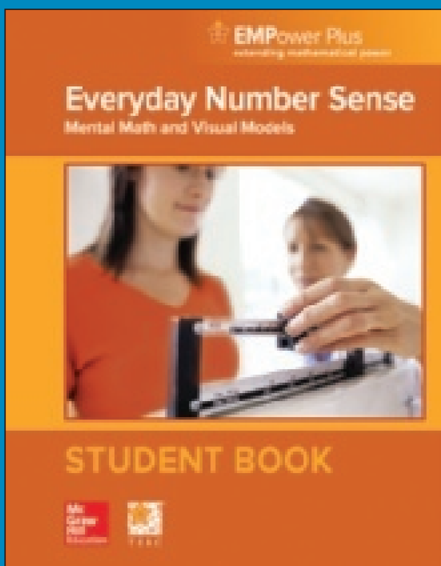
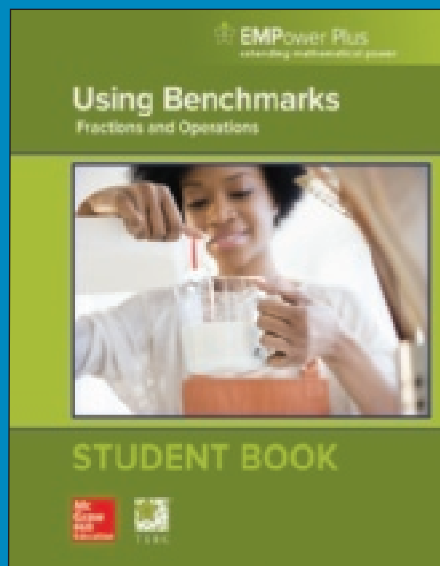
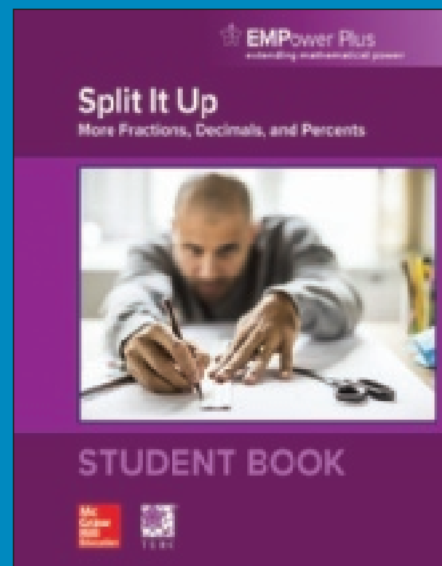
PROGRAM SAMPLER

EMPower Plus







A New Pathway to Mathematical Power

Mc Graw Hill Education

TERC

This Program Sampler includes:

- Sample lessons from each of the three new *EMPower Plus* units.
- An overview and synopsis of mathematical concepts covered for each title in the full *EMPower* series.

Dear Colleague:

When adults take the courageous step to return to the classroom, they deserve a high quality experience, one that respects their styles, intuitions, and experiences as well as one that acknowledges the roles they play as community members, workers, and parents. When youth who have fallen behind trust themselves and their teachers and give math learning another try, it is essential that they experience math instruction that respects their intelligence. Mathematics learning should be about developing connections, about making sense of mathematics as a system rather than as piecemeal processes and facts. Students deserve and need the best mathematics education possible, one that enables them to accomplish personal, lifelong-learning, and career goals in an ever-changing world.

EMPower Plus, like the original *EMPower* series, counters expectations of math class as a place for silent work to solve problems with one right answer. Learning with *EMPower* is different from the experience many of us had in traditional math classrooms. The differences are noticeable.

- The emphasis is on making sense of mathematics. Students think critically about the level of precision needed for different situations.
- Students gain flexibility, fluency, and accuracy.
- Students make and examine generalizations to support their understanding of the structure of number and operations.
- Students apply their number sense to explore everyday, algebraic, and geometric applications.
- Classrooms are learning communities, where participants share strategies, justify their reasoning, and interact with each other's ideas.
- The teaching experience is different. Teachers question and prompt generalizations. They uncover budding understandings and build from prior knowledge. This role is supported by many elements of the Teacher Book, an essential component of the *EMPower* program.
- *EMPower Plus* follows the research on the development of mathematical thinking to nurture mathematics learning. In addition, it aligns with updated emphases in new standards and high school equivalency tests.

EMPower's many contributors and its publisher invite you to join with teachers who wish to offer mathematically rich instruction that is personally relevant to students. We welcome teachers with varying levels of math background to join us in our ongoing endeavor to change the face of basic math teaching to a more active and empowering one for learners and teachers. You and your learners deserve the best!

Sincerely,

EMPower Plus Project Leaders

Donna Curry, Martha Merson, Myriam Steinback

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TERC is a not-for-profit education research and development organization dedicated to improving mathematics, science, and technology teaching and learning.

http://empower.terc.edu

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Contents

This Sampler provides you with an overview of the EMPower curriculum, and gives you Student and Teacher materials for one lesson from the three EMPower Plus units. You will also find an in-depth description of each unit, as well as a detailed outline of the mathematical concepts covered in each unit and lesson.

Contents

Getting Started with *EMPower Plus*

Background on *EMPower*

Extending Mathematical Power (EMPower) was the first math series that integrated mathematics education reform for educators, adult learners, and out-of-school youth. *EMPower* was designed especially for those students who return for a second chance at education by enrolling in remedial and adult basic education programs, high school equivalency programs, and developmental programs at community colleges. However, the currriculum is appropriate for a variety of other settings as well, such as high schoools, workplaces, and parent and paraprofessional education programs. *EMPower* builds interest and competency in mathematical problem solving and communication. The series serves as a model for a cohesive mathematics curriculum that offers content consistent with research and standards, including but not limited to the College and Career Readiness Standards and the Common Core Standards for Mathematical Practice. The curriculum fosters a pedagogy of learning for understanding; it embeds teacher support and is transformative yet realistic for multi-level classrooms.

New in *EMPower Plus*

This edition includes three fully updated books for students and teachers. Although *EMPower* users will recognize many of the activities from the first edition, we have also added new lessons and several opportunities for students to examine notation and algorithms (or methods) for solving problems with the four basic operations. In response to the increased mathematical rigor of the College and Career Readiness Standards for Adult Education and the new high school equivalency tests, the three updated *EMPower Plus* books—*Everyday Number Sense*, *Using Benchmarks*, and *Split It Up*—help students build a foundation of number and operation sense for algebraic thinking. The work of cognitive psychologists and mathematics education researchers is starting to show what adult math educators have long suspected: attention to conceptual understanding with opportunities to notice and talk about patterns and strategies increases students' flexibility with numbers and problem solving. In these updated *EMPower Plus* books, users will notice more opportunities for students to predict the results of operations and to see the connections between operations and their inverses (multiplying undoes division; square roots undo the effect of squaring). The excitement of learning math is in finding the meaning behind problem-solving steps that once seemed random. Recognizing mathematical properties at work has multiple benefits. Properties like the commutative property of addition and multiplication or the multiplicative identity make it possible for students to solve problems with understanding and with fewer errors. Recognizing the properties can lead students to value the strategies they use on a daily basis. Through *EMPower Plus* lessons, educators who have never thought much about operations, mental math, visual models, or benchmark numbers have opportunities to identify and encourage sturdy and reliable methods.

The Point of *EMPower*

EMPower consistently challenges students and teachers to extend their ideas of what it means to do math. The goal of *EMPower* is to help learners manage the mathematical demands of life by connecting situations and problems to mathematical principles. Situations include not only managing finance and commerce, but also interpreting news stories, applying health information, and facilitating family learning. *EMPower* is meant to be foundational, targeted to adults and young adults who test in the 4th-7th grade range who often have both areas of strength and gaps in their understanding. *EMPower* lessons introduce important mathematical ideas. They invite learners to identify patterns and to make connections. The lessons offer opportunities for students to explain their thinking and to justify their reasoning. Rather than focus on extensive, rote practice, *EMPower*'s main focus is to build learners' conceptual understanding, a critical platform from which they can explore more advanced concepts needed for future educational and career success.

In the following sections, you will read how *EMPower* shifts the culture of the classroom and how the lessons embody the College and Career Readiness Standards as well as the Common Core Standards for Mathematical Practice. The introduction provides an overview of the *EMPower* series as well as Frequently Asked Questions and Answers on both facilitation and the math content focus (number and operation sense with fractions) of the lessons within this book.

A student uses benchmark fractions as labels. The sketch shows evidence of reasoning about division of an 18-block walk into equal parts.

A Focus on Pedagogy and the Culture of the Classroom

Mathematics is meaningful within a social context. While mathematical truths are universal, the meaning and relevance of numbers change according to the setting and culture. *EMPower* classrooms become places where math ideas, strategies, hunches, and solutions are shared and discussed. The *EMPower* materials ask students to:

- Work collaboratively with others on open-ended investigations;
- Share strategies orally and in writing;
- Justify answers in multiple ways;
- Enter into and solve problems in various ways.

Key features of curriculum activities provide teachers with:

- Clear mathematical goals related to essential mathematics;
- Contexts that are engaging, challenging, and useful for adolescents and adults;
- Opportunities to strengthen learners' mathematical language and communication skills through productive struggle; and
- Puzzling dilemmas and problems that spark students' interest and motivate them to seek solutions.

These features make *EMPower* a resource for preparing students for tests of high school equivalency and community college coursework.

Students and teachers who experienced a traditional math education may find the expectations of *EMPower* take some getting used to. The chart highlights the contrast.

A rule-based approach emphasizes	*EMPower's* approach emphasizes
Computation, usually calculations by hand, with paper and pencil.	Mental math, visual models, and estimation using benchmark numbers, supported by a calculator when the numbers get unwieldy.
Procedures, often with limited attention to understanding; students practice following a given set of steps and then applying them to problems.	Conceptual understanding, sense-making, building on what students know, identifying patterns, and solving problems based on real or realistic contexts, all strengthened by connecting words, symbols, and visual models.
Completing the computational procedure correctly as evidence of understanding.	Being able to view the problem in a variety of ways, being able to communicate what the problem means or provide an example, and knowing what to expect as a sensible answer.
Applying the "correct" computational procedure to a problem.	Explaining and justifying their thinking, using strategies that illustrate flexibility and creativity when solving problems.

The Mathematical Practices and *EMPower*

EMPower's math background sections and unit introductions reference research in both K-12 and adult learning. In implementing *EMPower* lessons, teachers will foster the eight practices described in the Common Core and College and Career Readiness Standards for Mathematical Practice:

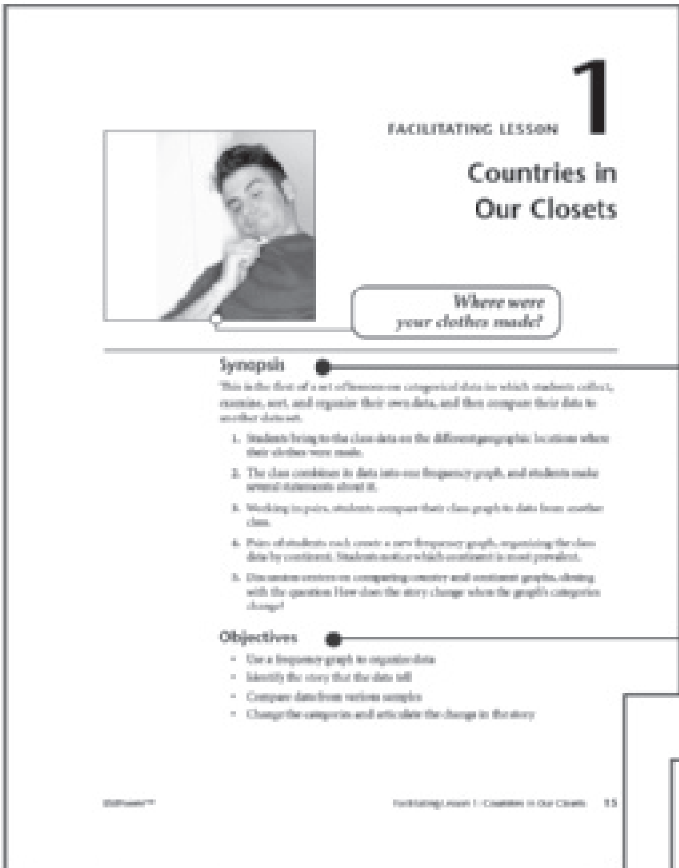
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Practices describe independent, proficient mathematical thinkers. The essence of the Practices is visible and audible in *EMPower* classes as students:

- generalize, to explain their reasoning;
- use tools such as number lines, arrays, fraction strips, Pattern Blocks, and calculators;
- work with estimates, rounding, and place value;
- use the structure of numbers, e.g., breaking numbers into 10's and 1's, to solve problems;
- examine algorithms, generalizing from patterns to form rules, reasoning, and justifying;
- make observations about notation and operations that help them solve problems more efficiently.

Overview of EMPower Units

Features of the Teacher Book


FACILITATING LESSON 1

Countries in Our Closets

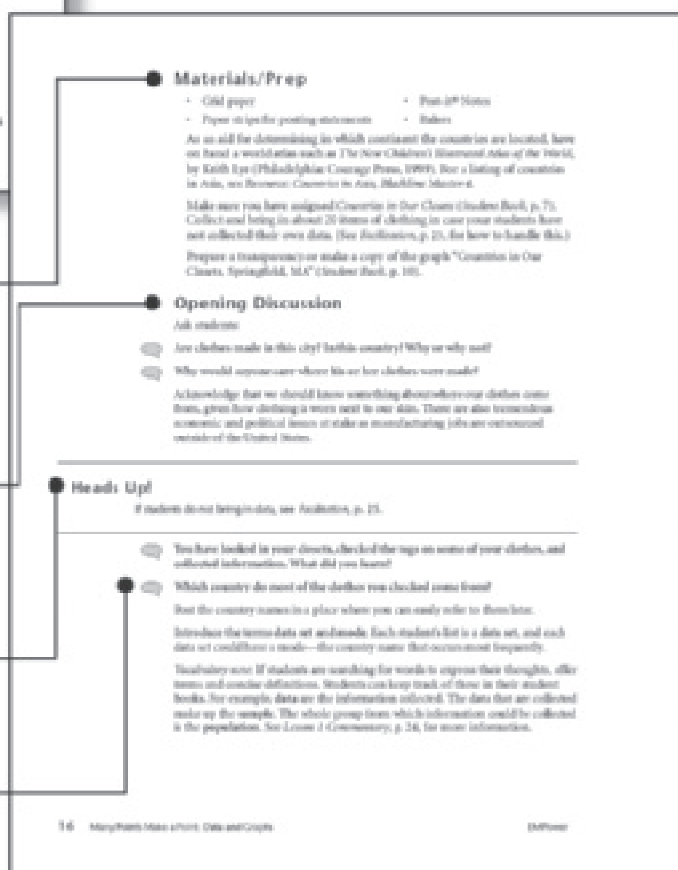
Where were your clothes made?

Synopsis

Objectives

Each lesson opens with a **synopsis** of the lesson activities.

Objectives summarize the skills students should gain from the lesson.


Materials/Prep

Opening Discussion

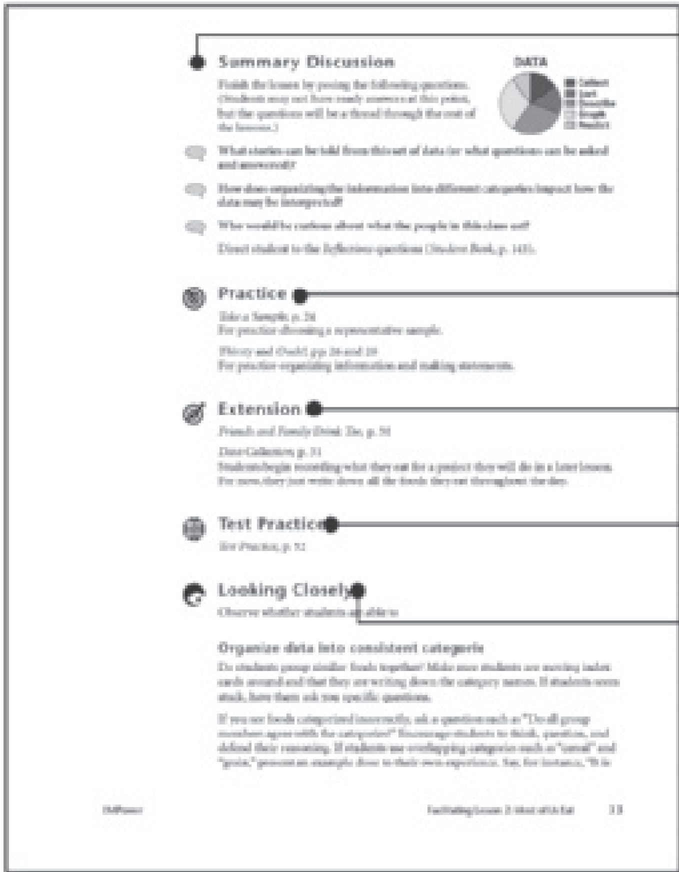
Heads Up!

The **Materials/Prep** feature tells teachers what materials they will need to prepare for the day's activities.

The **Opening Discussion** gives teachers ideas for engaging students at the beginning of the lesson.

Heads Up! alerts teachers to helpful hints and things to look for in facilitating the lesson.

Speech bubbles and bold text suggest statements and questions teachers can use to spark and guide class discussion.



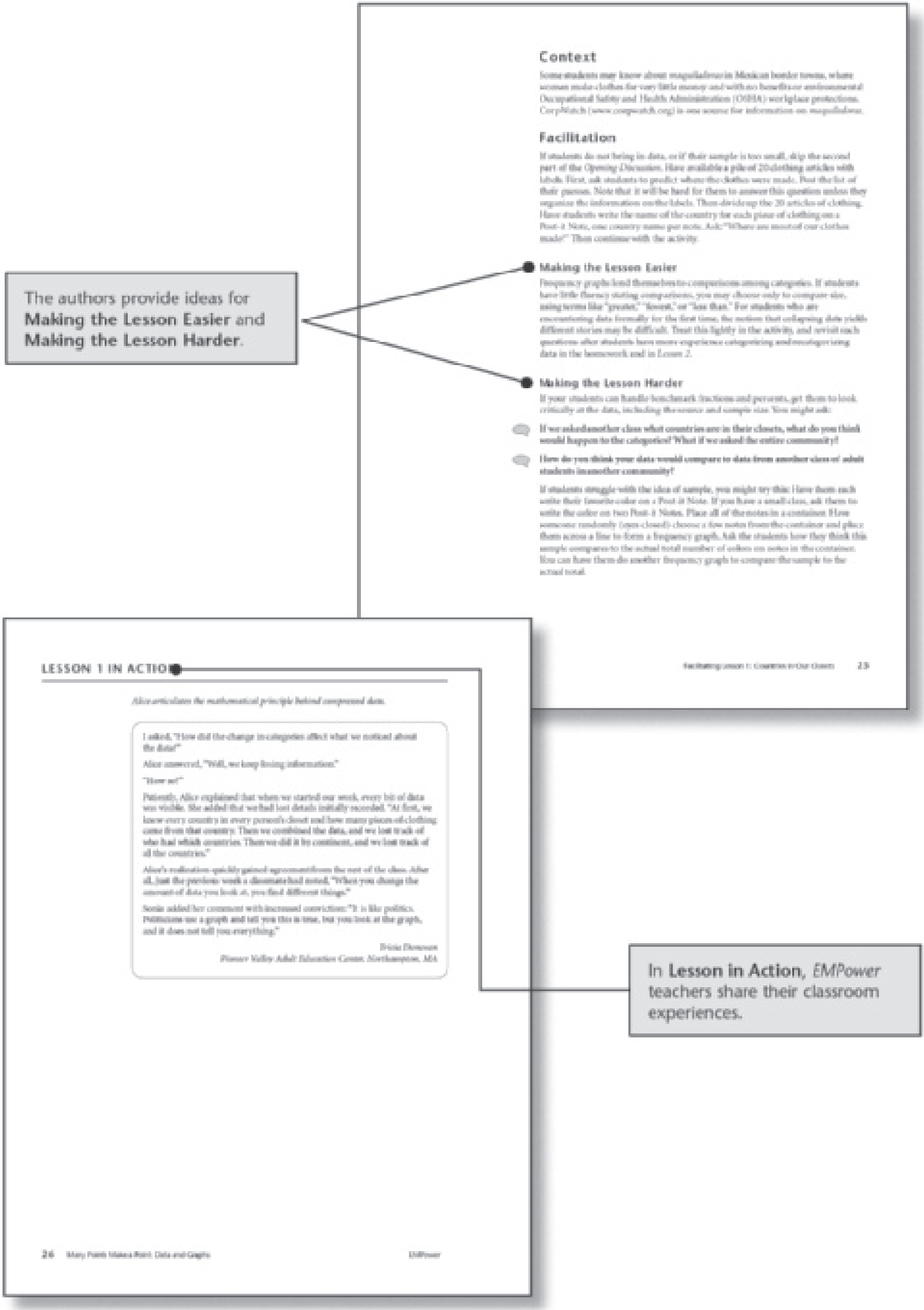
A **Summary Discussion** guides the conclusion of the lesson.

Practice, **Extension**, and **Test Practice** sections list and describe choices for additional work.

Math Inspections are an opportunity to examine a notation or structure, to name a pattern, or to test an idea.

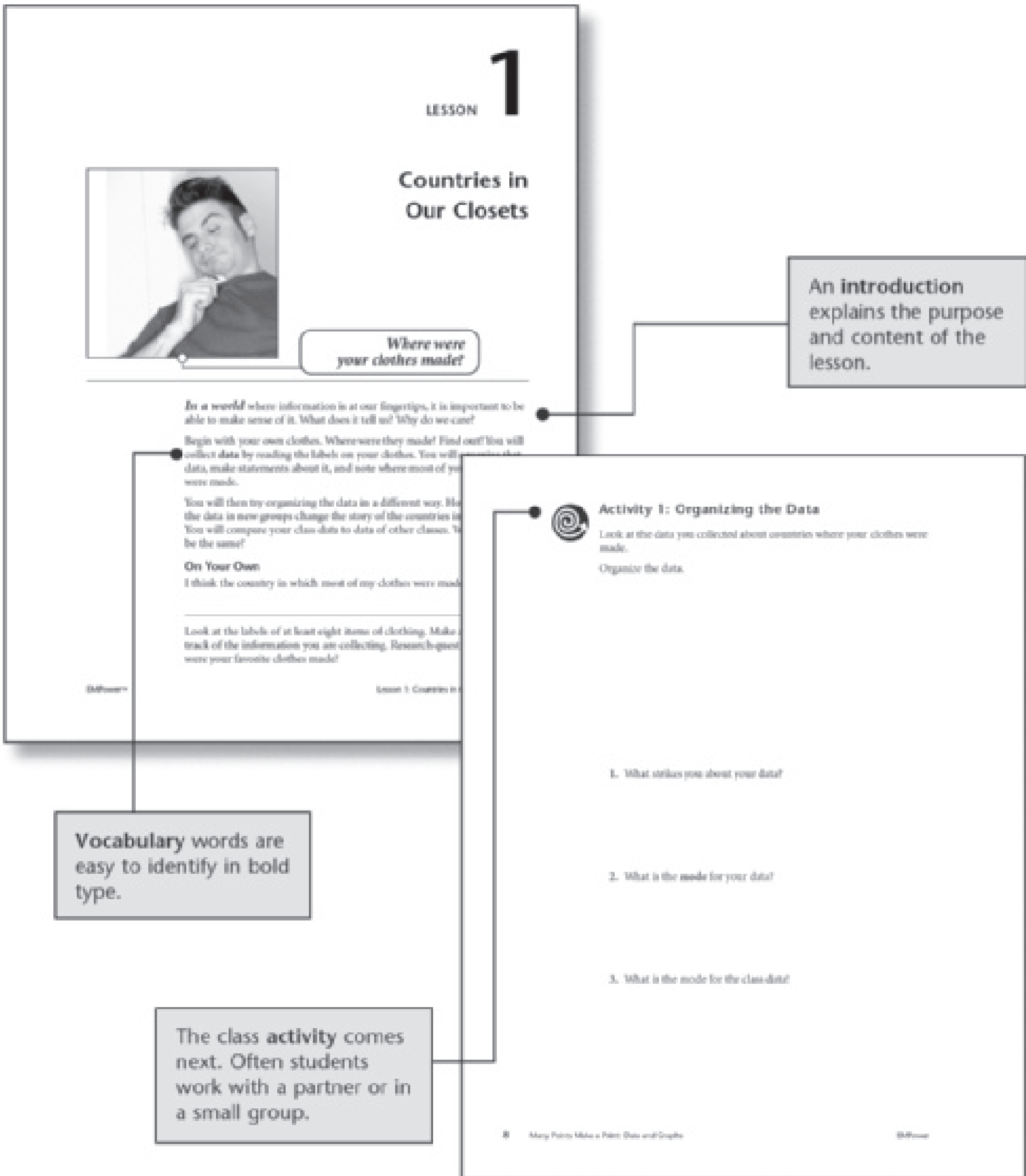
Looking Closely draws the teacher's attention to the objectives and suggests teaching strategies.

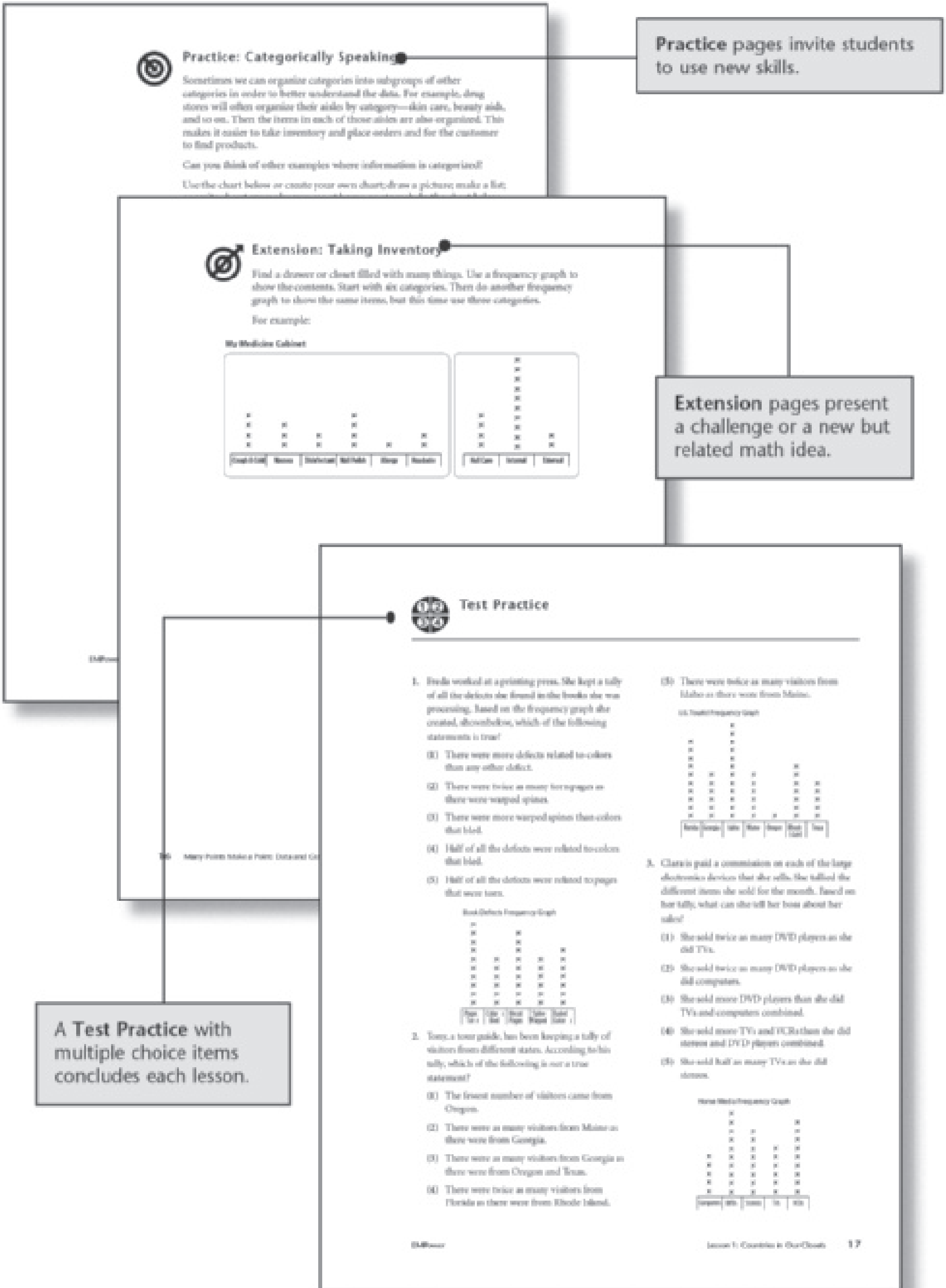
Math Background helps teachers deepen their understanding of mathematical concepts.


The authors provide ideas for **Making the Lesson Easier** and **Making the Lesson Harder**.
Context
Facilitation
Making the Lesson Easier
Making the Lesson Harder
LESSON 1 IN ACTION
Alice articulates the mathematical principle behind compressed data.
In **Lesson in Action**, *EMPower* teachers share their classroom experiences.

Overview of EMPower Units

Features of the Student Book




Practice: Categorically Speaking
Practice pages invite students to use new skills.
Extension: Taking Inventory
Extension pages present a challenge or a new but related math idea.
Test Practice
A Test Practice with multiple choice items concludes each lesson.

Changing the Culture

Teachers who use *EMPower* often face the challenge of transforming the prevailing culture of their math classrooms. *EMPower* teachers have offered these ideas for facilitating this transition:

- **Set the stage.** Engage students in drafting and agreeing to ground rules. Explicitly state that this is a space for everyone to learn. As one teacher said, "We are in this together. Share, even if you do not think you are right. Whatever you add will be helpful. It lets us see how you are looking at things."
- **Group your students.** Match students whose learning styles and background knowledge complement each other. Ask questions such as, "How did it go to work together? How did everyone contribute?
- **Allow wait time.** Studies have shown that teachers often wait less than three seconds before asking another question. Students need time to think.
- **Sit down.** Watch students and allow them to struggle a bit before interrupting to help them. Listen for logic and evidence of understanding. Follow the thread of students' thinking to uncover unconventional approaches. During discussions with the whole group, hand over the markers. Let students draw and make notes on the board.
- **Review written work.** Look beyond right and wrong answers to learn everything you can about what a student knows. Determine what seems solid and easy, as well as patterns in errors. If students are scattered, suggest ways they can organize their work. This is likely to lead to more efficient problem-solving and clearer communication.

Sequences and Connections

Any one of the *EMPower* books can stand alone, yet there are clear connections among them.

Some *EMPower* teachers alternate the books that focus on numbers with the books on geometry, data, and algebra. Particularly in mixed classes with some students at National Reporting System (NRS) Beginning Basic level or with older learners with minimal formal education, *Over, Around, and Within: Geometry and Measurement* works well as a starting point. The activities are concrete. Lessons introduce concepts like perimeter, area, and volume of two-dimensional shapes while keeping the computation focused on small whole numbers. Ratio and proportion unfold with scaffolding to insure that students can reason about rates and proportional relationships.

For teachers and students determined to become powerful problem-solvers with algebraic reasoning, begin with whole numbers and follow with fractions, decimals, and percents; ratio and proportion; and algebra. For this sequence, *Everyday Number Sense: Mental Math and Visual Models* is a starting point to develop whole number mental math skills and operation sense. *Using Benchmarks: Fractions and Operations* grounds students in part-whole relationships and operation sense with fractions. And *Split It Up: More Fractions, Decimals, and Percents* continues to expand students' repertoire with part-whole ratios in fractions, decimals, and percents. *Seeking Patterns, Building Rules: Algebraic Thinking* builds upon the tools and relationships used in *Keeping Things in Proportion: Reasoning with Ratios*. The work with fractions and decimals is useful in describing approximate relationships between data sets in *Many Points Make a Point: Data and Graphs*.

Book Descriptions

Over, Around, and Within: Geometry and Measurement
Students explore the features and measures of basic shapes. Perimeter and area of two-dimensional shapes and volume of rectangular solids provide the focus.

EMPower Plus

Everyday Number Sense: Mental Math and Visual Models
Students solve problems and compute with whole numbers using mental math strategies with benchmarks of 1, 10, 100, and 1,000. Number lines, arrays, and diagrams support their conceptual understanding of number relationships and the four operations.

EMPower Plus

Using Benchmarks: Fractions and Operations
Students use the fractions 1/2, 1/4, 3/4, and 1/10; the decimals 0.1, 0.5, 0.25, and 0.75; and the percents 50%, 25%, 75%, and 100% as benchmarks to describe and compare all part-whole relationships. Students extend their understanding of the four operations with whole numbers to fractions. They decide upon reliable procedures for the four operations with fractions.

EMPower Plus

Split It Up: More Fractions, Decimals, and Percents
Building on their command of common benchmark fractions, students build fluency finding tenths, 10%, and 1%. They work with decimals to the thousandths place. In analyzing the impact of the four operations on decimal numbers, they expand their repertoire and competence with part-whole relationships.

Seeking Patterns, Building Rules: Algebraic Thinking
Students use a variety of representational tools—diagrams, words, tables, graphs, and equations—to understand linear patterns and functions. They connect the rate of change with the slope of a line and compare linear with nonlinear relationships. They also gain facility with and comprehension of basic algebraic notation.

Keeping Things in Proportion: Reasoning with Ratios
Students use various tools—objects, diagrams, tables, graphs, and equations—to understand proportional and nonproportional relationships.

Many Points Make a Point: Data and Graphs
Students collect, organize, and represent data using frequency, bar, and circle graphs. They use line graphs to describe change over time. They use benchmark fractions and the measures of central tendency—mode, median, and mean—to describe data sets.

Frequently Asked Questions about Facilitation

Q: *Is there an ideal level for the* EMPower *series?*

A: The *EMPower Student Book* pages include situations and instructions that require some proficiency in written English. Students who test at NRS low and high intermediate levels or grades 4–7 grade level equivalency in mathematics are the best candidates for *EMPower*. Such students may have some familiarity with basic operations and know some number facts but might be unable to retain some procedures or perform them accurately and reliably—for instance, in the case of long division or of fraction operations. See the prerequisites for more information. Students who are higher level can benefit from *EMPower* if they have trouble getting started on a problem on their own, or if they are anxious and shut down when they see equations that look complicated. *EMPower* sets them up to be more independent, to test multiple solution paths, and to feel more confident in being flexible with numbers.

Q: *I have classes that are widely multi-level. Can this work?*

A: Many teachers see a wide range of levels within the group as an obstacle. Turn the range of levels to your advantage. Focus on students' representations (words, graphs, equations, sketches). This gives everyone the chance to see that answers emerge in several ways. Slowing down deepens understanding and avoids facile responses. Having calculators available can even the playing field. Implement the suggestions in *Making the Lesson Easier* and *Making the Lesson Harder* in the *Lesson Commentary* sections.

Q: *How do I deal with erratic attendance patterns?*

A: Uneven attendance can be disruptive. Students who miss class may feel disoriented; however, the lessons spiral back to the most important concepts. When the curriculum circles back, students will have a chance to revisit concepts and get a toehold.

Q: *What do I do if I run out of time, and there is no way to finish a lesson?*

A: Each activity is important, but reviewing it is equally important. It is better to cut the activity short so there is time to talk with students about what they noticed. Maximize the time by selecting a student or group whose work you feel will add to the class's understanding to report their findings. Be conscious of when you are letting an activity go on too long because the energy is high. Fun is good, but be sure important learning is happening. If you like to give time in class to reviewing homework, and you want to hear from everyone in discussions, you will run out of time. Schedule a catch-up session every three or four lessons.

Q: *How do I respond to comments such as "Can't we go back to the old way?"*

A: Change is unsettling, especially for students who are accustomed to math classes where their job is to work silently on a worksheet solving problems by following a straightforward example. Be clear about the reasons why you have chosen to de-emphasize some of the traditional ways of teaching in favor of this approach. Ultimately, you may need to agree to some changes to accommodate students' input. Meanwhile, reiterate for students what they have accomplished. When there is an "Aha!" moment, point it out.

Q: *My students don't have the time to go through a full curriculum. How can I convince students the value of using this program, even when their goal is to "get in and get out" as quickly as possible? Shouldn't I spend whatever time they have on a program designed to prepare them for college placement or high school equivalency tests?*

A: The National Center for Education and the Economy launched an intense study of the mathematics students need to be college and career ready. They determined that middle school math is vital for success in nine different programs offered at community colleges. They based their assessment on texts and exams from programs including nursing, accounting, and criminal justice. Though middle school math—fractions, decimals, percents, ratio, and proportion—are taught, they are not learned well. Teaching these concepts so that learners have a true foundation rather than a shaky, passing familiarity with a number of topics and procedures will enable students to meet their long-term goals.

Q: *My own math background is not strong. Will I be able to teach this curriculum?*

A: Yes! Most teachers tend to teach the way they were taught. Adopting a different stance requires support, and the more types of support, the better. This curriculum offers support in a few ways. The *Teacher Book* for each unit lists open-ended questions designed to keep the math on track. In the *Lesson Commentary* sections, the *Math Background* helps teachers deepen their understanding of a concept. In addition, the *Lesson in Action* sections provide examples of student work with comments that illuminate the underlying mathematics.

Expand your network of supportive colleagues by joining the Professional Learning Environment for EMPower. Post questions to the discussion board. Consider joining the Adult Numeracy Network, http://adultnumeracynetwork.org, and attend your regional NCTM conference. You can find face-to-face and on-line course offerings through the Adult Numeracy Center at TERC, http://adultnumeracy.terc.edu.

EMPower Plus
extending mathematical power

Everyday Number Sense

Mental Math and Visual Models

STUDENT BOOK

McGraw Hill Education

TERC

Everyday Number Sense: Mental Math and Visual Models

Learning about numbers and operations needn't produce a class of yawning faces or tense handgrips on pencils, as you will discover when students tackle the engaging problems presented in *Everyday Number Sense*. Problems involving travel distances, historical dates, temperature fluctuations, mortgage payments, shopping questions, calculator conundrums, and mathematical puzzles, allow students to build upon their own robus strategies for adding, subtracting, multiplying, and dividing. However, the problems presented also strengthen students' number and operation sense by encouraging them to solve problems using mental math strategies such as estimating and adjusting, as well as grouping, visualizing, and decomposing numbers. These strategies expose the structure of the number system in ways that lead to 'algebrafying' arithmetic and they help students more easily manage numbers. Along the way students see how mathematical tools–number lines, arrays, diagrams, and calculators–can ease mathematical problem solving.

Everyday Number Sense focuses on the whole number benchmarks of 1, 10, 100, and 1,000 in a variety of world-life situations where mental math, estimation, and calculator skills prove useful. Multiple strategies are key. The lessons guide students toward development of computational fluency, flexibility, and accuracy and an understanding of operation meanings. Students come to understand the relationship between addition and subtraction as they count and compare quantities, and to understand the multiplication and division relationship in terms of equal group problems. These unique lessons move students beyond the anxious world of remembered (or forgotten) algorithms and into the world of mathematical problem solving, reasoning, connecting, and communicating

Mathematical Concepts Covered for Everyday Number Sense: Mental Math and Visual Models

Book Description: Students solve problems with whole numbers using mental math strategies with benchmarks of 10, 100, and 1000. Number lines, arrays, and diagrams support conceptual understanding of number relationships and the four operations.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/Topics Covered**
Opening the Unit	Everyday Number Sense	• Personal math experiences • Mental math skills • Fluency with visual models and symbolic expressions • Number properties such as the commutative and distributive properties
Lesson 1	Close Enough with Mental Math	• Mental math strategies • The role of commutativity in addition of whole numbers
Lesson 2	Mental Math in the Checkout Line	• Totals computed mentally by rounding and then adjusting • Mental math processes notated mathematically • Generalizations about equations
Lesson 3	Traveling with Numbers	• Numbers on a number line located, put in order, and operated on • Numbers rounded to the nearest 10 and 100
Lesson 4	Traveling in Time	• Mental math strategies explained with a number line • Mental math and number-line actions recorded with equations • Counting by 10’s and 1’s to solve addition and subtraction problems • Generalizations about how addition and subtraction behave in equations
Lesson 5	Meanings and Methods for Subtraction	• Three models (or interpretations) for subtraction identified and used • Algorithms for addition and subtraction examined; why they work
Lesson 6	Extending the Line	• Negative and positive numbers located on a number line • Difference between two numbers determined
Lesson 7	Ups and Downs with Addition	• Visual models and symbolic notation to express addition with integers • Patterns identified that occur when adding integers (e.g., commutative and associative properties)

Lesson 8	Taking Your Winnings	• Composition of numbers in terms of 10's, 100's, and 1,000's examined and identified • Parentheses in expanded notation • Multiples of 10, 100, and 1,000 added and subtracted mentally • Mental calculations checked with calculators
Lesson 9	Patterns and Order	• Patterns for multiplying and dividing by 10, 100, 1,000 identified • Order of operations
Lesson 10	Picture this	• Connections between arrangements of objects in groups and arrays and written expressions • Equivalent expressions
Lesson 11	What's the Story?	• Situations represented with pictures and mathematical equations • Problem-solving strategies illustrated with equations and pictures • Squares and square roots of perfect squares • Multiplication with exponents
Lesson 12	Deal Me In	• Verbal language and symbolic notation for division matched to a concrete model • Mental math strategies for division applied to situations calling for splitting dollar amounts over time periods
Lesson 13	String It Along	• Direct measurements and scale used to find number of groups of a given size in a total • Mathematical symbols to express the action of division • Division related to multiplication and factors
Lesson 14	Making Do	• Units and precision for remainders • Remainders written and understood as decimals, fractions, and whole numbers • Paper-and-pencil division algorithms examined for why they work • Relationships among the four operations
Closing the Unit	Computer Lab	• Synthesis of the content • Areas of strength and weakness assessed

LESSON 7

Ups and Downs With Addition

Does addition always make things bigger?

Situations that can be represented by negative numbers are all around you. You have already extended the number line above and below zero, placed numbers on the line, and determined the distance between them. Temperatures can go below zero. And so can a bank account!

In this lesson, you will explore examples of how to add negative and positive numbers. You will use the number line and chips to explore what happens when you add positive and negative numbers. Based on your exploration of bank deposits and withdrawals, you should be able to make your own rules about adding integers.

Activity 1: Bank Balance Ups and Downs

With a partner, shuffle the 5 Bank Balance Cards. Beginning at zero, the day you walk in to open a checking account, track the addition of deposits and withdrawals, one step at a time. Then, show the actions on the number line.

0	+	________	=	________
Balance of ...	... then add ...		Altogether we have...	... a balance of ...
________	+	________	=	________
Balance of ...	... then add ...		Altogether we have...	... a balance of ...
________	+	________	=	________
Balance of ...	... then add ...		Altogether we have...	... a balance of ...
________	+	________	=	________
Balance of ...	... then add ...		Altogether we have...	... a balance of ...
________	+	________	=	________
Balance of ...	... then add ...		Altogether we have...	... a balance of ...

Activity 2: Show the Bank Activity with Objects

Return to *Activity 1*, where you showed a series of deposits and withdrawals. Using that same sequence, find a way to show withdrawals and deposits with objects like two-color chips. To make it easier, give a value of +$50 to one color and a value of -$50 to the other color.

Example:

Bank balance of $150, added with a withdrawal of $250, gives a total of -$100.

1. Keep track of each step.

2. Did you end up with the same balance as you did with the number line?

3. Which way—with the number line or with objects—makes it easier for you to see the addition of withdrawals and balances?

4. Write rules or steps for adding withdrawals and deposits in your own words.

Math Inspection: Check Both Sides of the Equal Sign

What is going on in these equations? Look at the first equation below. Determine why the change from the left side of the equal sign to the right side is acceptable. Do the same with the remaining two equations.

$$12 + (-8) = 12 - 8$$
$$600 + (-80) = 600 - 80$$
$$-4{,}000 + (-200) = -4{,}000 - 200$$

1. Write down what you notice happening in the three equations.

2. What do the equations have in common?

3. Make up another equation that follows that same idea.

4. How are the equations above examples of a rule?

5. Solve these equations.
Example: $90 + (-4) = 90 - 4 = 86$

a. $120 + (-42) =$

b. $-320 + (-180) =$

c. $-1{,}000 + -689 =$

d. $\$400 + -\$520 =$

Practice: Number of the Day

The number for today: -10

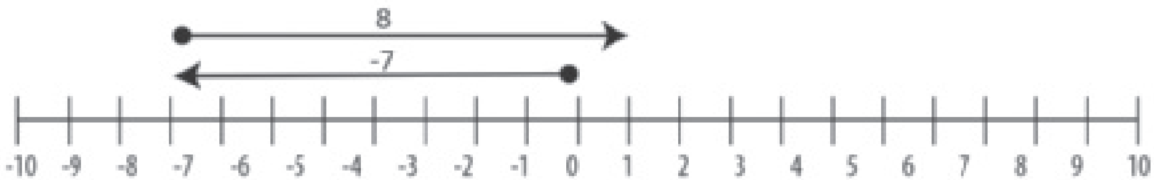
On the lines below, write as many expressions as you can, using the operation of addition, that have a value of -10.

Write expressions with all negative integers and some with a mix of both negative and positive integers. Be ready to say what patterns you notice.

Practice: More Practice Adding Integers

1. Solve the problems below. Show the addition. You can sketch lines for each problem as needed.

Example: -7 + 8 = 1



a. 3 + -5 =	4 + 7 =	-7 + 6 =	-8 + -3 =
b. 4 + -5 =	8 + -4 =	-1 + -5 =	-5 + 8 =
c. -9 + 8 =	-7 + 12 =	-10 + -12 =	-17 + 20 =
d. -8 + -5 =	6 + 12 =	-14 + 9 =	-13 + 12 =

2. Solve the problems. Then write a rule in your own words about what you notice about each of them.

a. -9 + 9 = 3 + -3 = -14 + 14 = 8 + -8 =

b. -20 + 20 = -2 + 2 = -1 + 1 = -7 + 7 =

3. Solve the problems. Then write a rule in your own words about what you notice about each of them.

a. -4 + 0 = 9 + 0 = 0 + -12 = 0 + 4 =

b. 8 + 0 = 0 + -3 = 10 + 0 = 0 + -1 =

Practice: Adding Integers in Different Situations

Read each problem below. Then write out the problem and solve it, using two-color chips or the number line.

1. Jaylen had $45 in his checking account. He wrote a check for $35, then deposited $75. Later he wrote another check for $105.

   a. How much money does he have?

   b. Which method did you use to solve the problem? Why?

2. Chloe had $78 in her checking account. She wrote two checks, one for $50 and the other for $46. She then deposited $104 into her account and wrote another check for $67.

   a. How much money is in her account now?

   b. Which method did you use to solve the problem? Why?

3. The Tiger Parks Department had 50 people on staff in January. They hired 4 new workers in March but then 7 workers retired in June. In July they hired an additional 8 new workers.

   a. Overall, did they gain or lose workers this year? How many?

   b. Which method did you use to solve the problem?

4. At year end Tiny City reports its total full-time workers.

Date	2011	2012	2013	2014	2015
Number of workers	40	35	20	30	55

   a. Compared to 2011 (40 workers), what is the biggest gain and when?

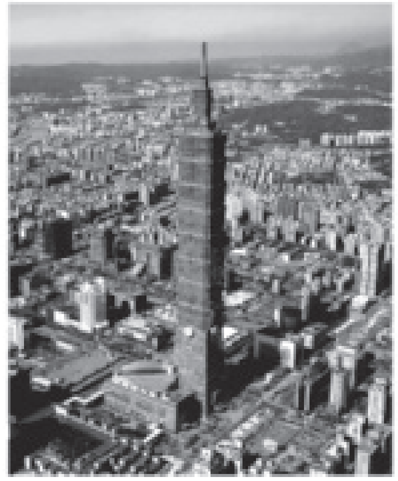
   b. Compared to 2011 (40 workers), what is the biggest decrease in workers and when?

   c. Which method did you use to solve the problem? Why?

4. William started with $30. He lost $10 the first game he played. He won $14 the second game, but then lost $5 two games in a row. How much money does he have now?

   a. Which method did you use to solve the problem? Why?

Practice: Up and Down the Elevator



The Taipei 101 building in Shanghai was the tallest building in the world from 2004 to 2010. It has floors above and below ground level (Floor 0). The first floor above ground level is Floor 1. That is just one example of how negative numbers can show up in real life.

Sketch a vertical number line that shows the following:

1. The Taipei 101 has 101 floors above ground level and another five floors below ground level. Make a sketch on a number line to show all the floors.

2. The Taipei 101 has a huge mall that covers the first five floors including the ground level (Floor 0) and one floor beneath ground level. Use a number line to show which floors are included in the mall in relation to all of the floors.

3. If you lived on the tenth floor, show some ways you could get to three floors below ground level, making two stops along the way. Write at least one equation to show the process and the end point of the trip.

FACILITATING LESSON 7

Ups and Downs with Addition

Does addition always make things bigger?

Synopsis

This lesson continues to explore integers, using both a number line and **zero-pairs** model. The focus is on visualizing the addition of signed numbers.

1. The class uses a classroom number line to track addition of bank deposits and withdrawals.
2. Students record movement back and forth on the number line with symbols.
3. Pairs use two-color chips and zero-pairs to model the bank deposits/withdrawals and other contexts.
4. The class makes generalizations about integer addition.

Objectives

- Use visual models and symbolic notation to express addition with integers
- Describe patterns that occur when adding integers (e.g., commutative and associative properties, the additive inverse, connecting subtraction of a positive amount with addition of a negative)

Materials/Prep

- Metric rulers or meter sticks
- *Blackline Master 6: Number Lines -1,000 to 1,000*, one per pair
- *Blackline Master 7: Number Lines -20 to 20*, optional resource for *More Practice*, one per student pair
- *Blackline Master 8: Bank Balance Cards*, five of the same cards from the full set per pair, cut apart
- *Blackline Master 9: Balance Ups and Downs*, optional resource for *More Practice*
- Two-color chips, or two colors of paper squares, several per student, or red and black markers along with paper squares
- Material for large number lines (masking tape, strips of paper, Post-it Notes)
- Play money for *Activity 2: Concentration*

Prepare several large number lines (at least two meters long, marked off in 10 cm intervals) on the floor using masking tape with sticky notes or two-inch wide paper. Label extending from -1000 to +1000, marked off in 100's. .

Heads Up!

If time allows, have student pairs make their own large number lines from -1,000 to +1,000 on long strips of flip chart paper or on adding machine tape. They can label increments of 100 or possibly 50. It is a valuable experience that requires planning equal intervals, as well as thinking about how the numbers progress on each side of the line.

Opening Discussion

Start by saying:

In the last lesson, you explored numbers below zero—the negative integers. Now we are going to look at how to add positive and negative integers. So, first let's discuss what you know about the operation of addition by looking at some statements.

Write four true-or-false statements on the board:

1. Addition, subtraction, multiplication, and division are the four mathematical operations.
2. Addition can be thought of as an act of combining amounts.
3. The answer in addition is called the sum, or total.
4. The sum, or total, is always larger than each of the parts that are added, or the addends.

Answers: 1. true 2. true 3. true 4. false

Ask small groups to discuss and come to agreement about the truth or falseness of each statement. Then ask for each group's decision, listening carefully to various perspectives. Expect disagreement, especially on the fourth statement, and ask for people's reasoning with examples. Record what people say. Acknowledge that it is not unreasonable to think statement 4 is true. In basic arithmetic, with positive numbers, the sum is generally larger than each of the parts ($4 + 10 + 7 = 21$, and 21 is larger than each addend). An exception occurs when at least one of two addends is zero ($9 + 0 = 9$, and 9 is equal to one of the addends). When you extend the numbers to include negative numbers, all bets are off!

Say:

In this lesson, you are going to explore addition with a number line including positive and negative numbers. You'll do that addition in two ways—with the number line and with color (chips, markers, or paper squares).

Activity 1: Bank Balance Ups and Downs

This activity begins with a -1,000 to +1,000 number line on the floor, large enough to walk along. If floor space is limited, it could be posted on the wall for all to see and to physically move along. This kind of physical movement creates some muscle memory.

Heads Up!

One goal of this activity is for students to view combining deposits and/or withdrawals as the operation of adding them. When you add a deposit, it brings your balance up. When you add a withdrawal, it brings the balance down. At some point, students will most likely assert, "Withdrawing just feels like subtracting to me!" Pause for emphasis at this point, saying "Exactly! You can think of adding a negative amount the same way you think about subtracting (taking away) a positive amount. (-150) added to 400 (400 + -150) is the same as 150 subtracted from 400 (400 – 150). They both bring the total down!" Encourage students to practice thinking of adding amounts, whether they are negative or positive.

The language of banking may not be familiar to all students. Words like balance, deposit, bounce, and check all have different meanings in other contexts. Explain them as needed.

Begin the activity by explaining that the number line on the floor/wall will be used to represent actions with a personal checking account.

Imagine that this line is a record of your bank account balance. What would the negative numbers represent? The positive numbers? Zero? Where should we place your account balance the day you walk into the bank, before you make your first deposit?

Demonstrate. Ask for a volunteer to stand at a number line at "before I make the first deposit." Then give directions and ask him/her to move the appropriate number of spaces in the appropriate direction. Encourage coaching from the group.

You begin with a balance of $0. Your first deposit is $400. Where are you now? [400 to the right of $0, 400 above zero, 400 positive, plus 400] **What did you do to get there?** [added 400 to 0] **Which direction did you move?** [toward the right (up) or positive direction]

(Write on the board)

We could record that as:

0 + 400 = 400

Now you write a check (withdrawal) to pay your cable TV bill of $150. Which direction did you move? [to the left, negative] **Where will that bring you?** [$250 above zero, positive 250, in the black 250]

After reaching agreement, write on the board.

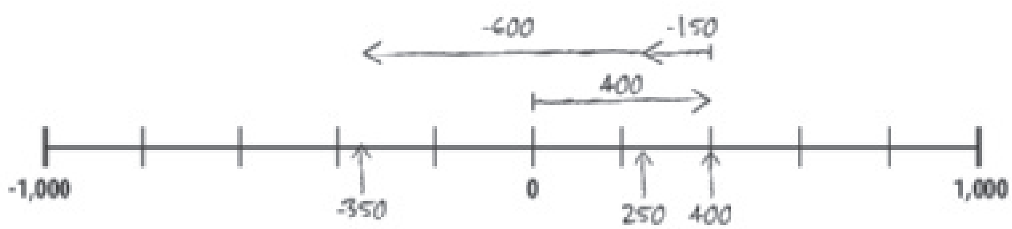
We could record that as:

400 + -150 = 250

Now you pay your part of the rent with a check for $600. Which direction will you move? [to the left, negative] **Where will that bring you?** [$350 below zero, negative 350, $350 in the hole]

250 + -600 = -350

We could show the three actions on the number line as:



What happens now, when you overdraw? Some banks just return (bounce) the check and also charge a fee. Some cover the check, but charge a fee. Some give you a safety valve (overdraft account) if you have a minimum in your savings. Let's assume this one is an overdraft account with no fees.

Once students are comfortable moving in the appropriate direction and have seen several examples written symbolically, ask them to complete *Activity 1* in pairs. Direct attention to *Activity 1: Bank Balance Ups and Downs* (*Student Book*, p. 108). Pass out five cards from the cut up set of *Blackline Master 8: Bank Balance Cards* to each pair, making sure each pile is shuffled. Also, distribute copies of *Blackline Master 7: Number Lines -1,000 to 1,000.*

As students are finishing up, ask two or three pairs to record their equations on the board.

Bring the whole class together to examine what students notice. They might say:

"We all got to the same balance in the end, even though we did in it different order" or "Order doesn't matter when you add."

Heads Up!

Be careful not to assign black/white as negative/positive. Better to start with red as negative ("in the red") and black or another color as positive.

Activity 2: Show the Bank Activity with Objects

Begin a discussion about the difficulty of "seeing" negative amounts of money. In this activity students use color and objects to count positive and negative amounts—to make these amounts concrete. This will give the class a way to visualize combining negative and positive numbers. Look for agreement and understanding among students—you can't hold negative amounts of money in your hands, even though you know debt is a heavy weight.

In this activity we are going to use markers or chips to "see" what positive and negative amounts of money look like. This will give you a chance to visualize what is happening when negatives and positives are combined.

Distribute two-color chips or two colors of markers to pairs of students. Explain that one is positive and the other is negative (if red is one of the color choices, it would be the side preferred for negative). Be sure there is agreement about which is positive and which is negative. Hold up the red, or negative item.

If this were money, what would it represent in your checking account? [withdrawing $1]

Ask the same about the other color. [depositing $1]

What if I put the two of these together? What do I have? [nothing—the positive and negative have cancelled each other out]

$$+1 \quad + \quad -1 \quad = \quad 0$$

Discuss the term *zero pair*: the positive and negative numbers which, added together, total zero. Then place four positive chips and four negative chips together where the students can see them.

How much money did I deposit? How much did I withdraw? Did I change the balance? Why or why not?

Write the following on the board and ask for responses, along with explanation of thinking:

\$70 + (?) = 0

-\$50 + (?) = 0

Discuss the term **additive inverse** of a number. -70 is the additive inverse of 70. Likewise, 50 is the additive inverse of -50.

Then model several examples of adding with positive and negative numbers. For example, model the situation of depositing \$5. Put out 5 positive chips. Then add a withdrawal of \$7. Put out 7 negative chips. You can find five zero pairs, which leave you with a balance of negative 2, or -2.

(+ + + + +) + (- - - - - - -) = (- -)

Direct students to *Activity 2: Showing the Bank Activity with Objects* (*Student Book*, p. 109) using the same six bank balance cards and the math they recorded in *Activity 1*. Ask the pairs to now use chips. Assign a convenient value to each color: positive \$50 = 1 color chip, and negative \$50 = 1 other color chip.

Ask them to see if they end up with the same balance as they did when they used the number line.

Suggest students add these terms to their vocabulary lists (*Student Book*, p. 255): zero pair, additive inverse.

Math Inspection: Check Both Sides of the Equal Sign

This inspection builds on the previous lessons—addition and subtraction with positive whole numbers. This inspection is another opportunity for students to see the connection between adding a negative amount and subtracting that same amount.

Ask students to work individually or in pairs to determine the solution to each problem. The three equations illustrate adding a negative integer on one side of the equation and writing equivalent expressions using subtraction of a positive integer on the other side.

When you debrief with the whole class, you want students to be able to state that adding a negative number is the same as subtracting the positive of that number. Use the term "additive inverse" as needed. The intent is for students to attach meaning to the concept and recognize it when it appears. With students, list some examples of when adding a negative shows up in daily life, e.g., working with debts or deficits that need to be taken into account.

Summary Discussion

Start the summary by reviewing the rules the class members generated about adding withdrawals and deposits (*Activity 2, Student Book*, p. 109).

Then remind the class that together you tried two ways to show addition with positive and negative numbers. Ask:

Which is easier for you to see, the two color chip model or the number line?

Listen for responses.

Let's go back to the four statements about addition we started with in this lesson. What do you agree with? How would you change the fourth statement to make it true?

Write down two more true statements about addition of integers that will be important to remember. Did you discover any shortcuts?

End with this statement:

When you study algebra, you will want to think about "addition of the opposite" rather than "take away." So when you see 40 – 30 you think 40 + (-30).

Give students a minute to tell a partner or the class in their own words why that is different.

Refer students to *Vocabulary* (*Student Book*, p. 255) to record their definitions and examples.

Practice

Number of the Day: -10, p. 111
For practice writing expressions that total to negative 10.

More Practice Adding Integers, p. 112
For practice adding positive and negative numbers. Provide copies of *Blackline Master 9* for students to use with this exercise.

Adding Integers in Different Situations, p. 114
For practice adding integers using number lines or chips in contextualized situations.

Up and Down the Elevator, p. 116
For practice adding integers on a vertical number line.

Test Practice

Test Practice, p. 117

Looking Closely

Observe whether students are able to

Use visual models (number lines or two color counters) and symbolic notation to express addition with integers

These models give all students, and particularly those who are visual learners, the chance to see what the action of combining looks like. With the two-colored counters, red may be negative, black may be positive. Pairing one of each color to cancel each other and make zero is very explicit. Likewise, moving along a number line, to the right for positive values and to the left for negative values, is a strong visual aid. Do students recognize the effect of combining amounts that cancel each other out?

Describe patterns that occur when adding integers

What language do students use to express how quantities change when negative or positive amounts are combined? See if they can anticipate in which direction the total is moving, toward a positive or negative extreme or toward zero, the neutral point. For example, listen for students to explain that adding negative numbers results in a number that is more extremely negative, further from zero on the number line. As an example, can they see that debt grows when you add negative amounts? You lessen debt by adding a positive number.

If students are able to attach meaning—real-world examples—to the mathematics they are working with, it will help them lay a sturdy foundation. Look for other ways to play with these ideas that involve falling and rising through a neutral point such as an elevator going to above and below ground levels of a building or parking garage or descending into and rising above the ocean from sea level.

Do students notice and use zero pairs to simplify their calculations (as exemplified in $5 + -5 = 0$)? Can they see the connection between adding a negative and subtracting a positive? These are some important number properties that lay the foundation for algebra.

Rationale

Negative numbers are as much a part of students' lives as positive numbers. Since students have a sense of what it means to go below zero, this lesson engages students in further developing their intuitive sense of the effect of combining positive and negative numbers. The lesson sticks with the concept of combining because the symbols, action, and meaning can then be explored and connected.

Math Background

This lesson continues to develop important mathematical ideas:

1. The effect of subtracting is the same as adding a negative;
2. The action of adding a negative can be visualized on a number line.

The properties of the operations underly what students do in arithmetic and in algebra. At this point students should be pretty confident that the order in which they add two numbers does not change the answer. This lesson is a chance to see whether this holds for combining negative and positive numbers. If 15 + 3 = 3 + 15, is this statement also true: 15 + -3 = -3 + 15. Why is this true and an allowable move when subtraction is not commutative?

When you rewrite 15 – 3 using addition, you have 15 + -3. The important point is the negative sign stays with the value 3. It signifies that 3 is less than zero on the number line, that any quantity in combination with it will be less 3. To the student it looks as though the operational sign of subtraction morphs into a negative sign.

To say this in words, 15 – 3 and 15 + -3 can both be interpreted as I have 15 with a loss of 3. They are equivalent expressions. Because addition is commutative, you can say, I had a debt of 3 in combination with 15. However, you cannot attach the negative sign to fifteen and make the 3 positive: that would change the meaning of the problem.

Another way to think about this is that the starting point matters when considering subtraction on the number line. However, the starting point does not matter with addition. One combines parts to make a total and these parts are easily interchangeable whereas one starts with a total and removes a part when subtracting. You could not reverse that and start with a part and remove a total.

Context

Apply new ideas to contexts like football (yards gained, yards lost) or breaking even when a certain dollar figure is reached.

Facilitation

Making the Lesson Easier

Slow down. Do a few more examples. Stick with the picture, connect with the story. Have students act out the story, if possible. If not, ask them to represent the story using a picture or number line. Students should have at least one model they understand and are able to use and check for themselves.

Two-color chips or a number line should make sense. At least one should be a tool they feel comfortable using to check themselves.

Making the Lesson Harder

Push students to articulate generalizations. Ask them to support their conjectures with examples. You could ask students to say what is the same or different about two problems with similar-appearing numbers, such as these:

-1,257 + 999

1,257 + -999

You might also ask students to evaluate expressions with messier numbers such as this one:

-1,099 + 257

EMPower Plus

extending mathematical power

Using Benchmarks

Fractions and Operations

STUDENT BOOK

McGraw Hill Education

TERC

Using Benchmarks: Fractions and Operations

Fractions may be interpreted many ways. In *Using Benchmarks*, students focus on the concept of fractions as representations of part/whole relationships. This opens the door for them to compare fractional quantities and make useful estimations about the size of amounts in a wide array of real-world situations. Understanding of the benchmark fractions, 1/2, 1/4, 3/4 and 1/10, develops gradually as students count and draw objects and answer the ever recurring questions: "What's the whole?" "What's the part?" As they move flexibly between finding a fractional part of an amount, as well as finding the whole when the part or fraction is known, students learn to use drawings and objects to support their reasoning and communication with others. By using the fundamental, familiar tools called benchmark fractions, decimals, and percents, students gain the dexterity necessary to explore the larger world of rational numbers.

Connections between the benchmark fractions and their decimal and percent equivalents are reinforced continually, so that students acquire flexibility, as well as fluency, with the use of benchmark terms. In addition, they learn to consider the part counted and the part remaining as complements that make a whole, so 3/4 is first introduced as the part left over when 1/4 is used and 9/10 is seen as the remaining amount when 1/10 is taken. Through a well-sequenced series of accessible lessons, students encounter deep mathematical ideas related to rational numbers. Along the way, they cement an understanding of the crucial and frequently used benchmark fractions.

Mathematical Concepts Covered for Using Benchmarks: Fractions and Operations

Book Description: Students use the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{1}{10}$; the decimals 0.1, 0.5, 0.25, and 0.75; and the percents 50%, 25%, 75%, 100%, and the multiples of 10% as benchmarks to describe and compare all part-whole relationships.

Lesson Number:	Lesson Name:	Mathematical Concepts/Topics Covered
Opening the Unit	Using Benchmarks	• Fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ • Halves as two equal parts of a whole • Amounts compared to one-half
Lesson 1	More Than, Less Than, or Equal to One-Half	• The part/whole relationship • Fractional amounts compared to $\frac{1}{2}$ • Multiple representations for $\frac{1}{2}$—decimal and percent • A 'whole' stated as a fraction
Lesson 2	Half of a Half	• Strategies for determining $\frac{1}{4}$ • Multiple representations for $\frac{1}{4}$ • Determining ¾ and one whole based on one-fourth
Lesson 3	Three Out of Four	• Strategies for finding $\frac{3}{4}$ based on $\frac{1}{4}$ • Using multiple representations for $\frac{3}{4}$ • Multiplying and dividing to find $\frac{3}{4}$
Lesson 4	Fraction Stations	• Fractions compared and described in relation to benchmarks for assessment purposes
Lesson 5	A Look at One-Eighth	• Strategies for finding $\frac{1}{8}$ of a given amount • Less familiar fractions such as eighths and sixteenths related to halves and quarters
Lesson 6	Equal Measures	• The meaning and value of any rational number written in fraction form $\frac{a}{b}$, where b is not zero • Equivalent fractions
Lesson 7	Visualizing and Estimating Sums and Differences	• Meanings of addition (combining) and subtraction (take away, difference, and missing part) to reason about fraction operations • Tools (e.g., fraction strips, rulers, Pattern Blocks) to reason about combining fractions and finding differences.

Lesson 8	Making Sensible Rules for Adding and Subtracting	• Procedures for adding and subtracting fractions • Procedures for simplifying fractions • Procedures expressed in words and symbols
Lesson 9	Methods for Multiplication with Fractions	• Understanding of multiplication is demonstrated in various ways • Reliable methods for multiplication with fractions • Mathematical properties (such as the commutative, associative, distributive, inverse, and identity properties) with fractions and whole numbers
Lesson 10	Fraction Division—Splitting, Sharing, and Finding How Many ____ in a ____?	• Various ways to demonstrate understanding of division • Reasonable procedures for division of fractions • Application of properties (such as the commutative, associative, distributive) to operations with fractions
Closing the Unit	Closing the Unit: Benchmarks Revisited	• Part-whole situations described with fractions • A variety of ways such as pictures and number lines are used to model part-whole situations and operations

LESSON 6

Equal Measures

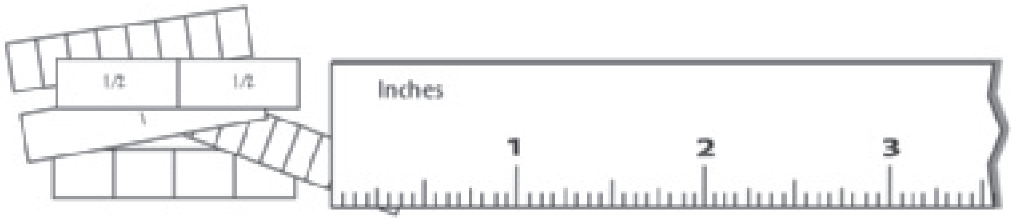
What do fashion design, carpentry, cooking, and computer graphics have in common?

Every number can be written in many forms. How many ways do you know to write a number that means the same as two and a half? How are you sure that a number is equal to three-fourths? When numbers look different, but have the same value, they are equivalent (like 1 and 100%). Sometimes numbers look almost alike, but do not have the same value (like \$250 and \$2.50). They are not equivalent.

People working in professions where measurement is important know this well. Errors can be expensive! Carpenters have a saying: Measure twice, cut once.

In this lesson, you will need to keep your eyes sharp and pay close attention. Consider the value of the quantities. Give yourself time to think about the meaning of the numbers.

Activity 1: Fraction Strips and Rulers—Tools to Think With



1. Think with the fraction strips and the inch ruler!

   a. Look at a set of fraction strips and the markings on a ruler. Write all of the fraction equivalencies you see.

   Example: $\frac{8}{16} = \frac{1}{2}$

2. With a partner, list examples for each fraction below.

   a. Five fractions that are equivalent to $\frac{3}{4}$. Show your thinking for one example with a picture.

   b. Five fractions that are equivalent to $1\frac{1}{2}$.

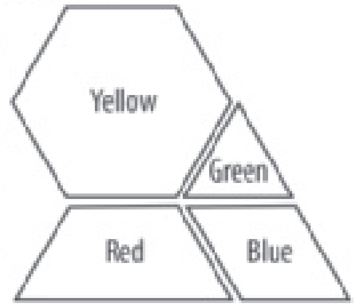
   c. Four fractions that are equivalent to $\frac{14}{16}$.

   d. Describe the strategy or strategies you used to create an equal fraction.

Activity 2: Pattern Blocks—Another Tool

You will need a pile of Pattern Blocks for this activity.

1. Counting the yellow hexagon as one, the whole, find the *fractional* value of
   a. the green triangle __________
   b. the red trapezoid __________
   c. the blue parallelogram __________



2. Examine the Pattern Blocks to answer each of the following questions. Show the equivalence with a picture, a math equation, and words.

   a. How many green triangles equal 1 yellow hexagon?

   The picture

   The math equation

   The math in words

   b. How many blue parallelograms equal 1 yellow hexagon?

   The picture

   The math equation

   The math in words

c. How many red trapezoids equal 1 yellow hexagon?

The picture

The math equation
The math in words

d. How many green triangles equal 1 red trapezoid?

The picture

The math equation
The math in words

e. How many green triangles equal $2\frac{1}{2}$ yellow hexagons?

The picture

The math equation
The math in words

f. How many blue parallelograms equal 1 red trapezoid?

The picture	The math equation
	The math in words

g. How many blue parallelograms equal 2 red trapezoids?

The picture	The math equation
	The math in words

3. Show two more equivalent statements.

Activity 3: Fraction Strips with Thirds and Sixths

1. Line up the following unit fractions in order from smallest to largest. Then describe the pattern you see.

   $\frac{1}{12}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{16}$ $\frac{1}{6}$ $\frac{1}{3}$

2. Now look at the fraction strips you created, including the three new ones. With your partner write all the fraction equivalencies you see. Be prepared to show your thinking.

   Example: $\frac{2}{4} = \frac{1}{2}$

3. When you divided $\frac{1}{2}$ into two equal smaller parts, you got a set of fourths. When you divided each of the fourths into two smaller parts, you got eighths.

   a. Describe the rule for breaking a fraction into two equal parts.

   b. Create some new fractions based on your rule.

   Example: $\frac{1}{16} = \frac{2}{32}$

4. Use your strips to explain why $\frac{1}{3}$ is closer to $\frac{1}{4}$ than $\frac{1}{2}$.

Activity 4: About Ones and Zeroes

1. True or false? Explain your reasoning with examples. Rewrite any false statements to make them true.

   a. Any number multiplied by 1 gives you that number. In other words, $a \times 1 = a$. True or false?

      Examples:

   b. Any number multiplied by 0 is 0. In other words, $a \times 0 = 0$. True or false?

      Examples:

   c. Any number added to 0 is zero. In other words, $a + 0 = 0$. True or false?

      Examples:

   d. Any number added to 1 is that number. In other words, $a + 1 = a$. True or false?

      Examples:

2. Use the true statements about 0 and 1 to make math easy.

   a. $\frac{3}{4} \cdot 0 =$

   b. $\frac{5}{16} \cdot 0 =$

   c. $\frac{5}{16} + 0 =$

   d. $\frac{5}{16} + 1 =$

   e. $\frac{3}{4}(4 - 3) =$

   f. $\frac{1}{2}(100 - 99) =$

   g. $\frac{1}{2}(\frac{1}{2} + \frac{1}{2}) =$

   h. $\frac{5}{16}(2\frac{1}{4} - 1\frac{1}{4}) =$

3. Create some of your own examples using the rules about 0 and 1.

4. You already know that $\frac{2}{2} = 1$, that $\frac{4}{4} = 1$, and that $\frac{8}{8} = 1$. Use that understanding to make the math below easy.

a. $\frac{1}{2} \times \frac{2}{2} =$

b. $\frac{1}{2} \times \frac{4}{4} =$

c. $\frac{1}{2} \times \frac{8}{8} =$

d. $\frac{1}{2} \times \frac{16}{16} =$

Do you agree or disagree with this statement about equation 4a.?

"*There are at least two correct answers:* $\frac{1}{2} \times \frac{2}{2} = \frac{1}{2}$ *and* $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$."

Explain your reasoning.

5. What strategies can you use for creating equivalent fractions?

Math Inspection: One in the Denominator

We've looked at many fractions and equivalent fractions. Many have had 1 in the numerator, but what about a 1 in the denominator?

Mark the statements True or False. Rewrite any false statements to make them true.

1. A fraction with a 1 in the denominator is equivalent to 1. True or false? _____

2. If a fraction has a 1 in the denominator, it is equal to $\frac{1}{2}$. True or false? _____

3. $\frac{1}{5} = \frac{5}{1}$. True or false? _____

4. $\frac{12}{1} = 12$. True or false? _____

5. $\frac{24}{1} = 2$. True or false? _____

6. $\frac{4}{1} = 4$. True or false? _____

7. A fraction with a denominator of 1 is equivalent to the number in the numerator. True or false? _____

8. Give a few examples of your own of fractions with a 1 in the denominator and an equivalent number.

   a. ____________ = ____________

   b. ____________ = ____________

   c. ____________ = ____________

Practice: Equivalent Fractions

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<tr><td colspan="8">$\frac{1}{2}$</td><td colspan="8">$\frac{1}{2}$</td></tr>
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</table>

Using the chart above or a set of fraction strips, fill in the numerators to make equivalent fractions.

$1 = \frac{\quad}{2} = \frac{\quad}{4} = \frac{\quad}{8} = \frac{\quad}{16}$ then $1 = \frac{\quad}{3} = \frac{\quad}{5} = \frac{\quad}{6} = \frac{\quad}{7} = \frac{\quad}{9} = \frac{\quad}{15} = \frac{\quad}{18} \cdots$

Complete the following equivalent fractions using the chart above.

a. $\frac{1}{2} = \frac{\quad}{4}$ **h.** $\frac{1}{2} = \frac{\quad}{8}$ **o.** $\frac{4}{16} = \frac{\quad}{4}$

b. $\frac{1}{4} = \frac{\quad}{8}$ **i.** $\frac{4}{4} = \frac{\quad}{8}$ **p.** $\frac{4}{16} = \frac{\quad}{8}$

c. $\frac{1}{8} = \frac{\quad}{16}$ **j.** $\frac{3}{4} = \frac{\quad}{8}$ **q.** $\frac{6}{16} = \frac{\quad}{8}$

d. $\frac{1}{4} = \frac{\quad}{16}$ **k.** $\frac{3}{4} = \frac{\quad}{16}$ **r.** $\frac{8}{16} = \frac{\quad}{4}$

e. $\frac{1}{2} = \frac{\quad}{16}$ **l.** $\frac{3}{8} = \frac{\quad}{16}$ **s.** $\frac{8}{16} = \frac{\quad}{2}$

f. $\frac{2}{4} = \frac{\quad}{16}$ **m.** $\frac{4}{8} = \frac{\quad}{16}$ **t.** $1 = \frac{\quad}{16}$

g. $1 = \frac{\quad}{4}$ **n.** $1 = \frac{\quad}{8}$ **u.** $1 = \frac{\quad}{2}$

Practice: Between $\frac{1}{3}$ and $\frac{1}{2}$

Are the fractions below between $\frac{1}{3}$ and $\frac{1}{2}$? Show with pictures or words.

1. $\frac{11}{20}$

2. $\frac{9}{16}$

3. $\frac{5}{12}$

4. $\frac{6}{16}$

5. $\frac{7}{14}$

6. $\frac{9}{20}$

7. $\frac{12}{15}$

8. $\frac{2}{15}$

9. $\frac{4}{9}$

Practice: Ratcheting Up (or Down) a Notch

A ratchet fits a range of sockets that from $\frac{1}{32}$" to $\frac{31}{32}$". Each socket is $\frac{1}{32}$" larger than the one before. Use this information to answer the questions below.

1. Sam and Dale are trying to tighten the bolts on their picnic table. Sam grabs a ratchet with a $\frac{5}{16}$" socket. He realizes that is not quite big enough. Which size socket should he try next? Why?

2. Kim is trying to tighten a bolt on a metal frame. She grabs a $\frac{17}{32}$" socket which is just a tad bit too large. Which size should she try next? Why?

3. Jerry is trying to loosen a bolt on his tractor. He tries to use a $\frac{5}{8}$" socket, which is too small. Which size should he try next? Why?

4. Danni is trying to tighten a bolt on her treadmill. She tries to use a $\frac{7}{16}$" socket, which is too big. Which size should she try next? Why?

5. John is trying to loosen a bolt on his grill. He tries a $\frac{7}{8}$" socket, which is too small. Which size should he try next? Why?

Practice: Where to Place It?

For each problem, mark the first given fraction on the line. Then circle the correct answer for whether the fraction is less than (<), equal to (=), or greater than (>) the fraction it is being compared to.

1. $\frac{17}{21}$ is less than (<) equal to (=) or greater than (>) $\frac{2}{3}$.

2. $\frac{11}{15}$ is less than (<) equal to (=) or greater than (>) $\frac{2}{3}$.

3. $\frac{16}{45}$ is less than (<) equal to (=) or greater than (>) $\frac{1}{3}$.

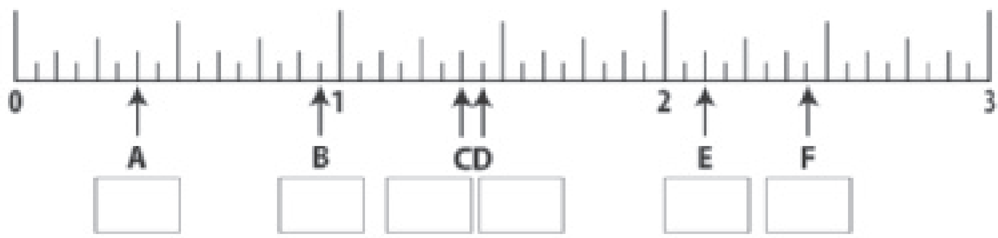
4. $\frac{35}{60}$ is less than (<) equal to (=) or greater than (>) $\frac{2}{3}$.

5. $\frac{13}{30}$ is less than (<) equal to (=) or greater than (>) $\frac{1}{3}$.

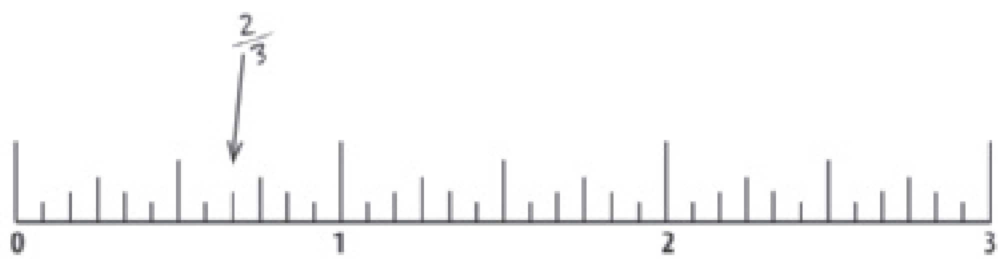
Practice: $\frac{2}{3}$ and $\frac{3}{4}$

1. Which is larger, $\frac{2}{3}$ or $\frac{3}{4}$? Show or explain how you know.

2. Name the points on the number line.



3. Show each point on the number line.



a. $2\frac{1}{3}$

b. $1\frac{5}{12}$

c. $2\frac{5}{6}$

d. $2\frac{1}{6}$

e. $\frac{7}{12}$

Mental Math Practice: Using Properties

How quickly can you mentally solve each of the problems below?

1. $\frac{3}{4}\ (\frac{2}{2} - \frac{4}{4})$
2. $12\ (\frac{1}{2} + \frac{1}{2})$
3. $4\ (\frac{1}{2}) + 6\ (\frac{1}{2} - \frac{1}{2})$
4. $8\ (\frac{6}{8} - \frac{5}{8})$
5. $-4 + 3\ (\frac{1}{2} + \frac{1}{2})$
6. $9 + (-3)(\frac{2}{4} - \frac{2}{4})$
7. $50\ (\frac{3}{4} - \frac{3}{4})$
8. $-10 + 5(\frac{1}{2})(2)$
9. $40 + (12)(\frac{1}{4})(4)$
10. $-2 + 7(\frac{1}{2})(4)$
11. ___ + $8\ \frac{1}{4} = 12 - 2$
12. three-quarters of a million + _____ = 1.5 million

Test Practice

1. Tom works 12-hour days, four days a week. Because they are long days, he thinks about how much work he has already finished in a day. So far today, he has worked 7 hours. What fraction of the day has Tom worked?
   (a) $\frac{4}{7}$
   (b) $\frac{5}{12}$
   (c) $\frac{7}{16}$
   (d) $\frac{7}{12}$
   (e) $\frac{1}{4}$

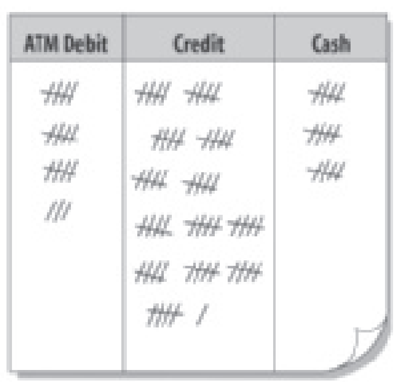
2. Sherry timed herself as she walked around the track. It took 20 minutes. What part of an hour does this represent?
   (a) $\frac{20}{1}$
   (b) $\frac{20}{40}$
   (c) $\frac{1}{3}$
   (d) $\frac{2}{3}$
   (e) $\frac{1}{20}$

3. Nate is trying to save \$3,000 for a used car. So far he has saved about \$1,400. About what fraction of the total has he saved?
   (a) almost $\frac{1}{4}$
   (b) almost $\frac{1}{3}$
   (c) almost $\frac{1}{2}$
   (d) almost $\frac{3}{4}$
   (e) almost $\frac{2}{3}$

4. Which of the following is not equivalent to $\frac{2}{3}$?
   (a) $\frac{4}{6}$
   (b) $\frac{6}{9}$
   (c) $\frac{8}{12}$
   (d) $\frac{9}{12}$
   (e) $\frac{10}{15}$

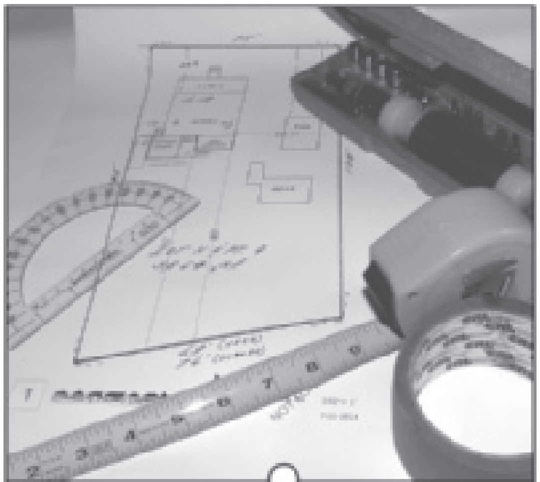
5. During a recent hurricane, about one-third of the population was without power. If the population is about 150,000, about how many people were without power?
   (a) 10,000
   (b) 50,000
   (c) 75,000
   (d) 100,000
   (e) 450,000

6. A small store tracked the payment type chosen by its customers during one day. According to the chart, about what fraction of purchases were made with credit cards?



FACILITATING LESSON 6

Equal Measures



What do fashion design, carpentry, cooking, and computer graphics have in common?

Synopsis

Students draw upon their prior knowledge of benchmark fractions to reason about fractional equivalents. They use a variety of tools to visualize fractions and equivalencies: fraction strips, inch-rulers, and Pattern Blocks. They reason and solve problems with the visual tools. Once their visual understanding is well grounded, students examine the formal mathematical approach to equivalent fractions, particularly the role the multiplicative identity (multiplication by 1) plays in generating equivalent fractions. They connect their visual models with the formal procedure.

1. Students review fraction equivalents with which they have become familiar (halves, quarters, and eighths), using a visual model.
2. Students create a set of fraction strips, add sixteenths to their repertoire and make connections to the lines on inch-rulers.
3. Students explore 1/2, 1/3, and 1/6 equivalencies with Pattern Blocks and then fraction strips.
4. Students use the multiplicative identity to generate equivalent fractions and connect the formal math to their visual models.

Objectives

- Interpret the meaning and value of any rational number written in fraction form *a/b*, where *b* is not zero
- Generate and demonstrate equivalent fractions

Materials/Prep

For *Activity 1*:

- Prepare paper strips (1" by 11"), in different colors. Make 5 unmarked strips per student.
- A plastic sheet protector containing one blank piece of paper, 1 per student
- Tape
- Colored markers (dry-erase), 1 per student
- 12-inch rulers marked to the nearest 1/16" or *Blackline Master 3*

For *Activity 2*:

- Pattern Blocks. Use only yellow, blue, red, and green or *Blackline Master 4*

For *Activity 3*:

- Prepare paper strips (1" by 11"), in colors different from those in *Activity 1*. Each student gets 3 unmarked strips, but make extras in case of error.

Heads Up!

Lessons 6 through *10* ground student reasoning in the informal and visual before presenting opportunities to use an algorithm or formal procedures to solve computation problems with fractions. Even if students remember procedures, ask them to put off using them in favor of solving problems visually so that everyone has the opportunity to make connections among the written problem, a visual representation of it, and strategies to solve it.

Opening Discussion

Offer some examples to remind students that another way to write 1, as in one whole, is as a fraction with the same number in the numerator and denominator.

Help me complete the story:

I had a total of $20 in my wallet and I lost it all. Not half, but all, so $20 out of $20. I can write the whole amount as … [20/20]**.**

My niece used up an entire 16 oz bottle of shampoo. Not half, but all, so 16 oz out of 16 oz. I can write the whole amount as … [16/16]**.**

I recycled every one of my 30 plastic bags in the house. Not 1/2, but all. So I can write that whole amount as … [30/30]**.**

Ask students to state the point in their own words. Offer your own summary along these lines:

There's more than one way to write 1 when it means ALL, the "whole." Fractions are flexible. It is a useful characteristic because it gives us a way to operate with them and simplify answers. But it also means they can be tricky. This lesson focuses on equivalent fractions. Because of equivalence, we can name part-whole fractions of the same value in many ways.

Write the following 17 fractions on the board in this random order:

0/8, 1/2, 3/4, 3/8, 2/2, 7/8, 4/4, 2/4, 5/8, 0/4, 1/4, 6/8, 8/8, 0/2, 1/8, 2/8, 4/8

Say:

I just wrote 17 benchmark fractions you have been working with on the board.

a. **Put them in order, from smallest to largest values.**

b. **What do you notice about the fractions? Which (if any) are the same? How do you know?**

Ask pairs to work together on creating a context and a visual to show how someone could "see" the relative values and get the point that some are equivalent. Emphasize the following as you give directions:

Use all 17 fractions. Be creative! The visual model can be anything you want—pizzas, cakes or other food, money, a number line, a group of objects—so long as you are clear about what "One Whole" (1) represents.

Make clear sketches or diagrams that others will be able to understand.

When pairs have completed their diagrams, ask them to exchange with another pair, answering the following questions:

- How clear is the other pair's drawing to you? Do you have suggestions to make it clearer?
- What is the same about the other pair's drawing and your drawing?
- What is different?
- Did you both come to the same conclusion?
- What is that conclusion?

Listen carefully to students' understandings.

Make sure there is sound understanding of the correct order and equivalencies, and that there are 9 different values represented by the 17 fractions

0/8= 0/4 = 0/2

1/8

2/8 = 1/4

3/8

4/8 = 2/4 = 1/2

5/8

6/8 = 3/4

7/8

8/8 = 4/4= 2/2

Transition to *Activity 1*.

Today we are going to focus attention on fractional equivalents—equal fractions. It's important that you "see" the equivalence and that you then think about some mathematical rules for equivalent fractions.

Activity 1: Fraction Strips with Rulers—Tools to Think With

For each student, have 5 strips of different colored paper, each 1" by 11", tape, clear plastic sheet protector, and colored markers. They will keep this fraction kit for the rest of the lessons.

Pass out 5 different colored strips to each student. Hold up the first piece and say:

Let's call this 1, or one whole. Mark it with a big "1."

1

Hold up a second piece and say:

I would like to fold this strip in half. How could I do this?

Make sure students fold width to width, not length to length. Have them fold and open the strip up and then say:

How do you know this is 1/2? What does 1/2 mean?

Students should say there are 2 equal parts, so one part out of two is 1/2. Direct them to label each 1/2 piece as 1/2 and mark the crease line with markers.

1/2	1/2

Take another strip of a different color and say:

I would like to fold this strip into fourths. How could I do this?

Students should recall half of a half, or fold it 2 times. Open it up and say:

How do you know this is 1/4? What does 1/4 mean?

Students should recall 4 equal parts, one part of the four is 1/4. Have them label every part with 1/4 and mark each crease line with markers.

1/4	1/4	1/4	1/4

Take a fourth strip and say:

I wonder what would happen if I fold this strip 3 times? How many parts will there be?

Listen for answers. Some students might say 6 thinking the segments are increasing by 2. Fold the strip 3 times (half of a half of a half) and open. There will be 8 sections. Ask:

What fractional parts are represented here? How do you know?

Like before, have them label each part as 1/8 and mark each crease line.

1/8	1/8	1/8	1/8	1/8	1/8	1/8	1/8

Pass out the last strip.

Do you see any patterns unfolding? How many times do you think we will fold this strip? How many equal parts do you think we'll find?

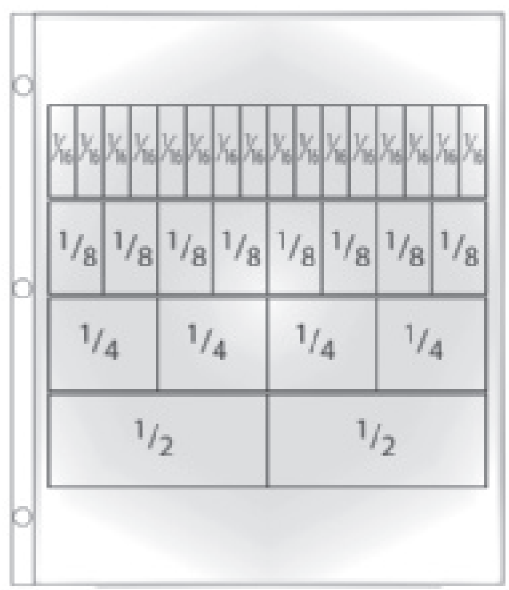
Listen for answers. Most should say 16 parts. Fold the strip 4 times. Again, unfold and label each part as 1/16.

1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16

Do you think we could continue to do this? Fold it 5, 6, or 7 times?

Most students will think it is impossible because of the thickness of the paper. If that didn't restrict us could we continue to do this? Yes. This is called density of fractions and there are an infinite number of fractions between 0 and 1.

Have students take the 5 strips and lay them one at a time in the sheet protector against a blank sheet of paper. Holding the protector with the hole punches facing up, lay sixteenths across the top edge, eighths underneath sixteenths, etc. Use tape only if a student needs it.



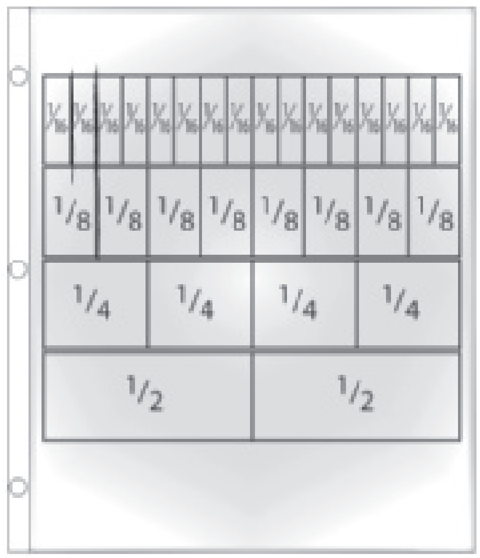
Using dry erase markers, tell students we are going to find fraction equivalencies and draw lines down the ends of ones that match. Start by drawing a line between the first two 1/16 pieces.

Say:

Look at 1/16. Does it match (line up) with any other fractions? (No.)

Look at 2/16. Does it match up with any other fractions? (Yes, 1/8.)

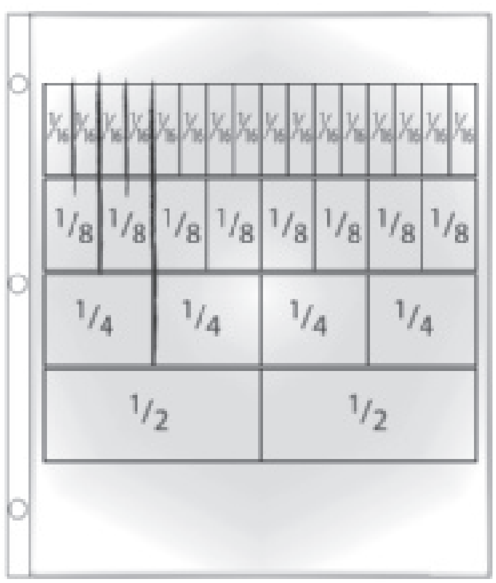
Draw a line up the edge of 2/16 to the edge of 1/8. See example:



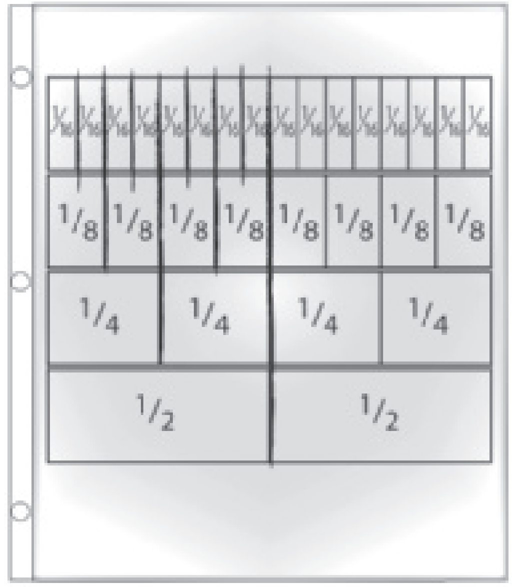
Does 3/16 line up with any other fractions? (No.)

Does 4/16 line up with any other fractions?

This time it will match both 2/8 and 1/4. Draw a line up the edge of 4/16, 2/8, and 1/4. See example:

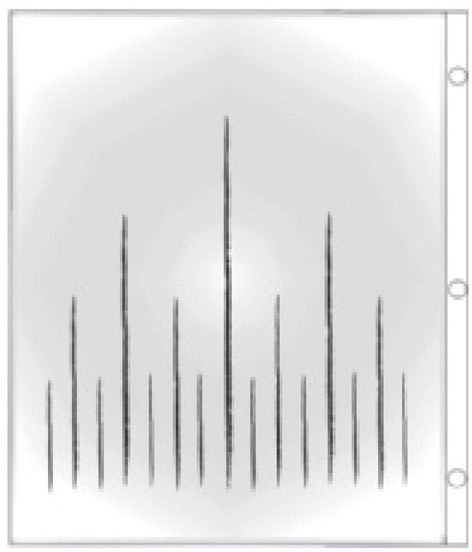


Continue with 5/16 (no matches), 6/16 (will line up with 3/8), 7/16 (no matches), and then 8/16 which will have the most matches. At this point it should look like this:



Continue this process all the way across the strip. When done, remove the fractions strips from the sheet protector.

What does this look like to you?



Write two sentences, each saying one thing you noticed as you were marking equivalent fractions.

Activity 2: Pattern Blocks—Another Tool

Form pairs. Distribute some Pattern Blocks to each pair, asking students to take a few yellow, red, blue, and green shapes. If actual Pattern Blocks are not available, make copies of *Blackline Master 4: Pattern Blocks* with colored markers.

Say:

In this activity, you are going to explore the fraction equivalents and values you "see" in these shapes.

Hold up the yellow hexagon and say:

For the purposes of this activity, the yellow hexagon is equal to 1 (one whole).

Direct students to *Activity 2* (*Student Book*, p. 89-91) and ask them to work together to answer the questions.

If some pairs finish sooner than others, challenge them with the question:

What are all the values in *Activity 2* if, instead of the hexagon being the whole, the red trapezoid is the whole (equal to 1)?

Activity 3: Thirds and Sixth Strips

Distribute more 1" by 11" strips. Students will add thirds, sixths, and one other strip to their kit of fraction strips. Ask them first to create the strip representing thirds. Say:

First, predict which benchmark fractions (1/2, 1/4, 3/4) you think one-third is between, then see if your prediction is correct. Based on your strips, which of the two benchmark fractions is 1/3 closer to? Why do you think that is so?

Direct students to look at all their benchmark fraction strips, including eighths and sixteenths and compare them to 1/3. Ask:

One-third is between which two fractions?

Have students mark their strips along the crease line and label each section 1/3.

Choose another color strip and ask students how they would create sixths.

Now, predict which set of benchmark fractions 1/6 falls between. Test out your prediction. What reasoning did you use to make your prediction?

Have students mark their strips in sixths. Ask students to think back to how they created their earlier strips, beginning with halves, then fourths. Hold up the remaining strip.

Predict the size of the fraction you would get when you divide sixths in half. Create and label the new strip (in twelfths).

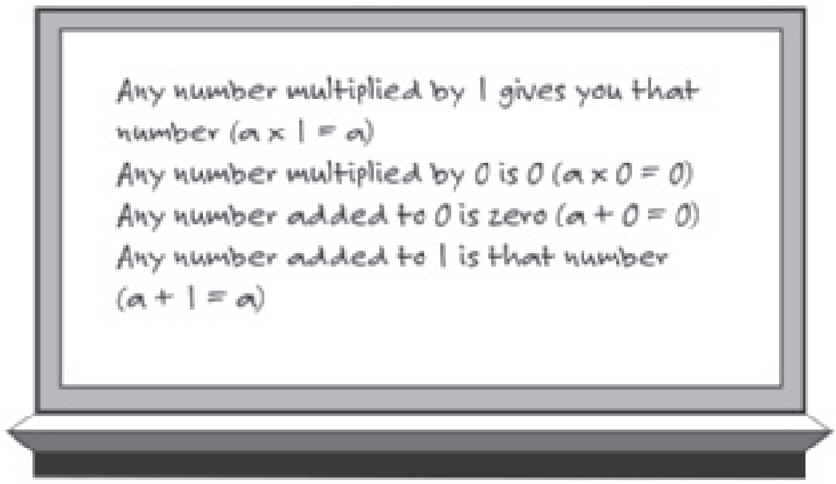
Students mark their strip in twelfths. If some students finish sooner than others, challenge them with the following:

Name and show two equivalent fractions between 1/3 and 1/2.

Add one-third to the class vocabulary list while students add one-third to their lists (*Student Book*, p. 166). Listen to make sure that students capture the idea of forming thirds by dividing by 3.

Activity 4: About Ones and Zeroes

Ask pairs to think about the following four statements (that you have posted).



Are these statements true or false?

After pairs have made decisions, ask for a vote, and facilitate a discussion. Finally, direct students' attention to *Activity 4: About Ones and Zeros* (*Student Book*, p. 93). The first section will be a place to record their notes of the discussion of the four statements. In the second section, they are asked to put those ideas into use.

Ask students to say what they are noticing in their own words. Write their words on the board verbatim. The take-home points from this investigation might be:

- 1 can be written in many ways. In fact, any number can be written in many ways.
- No matter the form, when one multiplies a number by 1, the product is that number. (Focus on this as the important idea that underlies the math of equivalent fractions.)
- If you start with a fraction, you can always get a fraction equal to it by multiplying by 1.
- 2/2, 3/3, 4/4 … , etc., are forms of 1 that are useful for generating equivalent fractions.

Math Inspection: One in the Denominator

This inspection is meant to help students understand the meaning of a fraction with a 1 in the denominator, distinguishing between the meaning of 1 in the denominator and 1 in the numerator.

Students often have a hard time understanding fractions greater than one. You might encourage students to focus on the size of the piece that makes the whole. For example, if the size of the piece is 1 and you have 5 of these then you have 5/1. You can also build on students' understanding that if they have a whole that is divided into quarters, they have 4/4, noting that the 4 in the denominator refers to the number of slices in a whole. An equivalent fraction can be written 1/1, whereas five times this amount can be written 20/4 or 5/1. This might help students feel more comfortable interpreting fractions with 1 in the denominator.

Ask students to explain each of their answers (True or False), not just the False ones. Problem 3 is one that often confuses students, and one that will allow you to assess their understanding of the meaning of a fraction. Listen for explanations of how 1/5 and 5/1 are different. Problem 7 is a generalization about the equivalence between a denominator of 1 and the number in the numerator. You may want to ask them to restate this idea using their own words.

Summary Discussion

Benchmarks

Ask each person to write down two or three statements about equivalent fractions. Then ask them to pass those statements to a partner to discuss each statement's accuracy. Finally, ask each pair to offer one statement they feel confident about, and any that they might be uncertain about. You might hear ideas like these:

- Any fraction can be turned into an equivalent fraction by splitting each piece that makes up the whole (the denominator) into two equal parts.
- The denominator for the new fraction will be twice the original fraction because there are twice as many pieces in the whole.
- The numerator for the new fraction will be twice the original numerator. To make it an equivalent fraction, we need to keep the value the same, and to keep the balance, there have to be twice as many pieces in the numerator as well as the denominator. For example, in 3/4 = 6/8, each of 4 pieces in the whole was split into two, so there are 8 pieces in the denominator. And each of 3 pieces in the numerator was also split into two, so there are 6.
- There are more pieces, but they are smaller.

If the above points are not mentioned, add them to the list.

Tell students to turn to *Reflections* (*Student Book*, p. 171), to record their thoughts about equivalent fractions.

Practice

Equivalent Fractions, p. 96
For more practice on finding equivalent fractions.

Between 1/3 and 1/2, p. 97
For practice using the benchmark fractions 1/3 and 1/2.

Ratcheting Up (or Down) a Notch, p. 98
For practice with sixteenths and thirty-secondths.

Where to Place It?, p. 99
For practice placing fractions on a number line.

2/3 and 3/4, p. 100
For practice placing fractions on a number line.

Mental Math Practice

Using Properties, p. 101

Test Practice

Test Practice, p. 102

Looking Closely

Observe whether students are able to

Describe the meaning and value of any rational number written in fraction form *a/b*

The language and sketches students have used to represent halves and fourths should be familiar and generalizable at this point, easily applied to any number in the form *a/b*, (numerator over the denominator). Still, students might be confused by fractions with similar numbers, like 3/5 and 5/3. Look for evidence that students can use visual tools to reason about fractions. Offer fraction strips, tiles, rulers, and Pattern Blocks to assist them in making sense of fractions of any amount.

Generate and demonstrate equivalent fractions

Before using equivalent fractions in operations, students should take time to compare fraction strips, to see how the same part-whole relationship can be labeled with a variety of fractions. It is essential that if students question or are confused by equality and equivalence, you affirm that the amounts are different, but the part-whole relationship is equivalent. Half of $40 is a larger amount of money than half of $10. But the relationship is equivalent. It's half in either case.

Rationale

Lessons 1–5 scaffold students' reasoning about part-whole relationships. By sketching and manipulating objects, students should have a grasp on halves and quarters and their equivalents before they begin doing formal mathematical procedures like reducing or simplifying fractions. Even if students remember procedures, asking them to solve problems visually first, then connecting their reasoning and steps to the math procedures, helps keep the meaning intact. We are guided by the writing of well-respected mathematics teacher educators, and believe their insights hold true for adults as well as younger learners:

> ... It makes sense to delay computation and work on concepts if students are not conceptually ready.
>
> Premature attention to rules for fraction computation has a number of serious drawbacks. None of the algorithms helps students think about the operations and what they mean. When students follow a procedure they do not understand, they have no means of assessing their results to see if they make sense. Second, mastery of the poorly understood algorithm in the short term is quickly lost. When mixed together, the differing procedures for each operation soon become a meaningless jumble.
>
> — Van de Walle, J., K. Karp, J. Bay-Williams, 2012

Math Background

So much about operations with fractions hinges on understanding equivalent fractions and how one comes by them. Equivalent fractions have the same value. The idea that a quantity can have more than one name is essential. Students can do this; they know calling someone by a first name, combination first name and last name, or last name with a title, like Ms. Cisneros, can communicate different messages, but it is still the same person. The relationship of half is still the same, whether we have 8 out of 16 or 50 out of 100. Calling on different names as well as finding and using equivalent relationships, like 2/4 instead of 1/2, makes it possible to combine and compare fractions.

It is easy to lose focus on the whole, but having that as a focus should enable students to use the identity 'principle' (property), $x/x=1$. If you start with a fraction, you can always find an equivalent fraction by multiplying by one.

In the next lessons students will look for "whole" or "one" within an improper fraction. They will find equivalent fractions to make combining and subtraction easier.

Context

Mention tools like ratchets, measuring cups, or other objects that come in a sequence of sizes labeled with fractions. Particularly listen for those with which students might be familiar.

Facilitation

Students often have a hard time understanding fractions greater than one. You might encourage students to focus on the number of pieces that make the whole. For example, if the number of pieces in one whole is one and if you have 5 of these, then you have 5/1. This builds on students' understanding that if they have a whole that is divided into quarters, they have 4/4, noting that the 4 in the denominator refers to the size of the slice. This might help students understand why we sometimes put 1 in the denominator.

When reviewing *Math Inspection: One in the Denominator* (*Student Book*, p. 95), ask students to explain each of their answers (True or False), not just the False ones. Problem 3 is one that often confuses students, and one that will allow you to assess their understanding of the meaning of a fraction—and the difference between 1/5 and 5/1. Question 7 is a generalization about the equivalence between a denominator of 1 and the number in the numerator. You may want to ask them to restate that using their own words.

Making the Lesson Easier

Keep fraction strips, Pattern Blocks, and number lines handy. Skip the 17 fractions during the *Opening Discussion*, but bring them up in the *Summary Discussion*. Skip sixths and come back to them later as a way to introduce fractions equivalent to one-third and two-thirds.

Making the Lesson Harder

For *Activity 3*, encourage students to find the decimal and percent equivalents for benchmark fractions. Using mental math, many students should be able to figure out the percent or decimal for one-eighth. A quarter is 25% and half of a quarter is one-eighth. By that token, half of 25% is 12.5%. While most of us might not be able to divide by 8 or multiply by .125 using mental math, many of us can in our heads, divide by half, half, and half again. So finding 12.5% of any number suddenly becomes very doable.

If students seem to grasp this idea easily, you may encourage them to explore the relationships among thirds and sixths and their decimal equivalents. You might need to remind them that if they don't know an equivalent off the top of their heads, one way to do this is to divide with a calculator the denominator into the numerator. Discuss relative size and the fact that in most situations with thirds, you round off with .33 or .66 because the decimal does not resolve. Following the pattern of half of one-quarter (25%) is 12.5%, see if they can find the percent equivalent for 1/6, which is approximately .15. These ideas can be revisited when students study thousandths.

During the opening discussion to put fractions in sequential order, I saw many groups of students use shapes such as circles and rectangles. Another group used one big number line, stacking equivalent fractions on top of each other to show they coincided on the number line. The groups using shapes did a lot of erasing and re-drawing when they discovered a new fraction that was in between the others. The group with the number line finished faster than the other groups. When the groups compared their results and methods, the groups with shapes all agreed that the number line method seemed easier to follow and understand than the various pictures of shapes.

Activity 1 is one I look forward to, but I notice many students are bored with it at first. The folding seems predictable and easy to them since we've done so much work with halves, fourths and eighths up to that point. However once we have taped the strips together and start finding equivalencies, the students light up. Most students appreciated having a visual way of finding equivalencies and planned to keep their plastic sleeve of fraction strips even as they continued on to new math classes.

A big eye opener for me was *Activity 3*, having the students predict where they thought one-third would fall on their fraction strips. Instead of using what they knew about part and whole to make the prediction, they seemed to be guessing. I am surprised by how many students say one-third will be smaller than one-fourth. Then we used the strips to fold thirds and compared to other fractions and students recalled that as the size of the pieces increases, the denominator gets smaller.

Jill Gorneau
Portland Adult Education, Maine

EMPower Plus

extending mathematical power

Split It Up

More Fractions, Decimals, and Percents

STUDENT BOOK

McGraw Hill Education

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Split It Up: More Fractions, Decimals, and Percents

Introduced in *Using Benchmarks*, 10% becomes the launch and a central math concept throughout *Split It Up*. This unit expands students' repertoire of fractions, decimals, and percents to include multiples of 10%, 1% or 1/100 and its multiples, as well as 1/8 and 1/3 and their multiples while maintaining the focus on fractions as representations of part/whole relationships and as models of the portion of an amount. Students puzzle over situations involving newspaper statistics, space allocations, taxes, material purchases, and nutrition labels, as they hone their rational-number mental math skills. The emphasis on mental math strategies allows students to reason about ways to combine and break apart amounts. For instance, they can regard 42% as the combination of four 10% amounts and two 1% amounts, or reason about 3/8 as 1/2 (50%) less 1/8 (12.5%), arriving at 37.5%.

Continued use of diagrams, manipulatives, and other visual models works to support reasoning. Students determine portions and determine the whole given a part. They calculate the percent of increase or decrease of whole numbers, compare fraction, decimal and percent amounts to benchmarks, and consider which form–fraction, decimal, or percent–seems best to use when solving problems. In *Split It Up*. Students learn these skills in ways that continue to serve them well in the world beyond the classroom. They come to rely on reasoning, not memorization, when solving mathematical problems that involve fractions, decimals, and percents.

Mathematical Concepts Covered for *Split It Up: More Fractions, Decimals, and Percents*

Book Description: Building upon their command of common benchmark fractions, students add 1/3's, 1/8's, and 1/100's, and their decimal and percent equivalents, to their repertoire of part-whole relationships.

Lesson Number:	Lesson Name:	Mathematical Concepts/Topics Covered
Opening the Unit	Split It Up	• Fractions, decimals, and percents in everday print materials • Problem solving with fractions and decimals and percents assessed
Lesson 1	One-Tenth	• One-tenth (and its multiples) related to benchmark fractions, particularly multiples of halves and thirds • Visual and numeric representations for one-tenth • Strategies for finding one-tenth of a quantity
Lesson 2	More About One-Tenth	• Representations equivalent to tenths • The role of place and the decimal point in a number's value
Lesson 3	What Is Your Plan?	• Strategies to determine multiples of 10% of an amount • The whole is equivalent to 100% • Arrays of 50 and 100 as a visual for percents • Multiples of 10% and their equivalent fractions
Lesson 4	One Percent of What?	• Strategies for finding 1% and its multiples of three- and four-digit numbers • Comparisons between 10% of an amount and 1% of another • The effect of the size of the whole on the size of a percent
Lesson 5	Taxes, Taxes, Taxes	• Multiples of 1% to find single-digit percentages • Multiples of 10% and 1% combined to find two-digit percentages
Lesson 6	Decimal Hundredths	• Visuals to show decimal place value in the tenths and the hundredths created • Fractions for decimal equivalents in the hundredths • Zeroes in numbers as optional or mandatory to expressing a number's value

Lesson 7	Smaller and Smaller	• Relationships among thousandths, hundredths, tenths, and ones • Expanded notation • Rounding decimals in the thousandths to the nearest 1, 0.1, and 0.01
Lesson 8	Adding and Subtracting Decimals	• Meanings for addition and subtraction operations with whole numbers and decimals • Place value to judge the soundness of answers to addition and subtraction problems involving fractions, decimals, and percents
Lesson 9	Multiplying Decimals	• Multiplication with whole numbers connected to fractions, especially to multiplication with decimal numbers • Reliable methods for multiplication with decimal numbers • Visual models and patterns for multiplication short-cuts with whole numbers and decimal numbers • Properties of arithmetic (e.g., commutative, distributive, associative) applicable to decimals
Lesson 10	Dividing Decimals	• Interpret division with decimals as splitting an amount or finding how many groups can “fit into” an amount • Matching verbal language and symbolic notation for division to a concrete model • Comparing and contrasting a/b with b/a
Lesson 11	Apply Decimal Learning	• Applying decimal operations and percents in real-life scenarios
Closing the Unit	Put It Together	• Identifying areas for future instruction • Problem-solving involving fractions, decimals, and percents • Reviewing conceptual understanding of operations involving decimals

LESSON **10**

Dividing Decimals

How do you think about division?

When faced with a division problem such as $12\frac{1}{2} \div 5$, you can think of it in one of two ways. If you think of division as a splitting action, you can picture $12\frac{1}{2}$ lbs. of food shared among 5 people. The result is $2\frac{1}{2}$ lbs. per person.

You can also look at it another way. You might think, "How many 5 lb. packages are in $12\frac{1}{2}$ lbs.?" The answer is there are two 5 lb. packages and $\frac{1}{2}$ of a 5 lb. package ($2\frac{1}{2}$). Think about these two meanings as you explore problems such as $8.214 \div 0.43$.

Use your estimation and mental math skills as you work with objects, diagrams, and measurement. Then test some methods you learned for dividing decimals and see why they work the way they do.

Activity 1: What Is the Message?

Symbols send a message. They can call for an action or state a relationship. For example:

$15 \div 3$, $\frac{15}{3}$, 15/3, and $3\overline{)15}$,

1. $3.25 \div 5 =$

   a. Circle the message.

   Split 3.25 into 5 parts. -or- How many 5's are in 3.25?

   b. Use words or draw a picture to represent the message you chose.

   c. Use numbers and words to explain a situation in everyday life that matches the problem.

2. $3.25 \div 0.5 =$

   a. Circle the message.

   Split 3.25 into 0.5 parts. -or- How many 0.5's are in 3.25?

   b. Use words or draw a picture to represent the message you chose.

c. Use numbers and words to explain a situation in everyday life that matches the problem.

3. 12.5 ÷ 100 =

a. Circle the message.

Split 12.5 into 100 parts. -or- How many 100's are in 12.5?

b. Use words or draw a picture to represent the message you chose.

c. Use numbers and words to explain a situation in everyday life that matches the problem.

4. 100 ÷ 12.5 =

a. Circle the message.

Split 100 into 12.5 parts. -or- How many 12.5's are in 100?

b. Draw a picture to represent the message you chose.

c. Use numbers and words to explain a situation in everyday life that matches the problem.

5. Compare Problems 1 and 2. How are they the same? How are they different?

6. Compare Problems 3 and 4. How are they the same? How are they different?

Activity 2: Methods for Dividing Decimals—They Have to Make Sense!

Part A: Whole Number Divisors

What do you remember about dividing decimals with paper and pencil? First make an estimate. Then solve the problem using a paper-and-pencil method of your choice.

1. Will shared 8.64 pounds of dog food equally among his three dogs. How much food did each dog eat?

   a. My estimate: ______

   b. With paper and pencil (show all work):

   c. Check with a calculator: ______

   d. The problem above has a whole number divisor. List the steps you took to divide a decimal by a whole number.

   e. Compare and discuss your method with a partner. In what ways are your methods the same? In what ways are they different?

   f. Do the methods work with these problems?

      $0.156 \div 3$ $\quad\quad$ $1.5 \div 30$

      Do the methods make sense to you? What do you understand? What is confusing? Explain.

Part B: Dividing by a Decimal

1. Johanna buys 8.3 lb. of hamburger. How many 0.5 lb. burgers can she make?

   a. My estimate: ______

   b. With paper and pencil (show all work):

   c. Check with a calculator: ______

   d. The problem above does NOT have a whole number divisor. List the steps you took to divide a decimal by a decimal.

   e. Compare and discuss your method with a partner. In what ways are your methods the same? In what ways are they different?

   f. Do the methods work with these problems?

      $0.156 \div 0.03$ $\quad\quad$ $1.5 \div 0.003$

      Do the methods make sense to you? What do you understand? What is confusing? Explain.

Activity 3: Weekly Expenses

A bank requires home mortgage applicants to fill out an expense report.

BANK INSTRUCTIONS: All expense figures must be listed by their *weekly* total. Do **NOT** list expenses by their *monthly* total. Divide the average monthly expense by 4.3 to compute the weekly expense. For example, if your average rent is $500.00 per month, divide 500 by 4.3. This will give you a weekly expense of $116.28.

1. Why does the bank tell its mortgage applicants to divide by 4.3?

2. Christina is very organized. She has kept a record for the past year of all her monthly payments. She will use this information to report her weekly total for each expense.

Category	Jan	Feb	Mar	Apr	May	Jun
Rent	$1,000	$1,000	$1,000	$1,000	$1,000	$1,000
Cell Phone	$68.64	$69.67	$67.89	$67.95	$70.12	$68.33
Heat (oil)	$ 171.99	$ 334.35	0	$ 232.78	0	0
Electricity	$ 27.59	$ 44.26	$ 55.10	$ 57.77	$ 39.79	$ 56.16

Category	Jul	Aug	Sep	Oct	Nov	Dec
Rent	$1,000	$1,000	$1,000	$1,000	$1,000	$1,000
Cell Phone	$71.03	$68.46	$69.78	$70.23	$70.52	$67.23
Heat (oil)	0	0	0	$324.90	$89.56	$203.12
Electricity	$ 30.54	$ 33.47	$ 43.12	$ 40.68	$ 26.49	$ 28.82

Complete the table below to figure out what Christina should report to the bank.

a. First, calculate the average monthly expense for each category.

b. Second, mentally estimate the weekly expense for each category.

c. Third, calculate a weekly amount from a monthly amount. Divide the average monthly expense by 4.3.

Category	Average Monthly Expense	Estimated Weekly Expense	Weekly Expense to Report to the Bank
Rent			
Cell Phone			
Heat (oil)			
Electricity			

d. Describe your estimation (mental math) method.

Practice: Four Ways to Write Division

Division can be written in several ways.

Be careful. The order of the numbers is important in division.

Complete the table. The first problem has been done as an example.

1. $8\overline{)56}$	$56 \div 8$	$\frac{56}{8}$	56 divided by 8
2.	$0.6 \div 8$		
3. $7.2\overline{)1.24}$			
4.		$\frac{0.8}{4}$	
5.	$1\frac{1}{2} \div 3$		
6.			$\frac{1}{4}$ divided by 10
7.			20.3 divided by 10
8. $10.5\overline{)1}$			

Practice: Target Practice 0.1, 0.01, 100

1. How can you divide a number to get 0.1 in each group? Can you solve the problem in three turns or fewer on the calculator? Try it.

Round 1: Start with 8. Your goal is to divide 8 by a number to reach 0.1.

First try $8 \div$ _____ $=$ _____

Second try $8 \div$ _____ $=$ _____

Third try $8 \div$ _____ $=$ _____

Round 2: Start with 42.

First try $42 \div$ _____ $=$ _____

Second try $42 \div$ _____ $=$ _____

Third try $42 \div$ _____ $=$ _____

Round 3: Start with 249.6.

First try $249.6 \div$ _____ $=$ _____

Second try $249.6 \div$ _____ $=$ _____

Third try $249.6 \div$ _____ $=$ _____

2. Try it again, this time with a target of 0.01.What number do you divide by to get 0.01 in each group? Look for a pattern.

$8 \div$ _____ $= 0.01$

$42 \div$ _____ $= 0.01$

$249.6 \div$ _____ $= 0.01$

3. Ask the question: "How many ___ in ____?" Try a few numbers. Do you see a pattern that helps you find a solution?

$250 \div \boxed{} = 100$

$1.5 \div \boxed{} = 100$

$0.1 \div \boxed{} = 100$

Practice: Which Is Not the Same?

Order is important in division. $\frac{1}{2} \div 5$ is not the same as $5 \div \frac{1}{2}$.

Use what you know about division notation and rules of order to choose the one expression that is not the same as the others.

(Reminder: The math in parentheses is calculated first.)

1. a. $2.8 \div 7$

b. 2.8 divided by 7

c. $2.8\overline{)\,7}$

d. $\frac{2.8}{7}$

2. a. $\frac{1.5}{1{,}000}$

b. $1{,}000 \div 1\frac{1}{2}$

c. $1.5\overline{)1{,}000}$

d. $\frac{1{,}000}{1\frac{1}{2}}$

3. a. 10 divided by $\frac{1}{6}$

b. $(14 - 4) \div \frac{1}{6}$

c. $10\overline{)0.6}$

d. $\frac{10}{0.167}$

4. a. $(72.8 + 7.2) \div 4$

b. $72.8 + (7.2 \div 4)$

c. $(72.8 \div 4) + (7.2 \div 4)$

d. $18.2 + 1.8$

5. a. $\frac{1}{2} \div \frac{1}{4}$

b. $0.25\overline{)0.5}$

c. $0.5\overline{)0.25}$

d. $\frac{0.5}{0.25}$

6. a. $\frac{2\frac{7}{8}}{3}$

b. $2.875 \div 3$

c. $\frac{(2 + \frac{7}{8})}{3}$

d. $2.875\overline{)\,3}$

Practice: Where's the Point?

Think of division as sharing. Place the decimal point in the correct spot in the underlined number.

1. $234.6 \div 22.58 = \underline{1\ 0\ 3\ 8\ 9\ 7\ 2\ 4}$

2. $2346.0 \div \underline{2\ 2\ 5\ 8} = 10.389724$

3. $234.6 \div 2.258 = \underline{1\ 0\ 3\ 8\ 9\ 7\ 2\ 4}$

4. $23.46 \div 22.58 = \underline{1\ 0\ 3\ 8\ 9\ 7\ 2\ 4}$

5. $6.5 \div 10 = \underline{0\ 0\ 6\ 5\ 0}$

6. $6.5 \div 100 = \underline{0\ 0\ 6\ 5\ 0}$

7. $6.5 \div \underline{1\ 0\ 0} = 6.5$

8. $15\frac{3}{4} \div 2.8 = \underline{5\ 6\ 2\ 5}$

9. $1.575 \div 2.8 = \underline{5\ 6\ 2\ 5}$

10. $\underline{1\ 5\ 7\ 5} \div 28 = 0.5625$

Practice: Multiplication and Division Patterns

1. Fill in the chart. Check your work with a calculator. The first problem has been done for you.

Original Amount	Multiplied by 2	Divided by 2	Multiplied by $\frac{1}{2}$	Divided by $\frac{1}{2}$
a. 7	$7 \times 2 = 14$	$7 \div 2 = 3\frac{1}{2}$	$7 \times \frac{1}{2} = 3\frac{1}{2}$	$7 \div \frac{1}{2} = 14$
b. 26				
c. 0.5				
d. $12\frac{1}{2}$				
e. 8.9				

2. Fill in the chart. Check your work with a calculator. The first problem has been done for you.

Original Amount	Multiplied by 4	Divided by 4	Multiplied by $\frac{1}{4}$	Divided by $\frac{1}{4}$
a. 7	$7 \times 4 = 28$	$7 \div 4 = 3\frac{3}{4}$	$7 \times \frac{1}{4} = 3\frac{3}{4}$	$7 \div \frac{1}{4} = 28$
b. 26				
c. 0.5				
d. $12\frac{1}{2}$				
e. 8.9				

Practice: Division Patterns

1. Solve each problem to continue the pattern.

   $10 \div 5 = 2$

   $9 \div 5 = 1.8$

   $8 \div 5 =$

   $7 \div 5 =$

   $6 \div 5 =$

   $5 \div 5 =$

   $4 \div 5 =$

   $3 \div 5 =$

   $2 \div 5 =$

   $1 \div 5 =$

2. What pattern do you notice? Why does it work?

Practice: Think Metric

1. How many 2.5 cm segments are in a 10 cm segment?
   a. Division equation:

   b. Picture or number line:

2. How many 0.1 liter portions are in 3 liters?
   a. Division equation:

   b. Picture or number line:

3. How many 3.5 kg portions are in 16 kg?
   a. Division equation:

   b. Picture or number line:

4. How many 0.75 km are in 5 km?
   a. Division equation:

   b. Picture or number line:

Calculator Practice: Decimal Division

Estimate the solution to each problem. Then calculate the exact answer. You can solve the problems in your head or with a calculator.

Problem	A Thoughtful Estimate	The Exact Answer
1. $1.11\overline{)37.74}$		
2. $0.9516 \div 4$		
3. $\frac{38.6}{2}$		
4. $199 \div 2.3$		
5. 16.8×2		
6. $16.8 \div 2$		
7. $6\frac{3}{5} \div 10$		
8. $\frac{(3\frac{1}{2} + 2\frac{1}{2})}{6.5}$		
9. $\frac{(5 - 2\frac{1}{5})}{1.25}$		

Calculator Practice: Way Under Average?

For each problem, write the numbers you used to estimate your answer or the numbers you entered on a calculator.

1. Sonya waits a quarter of an hour at the emergency room. How much under the average wait time is this? Base your average on the following recent wait times:

   55 minutes

   18 minutes

   95 minutes

   25 minutes

2. Shakita buys a house for $1.1 million. How much below the average house price is Shakita's house. Base your average on the following recent home sales:

   $1.5 million

   $2.3 million

   $1.9 million

   $2.2 million

   $2.1 million

   $1.9 million

3. Karla pays $3.59 for gas. Is she paying more or less than the average (based on the following recent gas prices)?

   $3.49

   $3.69

   $3.79

   $3.49

   $3.49

   $3.69

   What is the difference between the average and what Karla is paying?

4. Between December 2012 and February 2013, 0.56 inches of rain fell in Las Cruces, New Mexico.

   How does this amount of rainfall compare to the total of the averages for the same months since 2007?

Average		Recent	
December	0.60"	Dec. 2012	0.37"
January	0.32"	Jan. 2013	0.19"
February	0.17"	Feb. 2013	0.0"

5. How does this compare with the precipitation from December 2012 through February 2013 where you live?

Practice: Free Choice

Solve each problem. Then indicate the way you solved it, using: *mental math, paper and pencil,* or *a calculator*?

1. Set 1	Write your answer	How did you solve it?
a. $10 \div 2.5 =$		
b. $10 \div 0.25 =$		
c. $10 \div 0.025 =$		
d. $10 \div 0.250 =$		

2. Set 2	Write your answer	How did you solve it?
a. $4.5 \div 0.9 =$		
b. $0.45 \div 0.9 =$		
c. $4.5 \div 0.9 =$		
d. $4.5 \div 0.90 =$		
e. $0.045 \div 0.9 =$		

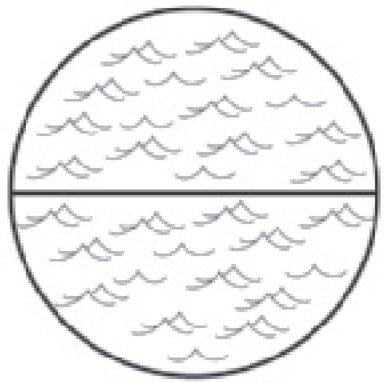
3. Set 3	Write your answer	How did you solve it?
a. $0.42 \div 6 =$		
b. $0.042 \div 6 =$		
c. $4.2 \div 6 =$		
d. $42 \div 6 =$		

4. Which set of problems was the easiest to do using mental math? Why?

Extension: Geometric Formulas

Farhad knows the distance across a circle (**diameter**) is $\frac{1}{3}$ of the distance around it (**circumference**).

1. a. Estimate the distance to swim across Moon Lake, which is 5 miles in circumference.



Moon Lake

Remember the diameter is approximately $\frac{1}{3}$ of the distance around a circle. You can also say a circle is approximately 3 times longer around than the distance through its center.

b. You can calculate a more precise answer using a calculator and the formula for finding the diameter of a circle. Use the value for pi (π), which is part of the formula:

circumference ÷ π = diameter

OR

π times the diameter = distance around

Moon Lake is 5 miles around. Divide by 3.14 to get the diameter of Moon Lake, which is 1.59 miles.

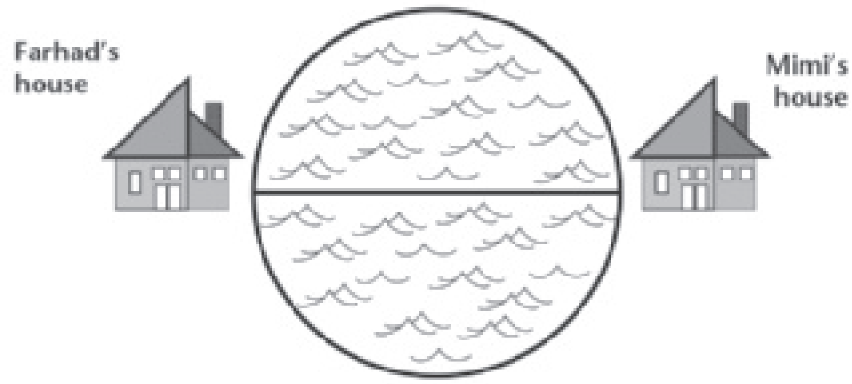
5 miles ÷ 3.14 or even more precisely, 5 miles ÷ 3.14159

Diameter of Moon Lake = 1.59 miles

How close is your estimate to the calculated distance?

2. Farhad swims halfway across the lake. How far did Farhad swim?

3. Mimi takes a boat from her house to Farhad's house and back to her house. How far does Mimi go by boat if she goes through the center of the lake?



4. Joy swam across the lake and back and thought it was a very hard swim. She decided to measure the distance around the lake and found that it is 5.5 miles. Mimi tells her that it is not such a big deal if the lake is 5 miles around or 5.5 miles around.

   a. What is the diameter of a 5.5 mile lake?

   b. Is swimming back and forth across a lake with a circumference of 5.5 miles a big deal compared to swimming across a lake with a circumference of 5 miles? Why?

Test Practice

1. $2.5 \div 0.25 =$
   (a) 0.1
   (b) 0.625
   (c) 1.0
   (d) 10
   (e) 100

2. Choose the number to complete the equation.

   $20 \div \boxed{} = 100$

   (a) 0.02
   (b) 0.2
   (c) 0.5
   (d) 2
   (e) 5

3. I saved \$9,000 over a period of five-and-a-half years. If I saved the same amount each year, how much did I save annually? Which of the following expressions represents how much I saved annually?
   (a) $9{,}000 \times 5.5$
   (b) $5.5\overline{)9{,}000}$
   (c) $5\frac{1}{2} \div 9{,}000$
   (d) $\frac{9{,}000}{12}$
   (e) $5.2 \times 9{,}000$

4. Which of the following expressions will have an answer that is not the same as the other answers?
   (a) $100 \times \frac{1}{5}$
   (b) $100 \div 5$
   (c) $(100 \times \frac{1}{10}) \times 2$
   (d) $\frac{100}{5} \times \frac{1}{5}$
   (e) 100×0.2

5. $\frac{1}{2} \div 2$
   (a) 0.1
   (b) 0.25
   (c) 0.4
   (d) 1.0
   (e) 2.5

6. Write the number in the box to complete the equation.

   $4.56 \div \boxed{} = 0.1$

FACILITATING LESSON **10**

Dividing Decimals

How do you think about division?

Synopsis

If students have studied division before, they may be aware that problem solvers can think of division either as 1) sharing, splitting, or partitioning an amount or 2) finding how many groups of a specific size or quantity are in an amount (How many 8's in 64?). In this lesson, students will review the two ways to look at division with decimal numbers. They will also explore why the traditional algorithm of the moving decimal point works.

1. Student pairs interpret and make a depiction of a numbers-only division problem, then create a real-life story to determine the answer.
2. Students share their remembered paper-and-pencil procedures for dividing decimals, then explore why the algorithms work.
3. Given a monthly expenditure, students calculate the weekly expenditure, explaining why division by 4.3 makes sense.

Objectives

- Interpret division with decimals as an act of splitting an amount or finding how many groups can "fit into" an amount, extending these ideas from the study of division with whole numbers
- Match verbal language and symbolic notation for division to a concrete model
- Compare and contrast *a/b* with *b/a* in various notation

Materials/Prep

- **IMPORTANT:** Read the *Rationale* and *Math Background* in *Lesson 10: Commentary*, pp. 119-120, before preparing this lesson
- Calculators
- Rulers
- *Blackline Master 10: 100-Block Grids*
- Fraction strips marked with equivalents

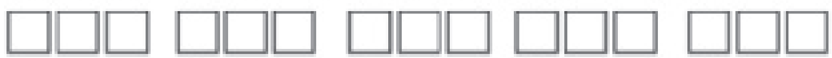
Opening Discussion

Open with the following question:

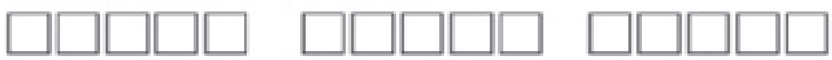
How would you explain the meaning of 15 ÷ 5?

Ask students to draw 15 ÷ 5. Looking for two different interpretations and ask volunteers to draw them on the board:

It means "split 15 into 5 groups. There are 3 in each group."



It means "how many 5's are in 15?"



If students do not offer both interpretations, make them explicit.

Summarize:

When you see division, you can use either way to make sense of the problem.

Which way helps you make sense of these equations? Which does not?

Write the following four problems on the board or display them on an overhead, and ask students to explain the meaning, either by using the idea of splitting or by asking "how many ___ in ___?"

1. 4.6 ÷ 4.6 =
2. 10 ÷ 2.5 =
3. 0.25 ÷ 2 =
4. 2.5 ÷ 10 =

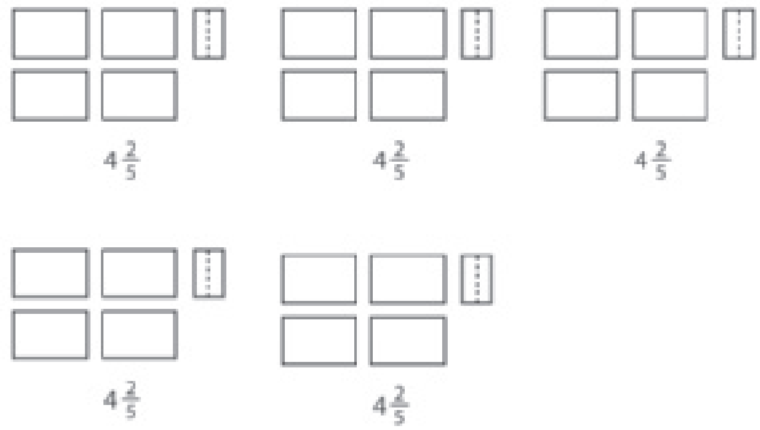
Invite discussion and ask students to make diagrams on the board. The first two problems are most likely interpreted as "how many ___ in ___?" The last two are more easily seen as splitting.

Say:

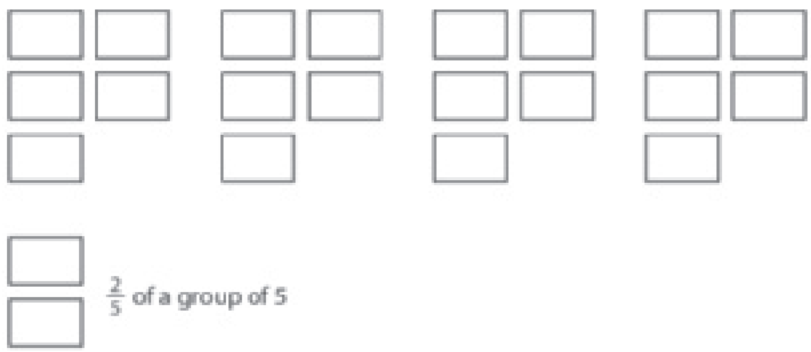
Let's draw a division problem.

Post the division problem 22 ÷ 5 = ___.

Solicit one volunteer to draw a picture of the problem, and ask the class whether they agree with the representation. This example is a partitioning situation, where the diagram shows five groups of 4 2/5 each.



Ask students for another way to interpret the problem. Students may show four whole groups of 5, and 2 left over, as in this example.



If students need another example, direct them to their lesson introduction and ask them to draw the two contextualized interpretations of 12 1/2 ÷ 5.

Keep both interpretations in mind as you work on the problems in this lesson.

Hand out rulers, copies of *Blackline Master 10: 100-Block Grids* and have fraction strips handy as thinking tools.

Activity 1: What Is the Message?

Set up the activity by saying:

You know that math symbols always mean something. You can show the meaning with a picture or diagram, or connect the math to a real-life situation.

Direct students to *Activity 1: What Is the Message?* (*Student Book*, p. 166). When students are done, have them share their diagrams publicly. Watch out for order (3.25 ÷ 5 ≠ 3.25 ÷ 0.5) and for understanding the difference between 12.5 ÷ 100 and 100 ÷ 12.5.

Activity 2: Methods for Dividing Decimals—They Have to Make Sense!

Refer students to *Activity 2: Methods for Dividing Decimals—They Have to Make Sense!* (*Student Book*, p. 169). Ask students to solve the two problems in Parts A and B with paper and pencil. One has a whole number divisor, the other has a decimal divisor.

As students work, walk around to see the computational strategies they use. At this point, you are just informally trying to determine who already uses a reasonable algorithm or method and who does not. Notice if there are some methods from other countries.

When everyone has had a chance to work through the problems, ask for volunteers to show the solutions at the board.

Part A: Whole Number Divisors

The first problem has a whole number divisor. Develop a method or a rule for dividing by a whole number:

When I want to divide a decimal number by a whole number, I ______________.

Part B: Dividing by a Decimal

Now turn to the second problem, with a decimal divisor. Pay special attention to see which students seem to be able to use the short-cut of moving the decimal point. Notice if there are some interesting methods from other countries. Some may find common denominators.

When everyone has finished, talk about their procedures.

Two problems (f.) have decimal divisors. Develop a method or rule for dividing by a decimal:

When I want to divide by a decimal number, I _________________.

Say:

Let's explore *why* these rules work.

Heads Up!

You may see two algorithms—procedures or methods—that people have been taught for paper-and-pencil decimal division. Even though these procedures appear different, they are based on the same principle: that is, equivalent ratios yield the same answer—or quotient.

METHOD 1

Some people *move* the *decimal point* in the divisor and then again in the quotient the same number of places, like this:

$.2\overline{)8} \longrightarrow 2.\overline{)80}$

Why does that make sense?

(80/2 is *equivalent* to 8/.2 because 8/.2 × (10/10) = 80/2 = 40, and multiplying by 10/10 is the same as multiplying by 1.)

METHOD 2

Some people *find common denominators* for the divisor and dividend, and then find it easier to think about what the problem means, like this:

8 /0.2 ⟶ 80/10 ÷ 2/10 = 40

(80/2 is *equivalent* to 8/0.2 because 8/0.2 × (10/10) = 80/2, and multiplying by 10/10 is the same as multiplying by 1.)

Ask students:

Which one of these methods is closest to your procedure?

Activity 3: Weekly Expenses

Refer students to *Activity 3: Weekly Expenses* (*Student Book*, p. 171).

Ask pairs of students to work together to think about why the bank instructions say to divide the monthly expense by 4.3. Share reasons.

Then ask students to work individually on the next part, using Christina's spreadsheet.

Given the monthly amounts for a year's expenses, the students' task is first to find the average monthly expenses, and then to estimate and to compute the exact weekly amounts to be reported on the form. Encourage calculator use to arrive at exact calculations.

After students have finished, ask them to compare answers with their partners and to share methods they used to do the mental estimations and the exact calculations. Then bring the class together and reinforce the variety of useable methods:

Who had an estimation method that worked well?

Spend time on the mental math methods. Ask individuals to come up to the board to explain and record their mental math strategies. Some may choose to reason with multiplication, others with division. There are many valid ways to do this. For example, some students might reason about splitting up the money this way:

- **Rent:** Because it is the same every month, $1,000, a good estimate is $200 to $250 per week. $250 would be a little too much and $200, a little too small.
- **Cell Phone:** The bill each month is between $67 and $72. The average each month is about $70, or $15-20 per week.
- **Heat:** Some months, there was no bill. Over the year, the cost was $1,400. If you spread that over 52 weeks, the weekly cost would be somewhere around $26.

Free Choice, p. 185
For extra practice calculating mentally, with calculators, or on paper.

Calculator Practice

Decimal Division, p. 182
For practice using the calculator to determine answers to division with decimals.

Way Under Average?, p. 183
For practice calculating averages and comparing to a new piece of data.

Extension

Geometric Formulas, p. 186
For applying understanding of decimals to geometric formulas.

Test Practice

Test Practice, p. 188

Looking Closely

Observe whether students are able to

Interpret division with decimals as an act of splitting an amount or finding how many groups can "fit into" an amount, extending these ideas from the study of division with whole numbers

Are students secure with the partitioning model for division? If students are unsure of division as an act of splitting, use manipulatives. Also, refer to the idea of unit price. If you know 5.4 lbs. of hamburger cost $11.89, what is the cost of one pound?

When faced with a situation of dividing $500 by 4.3, are students able to anticipate that the answer must be somewhere between $100 and $125? That is an important idea to develop in this lesson. As far as arriving at the exact answer, encourage a variety of strategies. A calculator is often most efficient, but for those students who prefer a paper-and-pencil procedure, that is fine as well. If you see students using the distributive property to simplify the problem, bring this to the awareness of the whole class. For example, $500 \div 4.3 = (430 \div 4.3) + (70 \div 4.3) = 100 + 16.27 = 116.27$.

Note: to use the distributive property when dividing, only the dividend can be decomposed. That is, $500 \div 4.3$ could not be solved with $(500 \div 4) + (500 \div .3)$ because the important unit here is the group size, which is 4.3.

To cement the connection between division and splitting the monthly payment into weekly amounts, use a calendar. The monthly amount must be split proportionately among the four-plus weeks.

- **Electricity:** This hovers around $40 per month, or somewhat less than $10 per week.

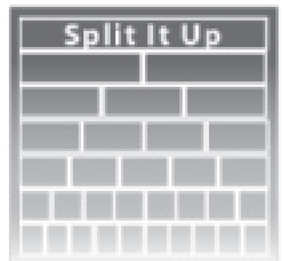
Acknowledge any paper-and-pencil methods. Depending on where and when they went to school, students may have learned different algorithms. Ask them to show those.

Who has a paper-and-pencil method that works well?

When you used the calculator, what did you have to keep in mind? When you estimated weekly expenses, did you think about the sharing model (or splitting or partitioning), or did you think about how many groups of _____ in _____?

Listens to student reasoning with examples from the categories of rent, cell phone, etc. Listen for the idea that order makes a difference when keying in the numbers. Summarize by remarking on the variety of ways a number can be divided or split.

Summary Discussion



Ask volunteers to share what they learned in this lesson about decimal division.

What is clear? What remains unclear?

How does working with decimal numbers in division seem the same or different from working with whole numbers?

Direct students to *Vocabulary* (*Student Book*, p. 206) and *Reflections* (*Student Book*, pp. 214-215), where students record what they want to remember.

Practice

Four Ways to Write Division, p. 173
For practice writing division problems using different symbols.

Target Practice: 0.1, 0.01, 100, p. 174
For practice using calculators to try to get close to the target number.

Which Is Not the Same?, p. 176
For looking at different ways to write division problems.

Where's the Point?, p. 178
For using reasoning and estimation to determine decimal point placement.

Multiplication and Division Patterns, p. 179
For practice looking for patterns with multiplication and division of decimals, then checking with calculators

Division Patterns, p. 180
For practice looking for patterns with simple division problems.

Think Metric, p. 181
For practice using the metric system to divide with decimals.

Free Choice, p. 185
For extra practice calculating mentally, with calculators, or on paper.

Calculator Practice

Decimal Division, p. 182
For practice using the calculator to determine answers to division with decimals.

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For practice calculating averages and comparing to a new piece of data.

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To cement the connection between division and splitting the monthly payment into weekly amounts, use a calendar. The monthly amount must be split proportionately among the four-plus weeks.

Spiral back to multiplication strategies. Though this lesson centers on ways to divide, the contexts can also be used to reinforce multiplication strategies by asking students to use multiplication to check their work (in addition to estimating the answers).

For more practice estimating the result of dividing an integer by a mixed number, assign *Practice: Where's the Point?* (*Student Book*, p. 178) and *Calculator Practice: Decimal Division* (*Student Book*, p. 182).

Match verbal language and symbolic notation for division to a concrete model

Are students consistently making accurate connections between the notation, a diagram or sketch, and words? This is one of the most important goals of the lesson—to see that the notation has meaning. Give students opportunities throughout the lesson to demonstrate understanding of division with decimals in various ways—in words, drawings, and with manipulatives.

As a warm-up, consider preceding *Activity 1: What's the Message?* with additional sketches of different division problems. Are students able to visualize the splitting of a fraction or decimal number into a number of equal parts? The abilities to make meaning of the notation, visualize it concretely, and apply it to a situation are the bedrock for anticipating the approximate answer. Again, allow for various procedures to arrive at an exact answer, either using a calculator or a paper-and-pencil method that students might have learned previously.

Compare *a*/*b* with *b*/*a* in various notation

Are students secure with the order in division notation? During the *Opening Discussion* session, emphasize the need for common understanding when someone writes or speaks about division. If, for instance, someone reads a problem as "5 divided into 7 1/2," he or she is commonly understood to mean, or 7.5 ÷ 5. A common mistake would be to write 5 ÷ 7 1/2.

Which number belongs inside the box when writing out a division problem? Which number is entered into the calculator first? Although such issues are less likely to arise when solving contextualized problems, students often encounter problems translating division expressions during testing situations. If students need practice with equivalent notation, assign the practices *Four Ways to Write Division* (*Student Book*, p. 173) and *Which Is Not the Same?* (*Student Book*, p. 176).

Other than noticing that the expressions a/b and b/a are not equivalent division problems (except when $a = b$), are students conversant with the notion of reciprocals? Are they able to see that 14 ÷ 2 yields the same result as 14 × 1/2 or 14 × 0.5? Assign *Practice: Multiplication and Division Patterns* (*Student Book*, p. 179).

Rationale

The *EMPower* authors recommend sticking to the meaning as essential grounding for students. If skipped or skimped on, students find themselves without a foundation for thinking through problems later. However, adults also need efficient ways to do the calculations. This lesson asks students to think about *why the procedures work* because that understanding will in turn highlight the properties of arithmetic, which are called upon in algebra. Interpreting the meaning of the problem, estimating with benchmarks, and attention to the math principles will shore up understanding and skill.

The goal is that every student end up with a reliable method he or she can use efficiently while understanding why the procedure works.

Get any group of adults in a room, ask them how they would do $8.214 \div 0.43$, and you will be surprised at the variety of methods. Some result in a correct solution—others do not.

Here are some we have seen:

- Punch numbers into the calculator, starting with 8.214 (OK!)
- Punch numbers into the calculator, starting with 0.43 (not OK!)
- Set up the "house" division $0.43\overline{)8.214}$ (OK if using conventional U.S. method)
- Set up the "house" division $8.214\overline{)0.43}$ (not OK if using conventional U.S. method)
- Ask, "Can I do it the way I did in my country?" $0{,}43\overline{)8{,}214}$ (OK if using convention from France, Spain, and many non-U.S. countries)
- Sigh and moan.

Then, since few people who work with numbers in their personal or working lives do long-hand division any more, hesitance and discomfort usually sets in as they start manipulating the numbers.

Traditional basic math workbooks for adults take a logical, step by step approach, starting with whole number divisors, stressing lining up points in the dividend and quotient. Then instruction moves on to moving points the same number of places. But what does decimal movement really mean? What principles of arithmetic permit that clever trick? **The essential idea is that we are rewriting the problem in another—equivalent—way.** And a way that is easier to work with.

$0.43\overline{)8.214}$ needs examination in light of the fact that any number times 1 is that number (multiplicative identity).

$$\frac{8.214}{0.43} = \frac{8.214 \times 1}{0.43} = \frac{8.214}{0.43} \times \frac{100}{100} = \frac{8214}{43}$$

This choice of 100/100 is contrived to have the new divisor be a whole number, but any fraction equal to 1 could be used to get an equivalent problem.

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Over, Around, and Within

Geometry and Measurement

STUDENT BOOK

McGraw Hill

TERC

Over, Around, and Within: Geometry and Measurement

Everyone has some experience with geometry and measurement. In this unit, students build upon their knowledge as they encounter increasingly complex dilemmas about the nature of shapes, the measures of perimeter, area, and volume, as well as linear-, square-, and cubic-unit measurements, both metric and U.S. customary. They learn to speak the language of geometry as they share secret designs and become increasingly familiar with shape attributes. Angles, and in particular right (90º) and straight angles (180˚) take center stage as students explore optimum reading angles and the use of protractors. They then proceed on a series of investigations regarding perimeter, area, and volume. Along the way, they learn about similar shapes, scale, and units of measure. The unit closes with an examination of surface area and volume. Assessments involve general review as well as practical applications of knowledge where, for instance, students plan to re-decorate their classroom or are asked to design a box fitting established criteria.

With its heavy emphasis on hands-on activities and mathematical discourse, *Over, Around, and Within* offers a welcoming context in which students develop a firmly grounded understanding of the often mysterious – angle relationships, unit differences and conversions, and interplay of dimensions in determining perimeter, area, and volume measures. Gradual shifts in emphasis allow students to move from intuitive to formal methods of measuring and comparing shapes and objects. No one memorizes formulas. Everyone understands them. Students will never see the world and its objects in the same way after completing this unit.

Mathematical Concepts Covered for Over, Around, and Within: Geometry and Measurement

Book Description: Students explore the features and measures of basic shapes. Perimeter and area of two-dimensional shapes and volume of rectangular solids provide the focus.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/Topics Covered:**
Opening the Unit	Geometry Groundwork	• Shapes identified and sketched • Angles introduced • Geometry vocabulary list started • Prior Geometry knowledge assessed
Lesson 1:	Sharing Secret Designs	• Two-dimensional shape characteristics identifed • 12 Basic Geometric Shapes identified and described
Lesson 2:	Get It Right	• Angles identified and described with conventional notation • Right angles introduced • Angle measurements estimated with 90° benchmark and determined precisely with protractors
Lesson 3:	Get it Straight	• Straight (180°) angles explored • Sums of angles in triangles and rectangles established
Lesson 4:	Giant-Size	• Similar shapes identified and described • Length and width dimensions introduced and measured • Perimeters determined by adding
Interim Assessment 1	Shapes and Angles	• Attributes of shapes' and angle measurements' knowledge assessed
Lesson 5:	Line Up by Size	• Area and perimeter distinguished
Lesson 6:	Combining Rectangles	• Rectangle area calculated in square centimeters • Composite shapes' areas and perimeters compared
Lesson 7:	Disappearing Grid Lines	• Formulas for area and perimeter derived • Missing dimension values determined • Area of a right triangle calculated
Lesson 8:	Conversion Experiences	• Standard English Units introduced • Linear unit conversions established

Lesson 9:	Squarely in English	• Square units – square inches, feet, and yards constructed and connected with area measure • Square unit conversions established
Lesson 10:	Scale Down	• Scale drawings made and steps for scaling analyzed
Interim Assessment 2	A Fresh Look	• Area, perimeter, measurements, and scale knowledge applied and assessed
Lesson 11:	Filling the Room	• Volume explored as capacity • Third dimension – height becomes apparent
Lesson 12:	Cheese Cubes, Anyone?	• Cubic inch introduced then used to measure volume • Volume formula derived
Lesson 13:	On the Surface	• Surface area and volume compared • Surface area and shape relationship generalized
Closing the Unit	Design a Box	• Geometry and measurement knowledge applied and assessed

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Keeping Things in Proportion

Reasoning with Ratios

STUDENT BOOK

McGraw Hill
TERC

Keeping Things in Proportion: Reasoning with Ratios

Proportional reasoning is an essential skill. Adults call upon this type of reasoning in everyday situations as well as in many areas of mathematics study. Traditionally, mathematics classes rush to cross-multiplication as the tool of choice for solving proportion problems. However, *Keeping Things in Proportion* begins by building on students' intuitive knowledge and the multiplicative relationships that are at the heart of proportionality. The hands-on lessons in this unit connect the central ideas of proportion across the spectrum of mathematics. Students work with rates and ratios in shopping, graphic design, and sampling situations that draw upon data and geometry knowledge while laying the groundwork for algebra study.

As students progress from concrete experiences with ratios to more challenging situations, they develop a bank of tools and strategies to solve proportional problems, and to examine the relationships *within* and *between ratios*. Tools include the rule of equal fractions, tables, graphics, unit rates, and cross-multiplication. Always, students are asked to use two solution methods to arrive at an answer. Non-proportional situations are considered as well. To facilitate conceptual development, numbers start out 'friendly' and turn 'messier' as the unit progresses. The numbers, however, prove less daunting to students as they apply their secure knowledge about proportion. Formal proportional reasoning evolves over time, and the lessons in this unit ensure that students are able to make proportional predictions and adjustments using a variety of tools effectively.

Mathematical Concepts Covered for Keeping Things in Proportion: Reasoning with Ratios

Book Description: Students use various tools—objects, diagrams, tables, graphs, and equations—to understand proportional and non-proportional relationships.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/Topics Covered:**
Opening the Unit	Comparing and Predicting	• Additive and multiplicative ways to compare amounts demonstrated • Ability to solve proportional and non-proportional problems assessed • Experiences with rate, ratio, and proportion shared
Lesson 1:	A Close Look at Supermarket Ads	• Ratios in everyday consumer advertisements identified • Equal ratios determined and equality demonstrated with diagrams • Mathematical rule for establishing equal ratios developed • Problems solved using equal ratio diagram or mathematical rule
Lesson 2:	It's a Lot of Work!	• Sample of work conducted and described over a period of time • Sample used to make a prediction for a larger amount by reasoning with equal ratios
Lesson 3:	Tasty Ratios *	• Ratios used to describe taste and visual comparisons • Ingredients adjusted so that proportions are correct
Lesson 4:	Another Way to Say It	• Two amounts compared using alternate but equivalent methods • Percents as ratios used to compare the part to the whole amount • Whole numbers rounded to make comparisons more manageable
Lesson 5:	Mona Lisa, Is That You?	• Reproductions in various sizes judged by eye to determine if proportional to an original • Measurements used to determine whether reproductions are proportional • Graph used as a tool to test for good reproductions (equal ratios) • Graph points connected with number pairs
Lesson 6:	Redesigning Your Calculator	• Rectangular shapes that are similar to one another drawn and measured • Fractions of a centimeter expressed as decimals • Area and perimeter changes are contrasted when length and width are doubled and halved
Interim Assessment	Checking In	• Sets of equal ratios created • Comparisons, predictions, and decisions made with ratios • Ratios in various formats written and interpreted

Lesson 7	Comparing Walks	• Speed described and quantified as relationship of distance and time • Strategy to find unit rate for *a/b* developed
Lesson 8	Playing with the Numbers	• Cross-product property introduced as another tool to check that two ratios are equal • True and false proportion equations examined • Estimation used to predict for more complicated numbers in proportions • Missing number in a proportion determined
Lesson 9	The Asian Tsunami	• Proportional reasoning concepts applied to international currency conversion • Estimation used to predict for difficult numbers in proportion problems • Exact (or nearly exact) answer determined for a missing number in proportion problem
Lesson 10:	As If It Were 100	• Percents used to make comparisons between data sets of different sizes, some with very large numbers • 1,000 used as a base for comparison between data sets of different sizes
Closing the Unit	Reasoning with Ratios	• Various tools used to address different proportionality situations assessed • Comparisons, predictions, and decisions made with ratios

EMPower
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Many Points Make a Point

Data and Graphs

STUDENT BOOK

McGraw Hill
TERC

Many Points Make A Point: Data and Graphs

The world of data sparks to life for students when they engage with numerous high-interest, real-world data sets, and construct as well as interpret a variety of graphs. Using personal data about the clothes they wear, the foods they eat, and the hours they spend watching television as well as social data about amusement parks, weather trends, and stock prices, students work individually, in pairs and in groups to make frequency, bar, circle, and line graphs. Along the way, they explore graph elements, such as axes, scales, and slope direction. They also encounter three summarizing statistics–mode, mean and median.

Throughout the series of carefully crafted lessons, students hone their graph/data interpretation skills and ability to connect the narrative story of a situation to its graphic display. They begin by making verbal statements about frequencies. As the lessons progress, they refine their observations. By the close of the unit, students are able to describe a graph or data set with benchmark fractions and percents as well as mean and/or median statistics. Concurrently, they develop the capacity to make reasoned arguments and decisions based on data. As well, they learn to question data and graphic representations.

Mathematical Concepts Covered for Many Points Make a Point: Data and *Graphs*

Book Description: Students collect, organize, and represent data using frequency, bar, and circle graphs. They use line graphs to describe change over time. They use benchmark fractions and percents and the three measures of central tendency—mode, median, and mean—to describe sets of data.

Lesson Number:	Lesson Name:	Mathematical Concepts/Topics Covered:
Opening the Unit	Many Points Make a Point	• Assess familiarity with graph formats, features, and purposes • Graph terms
Lesson 1:	Countries in Our Closets	• Categorize and compare data with frequency graphs • Identify graph 'story' • Change data display to see change in graph 'story'
Lesson 2:	Most of Us Eat	• Organize data for specific purposes • Describe data numerically with benchmark fractions and percents
Lesson 3:	Displaying Data in a New Way	• Bar and circle graph construction • Axes intervals • Bar and circle graph formats compared and contrasted
Lesson 4:	A Closer Look at Circle Graphs	• Parts and wholes in circle graphs • Benchmark percents to estimate circle graph portions • Circle Graph interpretation
Midpoint Assessment	The Data Say	• Bar and circle graph knowledge assessed
Lesson 5:	Sketch This	• Line graphs sketched • Correlation of graph line shape and graph story over time
Lesson 6:	Roller-Coaster Rides	• Line graph construction and description • Points of change
Lesson 7:	A Mean Idea	• 'Average' (mean) defined and determined given all values or missing values
Lesson 8:	Mystery Cities	• Multiple data lines • Scale variation impact • Graph and text alignment

Lesson 9:	Median	• Median detemined with odd and even data sets • Data set determined from given median
Lesson 10:	Stock Prices	• Tables connected to and generated from graphs • Scale generalizations
Closing the Unit	Stock Picks	• Application of graph knowledge for evaluations, recommendations, problem solving and presentations

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Seeking Patterns, Building Rules

Algebraic Thinking

STUDENT BOOK

McGraw Hill
TERC

Seeking Patterns, Building Rules: Algebraic Thinking

This unit demystifies basic algebra, as students explore the meanings revealed by tables, graphs, verbal rules, and equations. By investigating patterns in their own lives, In-Out tables, banquet table and patio tile arrangements, calorie-burning tables, graphs, and equations, the relationship between diameter and circumference, pay and accumulated earnings, gas price increases, or cell phone use-cost patterns, students learn to connect algebraic representations with the linear (and occasionally non-linear) patterns or functions they describe. They see that algebraic tools and symbols serve to describe and interpret a situation; the situation itself is always central. Early lessons introduce students to ways of 'reading' tables, graphs, and equations through construction of these representations. Graph, table, verbal rule, and equation conventions become familiar through varied and meaningful use.

Students gradually gain proficiency in representing situations graphically and symbolically then deepen their understanding, as they explore concepts and representational conventions related to rates of change. They come to recognize equivalent expressions and to compare expressions. Students grasp what a y-intercept, flat-line graph, straight- or curved-line graph, or a point of intersection reveal about situations. All of this learning occurs in a lively, practical way that takes the fear out of approaching algebra and replaces it with a sense of wonder and mastery.

Mathematical Concepts Covered for *Seeking Patterns, Building Rules: Algebraic Thinking*

Book Description: Students use a variety of representational tools—diagrams, words, tables, graphs, and equations—to understand linear patterns and functions. They connect the rate of change with the slope of a line and compare linear with nonlinear relationships. They also gain facility with and comprehension of basic algebraic notation.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/Topics Covered:**
Opening the Unit	Seeking Patterns, Building Rules	• Personal patterns described and term 'pattern' explored for assessment purposes • Algebra vocabulary list initiated • Prior algebra knowledge assessed
Lesson 1:	Guess My Rule	• Patterns/relationships between two variables in a visual pattern • Expressing patterns in equation form
Lesson 2:	Banquet Tables	• Tracking table data • Multiple representations of a situation to predict outcomes
Lesson 3:	Body at Work—Tables and Rules	• Verbal and symbolic rule practice
Lesson 4:	Body at Work—Graphing the Information	• Graph features identified and compared • Graph generated from tables and/or equations
Lesson 5:	Body at Work—Pushing It to the Max	• Graph construction and connections practiced • *x-y* relationships explored
Lesson 6:	Circle Patterns	• Diameter and circumference relationship explored • Rule and formula application
Midpoint Assessment	Using the Tools of Algebra	• Production and interpretation of representations assessed • Symbolic notation use assessed
Lesson 7:	What Is the Message?	• Translating equations • Equivalent expressions • Coefficients – meaning and representations

Lesson 8:	Job Offers	• Algebraic problem solving • Point of intersection • *y*-intercept
Lesson 9:	Phone Plans	• Features of graphs • Matching representations • Supporting decisions with mathematical information
Lesson 10:	Signs of Change	• Constant rate of change identified and compared in representations
Lesson 11:	Rising Gas Prices	• Linear and non-linear patterns/rates of change compared
Lesson 12:	The Patio Project	• Algebraic knowledge applied
Closing the Unit	Putting It All Together	• Algebraic knowledge assessed

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A New Pathway to Mathematical Power!

The EMPower Math curriculum was designed to help adult and adolescent learners study the mathematics needed to successfully manage math at home, at work, and in the community. With EMPower, students investigate mathematical dilemmas and puzzling problems set in engaging, real-world contexts. They work collaboratively and share ideas. Students think like mathematicians as they examine mathematical properties and common misconceptions to uncover multiple ways to solve problems.

Three updated **EMPower Plus** titles–*Everyday Number Sense*, *Split It Up*, and *Using Benchmarks* – help students build a strong foundation for algebraic thinking. This new edition of the EMPower Plus curriculum builds on the original series and includes new lessons, activities, and practice pages. The updated books emphasize the development of reasoning and operation sense, identifying patterns and formulating generalizations, and using benchmark numbers for making mental calculations—key foundational skills needed for success on high school equivalency tests, for higher education, and for the workplace.

Adult math educators at TERC and McGraw-Hill Education enthusiastically present this series to help students develop strategies for making decisions in everyday life and master the math needed to achieve their educational and career goals!

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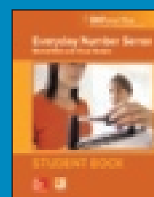
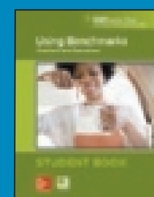
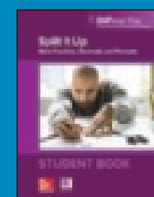
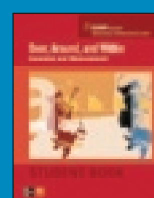




Number & Operation Sense: A Foundation for Algebra



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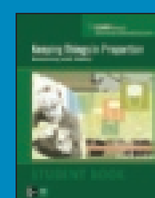

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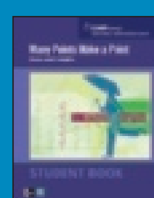

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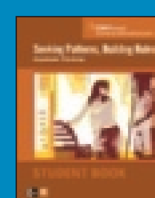

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