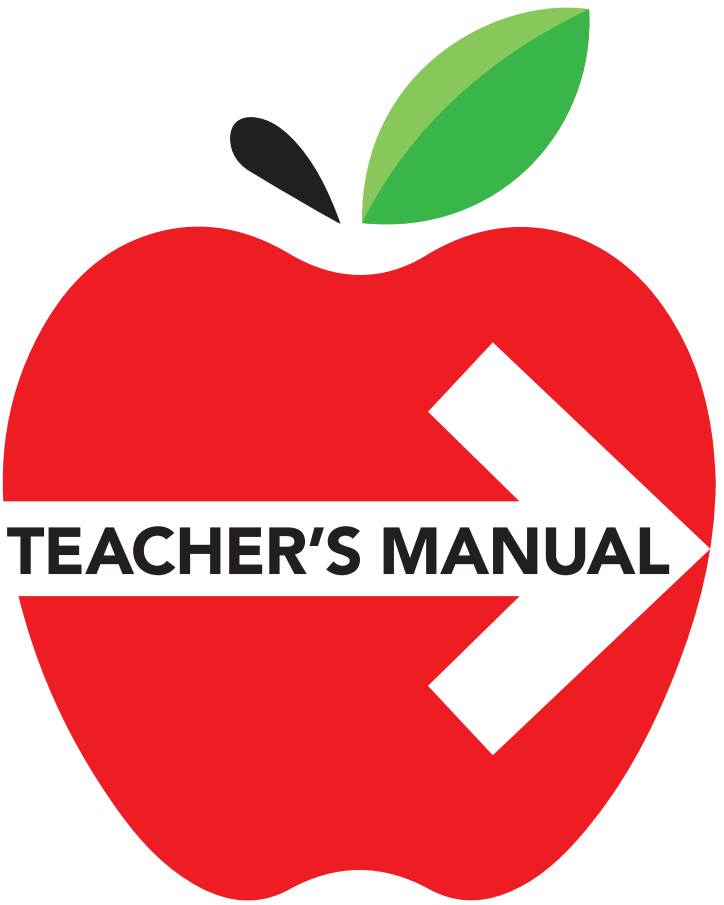
5 Steps to Teaching AP Precalculus



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Thanks to Greg Jacobs, an AP Physics teacher at Woodberry Forest School in Virginia, for developing the 5-step approach used in this teaching manual. Thanks also to Courtney Mayer, an AP Environmental Science teacher, for creating a sample teacher's guide that AP teachers could use to create their own guide. Finally, thanks to Deborah Klipp for helping me produce this manual.

Introduction to the Teacher's Manual

5 Steps to a 5 is pleased to be one of the first resources for teachers for their AP Precalculus class. No longer limited to just the teacher and the textbook, today's teachers can utilize online simulations, apps, computer-based homework, video lectures, etc. Even the College Board itself provides so much material related to the AP Precalculus exam that the typical teacher—and student—can easily become overwhelmed by an excess of teaching materials and resources.

This book is a vital resource for your class because it explains in straight forward language exactly what a student needs to know for the AP Precalculus exam and provides a review program students can use to review for the test. This teacher's manual will provide guidance on using this book in your class and advice on utilizing the best practices and best resources in your class.

This teacher's manual will take you through the 5 steps of teaching AP Precalculus. These steps are:

1. **Prepare a strategic plan for the course**
2. **Hold an interesting class every day**
3. **Evaluate your student's progress**
4. **Get students ready to take the AP exam**
5. **Become a better teacher every year**

I'll discuss each of these steps, providing suggestions and ideas that I use in my class. I present them here because, over the years, I found that *they work*. You may have developed a different course strategy, teaching activities, and evaluation techniques. That's fine; different things work for different teachers. But I hope you find in this teacher's manual something that will be useful to you.

STEP 1

Prepare a Strategic Plan for the Course

The Course and Exam Description (CED) from the College Board, which can be found at: **https://apcentral.collegeboard.org/courses/ap-precalculus/course**, lays out a suggested scope and sequence for the AP Precalculus class. The College Board has set it up in a way that topics and skills build as the year goes on. Over the years as I've worked with teachers at AP Summer Institutes, AP Workshops, and AP Readings, our suggestion has been to stick with the suggested scope and sequence, as it probably follows your current Precalculus textbook.

After you have taught the course for a few years and feel comfortable with the material, you may want to move topics around to better meet your classroom needs. One example is that in unit 3, I introduce vectors, which is found in the optional unit 4, topic 8. Vectors allow me to easily prove the Law of Cosines. Your local standards may require you to teach some topics in unit 4, such as Matrices or Conic sections, so feel free to sprinkle in those topics as you follow your current textbook layout or where you feel they naturally belong. You may also decide that a generic lesson at the beginning of the year focusing on transformations graphically or in tabular form can be helpful before completing specific transformations of polynomials, exponential or trigonometric functions fully introduced in unit 1, unit 2, or unit 3 respectively. This gives students the opportunity to practice and learn in an abstract form before applying the learning to known function and allows me the ability to

see if my students are struggling with the idea of transformations or a specific function, like a logarithmic function, is the problem.

The chart below shows the units and the time suggested for each unit in the CED. The number of class periods is based on a typical 45-minute class. If your school is on a form of block schedule or other non-typical schedule, you will need to adjust the pacing to fit your class needs.

As you plan your year, make sure to leave plenty of time for review. We like to leave at least one month of dedicated review time just before the test. Although you should be giving students practice with released multiple-choice and free response questions at the end of each unit, allowing students a full month of practice helps prepare them for the types of questions they will see on the AP exam.

TOPICS	PACING	5 STEPS TO A 5
Unit 1: Polynomial and Rational Functions	6 – 8 weeks	Chapter 5 Pgs. xx-xx
Unit 2: Exponential and Logarithmic Functions	6 – 9 weeks	Chapter 6 Pgs. xx-xx
Unit 3: Trigonometric and Polar Functions	7 – 10 weeks	Chapter 7 Pgs. xx-xx
Unit 4: Functions Involving Parameters, Vectors, and Matrices	7 weeks	Not currently available

STEP 2

Hold an Interesting Class Every Day

AP students should love coming to your class. Why? Because you should offer many opportunities and strategies to help students understand the material and internalize what they learn. I usually follow one of two models each day. When I introduce a new unit or concept, I use direct instruction, flipped instruction, or inquiry-based learning. I also give my students daily formative assessments to gauge their learning. These formative assessments are low-stakes opportunities or activities that require students to participate and collaborate with their peers.

- **Formative Assessments.** At the beginning of each class, instead of collecting the homework, my students do 3 or 4 questions that mimic the homework problems. If students are working individually, they are permitted to use their homework or notes to help guide them through the problems. When students are in groups, I require them to collaborate by allowing only one writing utensil, one calculator, and one piece of paper. At random, I will tell them to switch, so each student has a turn.
- **Direct Instruction.** I upload my notes to the students' learning management system and the students are required to download or print those notes on their own. This saves time, because I can address the important or often misunderstood sections of the notes, skipping over the things students can study and learn on their own. I hardly ever lecture for more than 15 minutes a day. The AP students can read and study my notes and can always come to me if they have any questions outside of class time.
- **Flipped Classroom.** I assign my students AP Daily videos to watch from AP Classroom or other sources for homework. As with direct instruction, I can address the important

or often misunderstood topics in class the following day.

- **Inquiry-Based.** This is used when the students are learning a new topic. Depending on the complexity of the lesson, the activity typically takes about half the class period, with time at the end of the period for another formative assessment opportunity. For more complex topics, the activity may take the entire period. The idea is to give the students hands-on experience doing math. I have a poster in my room that displays William Glasser thoughts on education: "We learn 10 percent of what we read, 20 percent of what we hear, 30 percent of what we see, 50 percent of what we see and hear, 70 percent of what we discuss, 80 percent of what we experience, and 95 percent of what we teach others." When students are struggling with the content of the course, all I do is refer to the activity and the main ideas come rushing back to them. Involve your students in the class and they will not only love the class but also excel on the AP exam.

If you have a classroom set of *5 Steps to a 5: AP Precalculus*, you can assign students homework of reading a few pages of the book that correlates to the topics you are teaching. You can assign them the review questions at the end of each chapter as homework. The book covers each unit and topic in the CED that are on the AP exam and all the review questions are aligned to the learning objectives.

The worksheets, classroom activities, and videos that you decide to use in your course will depend on your interests, time available, class composition, etc. However, there are some activities that are used in many AP Precalculus classrooms, and these are listed below by unit.

Here are some of my favorites:

TOPIC	ACTIVITY
Unit 1: Polynomial and Rational Functions	Desmos Activities: ▶ Polygraph: Twelve Functions ▶ 4.1-3 Rational Functions Other Activities: ▶ Always-Never-Sometimes ▶ Building the Model Solution
Unit 2: Exponential and Logarithmic Functions	Desmos Activities: ▶ Polygraph: Exponential & Logarithmic ▶ Match My Exponential Other Activities: ▶ Transformation Relay Game ▶ Logarithm War
Unit 3: Trigonometric and Polar Functions	Desmos Activities: ▶ Polygraph: Twelve Functions ▶ Graphing the Sine Function using Amplitude, Period, and Vertical Translation Other Activities: ▶ Odd One Out ▶ Polar Coordinate Battleship
Unit 4: Functions Involving Parameters, Vectors, and Matrices	

In addition, you might consider joining the AP Teacher Community or Facebook group that is specifically for AP Precalculus teachers. There you will find more ideas than you could possibly do in one year!

Looking for a fun way to prepare your students for the Free-Response Questions (FRQs)? Here are a few activities that have worked well in my classroom.

- **FRQ Mocktail Party:** Give each student a copy of the same FRQ. With only the question in their hand (no pens or pencils), have them walk around the room and mingle with their classmates, asking others about how they would approach the solution. After 10 minutes, students sit down and write out their solution individually. As a bonus, have snacks and beverages in your room for students to eat and drink as they walk around and mingle.
- **Relay Race:** Give each student a copy of the same FRQ and split the students into teams (3 on a team works best). Each team goes to a separate area of the classroom and has either a board to write on or a large pad of paper to write on. One student from each team has a writing utensil and goes to the board (or pad of paper). The next team member has a calculator. Those two teammates are not allowed to speak, only the teammate that does not have the pencil or calculator. Start a timer for 3 minutes and allow teams to write their solution to the FRQ. When the timer goes off, have teammates trade positions. The talker becomes the writer, the writer becomes the calculator, and the calculator becomes the talker. Continue to switch every three minutes until teams feel they are done with their solution. The first team to correctly write the solution to the FRQ wins. Players cannot look at another team's solution until the race is complete.
- **Building the model solution:** Using an FRQ, complete a step-by-step solution. Afterwards, write some steps with common mistakes that students make. Make copies of the correct steps and incorrect steps and cut them into strips. Have students in groups select the steps for the correct solution and have them piece the solution together.
- **Practice Grading FRQs:** At first, you grade your students FRQs using the scoring guidelines from College Board, pass the papers back to the students, and discuss how each point was earned/not earned. When students become familiar with the scoring guidelines, they should start grading their own solutions. Collect their graded papers and discuss any necessary discrepancies. You can also have students grade sample student responses that are found on the exam page of AP Central and they can grade responses from their classmates. Whether you use one of these grading techniques or all of them, the more familiar you and your students are with the scoring process, the more likely students will earn points for each part of their response. This is one of the most effective things that you can do to get the students ready for the exam. The days you spend doing this activity will make a huge difference in the way they approach the actual AP exam's FRQ.

STEP 3

Evaluate Your Students' Progress

As mentioned before, I am a big proponent of students doing. From day one, my students know about William Glasser's thoughts on education: "We learn 10 percent of what we read, 20 percent of what we hear, 30 percent of what we see, 50 percent of what we see and hear, 70 percent of what we discuss, 80 percent of what we experience, and 95 percent of what we teach others." My students are frequently in groups of at least 3 doing inquiry-based activities to discover a pattern rather than me standing in front of them reading from a textbook. I also believe that if my students are going to be successful on the AP exam, their grade in class should reflect their preparedness. It should be noted that to earn a 5 on an AP exam does not mean a student answered 90% of the questions correctly. While the first AP Precalculus exam has not been released, most AP classes score conversions indicate that approximately 75% or higher qualify for a 5 on the exam. That is in my class, a raw percent of approximately 80% is an A, and each subsequent cutoff for a grade level is approximately 10% to 12% lower than the previous one. Someone a long time ago decided that 90% was the cutoff for an A and that every subsequent grade is determined by subtracting 10%. I ask "WHY?"

I also believe that students crave structure, so I divide my grade into two categories: formative assessment and summative assessment. Formative assessments should be low stakes and a time for students to evaluate their progress, while summative assessments are when I as the teacher can see if my students have understood what I have been teaching. That is why my formative assessments are participation based daily 20-point grades that include items such as warm-ups, homework quizzes (I do not actually collect the students' homework), student inquiry activities, or tickets out the door that are not graded and recorded based purely on mastery of the material. For example, if I give a four-question homework check assessment, each question may be worth 3 or 4 points with the remaining points being participation points. Summative assessments are the only days my classes do not have a formative assessment grade and are culminating items such as chapter tests, semester projects, and the final exams. While I do not have a set percentage breakdown, formative assessments are typically between 35% and 40% of the overall grade and summative assessments are between 60% and 65% of the overall grade.

It is important to give students feedback in a timely manner. Giving them feedback on FRQs can be difficult to do especially if you have a large class. When a student must wait for the results the whole purpose of feedback is defeated. A student needs to gain confidence and develop study and test taking strategies. You need to know your student's strengths and weaknesses so you can adjust your lessons appropriately. One way to provide immediate feedback is to give students a FRQ in groups of two or three. This gives the students an opportunity to engage in academic conversion (see William Glasser's thoughts above) and gives you the opportunity to gauge your students' understanding while lighting the grading load. I also like to give students a FRQ individually and then have the students switch papers and grade their peers using the released grading guidelines. This allows the students an opportunity to learn how they will be graded at the AP Reading held in early June. When recording the grade for FRQs, recall that if you intend an A to be similar to a 5, you should consider the cutoff to be about 70 to 75% for an A, 55 to 60% a cutoff for a B, and 40 to 45% the cutoff for a C.

I recommend incorporating released AP questions from the College Board (found in AP Classroom) on your unit exams. You can also add questions from your textbook, from *5 Steps to a 5 AP Precalculus* book, or those you create yourself. However, there is no better resource than the people that write the test and there should be hundreds of questions in AP Classroom for you to use.

If you have a classroom set of *5 Steps to a 5 AP Precalculus*, you can assign a FRQ from one of the exams in the book that correlates to the unit you are teaching. You can go over the solution and grading of the problem with your class. Recall that the FRQ are similar each year and that question 1 and 2 require a graphing calculator. The first FRQ is Function Concepts which includes three parts and requires students to work with a variety of concepts. These may include function composition, inverse functions, function input-output values, zeros of a function, end behavior of a function, and identification of an appropriate function type to construct a function model. The second FRQ is Modeling a Non-Periodic Context using a real-life context. In Part (A), students use the given information to construct a function model by building a system of equations and finding the parameters using a method of choice. Function types include polynomial, piecewise-defined, exponential, and logarithmic. In Part (B), students calculate, apply, and reason with average rates of change and their units. In Part (C), students justify a conclusion about assumptions or limitations of the model. The third and fourth FRQ are non-calculator with the third Modeling a Periodic Context using a real-life context that is modeled by a sinusoidal function. In Part (A), students use the given information to identify coordinates of five labeled points on the graph of the sinusoidal function and its midline for two full cycles. In Part (B), students find the parameters of an analytical presentation of the sinusoidal function. Both Parts (A) and (B) require students to construct the sinusoidal model by using the context to determine the vertical dilation and vertical translation of the sine or cosine function (amplitude and vertical shift), and the horizontal dilation and horizontal translation of the sine or cosine function (period and phase shift). In Part (C), students answer questions about the behavior of the function and describe the change in the rate of change on a particular interval. The fourth FRQ is Symbolic Manipulations with several functions: exponential, logarithmic, trigonometric, and/or inverse trigonometric. Two parts of the question require students to solve equations using given functions. The third part of the question requires students to rewrite given function expressions in equivalent forms. In this question, students must (1) determine the exact values of expressions that can be obtained without a calculator (2) use algebraic methods and rules for exponents and logarithms to combine terms (3) show the work that leads to their answers in each part of the question.

It is vital that you have a FRQ style parts on each of your chapter exams. Getting students to practice responding to an open-ended question in the exam format that they will see on the AP exam is the best way to prepare them. Remember, the FRQs count 37.5% of the student's grade on the exam so practicing these on every unit exam you give is crucial.

STEP 4

Get Students Ready to Take the AP Exam

I have at least four weeks to dedicate to review for the exam in class. This means I have covered all three of the required units completely and we now have time to review. I want to emphasize, even though I am referring to reviewing for the exam, *I am really ensuring that they understand all the topics that are covered in a first-year Precalculus course in college.* The AP exam is simply the summative tool that is used at the end of the course.

In my AP classes, I start reviewing by giving a full-length exam found in AP Classroom to the students. For the first year of the AP Precalculus course, I intend to use a practice exam in AP Classroom. Since my class is only 57 minutes, it takes two days for them to take just the multiple-choice section, with 20 non-calculator questions on day 1. The next day, we finish the non-calculator section and then take the calculator section. The practice exam is not something I use as a summative exam; it is for diagnostic purposes only. You can have the students score their exam and fill out a chart like the one below. (This breakdown of questions by units comes from *5 Steps to a 5 AP Precalculus* Practice Exam 1.)

The student can simply put a check or an X over the number of the question if they got the question wrong. This allows the student to self-diagnose to which units they need to devote the most time, as well as determine if they struggle more with the calculator section or non-calculator section of the exam.

Unit 1 Non-Calculator	1	2	3	4	5	10	17	22	28		_____ correct/9
											% correct
Unit 2 Non-Calculator	6	7	9	11	13	14	20	25	26		_____ correct/9
											% correct
Unit 3 Non-Calculator	8	12	15	16	18	19	21	23	24	27	_____ correct/10
											% correct
Unit 1 Calculator	79	83	84	85							_____ correct/4
											% correct
Unit 2 Calculator	76	80	82	86							_____ correct/4
											% correct
Unit 3 Calculator	77	78	81	87							_____ correct/4
											% correct

I review using various activities in my class and allow the students to review in whatever ways work best for them. Here are some of my activities:

- **Activity 1: Review a Chapter in *5 Steps to a 5 AP Precalculus*.** If you have multiple copies of this book, students can use it to review. This is especially important if the diagnostic test showed there was a unit that they just didn't get the first time. The text in the book explains the subject, key terms are identified and defined, and review questions to check the student's understanding.
- **Activity 2: Make Your Own Review Card.** You know those laminated poster-like cards they sell to students to review? They can make their own now that they know what topics they need to study. Then they walk away with personalized study cards just for them. I provide the students with a blank file folder and a variety of markers.
- **Activity 3: Flash Cards.** I have a list of over 100 terms and formulas that are beneficial for students to know for the exam. A good place to start is the list of terms and their definitions at the end of each of the three units in *5 Steps to a 5: AP Precalculus*. The students then make flash cards on index cards of the words they need to review. An example is provided below:

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$

- **Activity 4: FRQs.** I have all of the released practice exam FRQs printed with the scoring guidelines attached (you can get this from AP Classroom). The students can pair up or work on their own reviewing the question, thinking about or writing a response, and then checking their work with the scoring guideline to see if they would have received each point. I will add the four released FRQs each year.
- **Activity 5: Released Multiple-Choice Questions.** Just like it sounds, I print every multiple-choice question that has been released on practice exams from the College Board and I sort them by unit. There is an envelope with each unit and type of problem written on the front, such as "Unit 1 non-calculator". Inside are all the questions ever asked that relate to Unit 1. Students draw a card, read it, see if they know the answer, and then flip the card over where I have written the answer and brief explanation on the back. Students can focus on the units and types of problem they found they need the most help with from their diagnostic exam. I will continue to add to the envelopes each year as questions are released from past exams in AP Classroom.
- **Activity 6: AP Classroom videos.** Students log into their own AP Classroom and watch the videos that correspond to the units and topics they found from their diagnostic exam that they needed the most help on. Students will need to use headphones, so they don't disturb others.
- **Activity 7: Circuit Training.** I love making circuit training worksheets and the students love doing them! The circuit worksheets are made with 12 to 20 questions. Students start with the first question and then proceed through the problems, placing #2 in the problem blank for the box with the answer to the first problem. They work on the questions and proceed until they reach the last question, which should have the answer from the first box. An example for part of the unit circle review is shown below.

Answer: $\frac{\sqrt{3}}{2}$ #1_____ $\tan\frac{2\pi}{3}$	Answer: $\frac{\sqrt{3}}{3}$ #_____ $\cos\frac{\pi}{2}$	Answer: $-\sqrt{3}$ #_____ $\sin\frac{\pi}{2}$
Answer: undefined #_____ $\cos\frac{5\pi}{3}$	Answer: $\frac{1}{2}$ #_____ $\sin\frac{2\pi}{3}$	Answer: 1 #_____ $\cos\frac{5\pi}{4}$
Answer: $-\frac{\sqrt{2}}{2}$ #_____ $\tan\frac{7\pi}{6}$	Answer: $-\frac{1}{2}$ #_____ $\tan\frac{3\pi}{2}$	Answer: 0 #_____ $\sin\frac{11\pi}{6}$

The question in the first box is $\tan\frac{2\pi}{3}$, which is $-\sqrt{3}$. Therefore, the student should put a #2 in the third box in the first row and then write the answer to $\sin\frac{\pi}{2}$, working their way through the problems until the answer is to the last question is $\frac{\sqrt{3}}{2}$. Circuits are formative for the student and provide instantaneous feedback to let the student know if they got the correct answer or not, as wrong answers are not listed.

- **Activity 8: Take a Practice Exam.** Students can practice using the practice tests found in AP classroom. If you have multiple copies of *5 Steps to a 5 AP Precalculus*, students can take the two full-length practice exams found at the end of the book and score themselves and read explanations for the ones they missed.

  Taking the diagnostic test found in chapter 3 of the book is especially useful for the students. A chart in the book, like the one provided above, allows them to reassess their weaknesses showing the progress they've made and the areas they still need to study. Since taking the test takes multiple days of class, a good option is to have the students do it at home and use the class time to evaluate their progress.

I'm sure you can come up with other activities to add to these, but these can help you get started. My philosophy is that the students know what they need and are mature enough to use this time wisely. I am also around to help with any individual questions they may have and to do a mini review of any topic they want my help with.

STEP 5

Become a Better Teacher Every Year

A good AP teacher tries to do better, regardless of how they measure success. If there is anything that didn't work as well as you had hoped this year, there's always next year to try to revise it or do something different. The message is the same whether you are a novice at AP or a veteran: Your goal is to become a better teacher every year.

A qualifying score on the AP exam is considered a 3 or better. I tell teachers that a 3 or better means "college done." They earned a score that, at most universities, will mean they get college credit and they do not have to take the class again. However, what about a 2? I say a 2 is "college ready." This means that the student that makes a 2 is ready to do very well in this course when they take it in college. They have the background knowledge, and, with just a little more time and practice, they will be ready to earn that A in their college precalculus class. Keep this in mind when you receive the score reports in July; you can be proud of those students who made a qualifying score but also celebrate those students who are college ready!

How do you judge success? There is no right or wrong answer to this. We all teach at different schools with different students. A few students come to you with strong Algebra 2 skills and, let's be honest, could probably pass the exam on day 1 with or without you depending on how in-depth their Algebra 2 course covered units 1 and 2. Most students may lack some of the skills or knowledge that is needed to do well on the AP exam. These students are the ones who need us most and, when they succeed, it is because of their effort and our support.

For all AP teachers, both new and experienced, the best thing you can do to improve is to use the Instructional Planning Report you receive after student scores are calculated. You can access this document in your AP Classroom. You get a breakdown of scores by unit, by question type (MCQ and FRQ), etc. This information is what you need to adjust your course for the next school year. If you notice that students struggled with a particular unit or skill, this is where you make changes. Perhaps you spend a little extra time on this unit or maybe you find new activities to use during class. Maybe you review this unit in class before next year's AP exam. What if students did well on the multiple-choice but not the free response questions? How can you get some additional professional learning to better teach them how to write? It may take a few years to see results, but with attention to the Instructional Planning Reports and with continuous adjustments to your class, your scores will soar!

It is also very important to attend an AP Summer Institute both as you begin to teach the course and every few years as you continue. To get new ideas and insights, I recommend that you take summer institutes from different instructors. I also encourage all AP teachers to apply to be a reader for the AP exam. The AP reading is the best professional learning experience you can receive. You can apply online through the College Board's website.

Additional Resources for Teachers

Make sure to always use the College Board's Course and Exam Description (CED) for the course (**https://apcentral.collegeboard.org/media/pdf/ap-precalculus-course-and-exam-description.pdf**). If the topic is in the CED, it will be on the AP test. If the topic is not in the CED, it is out of scope for the course and will not be tested. This page has links to other College Board resources including AP Central and the AP Classroom, where among other things, you'll find daily videos for each lesson. You can also share ideas and questions with other AP Precalculus teachers at **https://apcommunity.collegeboard.org/**.

Classroom Tools

Real-time formative assessment data: **https://plickers.com**

Exit-ticket: **https://socrative.com**

Kahoot!: **https://kahoot.com**

Quizlet: **https://quizlet.com search precalculus**

Desmos Activity Builder: **https://www.teacher.desmos.com search AP Precalculus**

Worksheets: **https://www.teacherspayteachers.com/search AP precalculus**

Calcmedic: **https://www.calc-medic.com/150-days-ap-pc**

I hope that you found this resource helpful. If you have any questions, suggestions, need additional information, or would like to report an error, I'd love to hear from you. E-mail me at **5stepsAPprecalc@gmail.com**