Curriculum Matters: What You Get is What You Teach

Analysis of data from the Third International Math and Science Study (TIMSS) provides evidence that many American students are greatly disadvantaged in both math and science.

TIMSS is the most extensive and far-reaching cross-national comparative study every attempted. It was conducted in 1995, with 42 countries participating in at least some part of the study. TIMSS tested three student populations:

- Those who were mostly nine years old (grades three and four in the U.S.)
- Those who were mostly 13 years old (grades seven and eight in the U.S.)
- Students in the last year of secondary school (12^{th} grade in the U.S)

At the fourth-grade level, the U.S. did reasonably well on the TIMSS exam and scored decently in mathematics. Our performance was above average, though not in the top tier of countries.

By eighth grade, however, the U.S. dropped to slightly below average in mathematics. In other words, just four years along in our educational system, our scores fell to below average. **Even our best students did not do OK.** A comparison of scores in 22 countries revealed that U.S. eighth-graders who scored at the 75^{th} percentile were actually far below the 75^{th} percentile in 19 of the other countries. The most dramatic results were in comparison to Singapore—a score at the 75^{th} percentile in the U.S. was below the 25^{th} percentile in Singapore.

This decline continues so that by the end of secondary school, our performance is near the bottom of the international distribution. In both math and science, our typical graduating seniors outperformed students in only two other countries: Cyprus and South Africa.

Part of the 12^{th}-grade TIMSS study involved advanced students, those taking courses like calculus or college-preparatory physics. The results are quite startling: We are near the bottom of this international distribution also.

Some people might ask, "What difference does it make if we can't do fancy math problems?" It does make a difference. A typical problem on the TIMSS 12^{th}-grade math test shows a rectangular wrapped present, provides its height, width, and length, as well as the amount of ribbon needed to tie a bow, and asks how much total ribbon would be needed to trace logically around the package. Students simply need to trace logically around the package, adding the separate lengths so as to go around in two directions and then add the length needed for the bow. **Only one-third of U.S. graduating seniors can do this problem, however.** This is serious.

Discussion of the TIMSS results has prompted researchers and policymakers to investigate look at the curricula in top achieving countries and try to distill how we can improve our curricula relative to others in the world.

One study, *A Coherent Curriculum* (2002), characterized the U.S. Curricula as "a mile wide, an inch deep." It found that math content in the U.S. relative to the rest of the world was different in four ways:

- **Our content is not focused**. You find more topics at each grade level than in any other nation.
- **Our content is highly repetitive**. We introduce topics early and then repeat them year after year. In contrast, top achieving countries focus on an early emphasis on arithmetic in grades one through four, gradually evolving to more advanced algebra and geometry beginning in grades seven and eight. To make matters worse, very little depth is added each time the topic is addressed because we devote much of the time to reviewing the topic.
- **Our intended content is not very demanding** by international standards, especially in the middle school. The rest of the world follows a three-tier pattern of complexity. The first tier places an emphasis primarily on arithmetic, including common and decimal fractions. It is covered in grades one through four. Grades five and six serve as an overlapping transitional tier with continuing attention to a few arithmetic topics, but also with an introduction to more advanced topics such as percentages, negative numbers, proportional concepts and problems. The third tier, covered in grades seven and eight, consists primarily of advanced number topics (primes and factorization, rational numbers and their properties). The curriculum structure also includes a small number of topics that provide continuity across all three tiers, such as measurement, that seem to support the overall curriculum structure.
- **Our intended content is incoherent**. Math is really a handful of basic ideas; but in the U.S., mathematics standards are long laundry lists of seemingly unrelated, separate topics.

The TIMSS achievement results also prompted policymakers to review state standards. *The State of State Math Standards* by six prominent mathematicians identified nine common problems in state standards and, by implication, the textbooks written to align with these standards. It is a testimony to the high standards of SRA's Direct Instruction authors, that our DI math programs, ***Connecting Math Concepts*** **(CMC)** and ***Corrective Mathematics*** **(CM)** address these problems.

Nine Common Problems in State Mathematics Standards	**How DI Math Programs Address Each Problem**
1. The standards put an excessive emphasis on calculator use. *Calculators enable students to do arithmetic quickly, without thinking about the numbers involved in a calculation. For this reason, using them in a high school science class, for example is perfectly sensible. But for elementary students, the main goal of math education is to get them to think about numbers and learn arithmetic. Calculators defeat that purpose.* *The State of State Math Standards*	**CMC** introduces calculators in Level C. Student are shown how to use calculators to work problems. They then use calculators to check their work. **CM** is a supplemental, intervention program. It assumes that the core instructional program includes adequate instruction on the use of calculators. Therefore CM modules do not address calculator skills.

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2. There is no expectation for students to memorize basic facts. *Memorizing the "basic number facts," i.e., the sums and products of single-digit numbers and the equivalent subtraction and division facts, frees up working memory to master the arithmetic algorisms and tackle math applications. Research in cognitive psychology points to the value of automatic recall of the basic facts. Students who do not memorize the basic number facts will founder as more complex operations are required, and their progress will likely grind to a halt by the end of elementary school. There is no **real** mathematical fluency without memorization of the most basic facts.* *The State of State Math Standards*	**CMC** teaches facts as a both a member of a number family and of a fact series. • Number families consist of 3 numbers that go together to form a basic fact. • Fact series present facts in order to show their relationship to counting. • Ample oral and written practice ensures that facts are memorized. • Mental arithmetic is used to support work with facts. • Optional BLM's for Levels A-D provide opportunities for additional practice. **CM** teaches facts as both a member of a number family and a fact series. • Number families consist of 3 numbers that go together to form a basic fact. • Fact series present facts in order to show their relationship to counting. • Fact Games, Timing Formats, BLM's and ensure that facts are memorized.
3. No expectation for children to learn the standard algorithms. *The standard algorithms are powerful theorems and they are standard for a good reason: They are guaranteed to work for all problems of the type for which they were designed. Knowing the standard algorithms, in the sense of being able to use them, is a foundational skill for elementary schools students. Students who master these algorithms gain confidence in their ability to compute. They know they can solve any addition, subtraction, multiplication, or division problem without relying on a mysterious black box, such as a calculator. Moreover, the ability to execute the arithmetic operations in a routine manner helps student to think more conceptually and are well positioned to understand the meaning and uses of other algorithms in later years.* *The State of State Math Standards*	**CMC** teaches coherent routines that allow students to handle a variety of computation problems. • Instruction is carefully sequenced and scaffolded to allow students to handle a wide variety of computation problems. **CM** teaches coherent routines that allow students to handle a variety of computation problems. • Instruction is carefully sequenced and scaffolded to allow students to handle a wide variety of computation problems. • Special routines help students deal with difficult problems such as borrowing from zero in subtraction.

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4. The standards pay too little attention to the coherent development of fractions. *In general, too little attention is paid to the coherent development of fractions in the late elementary and early middle grades, and there is not enough emphasis on pencil-and-paper calculations.* *The State of State Math Standards*	**CMC** introduces fractions in Level C. • Initial instruction relates fractions to number lines, thus building on number sense developed in Levels A and B. • Work with fractions is carefully linked to multiplication and division. • Starting in Level D, fractions are related to equivalent fractions, to whole numbers, to decimal notation and to ratios and proportions. • Coherent routines are introduced to teach students to add, subtract, multiply and divide fractions and mixed numbers. • Early work uses very simplified language that does not confuse students. **CM** pays special attention to the development of fractions and decimals in two modules: • *Basic Fractions* presents a robust model that conveys a precise meaning for fractions. • Visual examples help students understand what happens when fractions are added. • The module also teaches student to add and subtract fractions with common denominators. • *Fractions, Decimals, and Percents* teaches students to add and subtract fractions with unlike denominators and to multiply and divide fractions. • Fractions are related to decimals and percents. • Both modules provide careful teaching and systematic practice to create steady, measurable progress.

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5. There is too much attention given to patterns. *The attention given to patterns in state standards verges on the obsessive. Students are asked, across many grades to create, identify, examine, and find "the rule" for repeating, growing, and shrinking patterns, where the patterns may be found in numbers, shapes, tables, and graphs . . . The attention given to patterns is far out of balance with actual importance of patterns in K-12 mathematics.* *The State of State Math Standards*	**CMC** emphasizes the connections between related mathematical concepts, such as addition/subtraction and fractions/decimals/ratios rather than patterns. • Level A builds on the aspect of mathematics most familiar to young children, counting, and uses counting skills to teach concepts of more, less, place value, and addition. • Counting, place value, and more/less are integrated in the teaching of addition. • Money problems are used to extend numerical concepts (e.g. counting, addition, place value) to problems relevant to everyday experiences. **CM** is a supplemental, intervention program. It assumes that the core instructional program includes adequate work with patterns. Instead CM modules emphasize work with facts, calculations, and problem-solving, problems commonly identified in students needing remediation.
6. The use of manipulatives is overemphasized. *Manipulatives can be helpful in introducing new concepts for elementary pupils, but too much use of them runs the risk that students will focus on the mainpulatives more than the math, and even come to depend on them.* *The State of State Math Standards*	**CMC** Levels A-C include a manipulative activity that can be used to strengthen concepts and operations introduced in the program. The manipulative activities are not used for initial instruction. Instead they are presented as *additional* activities that help students relate what has been taught to different types of problem or conceptual formats. **CM** is a supplemental, intervention program. It assumes that the core instructional program includes adequate work with manipulatives and does not include work with manipulatives in modules.

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7. Overemphasis on estimation at the expense of exact arithmetic calculations. *Fostering estimation skills is a commendable goal shared by all math programs. However, there is a tendency to overemphasize estimation at the expense of exact arithmetic calculations. For simple subtraction, the correct answer is the only reasonable answer. The notion of "reasonableness" might be appropriate in connection with measurement, but not in connection with arithmetic of small whole numbers. The main goal of elementary school math is to get students to think about numbers and to learn arithmetic. Hand calculations force students to develop an intuitive understanding of place value and of fractions.* *The State of State Math Standards*	**CMC** teaches arithmetic calculations through carefully sequenced and strongly scaffolded lessons. This instruction is characterized by: • Skills developed in small steps. • Ample opportunities for practice so students become facile in applying new skills. • Introducing all necessary preskills before operational routines. • On-going review, practice and application. **CM** provides articulated steps for teaching standard algorithms • Cognitive processes required by students are taught overtly • Routines address various problem types students are likely to encounter • Students learn all necessary component skills (preskills) prior to the introduction of a routine
8. Probability and statistics introduced too early (often Kindergarten) instead of being delayed until students can build on their knowledge of fractions and ratios. *Sound math standards delay the introduction of probability until middle school, then proceed quickly by building on students' knowledge of fractions and ratios.* *The State of State Math Standards*	**CMC** introduces the concept of probability in Level D and is initially based on students' understanding of fractions. • Problem types are introduced that teach students to express probability as fractions and proportions. • Students develop a representational framework that allows them to make sense of statements such as: the odds are 3 to 2, or the probability of winning is 1 in 7,240,000. • They also see the power of making predictions and drawing accurate conclusions based on empirical observations. **CM** teaches ratios and equations but does not introduce probability and statistics.

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9. Failure to develop problem solving strategies in a straight-forward manner. *Problem-solving is an indispensable part of learning mathematics. Children should be able to solve single-step word problems in the earliest grades and deal with increasingly more challenging, multi-step problems as they progress. Too often, programs fail to develop important prerequisites before introducing advanced topics.* *The State of State Math Standards*	**CMC** provides students with an increasing repertoire of problem solving strategies that are fine-tuned to the types of problems they are required to solve. • In Levels A-C, students learn the concept of number family. • They learn to relate word problems to operations completed using the number line. required to solve the problem. • Starting in Level C, they learn a precise strategy for determining which operation is required and to represent word problems visually before determining the operation and solving the problem. • They learn to represent the values dealing with equal-sized groups as multiplication/division. • In Level D, students learn that multiplication and division situations may also be solved with a ratio and proportion strategy. • As they develop confidence in solving word problems, they also learn that there may be more than one method to solve problems that are equally successful, including using tables to organize and interpret data and working backwards. **CM** (like CMC) teaches a precise strategy for determining which mathematics operation is required by a given story problem—a feature not typically shared by other mathematics programs. • Work with word problems is taught as an extension of number families. • Students are taught a precise strategy for determining the operation to be performed—e.g. if the problem gives the big number, you subtract. • The problem-solving strategy is applied to a wide-variety of problem types. • Furthermore, specific preskills each problem type are carefully taught.