

# Dual Language/Bridging Lessons

## Unidad 3: Proporciones y Tasas

Gran idea: Generalizar con múltiples representaciones, modelar el mundo, relaciones fraccionarias

Conexión de contenido: Explorar cantidades cambiantes, Separar (o Descomponer) enteros, Unir partes

Pregunta esencial: ¿Cuándo utilizamos proporciones y tasas?

## Unit 3: Ratios and Rates

Big Idea: Generalizing with Multiple Representations, Model the World, Fraction Relationships

Content Connection: Exploring Changing Quantities, Taking Wholes Apart, Putting Parts Together

Essential Question: When do we use ratios and rates?

### Preguntas de enfoque por lección • Focus Questions by Lesson

Lección 1	<b>Comprender Proporciones</b> ¿Cómo se pueden usar las proporciones para resolver problemas y para describir la relación entre cantidades?
Lección 2	<b>Comprender las tasas y las tasas unitarias</b> ¿Cómo se puede describir una tasa unitaria como una tasa con 1 como segundo término?
Lección 3	<b>Determinar proporciones equivalentes usando tablas</b> ¿Cómo se pueden usar la multiplicación y la división para generar proporciones equivalentes en una tabla de proporciones?
Lección 4	<b>Gráficas de proporciones equivalentes</b> ¿Cómo se pueden identificar las razones equivalentes en una gráfica?

Lesson 1	<b>Understand Ratios</b> How can ratios be used to solve problems and to describe the relationship between quantities?
Lesson 2	<b>Understand Rates and Unit Rates</b> How can a unit rate be described as a rate with 1 as the second term?
Lesson 3	<b>Determine Equivalent Ratios Using Tables</b> How can multiplication and division be used to generate equivalent ratios in a ratio table
Lesson 4	<b>Graphs of Equivalent Ratios</b> How can equivalent ratios be identified on a graph?

Lección 5	<b>Comparar relaciones de proporciones</b> ¿Cómo se puede usar una tabla de proporciones para comparar relaciones de razón?	Lesson 5	<b>Compare Ratio Relationships</b> How can a ratio table be used to compare ratio relationships?
Lección 6	<b>Razonamiento de proporciones: Conversión de medidas dentro del mismo sistema</b> ¿Cómo se pueden resolver problemas matemáticos y del mundo real utilizando el razonamiento de proporciones para convertir medidas dentro del mismo sistema de medición?	Lesson 6	<b>Ratio Reasoning: Convert Measurements Within the Same System</b> How can real-world and mathematical problems be solved by using ratio reasoning to convert measures within the same measurement system?
Lección 7	<b>Razonamiento de proporciones: Conversión de medidas entre sistemas</b> ¿Cómo se pueden resolver problemas matemáticos y del mundo real utilizando el razonamiento de proporciones para convertir unidades de diferentes sistemas de medición?	Lesson 7	<b>Ratio Reasoning: Convert Measurement Between Systems</b> How can real-world and mathematical problems be solved by using ratio reasoning to convert units of different measurement systems?

## Explorar palabras • Explore Words

Las palabras de vocabulario de la unidad se pueden usar para ayudar a los estudiantes a transferir su conocimiento del contenido de un idioma al otro: del español al inglés o del inglés al español. Consulte las estrategias para *construir el lenguaje de las matemáticas* en la página 313 de la Edición para el docente. Se pueden encontrar estrategias adicionales en las páginas 8-12 de este Manual del estudiante multilingüe.

The unit vocabulary words can be used to help students transfer their content knowledge from one language to the other—from Spanish to English, or from English to Spanish. See the *Building the Language of Math* strategies on page 313 of the Teacher Edition. Additional strategies can be found on pages 8-12 of this Multilingual Learner Handbook.

## Cognados • Cognates

California Reveal Math	California Reveal Math
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Plano de coordenadas	Coordinate plane
Doble recta numerica	Double number line
Tasas equivalentes	Equivalent ratios
Pares ordenados	Ordered pairs
Proporción de parte a parte	Part-to-part ratio
Proporción de parte a todo	Part-to-whole ratio
Tasa	Rate
Termino	Term
Precio Unitario	Unit price
Tasa Unitaria	Unit rate

## Forming Comparative and Superlative Adjectives

In this unit, students will need to use comparative and superlative adjectives to compare and contrast two or more things. Remind students a comparative adjective compares two nouns, and a superlative adjective compares more than two nouns.

- Tell students that to change an adjective to the comparative or superlative form, they need to consider the number of syllables. They also need to check whether the adjective ends in a vowel + consonant or in a -y or -e.
- For one-syllable adjectives, adding the suffix -er will make the adjective comparative and adding the suffix -est will make the adjective superlative, such as high/higher/highest. If the adjective ends in a vowel + consonant, then the consonant is doubled before adding -er or -est, such as red/redder/reddest. If the adjective ends with -e, just add -r or -st, such as late/later/latest.
- For two-syllable adjectives, the comparative and superlative can be formed either by adding -er or -est or by using more or the most in front of the adjective. If the adjective ends in -y, change the -y to -i and then add -er.
- For adjectives that have more than two syllables, always use more and most. There are some frequently used adjectives that are irregular, such as good/better/best and bad/worse/worst.
- **Turn and Talk** Have partners preview the unit, discuss when they might use comparative and superlative adjectives, and ask questions about them. Prompt with questions, such as: How do you know when to just add -er or -est to change an adjective?

Select a question or set of questions to use for bridging content. Have students work in pairs or small groups to complete questions. A table of possible questions focused on representing information is presented below.

Lesson	Practice Questions Session 1	Practice Questions Session 2
3-1	#4-5	#1-3
3-2	#3	#1-3
3-3	#1-4,9-10	#1-5
3-4	#7-9	#1-4

3-5	#3,6,8	#1-2
3-6	#1-2	#3-5
3-7	#3-6	#1,3,5

## Multilingual Learner Scaffolds

**Emerging** Students may need support in identifying the number of syllables. Read the adjective aloud, pausing at each syllable break. Have them repeat the word after you. Give a short list of adjectives to students and have them change the forms to comparative and superlative.

**Expanding** Ask students to first make a list of adjectives. Then, ask them to write sentences using the base form of the adjectives from their list. Have them alter the sentences to compare two things using the comparative and again to compare three things using the superlative. If students are having a difficult time thinking of adjectives to use, encourage them to use a dictionary, preferably an English learner’s dictionary. Accept all reasonable responses.

**Bridging** Have students demonstrate their understanding by writing a sample sentence for one-syllable, two-syllable, and longer adjectives in both the comparative and superlative. Provide light support as needed. Rewrite as necessary.

## Formar adjetivos comparativos y superlativos

En esta unidad, los estudiantes necesitarán usar adjetivos comparativos y superlativos para comparar y contrastar dos o más cosas. Recuérdeles a los estudiantes que un adjetivo comparativo compara dos sustantivos y un adjetivo superlativo compara más de dos sustantivos.

- Diga a los estudiantes que para cambiar un adjetivo a la forma comparativa o superlativa, deben considerar el número de sílabas. También deben verificar si el adjetivo termina en una vocal + consonante o en -y o -e.
- Para adjetivos de una sola sílaba, agregar el sufijo -er hará que el adjetivo sea comparativo y agregar el sufijo -est lo hará superlativo, como high/higher/highest (alto/más alto/el más alto). Si el adjetivo termina en una vocal + consonante, entonces la consonante se duplica antes de agregar -er o -est, como red/redder/reddest (rojo/más rojo/el más rojo). Si el adjetivo termina en -e, solo agregue -r o -st, como en late/later/latest (tarde/más tarde/el más tardío).
- Para los adjetivos de dos sílabas, el comparativo y el superlativo se pueden formar añadiendo -er o -est, o usando more (más) o the most (el más) delante del adjetivo. Si el adjetivo termina en -y, cambie la -y a -i y luego agregue -er.
- Para adjetivos que tienen más de dos sílabas, siempre use more y most. Hay algunos adjetivos de uso frecuente que son irregulares, como good/better/best (bueno/mejor/el mejor) y bad/worse/worst (malo/peor/el peor).
- **Gira y habla** Pida a las parejas que revisen la unidad, discutan cuándo podrían usar adjetivos comparativos y superlativos, y hagan preguntas sobre ellos. Impúselos con preguntas como: ¿Cómo sabes cuándo debes simplemente agregar -er o -est para cambiar un adjetivo?

Seleccione una pregunta o un conjunto de preguntas para usar como conexión de contenido. Pida a los estudiantes que trabajen en parejas o en grupos pequeños para completar las preguntas. A continuación, se presenta una tabla con posibles preguntas centradas en la representación de la información.

Lección	Preguntas de Práctica Sesión 1	Preguntas de Práctica Sesión 2
3-1	#4-5	#1-3
3-2	#3	#1-3
3-3	#1-4,9-10	#1-5
3-4	#7-9	#1-4
3-5	#3,6,8	#1-2
3-6	#1-2	#3-5
3-7	#3-6	#1,3,5

## **Apoyo para Estudiantes Multilingües**

- Emergente** Permite que los estudiantes necesiten apoyo para identificar el número de sílabas. Lea el adjetivo en voz alta, haciendo una pausa en cada separación silábica. Pídales que repitan la palabra después de usted. Entregue a los estudiantes una lista corta de adjetivos y pídales que cambien las formas al comparativo y al superlativo.
- En Expansion** Pida a los estudiantes que, primero, hagan una lista de adjetivos. Luego, pídales que escriban oraciones usando la forma base de los adjetivos de su lista. Pídales que modifiquen las oraciones para comparar dos cosas usando el comparativo y, nuevamente, para comparar tres cosas usando el superlativo. Si los estudiantes tienen dificultades para pensar en adjetivos, anímelos a usar un diccionario, preferiblemente un diccionario para estudiantes de inglés. Acepte todas las respuestas razonables.
- En Transicion** Pida a los estudiantes que demuestren su comprensión escribiendo una oración de ejemplo para adjetivos de una sílaba, de dos sílabas y más largos tanto en el comparativo como en el superlativo. Proporcione apoyo ligero según sea necesario. Reescriba según sea necesario.

# Dual Language Connections

## Language Objectives

Students use adjectives to describe relationships between quantities. (Lesson 3-1)
Students use comparatives and superlatives related to travel choices: fastest, cheapest, best, slowest, lower/higher, etc. (Lesson 3-2)
Students ask and answer questions about comparisons using because clauses and familiar modal expressions. (Lesson 3-3)
Students use a variety of verbs and precise language to describe a process and demonstrate understanding of similarities and differences. (Lesson 3-4)
Students use the comparative form of adjectives and sequence words to describe processes and strategies. (Lesson 3-5)
Students will ask questions that justify answers to questions to express a possibility or make a suggestion. (Lesson 3-6)
Students learn to recognize and use the present progressive tense in statements. (Lesson 3-7)

## Communicate Your Understanding

Have students work with partners or in small groups to discuss the Essential question of the unit or the focus question for that lesson. Students can first discuss in Spanish, then rephrase or summarize in English.

## Transfer Learning from English to Spanish

Invite students to generate words, phrases, and sentences in English that express what they have learned about the Essential Question or lesson focus question. Record their ideas on the left side of an anchor chart like the one below. Then tell students they will now transfer what they learned from English to Spanish. Begin by having students work with partners or in small groups to come up with Spanish translations for each item in the left column. Record their translations in the right column of the chart, providing any Spanish terms that students are unable to translate.

When do we use ratios and rates?	¿Cuándo utilizamos proporciones y tasas?
Conversions Map scale Unit price Slope Proportional relationship Multiple representations	Conversiones Escala de mapas Precio Unitario Pendiente Relación proporcional Representaciones múltiples

## Language and Math Sentence Frames for Unit 3

Usamos tasas cuando medimos cuánto cambia una cantidad sobre otra, como al calcular _____ por _____.	We use rates when measuring how much a quantity changes over another, such as calculating _____ per _____.
Aplicamos el razonamiento de tasas para cambiar entre diferentes sistemas de medición, convirtiendo _____ a _____.	We apply rate reasoning to change between different systems of measurement, converting _____ to _____.

Usamos razones para comparar dos cantidades que comparten la misma unidad, como \_\_\_\_\_ a \_\_\_\_\_.

We use ratios to compare two quantities that share the same unit, such as \_\_\_\_\_ to \_\_\_\_\_

# Conexiones de Lenguaje Dual

## Objetivos de Lenguaje

Los estudiantes utilizan adjetivos para describir las relaciones entre cantidades. (Lección 3-1)
Los estudiantes utilizan comparativos y superlativos relacionados con opciones de viaje: <i>más rápido, más barato, mejor, más lento, más bajo/más alto, etc.</i> (Lección 3-2)
Los estudiantes hacen y responden preguntas sobre comparaciones usando cláusulas con "porque" y expresiones modales familiares. (Lección 3-3)
Los estudiantes utilizan una variedad de verbos y lenguaje preciso para describir un proceso y demostrar la comprensión de semejanzas y diferencias. (Lección 3-4)
Los estudiantes utilizan la forma comparativa de los adjetivos y palabras de secuencia para describir procesos y estrategias. (Lección 3-5)
Los estudiantes harán preguntas que justifiquen las respuestas a otras preguntas para expresar una posibilidad o hacer una sugerencia. (Lección 3-6)
Los estudiantes aprenden a reconocer y utilizar el tiempo presente progresivo en las declaraciones. (Lección 3-7)

## Comunica tu Comprensión

Pida a los estudiantes que trabajen con compañeros o en grupos pequeños para discutir la pregunta esencial de la unidad o la pregunta de enfoque de esa lección. Los estudiantes pueden discutir primero en español y luego reformular o resumir en inglés.

## Transferencia del Aprendizaje del Inglés al Español

Invite a los estudiantes a generar palabras, frases y oraciones en inglés que expresen lo que han aprendido sobre la Pregunta Esencial o la pregunta de enfoque de la lección. Registre sus ideas en el lado izquierdo de una tabla de apoyo como la que se muestra a continuación. Luego, diga a los estudiantes que ahora transferirán lo que aprendieron en inglés al español. Comience pidiendo a los estudiantes que trabajen con compañeros o en grupos pequeños para crear traducciones al español para cada elemento en la columna izquierda. Registre sus traducciones en la columna derecha de la tabla, proporcionando cualquier término en español que los estudiantes no puedan traducir.

When do we use ratios and rates?	¿Cuándo utilizamos proporciones y tasas?
Conversions Map scale Unit price Slope Proportional relationship Multiple representations	Conversiones Escala de mapas Precio Unitario Pendiente Relación proporcional Representaciones múltiples

## Marcos de Oraciones de Lenguaje y Matemáticas para la Unidad 3

Usamos tasas cuando medimos cuánto cambia una cantidad sobre otra, como al calcular _____ por _____.	We use rates when measuring how much a quantity changes over another, such as calculating _____ per _____.
Aplicamos el razonamiento de tasas para cambiar entre diferentes sistemas de medición, convirtiendo _____ a _____.	We apply rate reasoning to change between different systems of measurement, converting _____ to _____.

Usamos razones para comparar dos cantidades que comparten la misma unidad, como \_\_\_\_\_ a \_\_\_\_\_.

We use ratios to compare two quantities that share the same unit, such as \_\_\_\_\_ to \_\_\_\_\_.

## Written Communication

Students practice written mathematical communication by composing explanations of their problem-solving process, reasoning, or conceptual understanding. Students respond to a prompt that asks them to explain how they solved a problem, why a method works, what a concept means, or how mathematical ideas connect. Writing may include words, symbols, diagrams, and equations as appropriate.

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

## Multilingual Learner Scaffolds

**Emerging** Provide sentence frames with blanks for students to complete, a word bank with key mathematical vocabulary (with visuals or translations), and a model example. Frames should follow a simple sequence: "I used \_\_\_\_." "First, I \_\_\_\_." "Then, I \_\_\_\_." "The answer is \_\_\_\_." Students complete frames by filling in words, numbers, or short phrases from the word bank. They may add simple drawings or diagrams alongside their sentences. Accept writing that combines frames, labels, and visual representations. Students share their completed frames with a partner.

**Expanding** Provide sentence starters (not complete frames), a vocabulary word bank, and transition words (first, next, then, because, so). Starters guide structure but require students to complete ideas: "To solve this, I \_\_\_\_." "I chose this method because \_\_\_\_." "First, \_\_\_\_." "This shows \_\_\_\_." Students complete each starter and write 1-2 additional sentences incorporating mathematical vocabulary. Writing should include 4-6 sentences total explaining their process or reasoning. Students may include labeled diagrams. After writing, students exchange papers with a partner for peer feedback using a simple checklist: vocabulary used, steps clear, and reasoning included.

**Bridging** Provide an open-ended writing prompt specifying required components: explanation of process, use of precise mathematical vocabulary, reasoning or justification, and connections to concepts. Students may reference a vocabulary list, but no sentence frames or starters are provided. Students independently compose a complete written explanation (a paragraph or an organized response, 6-8+ sentences) with clear organization: what they did, how they did it, why it works, and what it means. Writing should demonstrate mathematical reasoning using academic language. After writing, students participate in a structured peer review: exchange papers, provide written feedback on clarity and mathematical accuracy, and ask questions. Students revise their writing based on feedback.

## Reading

Students rotate through reading stations, each containing a mathematical scenario or problem context. Students read the scenario using the scaffold appropriate for their language level and record their analysis:

(1) What information is provided? (2) What mathematical concepts or operations are suggested? (3) What strategy could be used to solve it? (4) How does this demonstrate mathematical reasoning or flexibility? Students share their analysis with a partner, comparing interpretations and solution approaches.

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

## **Multilingual Learner Scaffolds**

**Emerging** Provide simplified scenarios with key mathematical terms and numbers highlighted or bolded, accompanied by visual supports such as diagrams, pictures, or a word bank with images. Pre-read the scenario aloud, modeling think-aloud strategies while pointing to visuals and key words. Have students echo-read or follow along. Provide color-coding tools for students to mark numbers (one color) and mathematical action words/concepts (another color). Use a structured graphic organizer with sentence frames for each analysis component, such as: "The numbers are \_\_," "This shows \_\_," "I can use \_\_." Students share their color-coded text and completed frames with a partner.

**Expanding** Provide complete scenarios using brief paragraphs, with key mathematical vocabulary and concepts bolded or underlined. Include a vocabulary reference guide with definitions and visuals. Have students read the scenario independently first, then read it aloud to a partner for feedback and clarification. Provide a graphic organizer that breaks down the analysis into guided steps with sentence starters, such as: "The scenario describes \_\_," "The mathematical relationship is \_\_," "A strategy I could use is \_\_ because \_\_," "This shows \_\_ thinking because \_\_." Students complete each section and discuss their reasoning with their partner before moving to the next station.

**Bridging** Provide complex scenarios that include extra information not needed to solve the problem alongside the necessary data, incorporating academic mathematical vocabulary and multi-step reasoning. Have students read independently and annotate the text by: underlining essential information, crossing out irrelevant details, circling key vocabulary, and noting questions or connections in the margins. Students complete an open-response analysis identifying: the core mathematical problem, at least two different solution strategies, and an explanation of why multiple approaches are valid and how they demonstrate mathematical flexibility or connection between concepts. Students share their annotated text and written analysis with a partner, defending their reasoning and evaluating alternative approaches.

## Speaking

Students engage in structured one-on-one or small group interviews where they verbally explain their mathematical thinking, problem-solving process, or understanding of a concept. A partner, small group, or teacher asks guiding questions, and the student responds orally, elaborating on their reasoning. The interviewer may ask follow-up questions to deepen the explanation or clarify understanding.

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

## Multilingual Learner Scaffolds

**Emerging** Provide students with visual supports (their work, manipulatives, diagrams) to reference during the interview. Give both the interviewer and the respondent simple questions and sentence frames: "What did you use?" / "I used \_\_\_\_." "Show me \_\_\_\_." / "This is \_\_\_\_." "What is the answer?" / "The answer is \_\_\_\_." Students respond with 1-2-word answers, short phrases, or simple sentences using sentence frames, supported by pointing and gestures. Interviewer asks 2-3 basic questions. Accept responses combining words, gestures, and visual references. Partners switch roles so each student practices both asking and answering.

**Expanding** Provide students with a list of interview questions and sentence frames for responses: "How did you solve this?" / "I solved it by \_\_\_\_." "Why did you choose that method?" / "I chose \_\_\_\_ because \_\_\_\_." "What does \_\_\_\_ mean?" / "It means \_\_\_\_." Students respond using complete sentences (3-5 sentences total), incorporating mathematical vocabulary from a word bank. Interviewer asks 3-4 questions, including at least one follow-up: "Can you explain that more?" or "What happened next?" Students may reference their work or visuals while speaking. Partners switch roles.

**Bridging** Provide students with open-ended interview questions that require detailed explanations: "Explain your complete problem-solving process." "Why does this method work?" "How would you solve this differently?" "What connections do you see?" Students provide detailed explanations, using precise mathematical vocabulary and academic language, and speak for 1-2 minutes per question. Responses should include reasoning, justification, and examples. Interviewer asks follow-up analytical questions: "Can you elaborate on \_\_\_\_?" "What if \_\_\_\_?" "How does this relate to \_\_\_\_?" Students think aloud, building on questions without relying on frames. Partners or small groups switch roles, with each student practicing extended mathematical discourse.

## Listening

Students practice listening comprehension by identifying how quantities, values, or relationships change as they listen to the teacher read a mathematical scenario or problem context. Students demonstrate understanding through written responses, note-taking, annotations, or strategic marking of key information. The teacher provides a list of mathematical signal words or concepts to listen for (e.g., change indicators, operation clues, relationship words). After listening, students compare their understanding with a partner by sharing their notes or annotations.

*Use the Multilingual Learner Scaffolds listed below. Remind students to use the bilingual anchor chart as a resource for vocabulary and ideas.*

## Multilingual Learner Scaffolds

- Emerging** Read the scenario slowly, emphasizing key vocabulary with gestures (Total Physical Response) and visuals. Provide students with a printed copy of the scenario, model for them, pointing, circling, or highlighting the main vocabulary as the text is read. Then ask them to point, circle, or highlight the main vocabulary as you read the text. Have them share their responses with a partner.
- Expanding** Read the scenario at a moderate pace with clear enunciation and pauses. Provide students with a printed copy and preview 3-5 key signal words. As you read twice, students underline vocabulary, circle numbers, draw arrows for changes, and write brief margin notes. Afterward, they compare annotations with a partner.
- Bridging** Read the scenario aloud using academic vocabulary and complex structures. Provide students with a blank or note-taking template for key information, relationships, and changes. Students annotate independently, noting quantities, operation indicators, academic vocabulary, and questions. After listening once or twice, partners explain their understanding and discuss how language conveys meaning.