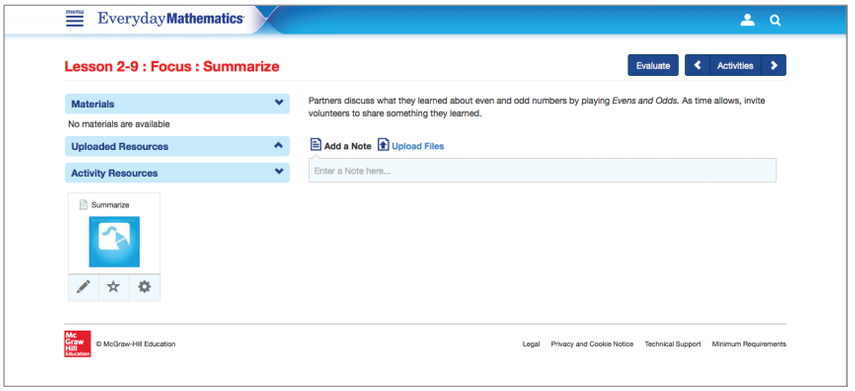
two hundred twenty-three 223

This *Math Journal* page includes student explanation, Grade 2 Lesson 9-2

Summarizing the Lesson

Each lesson suggests a summary statement or discussion question. Lesson summaries offer students a chance to bring closure to the lesson, reflect on the concepts and skills they have learned, and pose questions they may still have about the lesson content. This kind of metacognitive structure is particularly important for students who need more scaffolding.

Summarizing the lesson provides a structure for students to reflect about the lesson and is important for struggling learners, Grade 2 Lesson 2-9



Summary text from ConnectED Teacher Center.

10.4 Working with Advanced Learners

Most schools will include some students in each grade whose performance level significantly exceeds that of other students their age. These students routinely grasp concepts quickly, finish assignments early, and often can benefit from a more in-depth exploration of the mathematics of a lesson.

While much attention in differentiation focuses on increasing the accessibility of the lesson for students who struggle with the mathematics, addressing the needs of advanced students poses similar challenges and requires the same amount of intentionality that is needed in planning instruction for struggling learners. Advanced students sometimes feel that their strengths are not recognized or respected by the teacher. For example, comments such as, "I know you know the answer. I want to see if others understand." may inadvertently discourage advanced students from participating in class discussions. Advanced students also may complain about not getting enough challenging work and become bored while waiting for others to learn concepts. They may feel isolated from other students in the class and need guidance about how to appropriately interact with peers in small groups.

Success with advanced students first depends upon identifying and acknowledging their special abilities. This requires an understanding that students may be exceptional with one aspect of the mathematics and less so in others. Once the student's strengths are identified, appropriate instructional strategies need to be developed.

10.4.1 Challenging Advanced Learners within Lessons

With its focus on problem solving and using math in meaningful contexts, *Everyday Mathematics* offers many opportunities to appropriately challenge and engage students who need more stretch.

- Nearly all *Everyday Mathematics* lessons include a set of high cognitive demand tasks with mathematical challenge that can be extended. This allows students to take the task to a new level while keeping them engaged in the core lesson—rather than just piling on more problems and extra work unrelated to the lesson.
- Every regular lesson includes recommended enrichment activities related to the lesson content on the Differentiation Options page opposite the Lesson Opener. These activities often involve activity cards to be used by individual students or small groups in Learning Centers. Adjusting the Activity notes in the lessons sometimes include suggestions for extending the activity.
- *Everyday Mathematics* lessons incorporate varied grouping configurations, which enables the kind of flexibility that is helpful with advanced learners in heterogeneous classrooms.
- Progress Check lessons include suggestions for extending assessment items for advanced learners and additional Challenge problems.

For more information, see Section 10.2.1 *Differentiation Features in Lessons.*

For more information, see Progress Check Lessons in Section 10.3.2 *Effective Differentiation Begins with Good Assessment Practices.*

10.4.2 Acceleration with *Everyday Mathematics*

With the basic features described above, the great majority of advanced learners can be accommodated within the regular confines of daily *Everyday Mathematics* instruction. However, schools sometimes consider accelerating advanced students into more advanced mathematics classes, such as having a third-grade student attend mathematics class with fourth- or fifth-grade students. While this type of acceleration is appropriate in some cases, the "extendibility" of *Everyday Mathematics* lessons should diminish the need for such acceleration.

As you make decisions about whether to accelerate a student, consider that placement in a more advanced mathematics class typically means the student will miss the mathematical content in the grade being skipped. In programs such as *Everyday Mathematics*, where the learning trajectories are designed so that mathematical content advances in deliberate ways, skipping a grade runs the risk of not learning foundational content that scaffolds more advanced work. In particular, students might miss the conceptual development of important concepts that would benefit later learning. Plus, as indicated above, students who are gifted in one area of mathematics may not be ready for advanced work in other areas. Doing a careful assessment of a broad range of mathematical skills and competencies should be a part of any discussion involving accelerating a student to a more advanced mathematics class.

10.5 Working with English Language Learners in *Everyday Mathematics*

In every content area, teachers of English language learners are not just expected to present cognitively challenging content. They also must work to make that same content accessible to students who are still developing their English language proficiency. This edition of *Everyday Mathematics* incorporates a variety of strategies to increase accessibility of the lessons to English language learners.

A fundamental principle of *Everyday Mathematics* is that students learn mathematics best when they use it to solve problems in meaningful contexts. Similarly, languages are acquired more effectively when learned in conjunction with meaningful content and purposeful communication. Thus, instruction with *Everyday Mathematics* can serve two purposes for English language learners: helping them learn mathematics and helping them develop English language proficiency.

English language learners enter mathematics classrooms with many similarities and differences in the language spoken at home, previous school preparation, and academic background in English as well as in their first language. Grade level does not dictate English proficiency. For example, English language learners in higher grade levels may be at beginning English proficiency levels. Conversely, students in the early grades may be at higher levels of English proficiency. Some English language learners have extensive educational background, which includes the study of English. Others may have very limited formal school experiences, which may mean they lack literacy skills in their home language and English. Moreover, English proficiency does not determine mathematical proficiency.

Regardless of their background, English language learners will need scaffolding with English language in order to access the mathematics concepts involved in any lesson. Furthermore, effective scaffolding should attempt to address four language domains—listening, speaking, reading, and writing—as well as vocabulary usage, language forms and conventions, and discourse complexity.

For more information on developing academic language, see Section 5.1.2 *Academic Language Development*.

10.5.1 Supporting English Language Learners in *Everyday Mathematics*

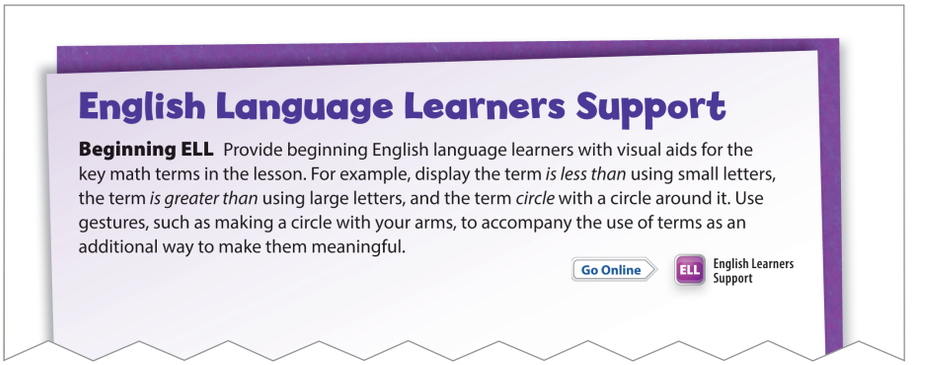
Some of the tools in *Everyday Mathematics* included to help teachers support English language learners will not be apparent to most teachers and students. For example, *Everyday Mathematics* authors have reviewed the written language in the student materials and have attempted to remove embedded clauses or other complicated sentence structures that often confuse English language learners. Visual representations of mathematical ideas were added to some pages to assist English language learners. These strategies increase accessibility for English language learners and other students without altering the mathematical goals or the cognitive demand of the lessons. *Everyday Mathematics'* attention to developing academic language for all learners will also help English language learners develop English proficiency even though this is not specifically called out as a support strategy for English language learners.

For more information on English language learners support, see Section 10.2.2 *Online Differentiation Support in Grades 1–6*.

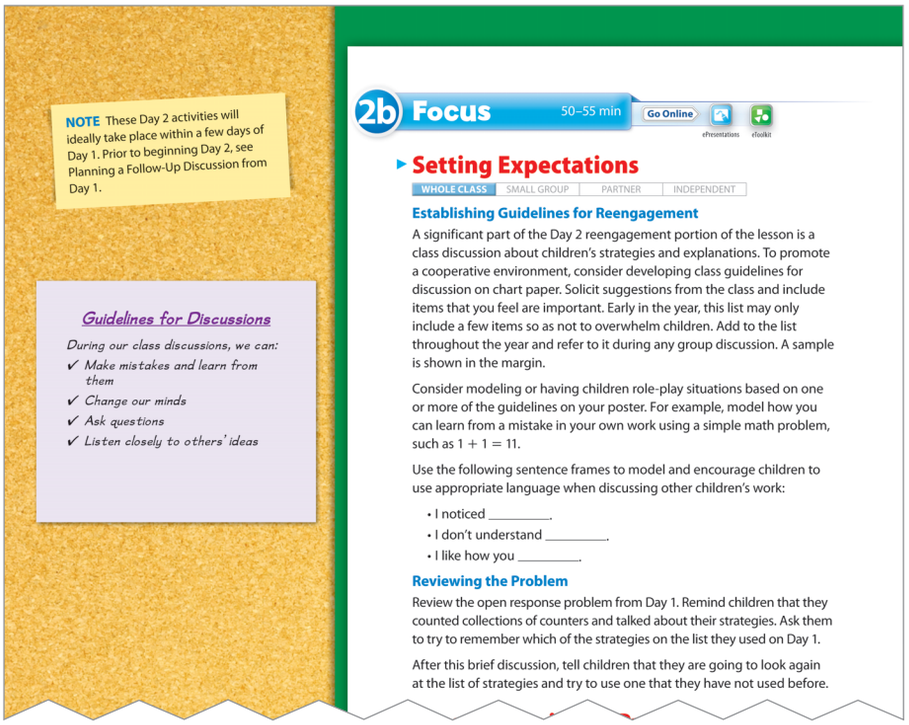
Other supports for English language learners are more explicitly called out in the teacher materials. These include:

- specific suggestions in the opening pages of each lesson for supporting beginning English language learners;
- expanded, lesson-specific suggestions and activities for English language learners in the online Differentiation Support pages that accompany every regular lesson; and
- Academic Language Development instructions in the *Teacher's Lesson Guide* that will support English language learners.

Taken together, this collection of instructional strategies and other supports provides teachers with a wide range of tools to assist English language learners with their work in *Everyday Mathematics*.



English Language Learners note, Grade 2 Lesson 1-11



NOTE These Day 2 activities will ideally take place within a few days of Day 1. Prior to beginning Day 2, see Planning a Follow-Up Discussion from Day 1.

Guidelines for Discussions

During our class discussions, we can:

- ✓ *Make mistakes and learn from them*
- ✓ *Change our minds*
- ✓ *Ask questions*
- ✓ *Listen closely to others' ideas*

2b Focus 50–55 min Go Online ePresentations eToolkit

Setting Expectations

WHOLE CLASS | SMALL GROUP | PARTNER | INDEPENDENT

Establishing Guidelines for Reengagement

A significant part of the Day 2 reengagement portion of the lesson is a class discussion about children's strategies and explanations. To promote a cooperative environment, consider developing class guidelines for discussion on chart paper. Solicit suggestions from the class and include items that you feel are important. Early in the year, this list may only include a few items so as not to overwhelm children. Add to the list throughout the year and refer to it during any group discussion. A sample is shown in the margin.

Consider modeling or having children role-play situations based on one or more of the guidelines on your poster. For example, model how you can learn from a mistake in your own work using a simple math problem, such as 1 + 1 = 11.

Use the following sentence frames to model and encourage children to use appropriate language when discussing other children's work:

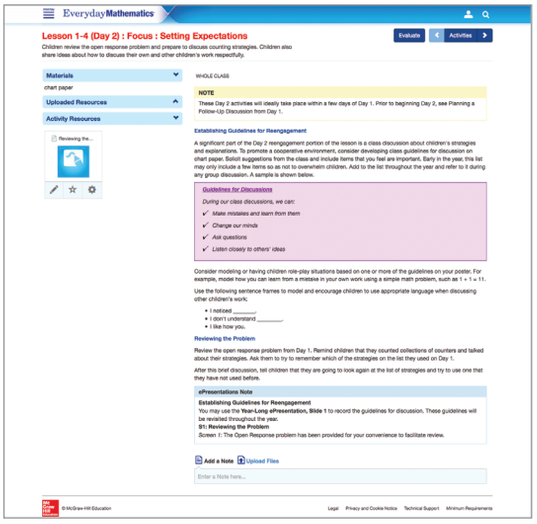
- I noticed _____.
- I don't understand _____.
- I like how you _____.

Reviewing the Problem

Review the open response problem from Day 1. Remind children that they counted collections of counters and talked about their strategies. Ask them to try to remember which of the strategies on the list they used on Day 1.

After this brief discussion, tell children that they are going to look again at the list of strategies and try to use one that they have not used before.

Suggestions for modeling and encouraging discussion, Grade 1 Lesson 1-4



Guidelines for Discussions as they appear in the ConnectED Teacher Center, Grade 1, Lesson 1-4

10.5.2 Sheltered Instruction

The Sheltered Instruction Observation Protocol (SIOP) Model was developed at the Center for Applied Linguistics (CAL) specifically to help teachers plan for the learning needs of English language learners. The model is based on the sheltered instruction approach, an approach for teaching content to English language learners in strategic ways that make the content comprehensible, while promoting English language development. The recommended components of the SIOP Model are summarized below. It provides a useful structure for thinking about using lessons with English language learners. While the list of components is long, most of the components of the SIOP Model are simply elements of good teaching that are already part of most teachers' practice. The SIOP Model creates

a structure to encourage teachers to look at the lesson through the lens of an English language learner and to adjust the language demands using a variety of strategies that already may be part of their teaching repertoire.

SIOP COMPONENTS AND FEATURES

Lesson Preparation
Content objectives clearly defined, displayed, and reviewed with students
Language objectives clearly defined, displayed, and reviewed with students
Content concepts appropriate for age and educational background level of students
Supplementary materials used to a high degree, making the lesson clear and meaningful (for example, computer programs, graphs, models, visuals)
Adaptation of content (for example, text, assignment) to all levels of student proficiency
Meaningful activities that integrate lesson concepts (for example, interviews, letter writing, listening, and/or speaking)

Building Background
Concepts explicitly linked to students' background experiences
Links explicitly made between past learning and new concepts
Key vocabulary emphasized (for example, introduced, written, repeated, and highlighted for students to see)

Comprehensible Input
Speech appropriate for students' proficiency level (for example, slower rate, enunciation, and simple sentence structures for beginners)
Clear explanation of academic tasks
A variety of techniques used to make content concepts clear (for example, modeling, visuals, hands-on activities, demonstrations, gestures, body language)

Strategies
Ample opportunities provided for students to use **learning strategies**
Scaffolding techniques consistently used, assisting and supporting student understanding (for example, use think-alouds and conversation starters as described below in Section 10.6.3.)
A variety of **questions or tasks that promote higher-order thinking skills** (for example, literal, analytical, and interpretive questions)

Interaction
Frequent opportunities for **interaction** and discussion between teacher and student and among students, which encourage elaborated responses about lesson concepts
Grouping configurations support language and content objectives for the lesson
Sufficient **wait time for student responses** consistently provided
Ample opportunities for students to **clarify key concepts in Lesson 1** (the student's home language) as needed with aide, peer, or Lesson 1 text

Practice and Application
Hands-on materials or manipulatives provided for students to practice using new content knowledge
Activities provided for students to **apply content and language knowledge** in the classroom
Activities integrate all **language skills** (for example, reading, writing, listening, and speaking)

Lesson Delivery
Content objectives clearly supported by lesson delivery
Language objectives clearly supported by lesson delivery
Students engaged approximately 90% to 100% of the period
Pacing of the lesson appropriate to students' ability levels

Review and Assessment
Comprehensive **review of key vocabulary**
Comprehensive **review of key content concepts**
Regular **feedback** provided to students on their output (for example, language, content, work)
Assessment of student comprehension and learning of all lesson objectives (for example, spot-checking, group processing) throughout the lesson

10.5.3 Developing and Reinforcing Vocabulary: Selected Accessibility Strategies for English Language Learners

Increasing English language learners' accessibility to lesson content involves a variety of strategies with the same basic principle: consider the language demands of a lesson and incorporate language-related strategies for helping students access the core *mathematics* of the lesson. In other words, provide students with enough language support so that their time with the lesson can focus on the mathematical ideas rather than interpreting the language.

As English language learners hear certain words in mathematics class, they will find it challenging to process and associate appropriate meanings. For example, the differences between these pairs of words are subtle: *width* and

with, great and *greater,* and *hole* and *whole.* Other challenges will come with distinguishing terms that have different meanings in social and mathematical contexts. For example, the words, *similar, change,* and *difference,* have special meanings in the mathematical context—students will encounter expressions and terms such as *make change, change diagram,* and *how much change?* In social situations, they may hear *change in my pocket* or *change clothes.*

Finally, slight alterations to a word can significantly change the meaning (for example, *four* and *fourths* or *withdraw* and *withdrew.* Often it is difficult for English language learners to discern these differences. Other situations that may be problematic include:

- words that may be used as different parts of speech, such as *to estimate* (v) and *an estimate* (n).
- collocations, which are terms made up of two or more words that are commonly used together, such as *slow down, right away, fill in,* and *count by.*
- idioms, which are expressions that are not predictable from the usual meaning of the words, for example, *as the crow flies.*
- cultural terms that English language learners may not be familiar with, such as *zip code*, *area code*, *leap year*, *in the ballpark,* and so on.

Selected accessibility strategies in *Everyday Mathematics* to address these issues are highlighted below.

Using Flexible Grouping

Small-group or partner activities give students an opportunity to practice English in context and support comprehension as students can ask for clarification. Students who speak the same home language may deepen their comprehension of concepts by sharing and clarifying their understanding, and then transferring their learning to English.

Using Visuals

As the mathematics concepts become more complex, the level of abstraction and academic complexity of the language students need in order to discuss the concepts increases. Using visuals along with verbal explanations strengthens comprehension. Look for ways to support concepts with visuals and encourage English learners to demonstrate their understanding by adding visuals to their answers.

Using Diagrams to Solve Problems

Diagrams are plans or drawings designed to explain how something works or to clarify the relationship between the parts of a whole. Diagrams are tools that help English learners build meaning for mathematical concepts. They also help English learners communicate their mathematical thinking.

Using Structured Routines

Routines are built into *Everyday Mathematics* to help students work efficiently and effectively. Routines support better comprehension for English learners because the predictability allows them to listen more closely to the new concept rather than having to focus on the procedural information.

Using Reference Materials

Encourage English learners to use the *Everyday Mathematics My Reference Book* in Grades 1 and 2 and the *Student Reference Books* in Grades 3–6 along with other reference materials in print and online, such as encyclopedias, almanacs, and dictionaries (including bilingual dictionaries). For Spanish speakers, note that technical terms used in *Everyday Mathematics* may be similar to the Spanish words, which may enhance Spanish speakers' retention of new terminology. In the appropriate context, list English and Spanish words for students to build meaning, but do not assume that students understand the meanings of the Spanish word. Some examples are: *angle/ángulo, circle/círculo, parallel/paralelo, interior/interior,* and *polygon/polígono.*

Role Playing

An excellent way to deepen understanding of concepts is to give students the opportunity to apply what they have learned to a familiar situation. In one lesson, students simulate a shopping trip using mock Sale Posters as visual references and play money as a manipulative to practice making change. In this example, English learners can take turns being the shopkeeper and the customer. This role play helps students learn and practice the phrases and vocabulary they need in real shopping situations while gaining familiarity with the language needed to access the mathematics content of the lesson.

Tapping Prior Knowledge

English learners sometimes feel that they must rely on others to help them understand the instruction and practice in school each day. However, English learners bring unique knowledge and experience that they should be encouraged to contribute to the classroom community. For example, working with metric measurement and alternative algorithms present excellent opportunities for English learners to share their expertise with the group. Those who have gone to school outside the United States may know the metric system or other algorithms well. Explore this asset in your class and utilize your students' expertise.

Building Background Knowledge

Some problem contexts will involve general knowledge commonly known among students who have grown up in the United States but is unfamiliar to others. For example, English language learners may not be familiar with the names of cities or states that are included in a data table, or with the sports or family activities that may be described in the problem. In order to access the mathematics, students first need to understand the problem context. Strategies described in this section, such as using visuals or role play, can help students learn the required background knowledge so that they can focus on the mathematics.

Using Gestures

One way to reinforce the meaning of instructions is to use gestures to emphasize a phrase, word, or mathematical idea. In one lesson, students talk about clockwise and counterclockwise turns. Gestures that illustrate the direction will connect the words with the clockwise/counterclockwise concept. Simple gestures, such as pointing, are also powerful tools for emphasizing connections in a lesson, such as connections between representations involving manipulatives, graphs, diagrams, and number models.

For an example of a 4-Square Graphic Organizer, see Providing Organizational Tools in Section 10.3.5 *General Differentiation Strategies*.

Using Graphic Organizers

Graphic organizers can support English learners as they work with rigorous mathematics content. A Venn diagram, flow chart, storyboard, or sequence chart allows students to fill in vital information and show their understanding. Teachers can also use graphic organizers to facilitate discussions about mathematical ideas: such as a Venn diagram to compare and contrast the attributes of different quadrangles. This edition of *Everyday Mathematics* often suggests using 4-Square Graphic Organizers to help develop and extend understanding of vocabulary terms. Students may be asked to complete one to show pictures, examples, non-examples, and their own student definitions of a particular term. Visual anchor charts are another type of graphic organizer. They are classroom displays that focus on a central concept or skill in a unit. Teacher and students co-create the displays and post them in the classroom to serve as a visual reference for describing procedures, strategies, and terms. All graphic organizers are also located in the eToolkit Backgrounds for work in the Student Learning Center.

Using Total Physical Response (TPR)

Total physical response is a language-teaching method developed by James Asher, and is often used with English language learners, especially beginning English language learners. Students make use of multiple senses, as they listen and watch an action modeled and named by the teacher, using a think-aloud statement, such as: *This is 23.* The teacher uses guided practice to model and have students imitate her actions, saying: *Let's find 23. Let's write 23.* Students are then prompted to follow simple commands stated by the teacher, such as: *Show me 23 on the number line.* Total physical response activities help students concretely experience use of terms, in addition to hearing the language and usage of terms. They also require students to interpret the commands and provide a limited, often non-verbal response. In *Everyday Mathematics*, total physical response activities are often followed with short questions, such as: *What number is this? Is this 23?* Such questions should be preceded by several examples of modeling, joint practice of teacher and students, and student solo practice.

Using Think-Pair-Share

Think-Pair-Share is a widely used strategy that is particularly useful with English language learners because it allows students time to process and practice a response and share it first with a partner, which may be less threatening than attempting to communicate with a larger group.

For more information, see Section 5.1 *Facilitating Discussions*.

Using Conversation Starters

Students often have difficulty getting started with their oral and written communication. Language is used differently in a formal academic setting, such as a mathematics class, than in everyday discussions. Sentence frames can help students begin their oral or written responses and can teach the accepted syntax of mathematics classes. Examples: "I think $\frac{3}{4}$ is greater than $\frac{3}{5}$ because _____. First I _____. Then I _____. These are alike because _____."

Going beyond sentence frames to conversation starters encourages students to engage in focused and sustained academic discourse, promoting skills such as knowing how to justify ideas, negotiate meanings, and seek clarification. For

example, teachers can structure sustained conversations between pairs of students by providing prompt-and-response sentence starters to explain and support their reasoning.

Partner A: "Can you explain why you _____?"

Partner B: "Since we have _____, I _____. Did you see it differently?"

Partner A: "Yes, I _____. Can you clarify why you _____?"

Partner B: "I used the rule, _____. What rule did you use?"

Such conversation starters are particularly beneficial to intermediate and advanced English language learners. They facilitate student use of academic language and go beyond their expertise with conversational English.

Using Think-Alouds

In Think-Alouds, the teacher shares what he or she is thinking while demonstrating a concept. With this technique, the teacher models the appropriate use and pronunciation of vocabulary and connects the oral language with words, numbers, and other representations of a mathematical situation. For example, a teacher may state: *I am going to exchange ten cubes for one long. I take away the ten cubes and add a long to my collection. Now I have three longs and seven cubes. I will write a 3 on my place-value chart…* While doing the Think-Aloud, the teacher is using gestures to connect the spoken words with the manipulatives, the place-value chart, and the written numbers. The teacher can also use voice inflection to emphasize important vocabulary.

Accepting Nonverbal Responses

Some English language learners who may be unable or reluctant to provide oral responses may be able to demonstrate their understanding with nonverbal responses, such as pointing to a preferred answer, writing their solution on a slate, or demonstrating an idea using manipulatives or drawings. Encouraging and accepting nonverbal responses can help English language learners develop confidence in their ability to participate in mathematics class.

Using Cognates

An important component of accelerating academic language acquisition for English learners is teaching strategies for learning new words and concepts in English. Most English learners begin to acquire English by comparing new words and elements to the language they already know. For students who speak a primary language other than English that shares Greek, Latin, or Arabic roots, there are many prefixes, suffixes, and cognate patterns that they can use to learn new academic English words.

Cognates are words that share roots across languages. They have the same meaning and may be spelled similarly in other languages. An example of cognates are words that end with *-tion* in English, which share the same root with Spanish words ending in *-ción,* such as the names of the mathematical operations: *addition/adición, division/división,* and *multiplication/mulitiplicación.* When students look for these word patterns, they can more quickly ascertain the meaning of the English word by applying what they know from a related language.

Building Academic Language

Academic language refers to language used in the learning of academic subject matters in formal school contexts. In contrast, social language refers to talk in other daily interactions. English learners generally become proficient in conversational English more quickly than in academic language. Teachers can accelerate students' acquisition of academic language by making it a focus of planning and intentional teaching.

Everyday Mathematics offers many suggestions for developing students' academic language. These suggestions, often found in Academic Language Development notes, are intended for all learners, not just English language learners.

For more information about how *Everyday Mathematics* supports development of academic language, see Section 5.1.2 *Academic Language Development.*

10.6 Summary

Effective differentiation is easily discussed but hard to enact. It requires careful assessment of students' progress, misconceptions, strengths, and needs. Teachers then need to translate that knowledge into a set of readily available strategies that enable students at different places in the learning continuum to access the core mathematics and, when feasible, extend it. Meaningful differentiation is not possible in the absence of a rich curriculum, such as *Everyday Mathematics*, that allows students to approach and solve problems in different ways and provides many ways to assess student understanding.

The strategies for differentiation discussed in this section are not a "magic bullet" that will make differentiation simple. Collectively, however, they create a sufficiently robust toolbox for teachers that make effective differentiation achievable every day.

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