

# Dual Language/Bridging Lessons

## Unidad 3: Relaciones proporcionales

Gran idea: Representación gráfica de relaciones, relaciones proporcionales, tasas unitarias en el mundo

Conexión de contenido: Explorando cantidades cambiantes

Pregunta esencial: ¿Cómo te ayudan las relaciones proporcionales a comprender situaciones cotidianas?

## Unit 3: Proportional Relationships

Big Idea: Graphing Relationships, Proportional Relationships, Unit Rates in the World

Content Connection: Exploring Changing Quantities

Essential Question: How do proportional relationships help you understand everyday situations?

### Preguntas de enfoque por lección • Focus Questions by Lesson

Lección 1	<b>Conectar proporciones, tasas y proporciones</b> ¿Cómo encontrar la tasa unitaria cuando uno o ambos términos tienen un valor fraccionario?
Lección 2	<b>Usar tablas para determinar la proporcionalidad</b> ¿Cómo determinar si una relación es proporcional analizando las cantidades en una tabla?
Lección 3	<b>Usar gráficos para determinar la proporcionalidad</b> ¿Cómo determinar si una relación es proporcional analizando la estructura de una gráfica?
Lección 4	<b>Representar relaciones proporcionales con ecuaciones</b> ¿Cómo se pueden modelar y resolver las relaciones proporcionales usando la ecuación $y = kx$ ?

Lesson 1	<b>Connect Ratios, Rates, and Proportions</b> How to find the unit rate when one or both terms have a fractional value?
Lesson 2	<b>Use Tables to Determine Proportionality</b> How to determine whether a relationship is proportional by analyzing the quantities in a table?
Lesson 3	<b>Use Graphs to Determine Proportionality</b> How to determine whether a relationship is proportional by analyzing the structure of a graph?
Lesson 4	<b>Represent Proportional Relationships with Equations</b> How can proportional relationships using the equation $y = kx$ be modeled and solved?

Lección 5	<b>Describir relaciones proporcionales</b> ¿Cómo describir una relación proporcional usando la constante de proporcionalidad en cada representación?	Lesson 5	<b>Describe Proportional Relationships</b> How to describe a proportional relationship using the constant of proportionality in each representation?
Lección 6	<b>Usar razonamiento proporcional para resolver problemas de proporciones de varios pasos</b> ¿Cómo se pueden usar las relaciones proporcionales para resolver problemas que involucran proporciones a través de diferentes representaciones?	Lesson 6	<b>Use Proportional Reasoning to Solve Multi-Step Ratio Problems</b> How can proportional relationships be used to solve problems involving ratios and across different representations

## Explorar palabras • Explore Words

Las palabras de vocabulario de la unidad se pueden usar para ayudar a los estudiantes a transferir su conocimiento del contenido de un idioma al otro: del español al inglés o del inglés al español. Consulte las estrategias para *construir el lenguaje de las matemáticas* en la página 297 de la Edición para el docente. Se pueden encontrar estrategias adicionales en las páginas 8-11 de este Manual del estudiante multilingüe.

The unit vocabulary words can be used to help students transfer their content knowledge from one language to the other—from Spanish to English or from English to Spanish. See the *Building the Language of Math* strategies on page 297 of the Teacher Edition. Additional strategies can be found on pages 8-11 of this Multilingual Learner Handbook.

## Cognados • Cognates

California Reveal Math	California Reveal Math
Constante de proporcionalidad	Constant of proportionality
Tasa constante	Constant ratio
Proporcion	Proportion
Relación proporcional	Proportional relationship
Proporcionalidad	Proportionality
Tasa unitaria	Unit rate

## Using Morphology to Determine Word Meanings

Throughout this unit, students will read, hear, and say several closely related words. Some are math-specific, while others represent general academic vocabulary. To expand receptive and expressive vocabulary, explain that words can be broken down into parts that have shared, or similar, meanings across words. By learning these meanings, students can determine the meanings of unknown words.

- **Turn and Talk:** Tell students that the first word part they should identify is the root or base word. Explain this distinction by saying that the former cannot stand on its own as a word (geology; geo) while the latter can (friendship; friend). Lead the group to brainstorm for common root and/or base words, writing them on the board. Ask:
  - What root or base word do you think will produce the greatest number of words?
  - How many words can you think of that include that root or base word?
- Use these secondary words as a springboard to introduce the concept of affixes. Highlight or circle these recurring prefixes or suffixes, encouraging students to deduce their meanings by observing how they consistently alter the meaning of a root or base. Make sure that these include common affixes such as re-, un-, or -ity. Show how such affixes can retain the overall meaning of the root or base word but change the part of speech of a word.
- **Wrap Up:** Focus on related words from the unit, encouraging students to define them and their individual parts. These could include relate/relationship, equation/equivalent, and proportion/ proportional/proportionality.

Select a question or set of questions to use for bridging content. Have students work in pairs or small groups to complete questions. A table of possible questions focused on representing information is presented below.

Lesson	Practice Questions Session 1	Practice Questions Session 2
3-1	#10-13	#1-4
3-2	#1-3	#3-4
3-3	#1-3	#1-2
3-4	#1-2,5	#3-4
3-5	#8-10	#1,4
3-6	#6-10	#4

Have students use counters, cubes, or arrays to represent the problems. Students explain their model in Spanish, then English.

### Multilingual Learner Scaffolds

#### Emerging

Allow students to analyze words in their home language. Work with them to list those that can be altered with affixes to form others. Point out that affixes are quite common; for example, the suffixes -s or -es create plurals.

**Expanding**

Have students write sentences using base words, and then add affixes to form new words for use in new sentences. Help students make the two sentences connect logically to highlight the shift in meaning or part of speech. Prior to writing, allow students to express these sentences orally to receive feedback from you or peers.

**Bridging**

Ask students to write sample sentences that highlight the math domain words shown above. They can look these up in a dictionary or skim the text to determine how the words are used in context. Help them articulate their own strategy for how knowledge of morphology can help them determine word meanings.

## Usar la morfología para determinar el significado de las palabras

A lo largo de esta unidad, los estudiantes leerán, escucharán y dirán varias palabras estrechamente relacionadas. Algunas son **específicas de matemáticas**, mientras que otras representan **vocabulario académico general**. Para expandir el vocabulario receptivo y expresivo, explique que las palabras se pueden dividir en partes que tienen significados compartidos o similares entre palabras. Al aprender estos significados, los estudiantes pueden determinar el significado de palabras desconocidas.

- **Gira y Habla** Diga a los estudiantes que la primera parte de la palabra que deben identificar es la raíz o la palabra base. Explique esta distinción diciendo que la primera no puede existir por sí sola como palabra (geología; geo) mientras que la segunda sí puede (amistad; amigo). Guíe al grupo a hacer una lluvia de ideas sobre raíces o palabras base comunes, escribiéndolas en el pizarrón. Pregunte:
  - ¿Qué raíz o palabra base creen que producirá la mayor cantidad de palabras?
  - ¿Cuántas palabras se les ocurren que incluyan esa raíz o palabra base?
- Utilice estas palabras secundarias como un punto de partida para introducir el concepto de afijos. Resalte o rodee estos prefijos o sufijos recurrentes, animando a los estudiantes a deducir sus significados observando cómo alteran consistentemente el significado de una raíz o palabra base. Asegúrese de que estos incluyan afijos comunes como re-, in-, o -dad (-ity). Muestre cómo tales afijos pueden retener el significado general de la raíz o palabra base, pero cambian la clase de palabra (part of speech) de la misma.
- **Cierre** Concéntrese en palabras relacionadas con la unidad, animando a los estudiantes a definir las y a definir sus partes individuales. Estas podrían incluir: relacionar/relación, ecuación/equivalente, y proporción /proporcional/ proporcionalidad.

Seleccione una pregunta o un conjunto de preguntas para usar como conexión de contenido. Pida a los estudiantes que trabajen en parejas o en grupos pequeños para completar las preguntas. A continuación, se presenta una tabla con posibles preguntas centradas en la representación de la información.

Leccion	Preguntas de Práctica Sesión 1	Preguntas de Práctica Sesión 2
3-1	#10-13	#1-4
3-2	#1-3	#3-4
3-3	#1-3	#1-2
3-4	#1-2,5	#3-4
3-5	#8-10	#1,4
3-6	#6-10	#4

## **Apoyo para estudiantes multilingües**

- Emergente** Permita que los estudiantes analicen palabras en su lengua materna. Trabaje con ellos para listar aquellas que pueden alterarse con afijos para formar otras. Señale que los afijos son bastante comunes; por ejemplo, los sufijos -s o -es crean plurales.
- En Expansion** Pida a los estudiantes que escriban oraciones usando palabras base y luego añadan afijos para formar palabras nuevas que usarán en oraciones nuevas. Ayude a los estudiantes a conectar las dos oraciones lógicamente para resaltar el cambio de significado o de clase de palabra. Antes de escribir, permita que los estudiantes expresen estas oraciones oralmente para recibir retroalimentación de usted o de sus compañeros.
- En Transicion** Pida a los estudiantes que escriban oraciones de ejemplo que destaquen las palabras del dominio matemático mostradas anteriormente. Pueden buscarlas en un diccionario o examinar el texto para determinar cómo se usan las palabras en contexto. Ayúdelos a articular su propia estrategia sobre cómo el conocimiento de la morfología puede ayudarles a determinar el significado de las palabras.

# Dual Language Connections

## Language Objectives

Students read text closely to explain relationships using compare/contrast organization. (Lesson 3-1)
Students express conclusions using academic language. (Lesson 3-2)
Students exchange ideas by asking and answering open-ended questions with because clauses. (Lesson 3-3)
Students use text features such as diagrams, tables, and annotations to comprehend complex informational text. (Lesson 3-4)
Students have collaborative conversations to describe relationships using models. (Lesson 3-5)
Students comprehend academic language to follow directions. (Lesson 3-6)
Students discuss how multiplication helps them represent and solve division problems. (Lesson 3-7)

## Communicate Your Understanding

Have students work with partners or in small groups to discuss the Essential question of the unit or the focus question for that lesson. Students can first discuss in Spanish, then rephrase or summarize in English.

## Transfer Learning from English to Spanish

Invite students to generate words, phrases, and sentences in English that express what they have learned about the Essential Question or lesson focus question. Record their ideas on the left side of an anchor chart like the one below. Then tell students they will now transfer what they learned from English to Spanish. Begin by having students work with partners or in small groups to come up with Spanish translations for each item in the left column. Record their translations in the right column of the chart, providing any Spanish terms that students are unable to translate.

How do proportional relationships help you understand everyday situations?	¿Cómo te ayudan las relaciones proporcionales a comprender situaciones cotidianas?
Unit cost Direct variation Conversions The relationship is proportional The constant of proportionality	Costo por unidad Variación directa Conversiones La relación es proporcional La constante de proporcionalidad

## Language and Math Sentence Frames for Unit 3

Las relaciones proporcionales establecen una tasa constante entre _____ y _____.	Proportional relationships establish a consistent rate between _____ and _____.
La constante (k) muestra cuánto cambia _____ cuando cambia _____.	The constant (k) shows how much _____ changes when _____ changes.
Las proporciones son esenciales para calcular un _____ por ciento de descuento en un artículo de \$_____.	Proportions are essential for calculating a _____ percent discount on a \$_____ item.

# Conexiones de Lenguaje Dual

## Objetivos de Lenguaje

Los estudiantes leen textos con atención para explicar relaciones utilizando la organización de comparación/contraste. (Lección 3-1)
Los estudiantes expresan conclusiones utilizando lenguaje académico. (Lección 3-2)
Los estudiantes intercambian ideas haciendo y respondiendo preguntas abiertas con cláusulas que contienen "porque" ( <i>because clauses</i> ). (Lección 3-3)
Los estudiantes utilizan elementos textuales como diagramas, tablas y anotaciones para comprender textos informativos complejos. (Lección 3-4)
Los estudiantes tienen conversaciones colaborativas para describir relaciones utilizando modelos. (Lección 3-5)
Los estudiantes comprenden el lenguaje académico para seguir instrucciones. (Lección 3-6)
Los estudiantes discuten cómo la multiplicación les ayuda a representar y resolver problemas de división. (Lección 3-7)

## Comunica tu Comprensión

Pida a los estudiantes que trabajen con compañeros o en grupos pequeños para discutir la pregunta esencial de la unidad o la pregunta de enfoque de esa lección. Los estudiantes pueden discutir primero en español y luego reformular o resumir en inglés.

## Transferencia del Aprendizaje del Inglés al Español

Invite a los estudiantes a generar palabras, frases y oraciones en inglés que expresen lo que han aprendido sobre la Pregunta Esencial o la pregunta de enfoque de la lección. Registre sus ideas en el lado izquierdo de una tabla de apoyo como la que se muestra a continuación. Luego, diga a los estudiantes que ahora transferirán lo que aprendieron en inglés al español. Comience pidiendo a los estudiantes que trabajen con compañeros o en grupos pequeños para crear traducciones al español para cada elemento en la columna izquierda. Registre sus traducciones en la columna derecha de la tabla, proporcionando cualquier término en español que los estudiantes no puedan traducir.

How do proportional relationships help you understand everyday situations?	¿Cómo te ayudan las relaciones proporcionales a comprender situaciones cotidianas?
Unit cost Direct variation Conversions The relationship is proportional The constant of proportionality	Costo por unidad Variación directa Conversiones La relación es proporcional La constante de proporcionalidad

## Marcos de oraciones de lenguaje y matemáticas para la Unidad 3

Las relaciones proporcionales establecen una tasa constante entre _____ y _____.	Proportional relationships establish a consistent rate between _____ and _____.
La constante (k) muestra cuánto cambia _____ cuando cambia _____.	The constant (k) shows how much _____ changes when _____ changes.
Las proporciones son esenciales para calcular un _____ por ciento de descuento en un artículo de \$_____.	Proportions are essential for calculating a _____ percent discount on a \$_____ item.

## Written Communication

Have students write a letter to a friend to explain the essential question: How do proportional relationships help you understand everyday situations?

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

### Multilingual Learner Scaffolds

- Emerging** Students can write short sentences in English with support words in their home language. Provide students with sentence starters for each part of the letter. "Dear..." (Greeting), "One thing I learned about the essential question (focus question) is..." (Body), "Your friend..." (Closing)
- Expanding** Students' letters should include at least 4 vocabulary words as well as language from the essential question or the focus question. Students can create a bilingual draft and then revise it in English.
- Bridging** Students write their letter in English, but can brainstorm in their home language. Remind students of academic connectors to support their letter writing (e.g., in conclusion, this shows that, an example is, as a result).

## Reading

Students read mathematical statements and contextual problems, then identify and justify appropriate solution strategies. Working in pairs, students analyze the problem, discuss which strategy would be most effective, select their approach, and document their chosen strategy on a recording sheet.

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

### Multilingual Learner Scaffolds

- Emerging** Provide visual supports by including pictures, diagrams, or manipulatives alongside the mathematical statements or word problems. Conduct a guided reading of the problem, pointing to key words and visuals while reading aloud. Have students echo-read or chorally read short phrases. In pairs, students point to and discuss visuals, select a strategy using the anchor chart, and record it on their sheet using provided sentence frames such as: "We will use \_\_\_\_" or "Our strategy is \_\_\_\_."
- Expanding** Provide mathematical statements and word problems with visual supports such as manipulatives, number lines, diagrams, or annotated text with key vocabulary highlighted. Have students read the problem with a partner, taking turns reading sentences aloud. In pairs, students discuss which strategy to use by completing sentence frames such as: "We chose \_\_\_\_"

because \_\_\_\_" or "This strategy works because \_\_\_\_." Students record their chosen strategy with a brief explanation on their recording sheet and share their reasoning with another pair using the sentence frames.

### **Bridging**

Provide mathematical statements and word problems with visual supports such as manipulatives, number lines, diagrams, or annotated text with key vocabulary highlighted. Have students read the problem with a partner, taking turns reading sentences aloud. In pairs, students discuss which strategy to use by completing sentence frames such as: "We chose \_\_\_\_ because \_\_\_\_" or "This strategy works because \_\_\_\_." Students record their chosen strategy with a brief explanation on their recording sheet and share their reasoning with another pair using the sentence frames.

### **Speaking**

Students engage in structured one-on-one or small-group interviews in which they verbally explain their mathematical thinking, problem-solving process, or understanding of a concept. A partner, small group, or teacher asks guiding questions, and the student responds orally, elaborating on their reasoning. The interviewer may ask follow-up questions to deepen the explanation or clarify understanding.

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

### **Multilingual Learner Scaffolds**

#### **Emerging**

Provide students with visual supports (their work, manipulatives, diagrams) to reference during the interview. Give both the interviewer and the respondent simple question and sentence frames: "What did you use?" / "I used \_\_\_\_." "Show me \_\_\_\_." / "This is \_\_\_\_." "What is the answer?" / "The answer is \_\_\_\_."

Students respond with 1-2-word answers, short phrases, or simple sentences using sentence frames, supported by pointing and gestures. Interviewer asks 2-3 basic questions. Accept responses combining words, gestures, and visual references. Partners switch roles so each student practices both asking and answering.

#### **Expanding**

Provide students with a list of interview questions and sentence frames for responses: "How did you solve this?" / "I solved it by \_\_\_\_." "Why did you choose that method?" / "I chose \_\_\_\_ because \_\_\_\_." "What does \_\_\_\_ mean?" / "It means \_\_\_\_."

Students respond using complete sentences (3-5 sentences total), incorporating mathematical vocabulary from a word bank. Interviewer asks 3-4 questions, including at least one follow-up: "Can you explain that

more?" or "What happened next?" Students may reference their work or visuals while speaking. Partners switch roles.

### **Bridging**

Provide students with open-ended interview questions that require detailed explanations: "Explain your complete problem-solving process." "Why does this method work?" "How would you solve this differently?" "What connections do you see?"

Students provide detailed explanations, using precise mathematical vocabulary and academic language, and speak for 1-2 minutes per question. Responses should include reasoning, justification, and examples. Interviewer asks follow-up analytical questions: "Can you elaborate on \_\_\_?" "What if \_\_\_?" "How does this relate to \_\_\_?" Students think aloud, building on questions without relying on frames. Partners or small groups switch roles, with each student practicing extended mathematical discourse.

### **Listening**

Students practice listening comprehension by hearing the teacher read a detailed explanation of a mathematical procedure, process, or concept. Students listen actively to understand the sequence of steps, key vocabulary, and reasoning involved. After listening, students demonstrate comprehension by engaging in a structured discussion in which they explain the process in their own words, compare their understanding with peers, and clarify any confusion.

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

### **Multilingual Learner Scaffolds**

#### **Emerging**

Provide students with picture cards for them to match as they listen to the explanation. Read the text slowly and emphasize important vocabulary. Use yes/no listening checks for each section of the explanation to check for understanding. Have students discuss with a partner.

#### **Expanding**

Provide students with a graphic organizer for them to complete as the explanation is read. Have them list each step in the organizer, and use quick yes/no listening checks to guide the pace of reading or to reread the section. After reading, students discuss in pairs or groups.

#### **Bridging**

Read the explanation aloud at a natural pace, and have students focus on listening to the passage. Read the explanation a second time and have them summarize the explanation in their own words, using an outline or writing a paragraph. Have them share their writing with others to discuss their summaries.