



PRECALCULUS

Second Edition



Julie Miller

Daytona State College

Donna Gerken

Miami Dade College





PRECALCULUS, SECOND EDITION

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About the Authors

Julie Miller is from Daytona State College, where she taught developmental and upper-level mathematics courses for 20 years. Prior to her work at DSC, she worked as a software engineer for General Electric in the area of flight and radar simulation. Julie earned a bachelor of science in applied mathematics from Union College in Schenectady, New York, and a master of science in mathematics from the University of Florida. In addition to this textbook, she has authored textbooks in developmental mathematics, trigonometry, and precalculus, as well as several short works of fiction and nonfiction for young readers.

“My father was a medical researcher, and I got hooked on math and science when I was young and would visit his laboratory. I remember doing simple calculations with him and using graph paper to plot data points for his experiments. He would then tell me what the peaks and features in the graph meant in the context of his experiment. I think that applications and hands-on experience made math come alive for me, and I’d like to see math come alive for my students.”

Donna Gerken was a professor at Miami Dade College where she taught developmental courses, honors classes, and upper-level mathematics classes for decades. Throughout her career she has been actively involved with many projects at Miami Dade, including those on computer learning, curriculum design, and the use of technology in the classroom. Donna’s bachelor of science in mathematics and master of science in mathematics are both from the University of Miami.

Dedications

In honor of Gus, Winston, Sine, Cosine, George, Pebbles, Lucy, Princess, Amber, Lady, Brownie, Aero, Chloe, Stanley, and Oliver for your past and present inspiration —Julie Miller —Donna Gerken



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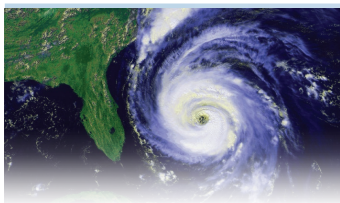
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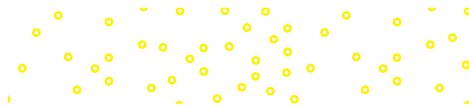
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Laboratory for Atmospheres, NASA
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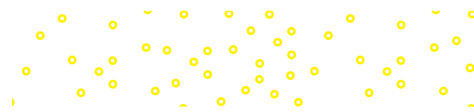
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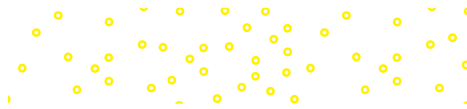
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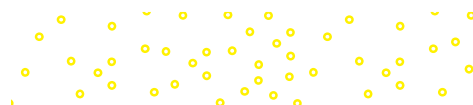
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Student Answer Appendix SA-1**Instructor Answer Appendix IA-1 (AIE only)****Subject Index I-1****APPENDIX A Additional Topics (Online)****Section A.1** Proof of the Binomial Theorem**Section A.2** Conic Sections Defined by a Fixed Point and a Fixed Line**Additional Online Content**

Detailed Chapter Summaries

Group Activities



Letter from the Authors

Precalculus serves as a gateway course for many students in disciplines across the college curriculum. For some students, it is the entrance into the higher mathematics needed for careers in science, technology, engineering, and mathematics. For others, precalculus serves as the primary resource needed to understand the complexities of a modern world awash with statistics, investment strategies, financial planning, and even medical decisions. With the broad scope of this foundational course in mind, we worked to make the textbook, the digital tools, and the supplements as clear, relevant, and accessible as possible.

As part of our revision plan, we modernized content with new data sets and contemporary topics that are germane to all of today's students. Inside, you will find data and information relating to the COVID-19 pandemic, health care costs and ethics, income tax rates, the development of modern computers, extreme weather, and the work of modern scientists and inventors. We have also placed an emphasis on inclusion and diversity so that *all* students feel connected to the content in this book and can envision themselves in the many fields and situations in life that embrace mathematics.

In the text, we have expanded our exercise sets with an emphasis on mixed exercises that require multiple tools to complete their solutions. Likewise, we've added additional "challenge" exercises to promote more critical thinking and less rote repetition. Throughout this revision we were especially cognizant of the struggles for both instructors and students in adapting traditional content to the digital style of teaching that emerged during the pandemic. To support the ongoing surge in online instruction and online homework, we have expanded our digital library of algorithmic exercises, enhanced the detailed solutions, and integrated our instructional videos into the homework.

We are excited for you, the student, to join an amazing journey in mathematics and hope it serves you well in your future.

Best to all,
Julie Miller
Donna Gerken

Key Features

Clear, Precise Writing

Because a diverse group of students take this course, Julie Miller has written this text to use simple and accessible language. Through her friendly and engaging writing style, students are able to understand the material easily.

Exercise Sets

The exercises at the end of each section are graded, varied, and carefully organized to maximize student learning:

- **Prerequisite Review Exercises** begin the section-level exercises and ensure that students have the foundational skills to complete the homework sets successfully.
- **Concept Connections** prompt students to review the vocabulary and key concepts presented in the section.
- **Core Exercises** are presented next and are grouped by objective. These exercises are linked to examples in the text and direct students to similar problems whose solutions have been stepped-out in detail.
- **Mixed Exercises** do *not* refer to specific examples so that students can dip into their mathematical toolkit and decide on the best technique to use.
- **Write About It** exercises are designed to emphasize mathematical language by asking students to explain important concepts.
- **Technology Connections** require the use of a graphing utility and are found at the end of exercise sets. They can be easily skipped for those who do not encourage the use of calculators.
- **Expanding Your Skills Exercises** challenge and broaden students' understanding of the material.
- **Point of Interest** boxes feature interesting topics in mathematics from a diverse and inclusive set of contributors. These essays promote critical thinking, discussion, and research and often include follow-up questions and exercises.

Problem Recognition Exercises

Problem Recognition Exercises appear in strategic locations in each chapter of the text. These exercises provide students with an opportunity to synthesize multiple concepts and decide which problem-solving technique to apply to a given problem.

Examples

- The examples in the textbook are stepped-out in detail with thorough annotations at the right explaining each step.
- Following each example is a similar **Skill Practice** exercise to engage students by practicing what they have just learned.
- For the instructor, references to an even-numbered exercise are provided next to each example. These exercises are highlighted with blue circles in the exercise sets and mirror the related examples. With increased demands on faculty time, this has been a popular feature that helps faculty write their lectures and develop their presentation of material. If an instructor presents all of the highlighted exercises, then each objective of that section of text will be covered.

Modeling and Applications

One of the most important tools to motivate our students is to make the mathematics they learn meaningful in their lives. The textbook is filled with robust applications and numerous opportunities for mathematical modeling for those instructors looking to incorporate these features into their course.

Callouts

Throughout the text, popular tools are included to highlight important ideas. These consist of:

- **Tip** boxes that offer additional insight into a concept or procedure.
- **Avoiding Mistakes** boxes that fend off common mistakes.
- **Instructor Notes** to assist with lecture preparation.

Graphing Calculator Coverage

Material is presented throughout the book illustrating how a graphing utility can be used to view a concept in a graphical manner. The goal of the calculator material is not to replace algebraic analysis, but rather to enhance understanding with a visual approach. Graphing calculator examples are placed in self-contained boxes and may be skipped by instructors who choose not to implement the calculator. Similarly, the graphing calculator exercises are found at the end of the exercise sets and may also be easily skipped.

End-of-Chapter Materials

The textbook has the following end-of-chapter materials for students to review before test time:

- Brief summary with references to key concepts. Detailed Chapter Summaries are available with the online resources.
- Chapter review exercises.
- Chapter test.
- Cumulative review exercises. These exercises cover concepts in the current chapter as well as all preceding chapters.

Supplement Package

Supplements for the Instructor

Author-Created Digital Media

Digital assets were created exclusively by the author team to ensure that the author voice is present and consistent throughout the supplement package.

- The coauthor, Donna Gerken, ensures that each algorithm in the online homework has a stepped-out solution that matches the textbook's writing style.
- Julie Miller created **video content** (lecture videos, exercise videos, graphing calculator videos, and Excel videos) to give students access to classroom-like instruction by the author.
- Julie Miller constructed over 50 **dynamic math animations** to accompany the text. The animations are diverse in scope and give students an interactive approach to conceptual learning. The animated content illustrates difficult concepts by leveraging the use of on-screen movement where static images in the text may fall short.

The Instructor's Resource Manual (IRM) is a printable electronic supplement put together by the author team. The IRM includes Guided Lecture Notes, Classroom Activities using Wolfram Alpha, and Group Activities.

- The Guided Lecture Notes are keyed to the objectives in each section of the text. The notes step through the material with a series of questions and exercises that can be used in conjunction with lecture.

- The Classroom Activities using Wolfram Alpha promote active learning in the classroom by using a powerful online resource.
- A Group Activity is available for each chapter of the book to promote classroom discussion and collaboration.
- The Problem Recognition Exercises are available as worksheets for students to work on as individuals or in groups to help them determine appropriate methods of solution for related problem types.

TestGen Among the supplements is a computerized test bank using the algorithm-based testing software TestGen® to create customized exams quickly. Hundreds of text-specific, open-ended, and multiple-choice questions are included in the question bank.

Annotated Instructor's Edition

- Answers to exercises appear adjacent to each exercise set, in a color used only for annotations.
- Instructors will find helpful notes within the margins to consider while teaching.
- References to even-numbered exercises appear in the margin next to each example for the instructor to use as Classroom Examples.

PowerPoints present key concepts and definitions with fully editable slides that follow the textbook. An instructor may project the slides in class or post to a website in an online course.

Our Commitment to Market Development and Accuracy

Acknowledgments:

Paramount to the development of *Precalculus* was the invaluable feedback provided by the instructors from around the country who reviewed the manuscript or attended a market development event over the course of the several years the text was in development. In particular, we want to thank Alina Coronel for her amazing work in bringing our content into ALEKS, and Jennifer Blue for her steadfast scrutiny of our manuscript.

A Special Thanks to All of the Event Attendees Who Helped Shape *Precalculus*.

Focus groups and symposia were conducted with instructors from around the country to provide feedback to editors and the authors and ensure the direction of the text was meeting the needs of students and instructors.

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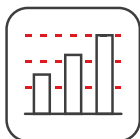
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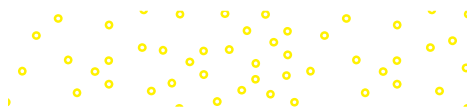
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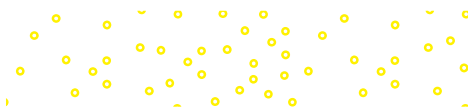
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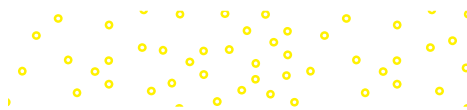
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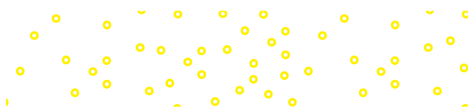
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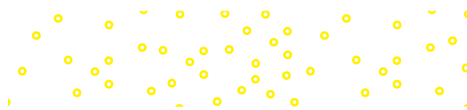
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