

Talking About Math: How K-12 Classroom Discourse Can Develop Mathematical Thinkers and Problem-Solvers

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Summary



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Introduction: A Shift to Discourse in the Classroom

For many adults, and even many of today's students, math class is a place where a teacher talks and students listen, responding only when called on to answer a question.

Much of this shift is attributable to differences in the nature of discourse in the classroom: Who talks, and what do they talk about? Who asks questions, and what kinds of questions do they ask? What are students thinking, and how do they back up their ideas? Such questions provide hints about the kind of classroom discourse that best supports student learning and how we can create an environment in which rich discourse can take place.

What Is Discourse, and Why Is It Important?

When we use the term “*classroom discourse*,” we are generally referring to the various types of written and spoken communication and discussions that happen in the classroom. But the ways in which we define discourse today go even deeper, and they include representing, thinking, interpreting, expressing, reflecting, agreeing and disagreeing, and even debating and arguing.

This kind of rich discourse offers students a way to express their ideas, reasoning, and thinking, and it also supports the deepening of that thinking as students must make conjectures and back up their ideas with evidence. Students learn to ask each other clarifying questions as they interact around mathematical ideas.

Accordingly, classroom discourse can be a central element of acquiring mathematical knowledge and understanding the nature of mathematics. Thus, it is worth exploring 1. the many ways students might communicate their thinking and reasoning, and 2. how teachers can structure discourse in the classroom to stimulate the development and extension of student learning.

“Standard for Mathematical Practice: Make sense of problems and persevere in solving them.”

Principles to Actions, National Council of Teachers of Mathematics (NCTM), 2014

25 years ago, NCTM described its vision of a teacher-structured, student-centered, discourse-rich classroom in its professional teaching standards. Today, that description has become even more important as we help students become mathematically proficient thinkers and problem solvers.

A teaching model that once seemed new to most people—one focused on students communicating their thinking and reasoning—is now showing up more and more in classrooms across the nation. Mathematics teachers and teaching experts have come to realize that for students to learn to think mathematically and be able to solve the range of problems they are likely to encounter, they must engage in productive, facilitated discussion and interaction around many different kinds of math tasks.

Who's Doing The Talking? Who Needs To Be?

There is growing consensus that students need frequent and regular opportunities to express their thinking and interact with others about their ideas. One way to ensure that this happens is by rethinking a teaching model used extensively in mathematics classrooms over the years.

Many educators were taught to learn to present a rule, procedure, or concept accompanied by examples, then work through guided practice with students before asking them to apply their new knowledge to one or more problems. This process is sometimes characterized as ‘I-We-You’—*I* (the teacher) explain, *We* (students and teacher together) work examples through guided practice, *You* (students) practice on your own.

Today, we know that students can benefit from wrestling with challenging problems. *Productively struggling* lays a foundation for thinking about mathematical ideas and developing their reasoning (Pasquale, 2015; Warshauer, 2014; NCTM, 2014; Seeley, 2015). If we start with a mathematically rich task or problem, we can turn the traditional ‘I-We-You’ model upside down to utilize a ‘You-We-I’ structure for teaching (Seeley, 2014), starting with a rich problem students don’t already know how to solve or a task they haven’t previously explored. In this kind of upside-down model, *You* (students) work on a problem, *We* (students with the teacher facilitating and probing) talk about what students think and their possible approaches to the problem, asking questions of each other and revealing their understanding, and *I* (the teacher) ensure that the resulting discourse culminates with students learning the intended mathematics of the lesson.

Providing students an opportunity to grapple with challenging ideas that they don’t already know, or problems that go beyond what they may have seen before, gives them something to communicate about. That communication, via written or spoken words or other representations, can clarify their thinking and reveal understandings and misunderstandings. Sometimes, in the process of talking through a solution or idea, a student may catch and often correct an error she or he has made. When a student develops a strategy or approach based on sound mathematical understanding, communicating what he or she did can increase the likelihood that the student will be able to use or adapt that strategy in future situations.

What About Students Who Are Not Proficient in English?

While all students can benefit from opportunities to talk about their thinking and reasoning, it may be even more important for students who are not strong in English to engage in discourse that goes beyond superficial language.

Dr. Susie Håkansson (2017) suggests that it is important for English language learners to engage in mathematical discourse focused both on receptive language functions (listening and reading) as well as productive language functions (speaking and writing). Having students who are not proficient in English use more language in the mathematics classroom, rather than less, may be counterintuitive. A teacher’s first inclination may be to give such students mathematics involving fewer words, focusing on numbers or symbols. But to do so would be to deny these students the opportunity to engage in rich problems and would put them at a disadvantage, potentially causing them to fall farther behind their English-proficient classmates.

Language experts remind us that teachers should learn to amplify and enrich, rather than simplify classroom language so that students have more opportunities to learn the intended concepts (Walqui & van Lier, 2010). Dr. Håkansson reiterates this call for amplification and suggests that it is important for teachers to go beyond simply focusing on words, phrases, vocabulary, and definitions to help students learn to use a full range of discourse tools to express and deepen their emerging mathematical understanding. She notes the importance of students using a variety of representations, including graphical and pictorial representations, as important components in their mathematical discourse. Representational tools can greatly aid all students—both those who are learning English and those who are proficient in English—in developing their mathematical language, as well as deepening their understanding of mathematical concepts, as described in NCTM’s representation standard (2000).

What Should Students Talk About?

The most productive discourse involves students thinking about, reasoning about, and making sense of mathematical ideas, problems, concepts, or even procedures. Engaging in mathematically rich tasks that challenge students somewhat beyond what they already know can set the stage for rich discourse around mathematical ideas.

If students are continually expected to share their thinking and listen to the thinking of others, they will become more accustomed to doing so. Over time, they become increasingly more sophisticated in their level of language and their depth of mathematical understanding (Chapin, O’Connor, & Anderson, 2013; Boaler, 2015). They get used to the notion that it’s ok to struggle a bit over a challenging problem, even to make a mistake, knowing that the discussion that evolves from their struggle is likely to lead to better understanding.

NCTM has suggested that the most effective student learning arises when tasks are carefully selected to elicit thinking and when the teacher poses questions that support the development of reasoning (NCTM, 1991; NCTM, 2014). However, they also note that tasks with a high cognitive demand may be challenging to teach and that teachers sometimes have a tendency to lower the level of the task during instruction (NCTM, 2014).

Some video examples of tasks that can engage students and elicit thinking are listed at the end of this paper. They include:

- Using a homemade video of a provocative situation to generate students’ questions and solutions (a mysterious hand reaches up to grab some cookies)
- Posing a fairly straightforward word problem (asking students to determine how many cups of red and blue paint are needed to make a certain quantity of a particular shade of purple)
- Analyzing each other’s work (involving solving linear equations)
- Determining whether there is a mistake in a teacher-presented situation (a pre-calculus graph)
- Asking students to make predictions about what might happen under certain conditions (changing the size of tires on a vehicle)

These examples demonstrate that there is no one particular kind of problem or task teachers should use as a foundation for rich classroom discourse. Effective tasks might be purely mathematical, might relate to something students could encounter outside of school, or might involve a clearly fanciful or invented situation. The teacher's role is to structure a task, or modify one, to ensure that it has multiple access points and the potential for varied approaches, while allowing students to productively struggle with challenging mathematical ideas that may go beyond their current understanding.

How Can Teachers Create Student-Centered, Discourse-Rich Classrooms?

Even before NCTM's teaching standards shined a light on the central role of rich tasks and classroom discourse, researchers had identified five important themes in teaching mathematics for long-term understanding (Lampert, 1989). Looking at these five themes today, they still ring true in thinking about creating a classroom community grounded in productive discourse. These include:

- The nature of the problems we pose (to be accessible yet challenging)
- Using multiple representations (for concepts, ideas, and problems)
- Engaging in dialogue (between students and the teacher and among students themselves)
- Diagnostic teaching (where teachers pay attention to students' learning on an ongoing basis)
- Collaboration among diverse participants (where new knowledge comes from a joint venture in the classroom rather than one-way dissemination of information)

Evidence continues to mount that a teacher's selection of tasks and the nature of discourse in the mathematics classroom, including the kinds of questions a teacher asks, have an effect on what students learn and on their depth of understanding (Hiebert & Wearn, 1993; Smith & Stein, 2011; NCTM, 2014). Thus, finding and modifying appropriate tasks and learning how to ask good questions and guide classroom discussions are central skills for helping students learn mathematics with lasting understanding.

Every day will not be the same in an effective mathematics classroom, with every lesson involving the same kind of task or the same extent of student discourse. But, every day should involve students making sense of what they're learning and communicating their mathematical understanding in some way. Throughout the year, there should be frequent and regular opportunities for students to engage in the kind of extended discourse around rich tasks that pushes them to extend their understanding as they learn new mathematics.

The teacher's role in orchestrating the flow of classroom discourse involves deciding when to tell, when to question, and when to correct, while determining when to intervene with guidance and when to let students wrestle with a mathematical situation (Kilpatrick, Swafford, & Findell, 2001). The teacher is the one who must determine how to keep the discourse productive by encouraging students to make assertions, offer solutions, address questions, and explain their thinking to their peers. It is the teacher who decides whose ideas might lead to the most productive discussion and learning, while creating an environment in which student's value every person's contributions.

Teachers can learn to ask questions that help students move through increasingly productive levels of mathematical discourse. To do so, teachers should focus on questions that stress mathematical thinking rather than just answers (“Why do you think so?” “Can you explain what you mean when you say that there will always be a leftover?”) and questions that probe for understanding (“How did you decide to multiply by 10?” “Where does the 7 in the equation show up in your graph?”). It is important to pay attention to both the quality of the discourse and also the content. The ways in which students share their ideas is important, but likewise discussion needs to be guided to address productive mathematical ideas (Bransford and Donovan, 2005).

If we want students to be willing to express their thinking and take the risk of sharing their ideas in front of their peers, we need classrooms where everyone’s contribution is valued and where it’s safe to take such risks. Creating such an environment calls for initiating community discussions about respecting each other in the classroom and establishing norms for how conversation and questioning will take place. Further, students need to accept that mistakes are likely to happen, and, in fact, that mistakes serve as a stimulus for rich discussions and powerful learning (Seeley, 2016). Jo Boaler (2015) reminds us that we learn more—and our brain grows more—when we make a mistake than when we get a right answer.

One particular challenge for teachers is how to balance the goal of accuracy in mathematics with helping students see the learning potential from mistakes. It is important to talk with students about both of these dimensions of our mathematics classes. There’s nothing wrong with striving for accuracy (within humane limits), while at the same time embracing mistakes. In fact, some mathematics problems, particularly in the area of mathematical modeling, may not yield a single, clear right answer. Too many adults today carry the belief that they’re just not ‘math people’ because of the fact that they have made mistakes in mathematics. We cannot underestimate the negative long-term effect of subjecting students to failure messages that interfere with their learning and their interest in learning mathematics. Nevertheless, getting to correct answers is a reasonable goal in mathematics. After all, mathematics is a discipline grounded in patterns and predictability, generating reliable and consistent answers to many problems.

How Can Others Support Teachers In Creating Discourse-Oriented Classrooms?

Many excellent teacher education and professional development programs help teachers learn skills in organizing classrooms rich in the kind of student discourse that leads to learning. We can recognize such excellent programs and expand other programs to support this kind of teacher learning.

Teacher evaluation and reward systems can be redesigned where necessary to value and encourage the kind of mathematics teaching expertise that pushes student-thinking while building a strong and deep foundation of conceptual and procedural knowledge.

Summary

Orchestrating rich, productive discourse in a mathematics classroom is certainly among the most important elements in mathematics teaching today. Classroom discourse about mathematics can be a tool for equity, a vehicle for developing reasoning, and an engine for lasting learning.

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Yet, learning to organize teaching around students' expression of, and interaction about, their thinking is not easy. Whether a teacher is coming straight from an excellent teacher education program or has been teaching for many years, it takes ongoing work and continued practice to implement this kind of meaningful teaching well. Students will also need to get used to a different kind of math class from what they may be used to, and as they gain experience expressing their ideas, the quality of their communication, the richness of their representations, and the level of their thinking will continue to improve.

If we want students to develop deep and lasting mathematical understanding, learn mathematics well, and gain a positive disposition to tackle mathematical problems, we must help teachers create classrooms in which students express, clarify, and justify their thinking on a regular basis.

Helpful Resources

Teachers can find more detailed guidance on how to create classrooms focused on meaningful understanding and how to orchestrate productive discourse from the sources listed below. The video resources offer concrete examples of the variety of ways research-based strategies for orchestrating discourse might be implemented.

Printed Resources:

5 Practices for Orchestrating Mathematics Discussions. Smith & Stein, 2011.

Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching. Boaler, 2015

Classroom Discussions in Math: A Teacher's Guide for Using Talk Moves to Support the Common Core and More, Grades K-6: A Multimedia Professional Learning Resource (3rd ed.) Chapin et al., 2013.

Video Resources:

<https://www.teachingchannel.org/blog/2016/05/13/modeling-with-math-nsf/>

A “Three-Act Lesson” for second grade on modeling subtraction after having students watch a brief homemade video showing a cookie monster eating some Oreos.

<https://www.teachingchannel.org/videos/ratios-and-proportions-lesson-sbac>

A sixth-grade lesson introducing ratios with a problem involving mixing purple paint. Produced by The Teaching Channel out of a collaboration between the Illustrative Mathematics Project and the Smarter Balanced Assessment Consortium.

<https://www.teachingchannel.org/videos/students-learn-from-mistakes-ccssmdc>

An eighth-grade lesson where students analyze and discuss each other’s mistakes on linear equations, representing a collaborative effort among teachers in planning, delivering, and reflecting on a lesson. Produced by The Teaching Channel out of a collaboration between the Illustrative Mathematics Project and the Smarter Balanced Assessment Consortium.

<http://mass.pbslearningmedia.org/resource/mtc13.pd.math.deb/encouraging-debate/>

A pre-calculus lesson led by a math teacher/debate coach, involving coordinates, including determining which parts of a graph were correct and which might have been incorrect. A PBS Learning Media high school video produced by WGBH/WGBY in Massachusetts.

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