

Dual Language/Bridging Lessons

Unidad 3: Funciones Lineales y No Lineales

Gran idea: Modelar con Funciones, Comparar Modelos, Componer Funciones

Conexión de contenido: Explorando Cantidades Cambiantes, Descomponer Totalidades, Unir Partes

Pregunta esencial: ¿Qué te puede decir una función sobre la relación que representa?

Unit 3: Linear and Nonlinear Functions

Big Idea: Modeling with Functions; Comparing Models; Composing Functions

Content Connection: Exploring Changing Quantities; Taking Wholes Apart, Putting Parts Together

Essential Question: What can a function tell you about the relationship that it represents?

Preguntas de enfoque por lección • Focus Questions by Lesson

Lección 1	Graficar funciones lineales ¿Cómo grafican los estudiantes las funciones lineales usando tablas de valores y las intersecciones en los ejes x e y?
Lección 2	Tasa de cambio y pendiente ¿Cómo se calcula e interpreta la tasa de cambio y la pendiente?
Lección 3	Forma pendiente–intersección ¿Cómo escribir y graficar ecuaciones en forma pendiente–intersección?
Lección 4	Transformaciones de funciones lineales ¿Cómo aplicar e identificar los efectos de las transformaciones en funciones lineales?

Lesson 1	Graphing Linear Functions How do students graph linear functions using tables of values and the x- and y-intercepts?
Lesson 2	Rate of Change and Slope How are rate of change and slope calculated and interpreted?
Lesson 3	Slope-Intercept Form How to write and graph equations in slope-intercept form?
Lesson 4	Transformations of Linear Functions How to apply and identify the effects of transformations to linear functions?

Lección 5	Secuencias aritméticas ¿Cómo escribir y graficar ecuaciones de secuencias aritméticas?	Lesson 5	Arithmetic Sequences How to write and graph equations of arithmetic sequences?
Lección 6	Funciones por partes y escalonadas ¿Cómo identificar y graficar funciones definidas por partes y funciones escalonadas?	Lesson 6	Piecewise and Step Functions How to identify and graph piecewise-defined and step functions?
Lección 7	Funciones de Valor Absoluto ¿Cómo identificar los efectos de las transformaciones en las gráficas de funciones de valor absoluto?	Lesson 7	Absolute Value Functions How to identify the effects of transformations of the graphs of absolute value functions?

Explorar palabras • Explore Words

Las palabras de vocabulario de la unidad se pueden usar para ayudar a los estudiantes a transferir su conocimiento del contenido de un idioma al otro: del español al inglés o del inglés al español. Consulte las estrategias para *construir el lenguaje de las matemáticas* en la página 112g de la Edición para el docente. Se pueden encontrar estrategias adicionales en las páginas 8-12 de este Manual del estudiante multilingüe.

The unit vocabulary words can be used to help students transfer their content knowledge from one language to the other—from Spanish to English, or from English to Spanish. See the *Building the Language of Math* strategies on page 112 of the Teacher Edition. Additional strategies can be found on pages 8-12 of this Multilingual Learner Handbook.

Cognados • Cognates

California Reveal Math	California Reveal Math
Funcion constante	Constant Function
Transformación	Transformation
Traslacion	Translation
Dilatacion	Dilation
Reflexion	Reflection
Secuencia	Sequence
Secuencia aritmética	Arithmetic Sequence
Función lineal por partes	Piecewise-Linear Function
Función de valor absoluto	Absolute Value Function

Strategies for Improving Collaborative Discussion

As students work through this unit, they will encounter opportunities to share ideas with partners. Introducing these techniques to improve speaking and listening will help students be more active in these discussions. You may choose to model these suggestions with students, or you may want to focus on one or two approaches, and then introduce the others later in the course.

- **Building Off a Classmate’s Ideas** Suggest to students that one way to continue the discussion is to add to what was said by supporting the idea or building from it. Encourage students to rephrase what their classmates have said, and then add their own ideas. Provide sentence frames such as I agree that..., and I also think that....
- **Asking a Clarifying Question** Encourage students to take initiative to be sure that they understand a classmate’s ideas or the task the group is working to complete. Remind them that asking questions can help avoid confusion and frustration. Provide sentence frames such as Can you explain what you mean by...? and I don’t understand the term.... Can you explain it to me?
- **Expressing Disagreement** As students work together to solve problems, there may be disagreement within groups about the right approach. Remind students to be respectful, but give them the tools to express disagreement. Provide sentence frames for support. For example: I don’t think that is right, I think it is.... and We may want to think about.... Can we look at this a different way?

Select a question or set of questions to use for bridging content. Have students work in pairs or small groups to complete questions. A table of possible questions focused on representing information is presented below.

Lesson	Practice Questions
3-1	#16-17,24,28
3-2	#5-7,24-25,37
3-3	#9-10,13,18,25,39
3-4	#10-12,19,27-32
3-5	#5-6,18-19,22,39
3-6	#8,18-19,22,26-30
3-7	#4,17,22,23-24,42

Multilingual Learner Scaffolds

Emerging Support students as they practice using these conversational scaffolds. Introduce a simple question on a topic that students may already know. For example: What is the best part about living in your town or city? Encourage students to use the sentence frames provided to discuss their ideas.

Expanding

Challenge students to use at least two of the three strategies in a discussion. Remind students that the words yes and no are useful in communicating agreement or disagreement, but should be followed by reasoning.

Bridging

Ask students to notice and take notes on other strategies they could use. Students may notice that their peers do not always speak in complete sentences, but that they support their ideas. They may notice that their peers invite participation by calling each other by name and asking: "What do you think?"

Estrategias para mejorar la discusión colaborativa

A medida que los estudiantes trabajen en esta unidad, encontrarán oportunidades para compartir ideas con sus compañeros. Introducir estas técnicas para mejorar la expresión oral y la comprensión auditiva ayudará a los estudiantes a ser más activos en estas discusiones. Puede elegir modelar estas sugerencias con los estudiantes o puede optar por centrarse en uno o dos enfoques y luego introducir los demás más adelante en el curso.

- Construir sobre las ideas de un compañero. Sugiera a los estudiantes que una forma de continuar la discusión es añadir a lo que se dijo, apoyando la idea o construyendo a partir de ella. Anime a los estudiantes a reformular lo que sus compañeros han dicho y luego agregar sus propias ideas. Proporcione marcos de oraciones como: *Estoy de acuerdo en que...* y *También pienso que...*
- Hacer una pregunta aclaratoria anima a los estudiantes a tomar la iniciativa y a asegurarse de que entienden las ideas de un compañero o la tarea que el grupo está trabajando para completar. Recuérdales que hacer preguntas puede ayudar a evitar la confusión y la frustración. Proporcione marcos de oraciones como: *¿Puedes explicar lo que quieres decir con...?* y *No entiendo el término.... ¿Puedes explicármelo?*
- Expresar desacuerdo A medida que los estudiantes trabajen juntos para resolver problemas, puede haber desacuerdo dentro de los grupos sobre el enfoque correcto. Recuérdales a los estudiantes que deben ser respetuosos, pero dales las herramientas para expresar su desacuerdo. Proporcione marcos de oraciones de apoyo. Por ejemplo: *No creo que eso sea correcto, creo que es...* y *Podríamos considerar.... ¿Podemos analizar esto de una manera diferente?*

Seleccione una pregunta o un conjunto de preguntas para usar como conexión de contenido. Pida a los estudiantes que trabajen en parejas o en grupos pequeños para completar las preguntas. A continuación, se presenta una tabla con posibles preguntas centradas en la representación de la información.

Lección	Preguntas de Práctica
3-1	#16-17,24,28
3-2	#5-7,24-25,37
3-3	#9-10,13,18,25,39
3-4	#10-12,19,27-32
3-5	#5-6,18-19,22,39
3-6	#8,18-19,22,26-30
3-7	#4,17,22,23-24,42

Apoyo para estudiantes multilingües

- Emergente** Apoye a los estudiantes mientras practican el uso de estos andamiajes conversacionales. Introduzca una pregunta simple sobre un tema que los estudiantes ya conozcan. Por ejemplo: ¿Cuál es la mejor parte de vivir en su pueblo o ciudad? Anime a los estudiantes a usar los marcos de oraciones proporcionados para discutir sus ideas.
- En Expansion** Desafíe a los estudiantes a utilizar al menos dos de las tres estrategias en una discusión. Recuérdales a los estudiantes que las palabras sí y no son útiles para comunicar acuerdo o desacuerdo, pero deben ir seguidas de un razonamiento.
- En Transición** Pida a los estudiantes que observen y tomen notas sobre otras estrategias que podrían utilizar. Los estudiantes pueden notar que sus compañeros no siempre hablan en oraciones completas, pero sí apoyan sus ideas. Pueden notar que sus compañeros invitan a la participación al llamarse por su nombre y preguntar: "¿Qué piensas tú?"

Dual Language Connections

Language Objectives

Students analyze and strengthen their use of mathematical language to support their ability to graph linear functions. (Lesson 3-1)
Students analyze academic vocabulary, math based terms, and everyday vocabulary to help interpret problems related to rate of change and slope. (Lesson 3-2)
Students explore mathematical terminology to support their work with equations in slope-intercept form. (Lesson 3-3)
Students analyze important action verbs, idioms, and word parts to help them identify the effects of transformations to linear functions. (Lesson 3-4)
Students will review mathematical terminology as they study equations of arithmetic sequences. (Lesson 3-5)
Students review math-based terms to help identify and graph piecewise-defined and step functions. (Lesson 3-6)
Students interpret math-based terms to identify the effects of transformations of the graphs of absolute value functions. (Lesson 3-7)

Communicate Your Understanding

Have students work with partners or in small groups to discuss the Essential question of the unit or the focus question for that lesson. Students can first discuss in Spanish, then rephrase or summarize in English.

Transfer Learning from English to Spanish

Invite students to generate words, phrases, and sentences in English that express what they have learned about the Essential Question or lesson focus question. Record their ideas on the left side of an anchor chart like the one below. Then tell students they will now transfer what they learned from English to Spanish. Begin by having students work with partners or in small groups to come up with Spanish translations for each item in the left column. Record their translations in the right column of the chart, providing any Spanish terms that students are unable to translate.

How do students graph linear functions using tables of values and the x- and y-intercepts?	¿Cómo grafican los estudiantes las funciones lineales usando tablas de valores y las intersecciones en los ejes x e y?
Linear Function $y = mx + b$ Table of values Y-intercept Slope	Función lineal $y = mx + b$ Tabla de valores Intersección de Y Pendiente

Language and Math Sentence Frames for Unit 3

Para graficar la función lineal, primero elegí los valores de x ____, ____, y ____.	To graph the linear function, I first chose the x-values ____, ____, and ____.
La intersección con el eje y es ____, lo que significa que la gráfica cruza el eje y en (0, ____).	The y-intercept is ____, which means the graph crosses the y-axis at (0, ____).
Para encontrar la intersección con el eje x, establecí que $y = 0$ y resolví para x, obteniendo $x =$ ____.	To find the x-intercept, I set $y = 0$ and solved for x, giving me $x =$ ____.

Conexiones de Lenguaje Dual

Objetivos de Lenguaje

Los estudiantes analizan y fortalecen su uso del lenguaje matemático para apoyar su capacidad de graficar funciones lineales. (Lección 3-1)
Los estudiantes analizan el vocabulario académico, los términos matemáticos y el vocabulario cotidiano para ayudar a interpretar problemas relacionados con la tasa de cambio y la pendiente. (Lección 3-2)
Los estudiantes exploran la terminología matemática para apoyar su trabajo con ecuaciones en forma de pendiente-intercepto (Lección 3-3)
Los estudiantes analizan verbos de acción importantes, modismos y partes de palabras para ayudarlos a identificar los efectos de las transformaciones a las funciones lineales. (Lección 3-4)
Los estudiantes revisarán la terminología matemática mientras estudian las ecuaciones de secuencias aritméticas. (Lección 3-5)
Los estudiantes repasan los términos matemáticos para ayudar a identificar y graficar funciones definidas a trozos y funciones escalonadas. (Lección 3-6)
Los estudiantes interpretan los términos matemáticos para identificar los efectos de las transformaciones de las gráficas de funciones de valor absoluto. (Lección 3-7)

Comunica tu Comprensión

Pida a los estudiantes que trabajen con compañeros o en grupos pequeños para discutir la pregunta esencial de la unidad o la pregunta de enfoque de esa lección. Los estudiantes pueden discutir primero en español y luego reformular o resumir en inglés.

Transferencia del Aprendizaje del Inglés al Español

Invite a los estudiantes a generar palabras, frases y oraciones en inglés que expresen lo que han aprendido sobre la Pregunta Esencial o la pregunta de enfoque de la lección. Registre sus ideas en el lado izquierdo de una tabla de apoyo como la que se muestra a continuación. Luego, diga a los estudiantes que ahora transferirán lo que aprendieron en inglés al español. Comience pidiendo a los estudiantes que trabajen con compañeros o en grupos pequeños para crear traducciones al español para cada elemento en la columna izquierda. Registre sus traducciones en la columna derecha de la tabla, proporcionando cualquier término en español que los estudiantes no puedan traducir.

How do students graph linear functions using tables of values and the x- and y-intercepts?	¿Cómo grafican los estudiantes las funciones lineales usando tablas de valores y las intersecciones en los ejes x e y?
Linear Function $y = mx + b$ Table of values Y-intercept Slope	Función lineal $y = mx + b$ Tabla de valores Intersección de Y Pendiente

Marcos de oraciones de lenguaje y matemáticas para la Unidad 3

Para graficar la función lineal, primero elegí los valores de x ____, ____, y ____.	To graph the linear function, I first chose the x-values ____, ____, and ____.
La intersección con el eje y es ____, lo que significa que la gráfica cruza el eje y en (0, ____).	The y-intercept is ____, which means the graph crosses the y-axis at (0, ____).
Para encontrar la intersección con el eje x, establecí que $y = 0$ y resolví para x, obteniendo $x =$ ____.	To find the x-intercept, I set $y = 0$ and solved for x, giving me $x =$ ____.

Written Communication

Have students write about what they learned in this unit.

Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.

Multilingual Learner Scaffolds

Emerging Students make a poster with drawings and captions showing what they learned in this unit. Students should label the drawings in both English and their home language. Captions can be in their home language, however, all vocabulary words should be in English.

Expanding Have students write down three of the focus questions from the unit and their answers. Students should show what they learned in the unit as part of their answers. In their responses, students can write their key points in English, and then write clarifying and supporting sentences in their home language.

Bridging Students write a paragraph summary explaining what they learned in the unit. Students will write their response in English, but can brainstorm and do their prewriting in their home language. Remind students to use academic connectors (e.g., therefore, this shows, as a result).

Reading

Students engage with instructional or explanatory mathematical text, reading independently or with a partner to extract key concepts, procedures, and reasoning. After reading, students respond to comprehension questions that target: (1) identification of mathematical definitions or forms, (2) interpretation of mathematical meaning or representation, and (3) analysis of strategic reasoning or connections. Students discuss their responses with a partner, and selected students share their thinking with the class.

Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.

Multilingual Learner Scaffolds

Emerging Provide a visual glossary or word bank that includes key mathematical terms from the text paired with images, symbols, diagrams, or native language translations. Before reading, have students preview the glossary and locate these terms in the passage by highlighting or circling them. Conduct a guided reading by reading the text aloud while students follow along, pausing to point out visuals and act out procedures when possible. Provide comprehension questions with visual supports and sentence frames with word banks, such as: "The ___ is ___" with options to choose from. Students complete responses by selecting from provided options or filling in single words/short phrases. Have students share

answers by pointing to their selected responses or reading their completed frames with partner support.

Expanding

Break the text into manageable chunks or sections and provide a matching or sequencing activity where students connect each chunk to its corresponding main idea, purpose, or step in a process. Include sentence starters for each chunk, such as: "This part explains __," "This step shows how to __," or "This describes __." After matching, have students read each chunk aloud with a partner, taking turns and discussing why each match makes sense. Provide comprehension questions with sentence frames that require students to combine information from multiple chunks, such as: "First, you __, then __," "The __ represents __ because __," or "This strategy works because __." Students write responses using the frames and share their reasoning with their partner and the class.

Bridging

Have students read the complete text independently, annotating as they read by: underlining key concepts, circling important vocabulary, and noting questions or connections in the margins. Provide sentence frames or paragraph starters that guide more extended written responses to comprehension questions, such as: "A __ is defined as/written as __," "The __ represents/tells me __ because __," or "Using __ helps/is effective because __." Encourage students to extend beyond the frames by adding examples, making connections, or providing additional reasoning. Students share their written responses with a partner, comparing interpretations and elaborating on their thinking before sharing selected responses with the class.

Speaking

Students work in small groups to prepare and deliver a short presentation explaining a mathematical concept, procedure, solution, or reasoning to peers. Using visual supports such as posters, diagrams, models, slides, or anchor charts, students collaboratively organize their thinking and communicate mathematical ideas orally. Presentations are suitable for student language levels and may range from demonstrating and naming key elements to providing detailed explanations with justification.

Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.

Multilingual Learner Scaffolds

Emerging

Small groups create a visual presentation (poster, labeled diagram, or model) that represents the mathematical concept using clear images, numbers, or symbols. Students can use their native language while preparing the presentation. Provide sentence frames: "This is __." "I see __." "This shows __."

Groups practice multiple times, with each member pointing to a part of the visual while speaking. During the presentation, each student uses 1-3

simple sentences with frames to identify key elements. Accept presentations combining pointing, gestures, and short phrases. Groups present to another group or class.

Expanding Small groups create a visual presentation (poster, slide, or labeled diagram) that shows the concept with organized steps or components. Provide connected sentence frames: "This shows ____." "First, ____." "Next, ____." "This is important because ____."

Groups outline their explanation, divide speaking roles, and practice together before presenting. During the presentation, each member delivers part of the multi-sentence explanation (4-6 sentences total), incorporating mathematical vocabulary from a word bank. Groups present to another group or class.

Bridging Pairs or small groups create a comprehensive visual presentation (slides, an annotated diagram, multiple representations) that illustrates the concept with detailed components and connections. Groups independently plan presentation structure: introduction, explanation with precise vocabulary, and conclusion, dividing responsibilities among members.

During the presentation, group members deliver a cohesive explanation (1-2 minutes total) using academic language and making explicit connections between visuals and mathematical reasoning. Students explain what, why, and how concepts work. Groups present to another group or class.

Listening

Students practice listening comprehension by hearing the teacher read a detailed explanation of a mathematical procedure, process, or concept. Students listen actively to understand the sequence of steps, key vocabulary, and reasoning involved. After listening, students demonstrate comprehension by engaging in a structured discussion in which they explain the process in their own words, compare their understanding with peers, and clarify any confusion.

Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.

Multilingual Learner Scaffolds

Emerging Provide students with picture cards for them to match as they listen to the explanation. Read the text slowly and emphasize important vocabulary. Use yes/no listening checks for each section of the explanation to check for understanding. Have students discuss with a partner.

Expanding Provide students with a graphic organizer for them to complete as the explanation is read. Have them list each of the steps in the organizer, and use quick yes/no listening checks to guide the pace of reading or to reread the section. After reading, students discuss in pairs or groups.

Bridging

Read the explanation aloud at a natural pace, and have students focus on listening to the passage. Read the explanation a second time and have them summarize the explanation in their own words, using an outline or writing a paragraph. Have them share their writing with others to discuss their summaries.