



Arizona Reveal
MATH[®]

Reveal the Full Potential
in Every Student

Reveal the Mathematician in Every Student

Arizona Reveal Math helps students develop the positive mindset, confidence, and skills to become problem solvers and mathematical thinkers. The program works by incorporating both inquiry-focused and teacher-guided instructional strategies within each lesson. Informed by the latest research on how they learn best, *Arizona Reveal Math* ensures students don't just meet the standards—they master them!

Our Powerful Program:



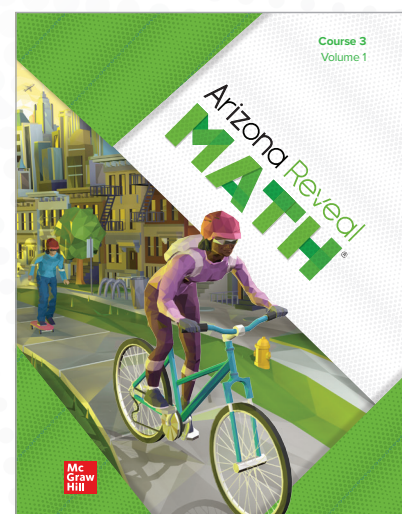
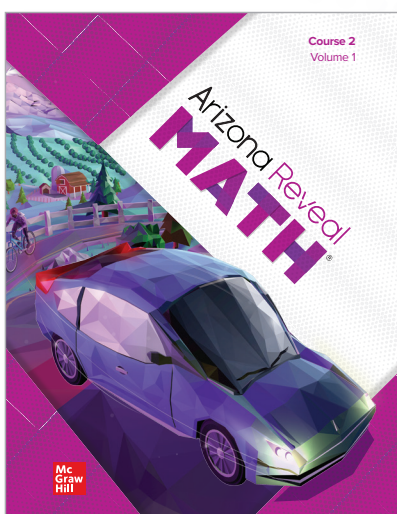
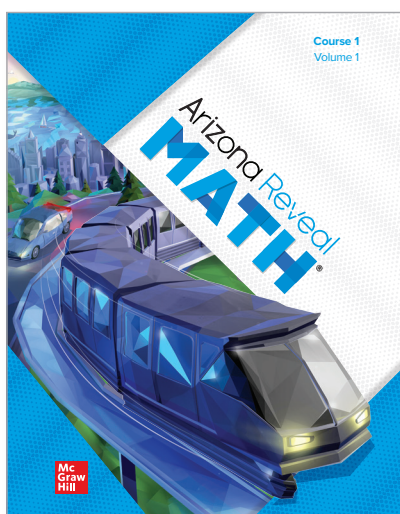
Champions a positive classroom centered on curiosity, connection, and a mathematical mindset.



Offers a flexible lesson design that provides access to rigorous instruction with robust teacher supports and scaffolds.



Tailors instruction for each student through data-driven insights and purposeful, personalized differentiation.



Arizona Reveal Math Authorship

McGraw Hill learning scientists teamed up with expert authors to create a program guided by validated academic research and classroom best practices.



Sarah Bush, Ph.D.

Expert in both theory and practice for middle school math instruction



John SanGiovanni, M.Ed.

Leader in understanding the mathematics needs of students and teachers



Annie Fetter

Advocate for student ideas and thinking that foster strong problem solvers



Cathy Seeley, Ed.D.

Thought leader and facilitator of high-quality mathematics education for all



Linda Gojak, M.Ed.

Expert in both theory and practice of strong mathematics instruction



Raj Shah, Ph.D.

Champion of perseverant problem-solvers and student curiosity in mathematics



Christa Jackson, Ph.D.

Advocate for strong STEM education and equity for middle school students



Cheryl Tobey, M.Ed.

Facilitator of strategies that drive informed instructional decisions



Georgina Rivera, M.Ed.

Expert in building student agency through culturally responsive teaching



Dinah Zike, M.Ed.

Creator of learning tools that make connections through visual-kinesthetic techniques



George Roy, Ph.D.

Expert in integrating technology into middle school instruction

Program Design Influenced by Teachers, Research, and Industry Experts

To design the program, our expert authorship consulted rigorous educational research. Foundational texts include *Principles to Actions* (NCTM) and *Making Sense of Math* (Cathy Seeley) as well as learning models such as Bloom’s Taxonomy and Webb’s Depth of Knowledge Guide. We then called upon our most trusted collaborators, hundreds of teachers across the country, for instructional insights to bring this research to life.

Major Focus Areas:

A Supportive Classroom Culture for All Students

Learner-focused practices develop a classroom designed for equitable learning.

Rich Mathematical Discourse

Instructional options and supports focus on student discourse while emphasizing academic and math vocabulary.

Productive Struggle

Opportunities to explore and engage with challenging mathematical ideas and relationships build deep understanding.

Sense-Making

Support for the development of sense-making and critical thinking skills develops proficient problem solvers.

Fluency

Flexible strategies help students to practice math content and achieve automaticity.

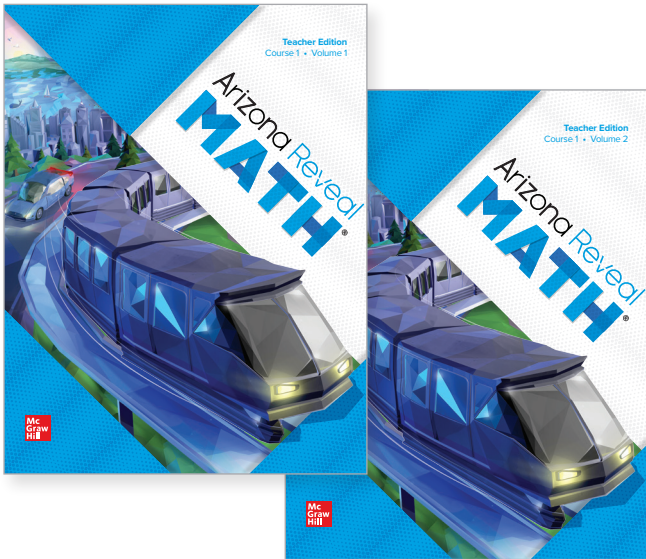
Instructional Routines

Structures and expectations create productive classroom interactions with students. Read more about Math Language Routines (MLR) on page 17.

Metacognition

Student reflection promotes math learning.

Teacher Resources



Print Resources

Teacher Edition, 2 Volumes

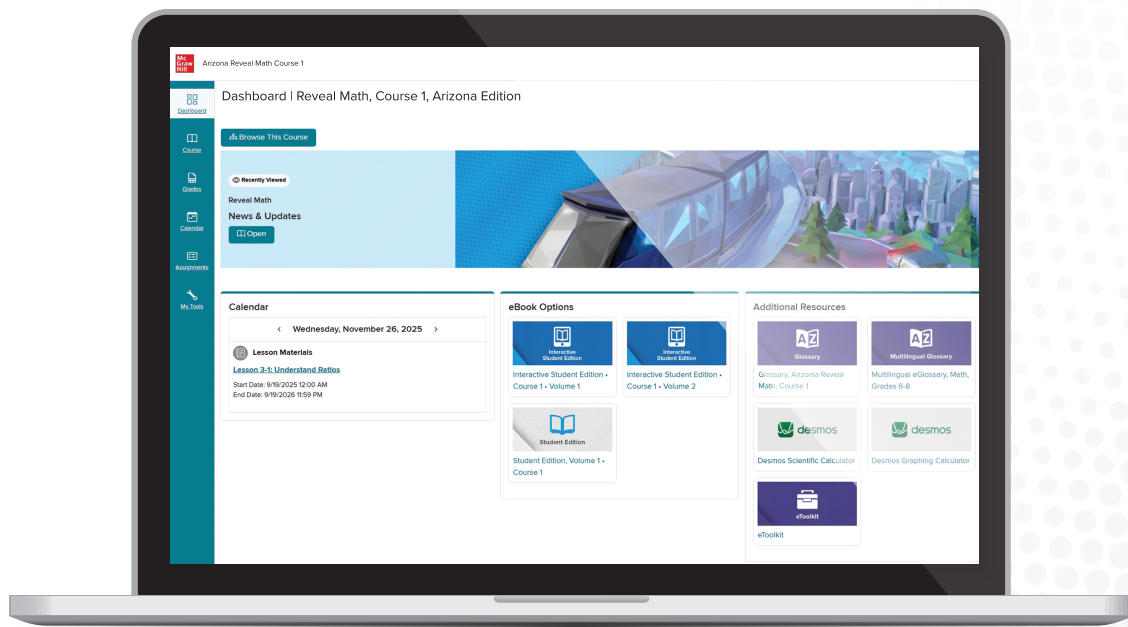
These spiral-bound Teacher Editions provide the essentials to plan and implement high-quality math instruction. Inside, you will find instructional supports including NCTM's Effective Teaching Practices, Math Language Routines, Multilingual Learner (ML) Language Scaffolds, and differentiation recommendations.



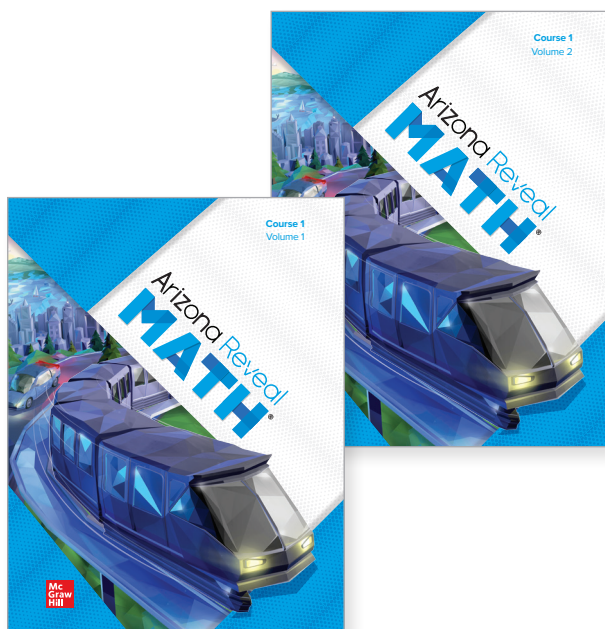
Digital Teacher Center Resources

Teachers have access to an intuitive and easy-to-use platform for planning, teaching, and assessment. The teacher digital experience includes:

- Engaging, interactive, prebuilt lesson presentations with embedded technology.
- Editable lesson presentations in PowerPoint allowing teachers to customize and share slide content or teach offline.
- An expansive library of professional learning workshops and videos, including expert insight videos.
- Digital practice and assessments item banks to build custom assignments.
- A robust selection of engaging differentiation resources.
- Dynamic unit practice.
- Digital exploration activities powered by Web Sketchpad®.
- Anytime access to the eToolkit.
- Practice and assessment PDFs.
- Teacher and administrator data and reporting.
- Classroom management and grouping tools.
- The ability to add resources, including presentations and website links.



Student Resources



Print Resources

Student Edition, 2 Volumes

Available in print and interactive formats, the Student Editions are write-in, three-hole-punched, and perforated for easy organization in a binder. Students engage in learning through the use of problem-solving, discourse, and reflection.

Spanish Student Edition, 2 Volumes

For students who need to access learning in their first language, a fully translated Spanish Student Edition is available.

Digital Student Center Resources

Students have access to a robust set of engaging digital tools and interactive learning aids, including:

- An Interactive Student Edition.
- Daily, interactive practice with embedded learning aids and dynamic (algorithmic) items.
- Dynamic, exploratory activities powered by Web Sketchpad®.
- Anytime access to a robust eToolkit (Virtual Manipulative Suite).
- Rich, exploratory STEM Adventures.
- Online assessments with interactive item types.
- Math Replay videos to review lesson content.
- Digital games designed for purposeful practice.

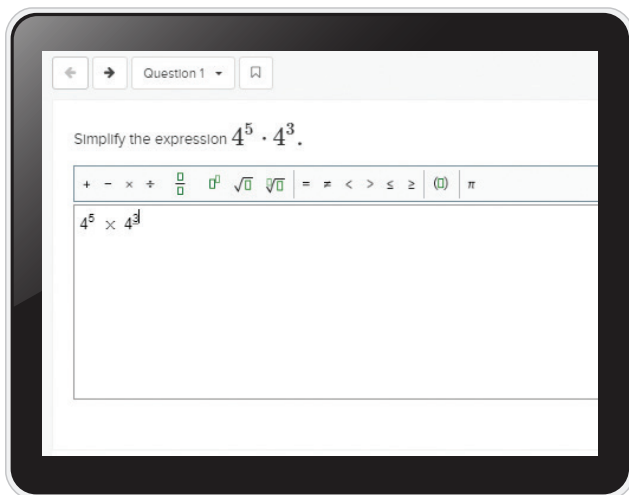
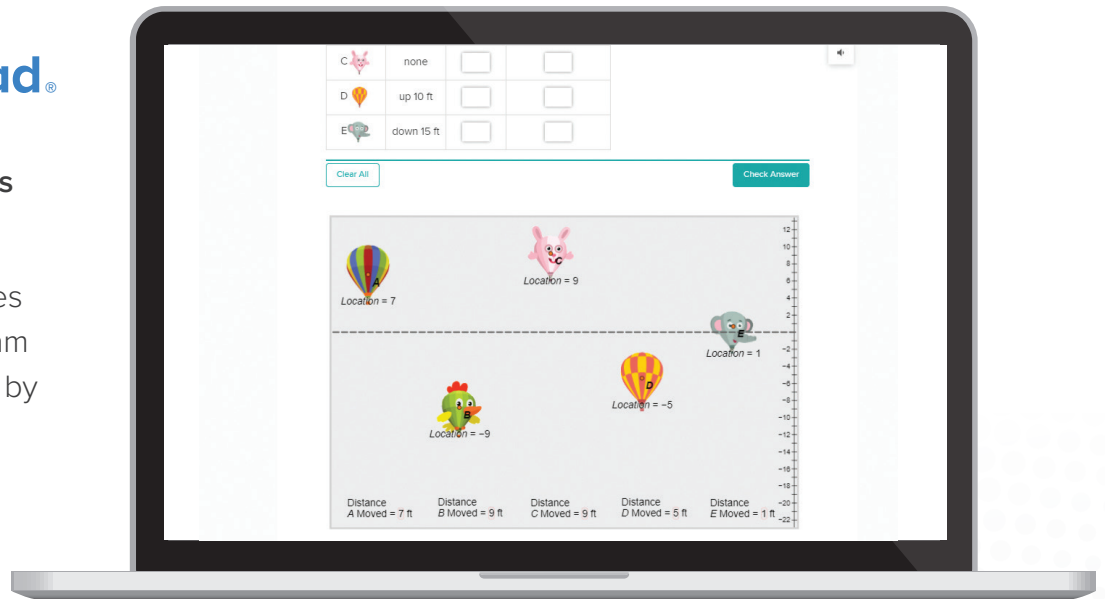
Where Technology Meets Math

Arizona Reveal Math supports both low-tech and high-tech classrooms. The blended print and digital instructional model captures the best of both modalities and brings them together in a seamless experience that makes math meaningful for your students.



Visualize Math Concepts in Action

Web Sketchpad® activities included with the program enhance understanding by demonstrating dynamic math concepts.



Prepare Students for Computer-Based Testing

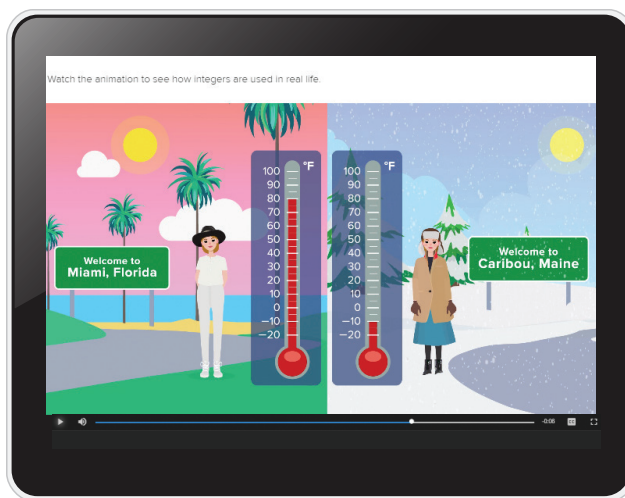
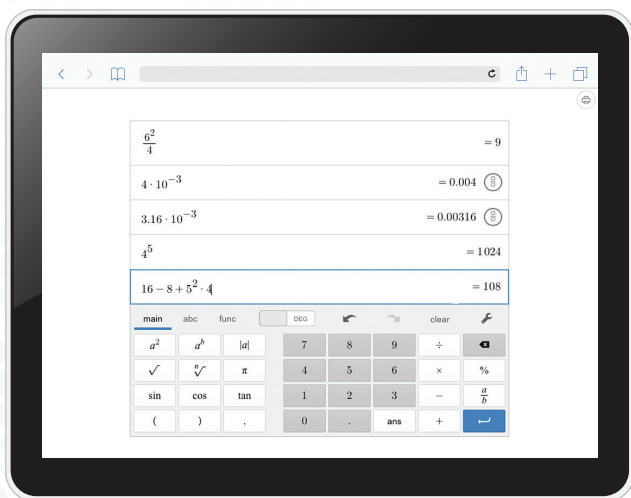
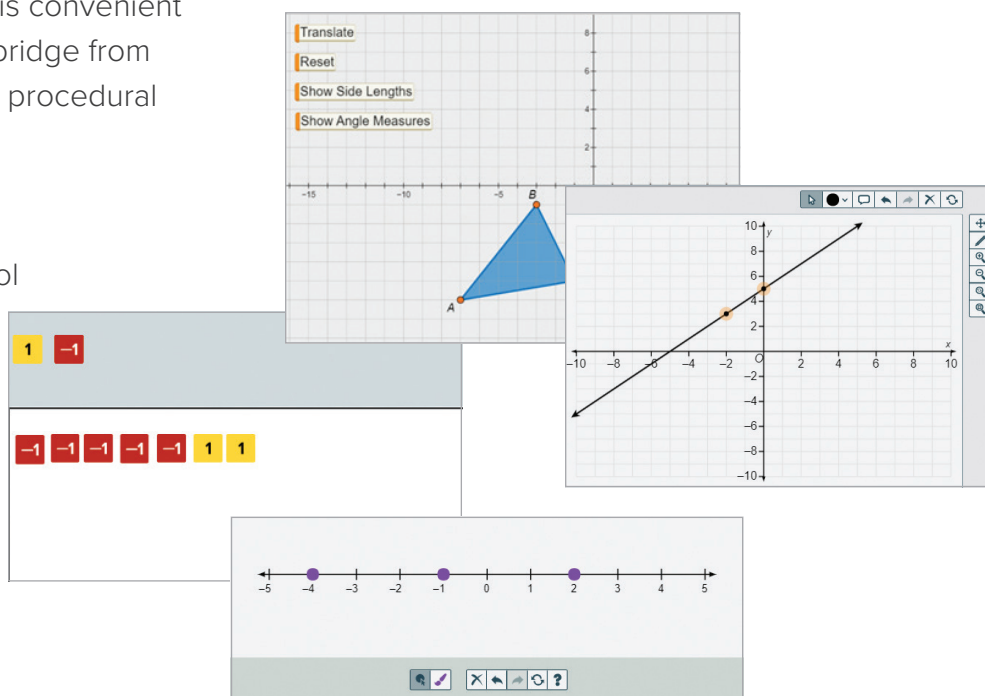
Technology-enhanced items provide students the valuable practice they need to master computer-based assessments. These items include:

- Drag-and-drop
- Equation editor problems
- Multiselect
- Open response

Utilize Digital Tools for Problem Solving

Embedded within lessons, this convenient collection of eTools builds a bridge from conceptual understanding to procedural fluency. It includes:

- Number Line Tool
- Coordinate Graphing Tool
- Transformations Tool
- Algebra Tiles Tool



Explore, Model, and Apply Math

The best-in-class Desmos scientific calculator, easily accessible in *Arizona Reveal Math*, allows students to practice using the same resource that appears on many common standardized tests.

Motivate with Truly Enjoyable Technology

Designed with student engagement in mind, the digital resources in *Arizona Reveal Math* include animations, videos, and interactive problems to enhance context and learning.

Spark Curiosity and Problem-Solving



Raj Shah, Ph.D.

Contributing Author



Curiosity Makes Math Irresistible

Making math irresistible for students starts with sparking their interest and curiosity. Each *Arizona Reveal Math* unit has a unit topic that students explore through the Unit Opener and the Ignite! activity.

Ignite! activities are collaborative activities designed by *Arizona Reveal Math* contributing author Raj Shah, Ph.D., to engage students and spark their mathematical curiosity prior to starting a unit. In each unit, the Ignite! activity:

- Follows a theme that relates to the STEM scenario in the Unit Opener.
- Poses interesting questions to motivate students to problem solve.
- Supports productive struggle and facilitates discourse.
- Recurs at the end of the unit as part of the Mathematical Modeling task.

Name _____ Date _____ Period _____

IGNITE!

Determining the U.S. Population

Every ten years since 1790, U.S. Government has conducted a census. At that time, census takers visited every house in the U.S. States to record the number of people living in the house.

For exercises 1-4, answer each question.

1. How might the methods used to conduct the census have changed since 1790?
2. What could be one method conducting a census of the people in your school?
3. Would the same method work for conducting a census of the people in your city or town? Explain your thinking.
4. You are part of a team creating a new school council for your school that includes everyone in the school—students, teachers, staff, and administrators. What would you propose for deciding the number of representatives each group would have? Explain your thinking.

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Unit 3 • Proportional Relationships 3

Bring the World Into Focus Through STEM

With STEM-focused unit topics, *Arizona Reveal Math* helps students observe and understand the math taking place all around them.

- Students engage in rich, contextual problem-solving tasks to explore real-world concepts.
- Modeling tasks help students apply the math they know to solve problems in their everyday lives.


Think About It

Under what conditions would air purification be necessary?

Unit 3
Proportional Relationships

Essential Question
How can you determine proportionality between two varying quantities?

Explore Through STEM
Air in Flight
Airlines have systems in place to create a healthy cabin environment for their passengers where the air is replaced every few minutes. Filters are used to prevent viruses and bacteria from spreading. The air flow design has most air leaving the cabin in the same row in which it enters the cabin.



Think About It
Under what conditions would air purification be necessary?

3 • Proportional Relationships

Unit Opener

Essential Question
How can you determine proportionality between two varying quantities?
Encourage students to share their initial responses to the Essential Question. Because students have not yet been formally introduced to proportional relationships, they may not have much to share. Then revisit the question throughout the unit, and ask students to expand on or revise their initial responses.

Explore Through STEM
Air in Flight
Throughout this unit, students will explore the proportional representation that informs the use of air sensors and air purifiers. Have students notice and wonder about the image. Ask:
• How are the passengers on the plane able to breathe when flying at high altitudes?
• How could we ensure we have enough oxygen for all the passengers on a long flight?
• How might we determine the air is safe to breathe on an aircraft?

Think About It
Under what conditions would air purification be necessary?
To facilitate discussion, use these prompts:
• How might an airline determine how much air circulation is needed on a given flight?
• How might air be evaluated and purified throughout a flight?
• What might be limitations to ensuring air quality on an aircraft? When might air sensors and purifiers not be effective?
Consider having students think about whether the size of the aircraft changes the amount of air circulation needed on the flight. They could explore what considerations need to be made for different numbers of passengers on various flights.

Preparing for the Explore and Discuss
How Do I Choose?
To decide which exploration to implement, consider the **Activity-Based Explorations (ABE)** are designed to engage students in deep understanding through research-based teaching practices. In Lessons 3-1 and 3-5 offer unique learning opportunities.
• Students are introduced to the concept of proportionality in Lesson 3-1. The ABE for this lesson is designed to dig into what a proportion is and how it is used to solve problems.
• In Lesson 3-5, students explore tools that are used to represent proportional relationships. The ABE is designed to explore the relationship between two quantities through which tools make the relationship more concrete.
• Consider the grouping of students. Because students are expected to work in groups, plan groups of students to provide the opportunities for everyone in the group to engage in the mathematics.

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Teacher Supports for Rich STEM Discourse

A teacher facilitation guide within the Teacher Edition supports rich discourse around the STEM topic of each unit, including visuals and prompts, as part of the Unit Opener. Available both in print and online formats of the Teacher Edition!



Flexible Lesson Design

Balanced Instructional Design

Arizona Reveal Math was built to align with the three pillars of a high-quality mathematics curriculum: focus, coherence, and rigor. This intentional instructional design ensures that students develop deep understanding, make connections, build conceptual proficiency, and understand the “why” behind mathematics. There is a well-defined learning progression that builds upon previous lesson content and anticipates future lessons. Additionally, each lesson prioritizes one or more aspects of rigor that align with content standards.

Focus

- Math Objectives
- Language Objectives

Coherence

- Previous
- Now
- Next

Rigor

- Conceptual Understanding
- Procedural Skill & Fluency
- Application

Instructional Choice

Arizona Reveal Math 6–8 offers teachers two instructional options: **Activity-Based Exploration** and **Guided Exploration**. They can choose whichever best meets the learning and pedagogical needs of their students and their instructional preferences.

Explore – Session 1 20 min

Launch Explore Assess Practice

CHOOSE YOUR OPTION

Activity-Based Exploration

Proportional Reasoning

ETP Implement Tasks That Promote Reasoning and Problem Solving
Students explore ratios, proportions, and rates. The goal is to have students understand that a proportion can be used to determine unknown values.

Materials
Digital: *Activity Exploration Journal*, pp. AEJ31–AEJ32, 1 per student
Hands-On: *Dog Breed Cards and Diagnosis and Medication Cards Teaching Resource*, 1 per student-group, *Activity Exploration Journal*, pp. AEJ31–AEJ32, 1 per student

Directions
Have students respond to the **Introductory Question** in their *Activity Exploration Journal*.

- How can you use a ratio relationship to determine unknown quantities?

Group students in pairs or small groups to work on this activity.

- Today we will explore proportions.

Digital: Students start and stop the crank and change the numbers in the noodles and seconds boxes. Using Websketch™, students explore the relationship between the number of noodles and the time it takes for a machine to make them.

Hands-On: Have each group select a dog card and a diagnosis/medication card. The students are working at a veterinarian's office. The medication information is given as ratios comparing the amount of medicine to bodyweight. The ratios are given in decimals, fractions, and mixed numbers. Students will need to determine how much medication the vet will need for their dog. Time permitting, students may want to draw another set of cards.

ETP Support Productive Struggle
As student-pairs explore the activities, check that all pairs understand the task. If students need guidance or support, ask:

- How can you find equivalent ratios?
- How can you use equivalent ratios to solve for unknown quantities?
- How can you use equivalent ratios to find a unit rate?

Math is... Making Connections

- How is this problem like something you have done before?

SMP As students complete the activity, encourage them to think about aspects of the real-world problem that are familiar. Students who make connections to previous work will be able to draw from their earlier experiences.

Have students complete the **Concluding Questions** in their *Activity Exploration Journal*.

How can you find a unit rate for a rate that contains fractions, mixed numbers, or decimals?
How can you use a proportion to solve for an unknown value?

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Activity-Based Exploration

Student Screen

Activity-Based printable available for this lesson.

Guided Exploration

Salt-Water Aquarium

Students explore the concepts of *equivalent ratios* and *proportions* in a problem about constructing a salt-water aquarium.

Materials
none

Introduce the problem situation on Student Edition p. 122.

ETP Pose Purposeful Questions

- How can you write ratios that compare the amount of salt to the amount of water?
- How could you use those ratios to find the total amount of salt the aquarium needs?

Collaborate and Connect

- How do you think you could solve this proportion?
 $\frac{35}{1} = \frac{x}{75}$

Have students work in small groups to solve the proportion. Have the groups compare their approaches and solutions.

MLR Collect and Display: Gather and Show Student Discourse
As students discuss the situation, write key words and phrases you hear, such as *rate*, *ratio*, *equal*, and *equivalent*. Display the words and phrases for the students to reference throughout the lesson. Update the collection with other relevant terms and new understanding as the lesson progresses.

Math is... Making Connections

- How is this problem like something you have done before?

SMP Invite volunteers to describe the aspects of the problem that are familiar. Students may recognize notation, terminology, or structure. Helping students make connections to previous work will allow them to draw from their earlier experiences.

Let's Explore More
Students work with partners or in small groups to complete the questions.
Check that students understand that two equivalent ratios written as an equation is a proportion. Students should also understand that the amount of salt will be different, but that the ratio of salt to water is the same.

Let's Explore More

How much salt will the need if she maintains the same salinity level?
4,200 grams of salt

Student Edition, p. 122

MLR Multilingual Learner Scaffolds

Entering/Emerging Preteach content-area vocabulary, such as *salinity*, and other unfamiliar words, such as *elapsed*. Explain that words such as *table* and *record* have more than one meaning. Provide a chart in which students can draw pictures illustrating the different meanings.

Developing/Expanding Remind students that words sometimes have more than one meaning, and have them identify these as they arise during the lesson. Ask students to use the different meanings of those words verbally in different sentences.

Bridging/Reaching Have students write paired sentences for each of the multiple-meaning words they encounter in the lesson. Using the word *bank* as an example, have them explain their thought process when eliminating meanings that do not make sense in context.

Lesson 3-1 • Connect Ratios, Rates, and Proportions 255

Exploration pages in the Teacher Edition

Effective Teaching, Active Learning

Effective Teaching Practices provide opportunities for teachers to elicit and use evidence of student thinking to help target their instruction effectively. Understanding student reasoning by asking them to explain their answers allows teachers to reinforce and enhance student understanding or target and address misconceptions.

Concept exploration, collaboration, and choice of problem-solving strategy help students develop and exercise agency during learning. Additionally, students begin to understand that mathematics is a language of its own in which they can communicate effectively to deepen their understanding.

ETP Effective Teaching Practices

Elicit and Use Evidence of Student Thinking

As students progress through the unit, ask them to explain their reasoning. Understanding the reasoning for their answers—whether they are correct or incorrect—allows for targeted instruction to reinforce and expand or enhance their understanding or address misconceptions and misunderstandings in a timely way.

As students learn about proportionality, there are multiple possibilities for errors in execution. Students may have misconceptions about

- the difference between proportional and nonproportional relationships;
- linear graphs that do not pass through the origin;
- common factors that define equivalent ratios and proportions.

Ask frequent questions, especially those that require reasoning. Use students' responses to inform instruction and determine what kinds of practice and review might be necessary.

For example, if students struggle to determine equivalent ratios or to identify the constant of proportionality, spend some time revisiting multiplication and division of fractions.

In Lessons 3-3, 3-4, and 3-5, students are introduced to proportional relationships. Monitor closely students' responses and thinking in these lessons to ensure they are understanding proportional relationships accurately.

The screenshot shows a lesson plan page titled "Develop - Session 2" with a 20-minute duration. It includes a navigation bar with "Lesson", "Develop", "Summarize & Apply", "Assess", and "Practice" buttons. The main content is under "Activity-Based Exploration" and includes an "Activity Debrief" section with three numbered items: 1. Critique, Correct, and Clarify; 2. Facilitate Mathematical Discourse; 3. Elicit Evidence of Student Understanding. There are also sidebars for "Math Is..." and "Multilingual Learner" with additional instructions.

ETP Facilitate Mathematical Discourse

Facilitate a whole-class discussion of the activities and Concluding Questions, reinforcing these key terms: *proportional*, *nonproportional*, and *constant of proportionality*. Ask:

- How can you identify the constant of proportionality from a graph?
- How can a graph tell you whether a relationship is proportional or not?
- What did you notice about the ratio of y to x ?
- What does the point $(0, 0)$ tell you about quantities that are in a proportional relationship?

ETP Elicit Evidence of Student Understanding

As students discuss their thinking, listen for understanding of:

- For every point (x, y) on the graph of a proportional relationship, the ratio of $\frac{y}{x}$ is constant.
- Two quantities are in a proportional relationship if the graph of the ordered pairs is a straight line through the origin.
- On the graph of a proportional relationship, the constant of proportionality is both the constant ratio $\frac{y}{x}$ and the value of r in the point $(1, r)$.
- The meaning of $(0, 0)$ in the graph of a proportional relationship: When there is zero of one quantity, there will zero of the other quantity.

Language of Math

Help your students develop the language of math with *Arizona Reveal Math* Math Language Routines and comprehensive vocabulary support. These embedded resources support teacher facilitation and student acquisition of mathematical language and vocabulary.

MLR Collect and Display

As students discuss the questions, listen and write on the board any key words they use. Display the words and phrases for student reference. Use the student-generated expressions to help make connections between student language and math vocabulary. Update the collection with new understandings as the lesson progresses.

Math Language Routines embedded within every lesson provide a framework for teachers to seamlessly promote mathematical language development as part of their everyday instruction to support all types of learners.

LOM Language of Mathematics

Vocabulary

Students will be using these key terms in this unit.

- **Constant of proportionality** (Lessons 3-2, 3-3, 3-4, 3-5): This is a new term. Students may be familiar with the term *constant* describing a situation that occurs continuously over a period of time. The mathematical meaning in this context is similar—it describes a quantity or parameter that does not change. The *constant of proportionality* describes the relationship that does not change between values in two sets of values. See entry for proportionality.
- **Constant ratio** (Lessons 3-2, 3-3, 3-4, 3-5): This is a new term. Students were introduced to the term *ratio* in Grade 6 during their study of ratios and rates. They determined equivalent ratios to solve problems during their Grade 6 study. The *constant ratio* describes the ratio that can be used to determine equivalent ratios.
- **Origin** (Lesson 3-7): Students were introduced to this term in Grade 5 when they were introduced to the coordinate plane.
- **Proportional relationship** (Lessons 3-1, 3-2, 3-3, 3-4): This is a new term. A proportional relationship is one for which all the values in two sets are equivalent ratios.
- **Proportionality** (Lessons 3-2, 3-3, 3-4, 3-5): This is a new term. A proportionality indicates equality between two or more ratios.
- **Ratio** (Lessons 3-1, 3-2, 3-3, 3-4, 3-5, 3-6): Students were introduced to this term in Grade 6 during their study of ratios and rates.
- **Unit rate** (Lesson 3-1, 3-4, 3-5): Students were introduced to this term in Grade 6 during their study of ratios and rates.

Building the Language of Mathematics

As students work through each lesson, have them complete the graphic organizer to build understanding of and proficiency with key mathematical terms and concepts.

Encourage students to come up with their own definitions and descriptions of terms. When students generate their own definitions or descriptions of terms, they are more likely to remember them long term.

Word Wall

If there is a Math Word Wall in the classroom, ask students to add their words, examples, and counterexamples of proportional relationships to the wall. As they share them, have each student explain their entry.

Building the Language of Mathematics

Complete the graphic organizer as you work through the unit.

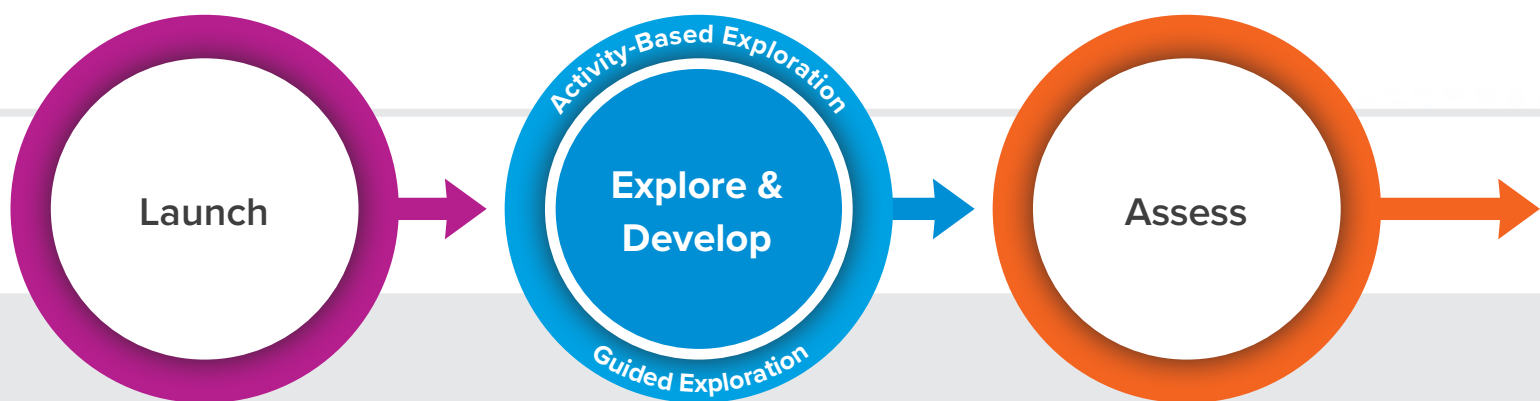
Proportional Relationships

Way to Represent	Example	Non-example
Table		
Graph		
Equation		



Lesson Model

The *Arizona Reveal Math* lesson model keeps sense-making and exploration at the heart of learning. Every lesson provides **two instructional options** to develop the math content and tailor the instruction to students' learning needs.



Every lesson begins with **Be Curious**, a sense-making activity:

- Students focus on noticing and wondering, not problem-solving.
- Teachers foster students' thinking through meaningful discussion.

Explore & Develop unpacks the lesson content through either an Activity-Based or Guided Exploration:

- Students explore concepts in small groups during which they can formalize their emergent ideas.
- Teachers facilitate the exploration of concepts through rich discourse.

Each *Arizona Reveal Math* lesson includes two opportunities to gauge student learning:

- The **Exit Ticket** is completed after Session 1 and helps to inform instruction for Session 2.
- The **Lesson Quiz** is completed after Session 2 and helps inform differentiation.

Create Consistency in Learning

Instructional routines are embedded within every *Arizona Reveal Math* lesson to help students become proficient doers of mathematics.

Build Fluency

Number Routines

Support the development of flexibility with numbers and fluency with operations.

MLR

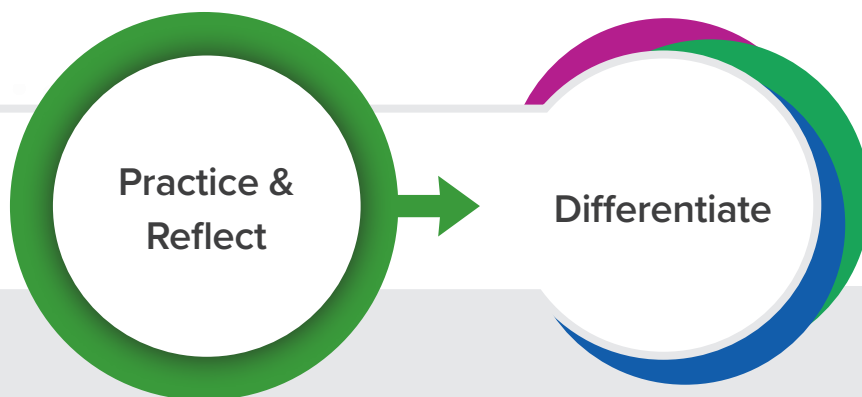
Math Language Routines

Promote mathematical language development as part of instruction.



Sense-Making Routines

Build sense-making as a foundation for problem-solving and mathematical modeling.



The **Practice** pages offer students opportunities to engage with the math and reflect on their learning:

- Students practice lesson concepts by completing the exercises.
- Teachers monitor progress and have students reflect on the lesson learning targets.

Lesson **Differentiation** supports all students in their path to understanding:

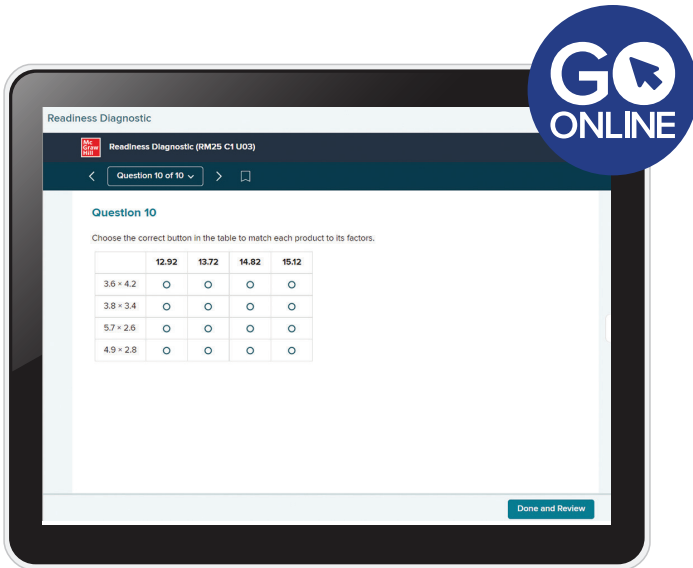
- Students work on differentiated tasks to reinforce their understanding, build their proficiency, and/or extend their thinking.

Types of Differentiation

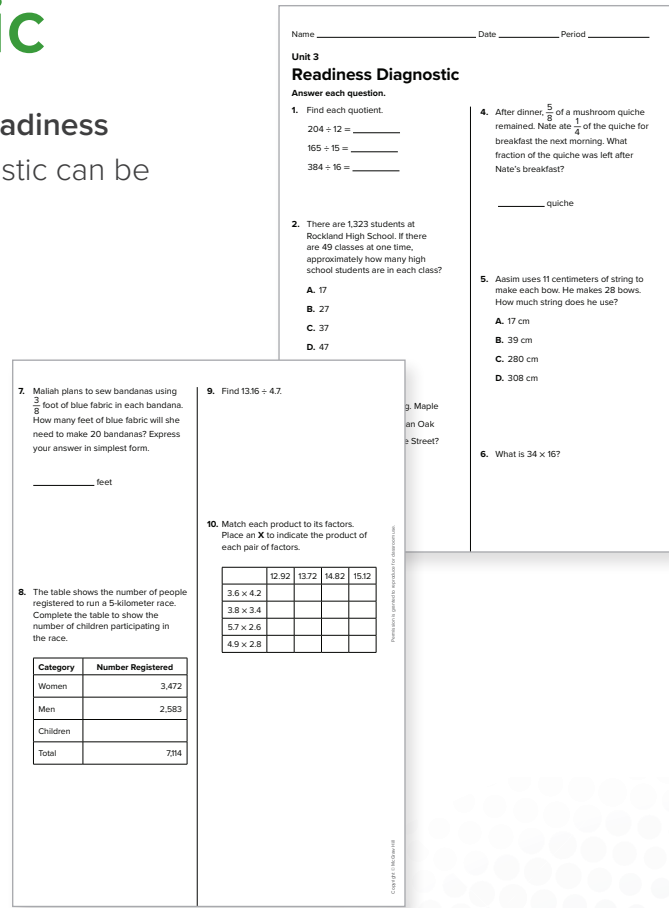
- R Reinforce Understanding**
Resources designed to revisit lesson concepts.
- B Build Proficiency**
Resources to build proficiency with lesson skills.
- E Extend Thinking**
Resources to enrich lesson concepts.

Readiness Diagnostic

Before beginning the unit, students complete the **Readiness Diagnostic** to identify any learning gaps. The diagnostic can be delivered online or downloaded and printed.



Online Diagnostic



Print Diagnostic

Provide Targeted Intervention

The Teacher Edition includes an **Item Analysis** table which recommends **Guided Support Intervention Lessons** for students who need them. These lessons are assignable through the Digital Teacher Center.

Targeted Intervention

Use Guided Support intervention lessons available in the Digital Teacher Center to provide targeted intervention.

Item Analysis

Item	DOK	Skill	Guided Support Intervention Lesson	Standard
1	1	Count to 3	Count to 3 with Dots as Objects	K.CC.B.5
2	1	Represent 5	Identify Sets of 1 to 5 Objects	K.CC.A.3
3	3	Compare groups	Same or Different with Two Groups	K.CC.C.6
4	2	Count to 3	Count to 3 with Dots as Objects	K.CC.B.5
5	2	Compare numbers	Less than, Greater than, or Equal to 5	K.CC.C.7
6	1	Represent 3	Identify Sets of 1 to 5 Objects	K.CC.A.3

STEM Connections

Explore unit concepts through recognizable STEM scenarios.

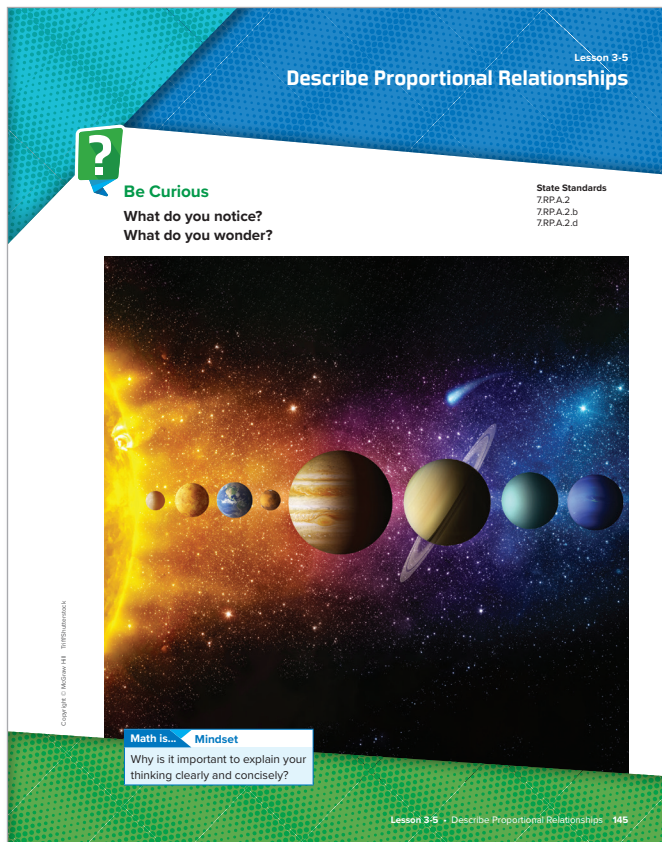
- **Explore Through STEM** identifies a STEM scenario in the Unit Opener that sets the theme that will be revisited throughout the unit.
- **STEM Adventures** are digital activities where students can engage in experiments, make hypotheses, and apply mathematical knowledge to analyze data.
- The **Mathematical Modeling** tasks at the end of each unit tie back to the STEM scenario in the Unit Opener.



Use Questions to Promote Student Ideas

Be Curious

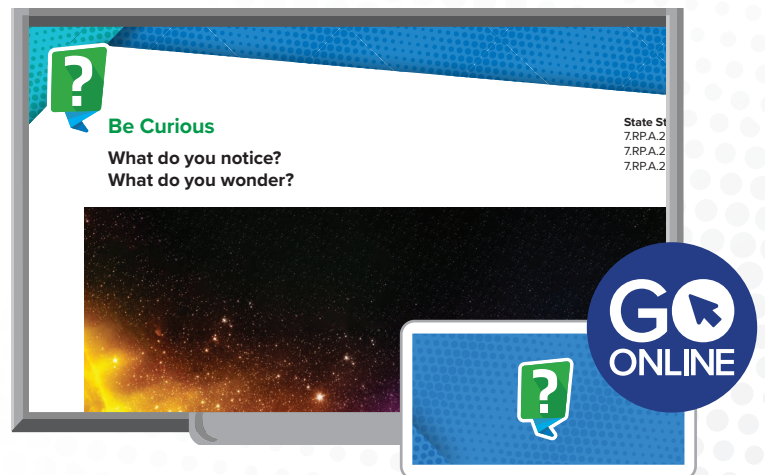
Be Curious, written by Annie Fetter, launches each session using a high ceiling, low floor sense-making activity with multiple entry points to help create an equitable classroom culture where all ideas are welcome and respected.



Print



Annie Fetter
Contributing Author



Digital

Number Routines

More or Less Than

Students build fluency with estimation strategies as they determine whether the value of a given expression is more or less than the target number.

These prompts encourage students to talk about their estimates:

- What numbers helped you think about your solution?
- How did you reason about the results?
- How does your strategy compare to ___'s?

More or Less Than...

In this routine, students estimate a result using any strategy and then compare their estimate to a given value. The intent is for them to estimate results rather than determine the exact result.



Number String Matrix

Students build fluency with operations as they use the solution to an equation to solve equations with the same digits with different base ten values.

These prompts encourage students to talk about their estimates:

- How are the factors related in the rows or columns?
- How does knowing $[9] \times [5]$ help you think about $[90] \times [500]$?
- What new problems could you solve because you know $[9] \times [5]$?

Number String Matrix

A Number String is a list of related equations. Students use the solution strategy for the first equation to solve the subsequent equations. A number string matrix is a set of related problems that are presented in rows and columns. Students pick a row or a column and solve the equations.



Number Routines

Every lesson includes two **Number Routines**, written by John SanGiovanni, M.Ed., that help students build number sense and proficiency with numbers. This supports their ability to fluently and flexibly apply strategies to solve problems.



**John SanGiovanni,
M.Ed.**

Contributing Author

Mathematical Discourse

In every lesson, students have the opportunity to engage in discussion about the math concepts from the lesson to build deeper understanding.

Orchestrating Rich Mathematical Discourse

In this lesson, students are introduced to and explore concepts related to ratio reasoning, a big idea in middle school mathematics. It's important that students have opportunities to engage in discussion about these concepts as they build their understanding of them. These suggestions from can help optimize the discussion of ratio concepts during either the Activity-Based or Guided Exploration.

1. Anticipate likely student responses.

- Activity-Based Exploration: As you plan for the lesson, think about the strategies your students are likely to use and misconceptions some students may have.
- Guided Exploration: As you plan for the lesson, review the questions in the teacher presentation and anticipate student responses to those questions. Think about which questions may pose challenges for students.

5 Practices for Orchestrating Productive Mathematical Discourse

Smith and Stein (2011)

1. **Anticipate** likely student responses.

2. **Monitor** students' thinking

3. **Select** student thinking to feature

4. **Sequence** student responses

5. **Connect** student thinking

Build Understanding Through Exploration

Teachers have their choice of two instructional strategies to facilitate student learning during the **Explore & Develop** phase:

Activity-Based Exploration

Students work together to explore concepts, develop and test hypotheses, and—most importantly—engage in productive struggle as they problem solve and generalize learning. Options for hands-on or digital activities are provided.

Guided Exploration

Teachers facilitate exploration through thoughtful discourse and collaboration using an interactive, digital presentation.

Explore - Session 1 20 min

Launch Explore Assess Practice

CHOOSE YOUR OPTION

Activity-Based Exploration

Mixing Paint

181 Implement Tasks the Promote Reasoning and Problem Solving
Students explore different shades of green that can be made from mixing different amounts of blue and yellow paint. The goal is to have students notice that the ratio of blue to yellow paint will dictate the shade of green paint.
Students can work in pairs or small groups to carry out the task that they can complete in either the Digital Student Center or with paper and pencil.
Materials two-color counters (optional)

182 Support Productive Struggle

- What are some ways of getting different shades of green?
- How might the amount of blue and yellow paint mixed affect the shade of green?
- What do you notice about the amount of blue paint and yellow paint for each shade of green?

Math is... Choosing Tools

- What tools might help you track the different shades of green?

183 Encourage students to record and track their different combinations of blue and yellow paint in a tool of their choosing.

184 Elicit Evidence of Student Thinking
Some students may approach this task by completing a table with descending values for the blue paint in one column and ascending values for the yellow paint in a second column. Make sure students see the corresponding amounts of blue and yellow paint as related and defining of a particular shade of green.

Activity Debrief

185 Collect and Display
As students share their findings and their tools, listen for and write on the board any key words they use. Display the words and phrases for student reference. Use the student-generated expressions to help make connections between student language and math vocabulary.

186 Facilitate Mathematical Discourse
As students come up with different shades of green, ask them to look for patterns with the amount of blue paint and yellow paint for each shade of green.

- How can you describe a specific shade of green?
- Why do you think different amounts of blue paint result in different shades of green?
- What would you need to do to make a lot of paint that is a specific shade of green?

Introduce the term *ratio*, *part-to-part*, and *part-to-whole* in the debrief. Show bar diagrams to represent the part-to-part and part-to-whole ratios. Say:

- For this shade of green, the ratio of blue paint to yellow paint is [1] to [2]. This bar diagram shows that relationship. What is the ratio of blue paint to yellow paint for this shade of green? These are both part-to-part ratios. That is, both blue paint and yellow paint are parts of the green paint.
- For this shade of green, the ratio of blue paint to green paint is [1] to [3]. This bar diagram shows that relationship. What is the ratio of blue paint to green paint for this shade of green? These are both part-to-whole ratios. That is, the blue paint is part of the green paint.

Guided Exploration

Pedro's Paint Mixture

Students explore the concept of ratios through a paint mixing situation. Introduce the problem situation. Have students consider why many hardware stores carry only white paint and create the exact color that a customer wants on the spot.

187 Pose Purposeful Questions

- Why might it be advantageous for a hardware store to carry only white paint and create specific color mixes that customers request?
- How would the hardware store be able to know that a color mixture matches the customer request?

188 Collaborate and Connect
For each of the different combinations of pieces of wood, have students work with a partner to determine whether the three pieces of wood can make a triangle. Ask:

- How do the three pieces of wood relate?

189 Collect and Display
As students discuss the questions, listen and write on the board any key words they use. Display the words and phrases for student reference. Use the student-generated expressions to help make connections between student language and math vocabulary. Update the collection with new understandings as the lesson progresses.

190 Use and Connect Mathematical Representations

- How does the blue paint relate to the green paint?
- What tools or models do you know that can represent this relationship?
- What do you think the bar diagram might look like? Draw the bar diagram.
- How are the different ways to write ratios related?

Students should notice that the terms are always in the same order.

191 Facilitate Meaningful Mathematical Discourse

- How can we describe a ratio?
- Explain why the blue paint to green paint is a part-to-whole ratio.
- How is the blue to yellow paint relationship different from the blue to green paint relationship?

Let's Explore More
Students work in partners or small groups to complete the questions. Check that students understand the difference between the blue to green and yellow to green ratio. Both are part-to-whole ratios, but use different parts.

192 Multilingual Learner Scaffolds
Entering/Emerging Support students in understanding the meaning of bar diagrams. Show a picture of bar graphs, both horizontal and vertical and ask students to point to the bars in the graph. Then have them point to the bars in the bar diagrams.
Developing/Expanding Check that students understand the meaning of bar diagrams. Show them some bar diagrams, both horizontal and vertical and ask them to describe the graphs. Encourage them to use bars in their descriptions. age them to use bars in their descriptions.
Bridging/Reaching Check students' understanding of bar diagrams. Ask them about other mathematical representations or models that have bars in them (bar graph) and to explain how the bars in the bar diagram are similar to those in the bar graph.

193 Multilingual Learner Scaffolds
Entering/Emerging Support students in understanding the meaning of bar diagrams. Show a picture of bar graphs, both horizontal and vertical and ask students to point to the bars in the graph. Then have them point to the bars in the bar diagrams.
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18 Unit 3 - Proportional Relationships

Unit 3 - Proportional Relationships 19

ETP Effective Teaching Practices

MLR Math Language Routines

MPP Math Practices and Processes

MLL Multilingual Learner Scaffolds

How Do I Choose?

Teachers can reference guidance in each Unit Opener or online in the Digital Teacher Center Unit Resources to help them decide which exploration to implement for lessons in a unit. The How Do I Choose? print and digital resources provide considerations for student engagement, scheduling, personal preference, and a variety of pairings or groupings.

How Do I Choose?

To decide which exploration is best for your class, consider the following:



▶ Activity-Based Exploration

- My students need practice working in pairs.
- During the **Be Curious** conversation, my students demonstrated they have the mindset to explore the concept on their own.

▶ Guided Exploration

- My students are engaged during class conversations.
- My students need practice presenting ideas to the entire class.
- My students struggled to see the math in the **Be Curious** conversation and need extra support to make connections during the **Explore & Develop**.



Digital Activity-Based Exploration

Digital Guided Exploration Presentation

Go Online and Explore!

Log in to the Digital Teacher Center to assign an interactive, digital Activity-Based Exploration to all or selected students. They can record their observations and findings in their Activity Exploration Journal.

If Guided Exploration is more suitable to class needs for the lesson, log in to the Digital Teacher Center to launch an interactive, digital presentation.

Assess to Inform Instruction and Differentiation

Name _____ Date _____ Period _____

Lesson 3-5
Exit Ticket

For item 1, use the information in the tables to complete the exercise.

The ratio tables show the number of red circles that Anna and Ruth each used in a design.

Anna			
Red Circles	3	6	9
Shapes	5	10	15

Ruth			
Red Circles	5	10	15
Shapes	8	16	24

1. Who has the greater ratio of red circles to shapes? Explain how you determined the answer.

Exit Ticket

At the end of Session 1, students demonstrate their understanding of lesson concepts by completing the **Exit Ticket**. Data from the Exit Ticket will help teachers inform instruction for the next session of that lesson.

Name _____

Lesson 3-5
Lesson Quiz

For item 1, use the tables to answer the question.

1. Each table represents an equivalent ratio. Complete the sentences.

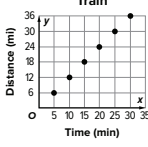
Orange Juice			
Size (fl oz)	12	16	32
Cost (\$)	\$1.80	\$2.40	\$4.80

Grapefruit Juice			
Size (fl oz)	8	20	40
Cost (\$)	\$1.44	\$3.60	\$7.20

Based on the cost per fluid ounce, _____ juice is the less expensive drink.
It is _____ cents per fluid ounce less expensive.

For item 2, use the graph to answer the question.

2. In the last 30 minutes, a car has traveled at a constant speed of 65 miles per hour on a highway. The graph shows the distance a train has traveled in the last 30 minutes.



Complete the sentence.
The _____ is traveling at a greater constant speed by _____ miles per hour.

For items 3 and 4, use the tables that show the ratio of chilies to all ingredients in two hot sauce recipes.

Recipe 1			
Chilies (g)	3	6	9
All Ingredients (g)	5	10	15

Recipe 2			
Chilies (g)	7	14	21
All Ingredients (g)	12	24	36

3. Which can you use as the second term in the ratio to compare the ratios of both recipes?
A. 24 B. 30 C. 42 D. 60

Lesson Quiz

At the end of Session 2, students complete the Lesson Quiz. Quiz data informs decisions for differentiation using the **Lesson Quiz Skill Tracker**.

Lesson Quiz Skill Tracker

The Lesson Quiz Skill Tracker in the Teacher Edition identifies Depth of Knowledge (DOK) and Standards covered by the Lesson Quiz to help teachers determine the next steps for each student based on quiz performance.

Lesson Quiz Skill Tracker

Item	DOK	Skill	Standard
1	1	Understand ratio relationships	6.RP.A.1
2	2	Use ratio reasoning	6.RP.A.3
3	2	Use ratio reasoning	6.RP.A.3
4A	1	Understand ratio relationships	6.RP.A.1
4B	3	Use ratio reasoning	6.RP.A.3

Purposeful Practice

Practice & Reflect

Practice & Reflect provides students with opportunities to solidify their understanding of the lesson concepts through independent practice pages. Two practice pages can be completed in the Interactive Student eBook or in the print Student Edition. Additional practice exercises are available online in Extra Practice with algorithmic question functionality, which changes question values upon attempt and includes learning aids.

For exercises 5 and 6, use a graph, table, or equation to represent proportional relationships. Then answer the questions.

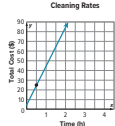
5. A bagger at a grocery store earns \$48.75 for working 3 hours. How much would she earn for working 1 hour? 12 hours?

6. At the farmer's market, Hans paid \$3.99 for 3 pounds of bananas. How much would he pay for 5 pounds of bananas?

For exercises 7 and 8, answer the questions.

7. The equation $y = 2.54x$ represents the relationship between centimeters and inches. What is the constant of proportionality and how could you use it to find the number of centimeters in 50 inches?

8. **Error Analysis** The graph shows the fee scale for a cleaning service. A sales representative says that the service charges \$25 per hour for cleaning. What feedback would you give the sales representative?



Reflect
Explain to a classmate how to determine if a relationship is proportional.

Math In... Mindset
Why was it important to explain your thinking clearly and concisely?

100 Unit 3 • Proportional Relationships

Dynamic Practice

Name _____ Date _____ Period _____

Practice

For exercises 1-4, answer the questions.

1. The relationship between the recommended diameter of an exercise hoop and a person's height is shown. How do the diameters compare to the different heights?

Height (in.)	Diameter (in.)
58	29
60	30
62	31
64	32

2. Aspect ratio is the ratio of width to height of a television screen. The table shows the width and height of several different televisions. How does the width to height compare?

Width (in.)	Height (in.)
27.9	15.7
37.5	21.1
43.6	24.5
56.7	31.9

3. The number of points Imani earns while playing a video game is proportional to the number of stars that she collects. The graph of this relationship passes through the points (0, 0) and (2, 600). How can you use this information to determine the number of points Imani earns for collecting one star?

4. **STEM Connection** An engineer is studying the amount of time it takes to charge the battery for three different electric cars and the distance the car can travel on a fully charged battery. Is the relationship between the time and distance proportional? Explain.

Car	Hour charge time	Miles on full charge
Car A	8 hour charge time	240 miles on full charge
Car B	6 hour charge time	120 miles on full charge
Car C	8 hour charge time	200 miles on full charge

Lesson 2.5 • Describe Proportional Relationships 149

Print Practice



Dynamic Digital Practice

Every lesson includes a range of practice sets with interactive question types, helpful tools, hints, examples, and multiple attempts allowed to support student success.

Additional Practice: Compare Ratio Relationships

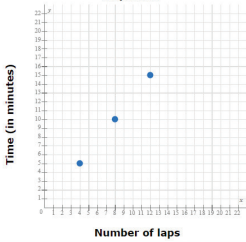
03-05 Additional Lesson Practice (RM25 C1)

Question 11 of 15

Question 11

Two bicyclists are traveling around the same track. The number of laps completed by bicyclist A and the time it took in minutes is shown in the table. The number of laps completed by bicyclist B and the time it took in minutes is shown in the graph.

Bicyclist A	
Number of laps	2 4 6
Time (in minutes)	5 10 15



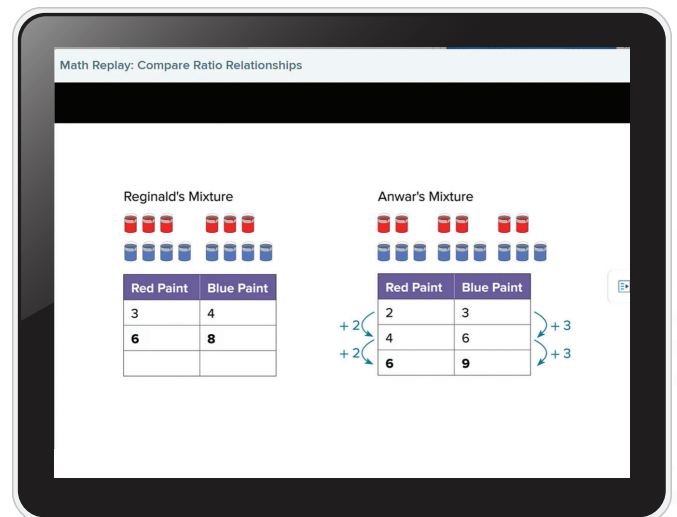
Applied Learning

Students complete exercises related to the lesson content. The exercises for each lesson target students' understanding of the concept or skill, their proficiency (fluency) with the skills, and include opportunities to apply the concepts and skills to new or unfamiliar situations. The section ends with a Reflect question that has students reflect on either the lesson concepts or specific mathematical thinking habits.



Math Replay Videos

Every lesson contains a one- to two-minute video explanation of the lesson concept for students to reference as they complete independent practice assignments.



To review today's lesson, have students watch the Math Replay video in their Digital Student Center.

Assign the On My Own practice to students from the Digital Teacher Center.



Differentiation for Diverse Learners

Robust differentiation resources help teachers meet the learning needs of students who would benefit from enrichment to extend learning or provide additional reinforcement for students requiring support.

B Build Proficiency

Assign

Interactive Additional Practice

Assign students either the print or digital assignment to practice lesson concepts. The digital assignment includes algorithmic exercises.

Assign

Spiral Review

Assign students either the print or digital version to review these concepts and skills.

- Recognizing Place Values and the Relation to Each Other

GO ONLINE

INDEPENDENT WORK

E Extend Thinking

Assign

STEM Adventures

In this STEM Adventure, students display, describe, and analyze data about daily household water consumption. Then they use statistics to investigate how water consumption is affected by wasteful or conserving behaviors and compare data sets.

GO ONLINE

INDEPENDENT WORK

R Reinforce Understanding

Assign

Take Another Look Lessons

Assign the interactive lessons to reinforce targeted skills.

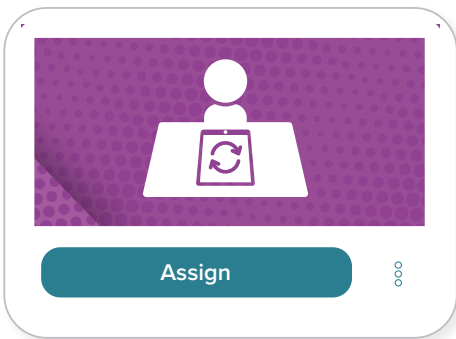
- Statistical Questions
- Create Line Plots and Dot Plots
- Describe a Data Distribution

GO ONLINE

INDEPENDENT WORK

Enrich Learning with Differentiated Resources

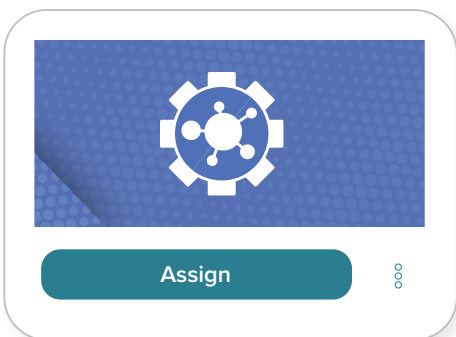
Arizona Reveal Math offers a variety of engaging, multi-modal activities with different delivery options to meet the individual needs of all students.



Take Another Look

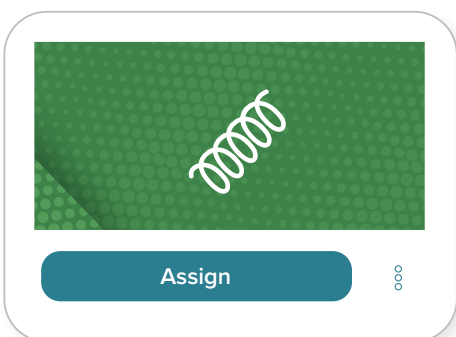
On-Level Reteach Mini-Lessons

Self-paced, digital mini-lessons consist of a three-part, gradual release activity: Model, Interactive Practice, and Check.



Extend Thinking Activities

Extend Thinking Activities challenge students who are ready to learn more. STEM Adventures is one Extend Thinking activity that involves students conducting experiments, making hypotheses, and analyzing data.



Spiral Review

Use the Spiral Review assignments at the end of a lesson to practice concepts presented in prior lessons.



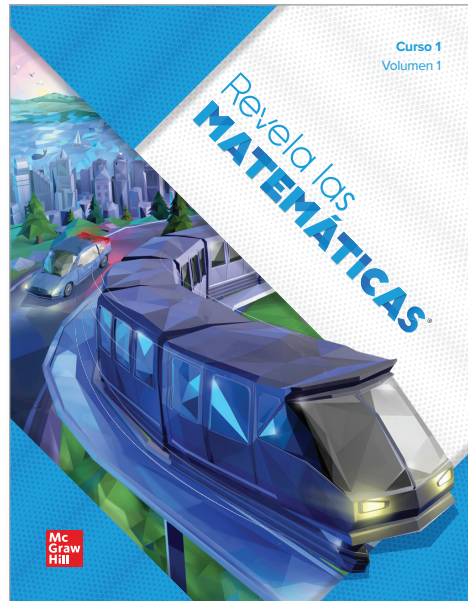
Digital Game Center

Digital Games help students become proficient with grade-level concepts in a fun and engaging practice environment.

Support for Multilingual Learners

In addition to Multilingual Language Scaffolds found in the Teacher Edition for each lesson, *Arizona Reveal Math* includes these components and resources to assist multilingual learners as they build language and mathematical proficiency:

- Spanish Student Edition
- English/Spanish Glossary
- Audio to improve listening comprehension skills
- ALEKS bilingual courses in Spanish



Spanish Student Edition

Math Language Development

Arizona Reveal Math is rife with mathematical language and specialized terms that may be new to students. Built-in academic language and text features help them grow their mathematical vocabulary and master key terms they are expected to know.

Math Language Development

Language Development - Academic Language

These mini-lessons focus on the academic terms listed in the Unit 3 planner.

Emerging/Entering

Write this sentence on the board and then read them aloud for the group.

*There are about [500] people in the photo on the Unit opener. Ask: Do we know the number of people in the photo? [No]. We can make a guess. A guess is an **estimate**. Let's estimate the number of [leaves on a tree/stars in the sky/people in a stadium]. Have students use this sentence frame: I estimate the number of... to be... Students can ask one another questions that lend themselves to estimates, such as, "Can you estimate the cost of...?" "I estimate the cost to be..."*

Developing/Expanding

Direct students to the Be Curious image in Lesson [x-x]. Say, *Let's analyze the different springs. What do we do when we **analyze** something?* [We look closely at it.] *What kinds of statement might we make when analyzing the springs?* [the number of values in each category] Write down students' ideas on the board or white board. Then have students work with a partner to analyze the data and then share out with the groups.

Bridging/Reaching

Display these two words: **estimate** and **predict**. Have students decide whether the statements below reflect estimating or predicting. *If I want to buy new sneakers and headphones, I'll need about \$200. I think our team will win the game tomorrow. It will probably take us 40 minutes to run 5 miles.* Have students discuss the difference between estimating and predicting.

Math Probes by Cheryl Tobey

Target Common Misconceptions

Math Probes, written by Cheryl Tobey, a leading expert in formative assessment, are designed to uncover students' mathematical misconceptions. These formative assessments, placed at the point-of-use in every unit, allow teachers to make sound instructional choices while teaching students that mistakes are an opportunity for growth.



Cheryl Tobey, M.Ed.
Contributing Author

Short, Formative Assessment

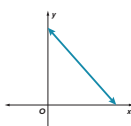
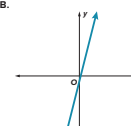

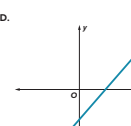
Each **Math Probe** features three to four items that assess students' conceptual understanding. Each item consists of two parts:

- **Part One** assesses students' understanding of concepts.
- **Part Two** asks students to share their thought process and ideas.

Name _____ Date _____ Period _____

Proportional Relationships

Circle the item(s) in each exercise that show a proportional relationship. You may select more than one item. Select none of the above if none of the items show a proportional relationship.

Circle your choice(s).	Explain your choice(s).
<p>1.</p> <p>A. $c = 3.75n$</p> <p>B. $p = 4m$</p> <p>C. $x = y$</p> <p>D. $y = x + 4$</p> <p>E. none of the above</p>	
<p>2.</p> <p>A. </p> <p>B. </p> <p>C. </p> <p>D. </p> <p>E. none of the above</p>	

Math Probe • Lesson Title: 151

Circle the item(s) in each exercise that show a proportional relationship. You may select more than one item. Select none of the above if none of the items show a proportional relationship.

Circle your choice(s).	Explain your choice(s).																														
<p>3.</p> <p>A. Aiden and Isabella are running at the same rate around a track. Isabella had already run 4 laps before Aiden started. How many laps had Aiden run after Isabella had run n laps?</p> <p>B. Riley can type 30 words in 14 seconds on her phone. At this rate, how many words can she type in m minutes?</p> <p>C. Javier paints 50 square feet in 1.25 hours. How long will it take Javier to paint x square feet?</p> <p>D. none of the above</p>																															
<p>4.</p> <p>A. <table border="1" data-bbox="812 1617 990 1659"> <tr><th>Time (h)</th><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><th>Number of Items</th><td>4</td><td>8</td><td>12</td><td>16</td></tr> </table></p> <p>B. <table border="1" data-bbox="812 1669 990 1711"> <tr><th>Number of Items</th><td>2</td><td>4</td><td>6</td><td>8</td></tr> <tr><th>Cost (\$)</th><td>5</td><td>10</td><td>12</td><td>15</td></tr> </table></p> <p>C. <table border="1" data-bbox="812 1722 990 1764"> <tr><th>Gallons</th><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><th>Miles</th><td>25</td><td>50</td><td>75</td><td>100</td></tr> </table></p> <p>D. none of the above</p>	Time (h)	1	2	3	4	Number of Items	4	8	12	16	Number of Items	2	4	6	8	Cost (\$)	5	10	12	15	Gallons	1	2	3	4	Miles	25	50	75	100	
Time (h)	1	2	3	4																											
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Gallons	1	2	3	4																											
Miles	25	50	75	100																											

152 Unit 3 • Proportional Relationships

Take Action

The teacher support materials that accompany the **Math Probes** are designed around a three-part ACT cycle:

- **Analyze** the probe.
- **Collect** and assess student work.
- **Take Action** to correct misconceptions quickly and efficiently.

Math Probe

A

Analyze the Probe ✓ Formative Assessment

Review the probe prior to assigning it to your students. In this probe, students will determine which item(s) in each set show a proportional relationship and explain their choices.

Targeted Concept Understand proportional relationships in equations, tables, and verbal descriptions in which there is a constant ratio between two quantities.

Targeted Misconceptions

- Students may not recognize a proportional relationship when given a form other than $y = mx + 0$.
- Students may incorrectly assume that any graph that forms a straight line is proportional.

Authentic Student Work

Below are examples of correct student work and explanations.

Correct Example A

Set 4	a)	<table border="1"> <tr><td>Time (hour)</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>Number of items</td><td>4</td><td>8</td><td>12</td><td>16</td></tr> </table>	Time (hour)	1	2	3	4	Number of items	4	8	12	16	<p>a + c are both through (0,0) and constant rate.</p> <p>b doesn't seem to have a constant rate that I can find.</p>
	Time (hour)	1	2	3	4								
	Number of items	4	8	12	16								
	b)	<table border="1"> <tr><td>Number of items</td><td>2</td><td>4</td><td>6</td><td>8</td></tr> <tr><td>Cost (\$)</td><td>2.24</td><td>4.48</td><td>6.72</td><td>8.96</td></tr> </table>	Number of items	2	4	6	8	Cost (\$)	2.24	4.48	6.72	8.96	
Number of items	2	4	6	8									
Cost (\$)	2.24	4.48	6.72	8.96									
c)	<table border="1"> <tr><td>Gallons</td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>Miles</td><td>2.5</td><td>5.0</td><td>7.5</td><td>10.0</td></tr> </table>	Gallons	1	2	3	4	Miles	2.5	5.0	7.5	10.0		
Gallons	1	2	3	4									
Miles	2.5	5.0	7.5	10.0									
d)	none of the above												

Correct Example B

Set 1	a)	$c = 3.75n$	<p>$c = 3.75n$ $0 = 3.75(0)$ $0 = 0$</p> <p>$p = 4n$ $0 = 4(0)$ $0 = 0$ did (0,0) test</p> <p>$x = y$ $0 = 0$ $y = x + 4$ $0 = 0 + 4$ $0 \neq 4$</p>
	b)	$p = 4m$	
	c)	$x = y$	
	d)	$y = x + 4$	
	e)	none of the above	

C

Collect and Assess Student Work

IF the student selects...	THEN the student likely...	Sample Misconceptions
1. d. Does not select a, b, and/or c	does not understand that for a relationship to be proportional, y must equal 0 when $x = 0$, or may not recognize a proportional relationship when given a form other than $y = mx + 0$.	
2. a and/or d	assumes that all linear relationships are proportional.	In this case, the student includes linear relationships shown in a and d.
3. a and/or c	assumes that a constant rate of change automatically implies a proportional relationship.	In this case, the student assumes a constant rate for square feet per minute.
4. b or d; Does not select a and/or c	does not understand that for a relationship to be proportional, each ratio has the same unit rate.	

*Combinations of correct and incorrect responses: Many of the above difficulties result in a combination of correct and incorrect responses. For correct responses, be sure to check for sound reasoning.

T

Take Action

- Choose from the following resources or suggestions.
- Provide opportunities for students to see the same proportional context in a variety of representations in order to make connections between the various components of each representation in Lesson 5.
 - Have students compare and contrast proportional relationships with non-proportional relationships to help them generalize the characteristics of a proportional relationship. Focus on the key features of the proportional relationships in Lesson 3.
 - To help solidify concepts, have students create, represent algebraically, solve, and share their own problems involving contexts of proportionality in Lessons 4 and 5. This gives students the opportunity to observe key features in others' representations.
- Technology can enhance learning opportunities and allow students to concentrate on the connections between the various representations. Students will begin to see that key features of a graph look different based on the form of the written equation in Lesson 3.
- Revisit the Probe** After additional instruction, have students review their initial answers to the probe. Use these questions for discussion.
- What answers, if any, would you like to change? Why might you want to change them?
 - What questions do you still have about any of the items in this probe?

Unit Review

Resources for Assessment Preparation

Teachers can select the appropriate review activities to prepare students for unit assessments.

Name _____ Date _____ Period _____

Unit Review

Revisit the **What Do I Already Know?** page and complete the **After** section of the chart.

Vocabulary Activity

Use the words from the word bank to complete each sentence. Some words may be more than once.

- constant of proportionality
- nonproportional
- proportion
- proportional
- proportional relationship
- ratio
- unit rate

- In a proportional relationship, the constant ratio is called the _____.
- Two quantities that do not relate by a constant multiple are _____.
- The relationship between two quantities is _____ if the ratios comparing the two quantities are equivalent.
- The constant of proportionality is also the _____.
- A _____ is an equation stating that two ratios or rates are equivalent.
- A _____ is a comparison between two quantities, in which for every a units of one quantity, there are b units of another quantity.
- There is no _____ in a nonproportional relationship.
- Two quantities are in a _____ if the two quantities vary and have a constant ratio between them.

Unit 3 • Proportional Relationships 15

Students can use the **Vocabulary Activity** in the Student Edition to review mathematical language and terminology.

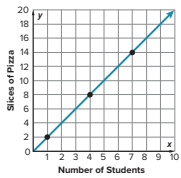
Review

For exercises 9-10, solve each problem.

- Last week, Karen sold 15 bracelets and made a profit of \$71.25. Assuming her costs and selling price stay the same, how much profit will she make from selling 25 bracelets? (Lesson X)
- Approximately how many pounds does a 2-kilogram jellyfish weigh? Round your answer to the nearest tenth. (Lesson X)

For exercise 11, select all statements that are true.

- The relationship between the number of slices of pizza purchased and the number of students served is shown in the graph. Which of the statements are true? (Lesson X)



- The relationship is proportional.
- The point (9, 18) satisfies this relationship.
- The constant of proportionality is $\frac{1}{2}$.
- The constant of proportionality is 2.
- The graph shows that every two student had one piece of pizza.

For exercises 12-13, use the scenario below.

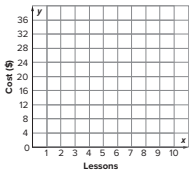
Mrs. Jameson paid \$202.50 for a group of 9 students to visit an amusement park.

- Write an equation that can be used to determine the cost for x students to visit the amusement park. (Lesson X)
- What would be the total cost if four more students wanted to join the group? (Lesson X)

For exercises 14-15, use the scenario below.

The cost of dance lessons is \$12 for 1 lesson, \$22 for 2 lessons, and \$32 for 3 lessons.

- Graph the ordered pairs on the coordinate plane. (Lesson X)



- Is the cost of dance lessons proportional to the number of lessons? Explain your reasoning. (Lesson X)

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Review exercises prepare students for assessments with practice targeted to mathematical content standards.

Item Analysis Tables in the Teacher Edition align lesson content to Depth of Knowledge (DOK) levels and the Math Content Standard for each item.

Review

Item Analysis

Item	DOK	Lesson	Standards
9	2	2-1	7.G.A.1
10	2	2-1	7.G.A.1
11	2	2-5	7.G.A.3
12	3	2-2	7.G.A.2
13	2	2-4	7.G.B.5
14	2	2-4	7.G.B.5
15	2	2-2	7.G.A.2

Fluency Progression and Practice

The Fluency Objective and Progression at the close of each unit helps teachers evaluate student progress. Fluency Practice provides students with opportunities to build procedural fluency.

Build Fluency Objective Students build fluency with division with multi-digit decimals. As students work to develop fluency with division with multi-digit decimals, have them reflect on and share with classmates the strategies they find the most useful.

Fluency Progression

Unit	Skill	Standards
1	Division with Multi-Digit Decimals	6.NS.B.2
2	Fraction Multiplication and Division (no negative rational numbers)	6.NS.A.1
3	Apply Operations with Multi-Digit Decimals	6.NS.B.3
4	Finding Unit Rates Including Terms with Fractions	7.RP.A.1
5	Percent Increase and Percent Decrease	7.RP.A.3
6	Equations in Proportional Relationships	7.RP.A.2.C
7	Adding and Subtracting Positive and Negative Rational Numbers	7.NS.A.1
8	Multiplying and Dividing Positive and Negative Rational Numbers	7.NS.A.2
9	Two-Step Equations ($px + q = r$)	7.EE.B.4.A
10	Solving $p(x + q) = r$	7.EE.B.4.A

1. Fluency Strategy

Students review the mathematical strategies.

2. Fluency Check

Students complete the practice.

3. Fluency Talk

Students explain the mathematical strategy.

Name _____ Date _____ Period _____

Fluency Practice

Fluency Strategy

Add or subtract decimals.

Align the decimal points. Annex zeros, if needed. Add or subtract as with whole numbers.

$$\begin{array}{r} 1 \\ 4.560 \\ + 13.246 \\ \hline 17.806 \end{array} \quad \begin{array}{r} 6 \text{ } 10 \\ 26.70 \\ - 3.45 \\ \hline 23.25 \end{array}$$

Multiply decimals.

Multiply. To place the decimal point, find the sum of the number of decimal places in each factor. The product has the same number of decimal places.

$$\begin{array}{r} 3.4 \\ \times 0.56 \\ \hline 204 \\ + 1700 \\ \hline 1.904 \end{array}$$

Divide decimals.

Multiply both the divisor and dividend by a power of 10 so that the divisor is a whole number. Divide. Place the decimal in the quotient directly above the decimal in the dividend.

$$\begin{array}{r} 3.4 \\ 12 \overline{) 408} \\ \underline{-36} \\ 48 \\ \underline{-48} \\ 0 \end{array}$$

1

2

3

Fluency Check

Add, subtract, multiply, or divide.

- $5.1 + 8.2 =$ _____
- $7.68 - 1.49 =$ _____
- $2.3 \times 1.4 =$ _____
- $55.9 \div 13 =$ _____
- $2.74 + 3.029 =$ _____
- $2.5 - 0.586 =$ _____
- $0.85 \times 0.09 =$ _____
- $3.6 \div 0.09 =$ _____

Fluency Talk

How would you describe the differences between operations with whole numbers and operations with decimal numbers to a classmate?

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Unit Review

Performance Task

For Part A through C, answer the question and include justifications.

Miranda and Juan want to rent bicycles for the afternoon. They will rent from either City Cycles or Biking Adventures. The rental rates are shown in the posters.

Part A

Which company should they rent from if they plan to rent bicycles for 2 hours or less?



Part B

Which company should they rent from if they plan to rent bicycles for 5 hours or less?



Part C

Miranda and Juan decide to rent for 3 hours. They find out that City Cycles charges a flat fee of \$2.50 to rent a bicycle helmet, but Biking Adventures includes helmet rental in the rental cost. From which store should they rent if they want to pay the lower price?

Unit Reflect

What helps you recognize a proportional relationship?

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Performance Task

Each unit culminates in two Performance Tasks that challenge students to apply skills from the current unit in real-world settings.

For each unit, one Performance Task is available in the Student Edition. A second, secure Performance Task is available in the Teacher Digital Center for use as an assessment. Teachers can navigate to the Assess section for the specific unit to assign the Performance Task.

Mathematical Modeling

The Mathematical Modeling tasks wrap up each unit with a real-world scenario related to the STEM unit focus and incorporating the Standards for Mathematical Practice. Students are provided with the opportunity to model with mathematics while utilizing appropriate tools to solve real-world problems and constructing viable arguments to present to their peers.

Students can choose between two different projects, increasing engagement and developing student agency. Teacher support is provided, including a guide for project development and facilitation.

Name _____ Date _____ Period _____

Mathematical Modeling

Measuring and Comparing Air Quality

The Air Quality Index (AQI) is a scale that informs the public on the quality of the air for the day. The AQI looks at five different pollutants in the air to determine air quality.

Air Quality Index (AQI) Values	Levels of Health Concern
0 to 50	Good
51 to 100	Moderate
101 to 150	Unhealthy to Sensitive Groups
151 to 200	Unhealthy
201 to 300	Very Unhealthy
301 to 500	Hazardous

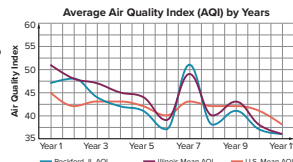
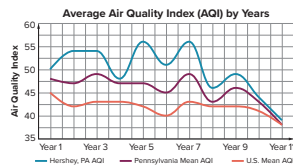
Choose one of the projects to complete.

Project One

The U.S. Olympic Committee is planning to build a new training facility for the track and field team. The committee is considering two possible locations: Hershey, Pennsylvania or Rockford, Illinois. Among the considerations for the new site is the average air quality. Good air quality is important for athletes, especially runners.

The graphs show the average air quality index for each site over a period of 11 years.

You are part of the site selection team and your team will make a recommendation to the U.S. Olympic Committee. Your task is to analyze the data in the graphs for the team. Consider the change in air quality over the ten-year period, noting any trends that you think might continue. Also consider the differences in the air quality for each specific site, for the state in which the site is located, and the country.



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Project Two

The superintendent in your school district has received a complaint of poor indoor air quality in one of the district schools. Four key indicators of indoor air quality with maximum acceptable levels are described in the table.

Pollutant	Source	Level
Carbon Dioxide (CO ₂)	Exhaled air from human breathing; combustion processes of carbon fuels	< 1,000 parts per million (ppm)
Carbon Monoxide (CO)	Improperly vented furnaces, malfunctioning gas ranges,	< 9 ppm
Particle Pollution PM ₁₀	Particulates with a diameter of 10 micrometers or less. Includes dust from soil, pollen, mold, burning of wood, oil, or coal	< 50 mg/m ³ for one hour
Particle Pollution PM _{2.5}	Particulates with a diameter of 2.5 micrometers or less. Includes soot from diesel engines in trucks and buses	< 12.0 mg/m ³

The superintendent has asked you and your classmates to analyze the results of the air quality tests. What do the results shown in the table suggest about the indoor air quality at the school in question?

Pollutant	Reading 1	Reading 2	Reading 3	Reading 4
Carbon Dioxide (CO ₂)	729 ppm	1030 ppm	956 ppm	1106 ppm
Carbon Monoxide (CO)	4 ppm	5 ppm	3 ppm	5 ppm
Particle Pollution PM ₁₀	52 mg/m ³	56 mg/m ³	62 mg/m ³	58 mg/m ³
Particle Pollution PM _{2.5}	10 mg/m ³	10 mg/m ³	9 mg/m ³	10 mg/m ³

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Flexible Assessments for Growth

Arizona Reveal Math® assessments allow students to demonstrate understanding through multimodal responses—verbal, written, visual, and hands-on. Assessments were designed with **Universal Design for Learning (UDL)** principles to support flexible, inclusive, and meaningful assessment.

Diagnostic Insights

Measure student understanding of prerequisite skills at the start of the year and before each unit with:

- The *ALEKS* Initial Knowledge Check—administered digitally at the beginning of the year.
- Unit Readiness Diagnostics—available in print or digital formats.

Summative Assessments and Performance Tasks


Evaluate student understanding, application, and progress aligned with the Arizona Mathematics Standards through:

- Unit Assessments
- Unit Secure Performance Tasks
- Benchmark Assessments
- Summative Assessments

Targeted Intervention

Access ready-made intervention resources aligned to diagnostic and summative assessment results:

- **Guided Support:** Teacher-facilitated, small-group mini-lessons
- **Skills Support:** Skill-based practice for targeted review
- **Take Another Look:** Digital mini lessons with gradual release activities

 Spark student engagement with ready-to-play Kahoot! quizzes for each unit. You can easily launch each kahoot from the McGraw Hill platform—no extra accounts needed!

Name _____

Unit 3
How Ready Am I?

1. Which number makes the equation true?
 $5 + 4 = 4 + ?$

A. 3 B. 4 C. 5 D. 6

2. Cara bought a package of toy cars for each of her 5 friends. Each package has 4 cars. Which equation can be used to find the total number of cars Cara bought?

A. $5 + 4 = ?$ C. $4 + 4 + 4 + 4 = ?$
B. $5 + 5 + 5 + 5 + 5 = ?$ D. $4 + 4 + 4 + 4 + 4 = ?$


3. Marco has 3 shelves in his room. There are 3 trophies on each shelf. How many trophies does Marco have?

A. 3 B. 6 C. 9 D. 12

4. Maria's dog buried 15 bones. Maria found 6 bones. Maria wrote the subtraction equation $15 - 6 = ?$ to find out how many bones are still buried. Which equation could Maria use to help solve her equation?

A. $15 + 6 = 9$ C. $6 - 15 = 9$
B. $6 + 9 = 15$



 **Administrator Reports:**
Access key insights and assessment results with ease.

Actionable Insights to Track Progress

The *Arizona Reveal Math*® data system, powered by the **Standards and Skills Graph**, serves as the hub for real-time class and student performance insights, tracking progress over time and highlighting past achievements across grade levels.

Two Inner Rings represent data from interim assessments. This nationally normed data is updated throughout the year and tracks overall student progress by grade and domain.



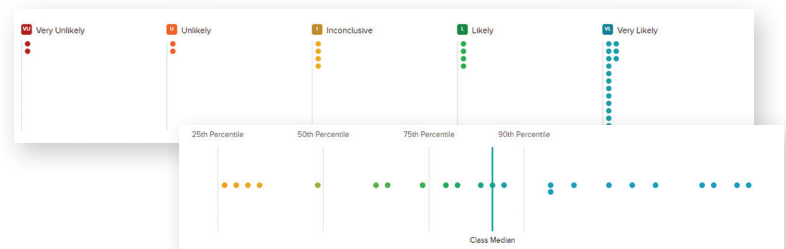
Two Outer Rings display students' proficiency in standards and skills within a specific domain or strand, updated daily with data from *Arizona Reveal Math* assessments, practice, *ALEKS*®, and intervention data from McGraw Hill's *Number Worlds*®.

Data Sources:

The **Standards and Skills Graph** is populated with student interim assessment data as well as data from McGraw Hill's core, supplemental, and intervention programs.



Distribution Charts provide real-time insights into student readiness, displaying performance data by grade, domain, and standards at a glance. Each dot represents a student's progress, simplifying lesson planning and guiding instruction.



Five Levels of Tailored Recommendations at Your Fingertips

All data captured in the Standards and Skills Graph automatically groups students and generates up to five levels of tailored recommendations, including personalized practice and small-group mini-lessons. Built-in support for intervention and acceleration is all assignable with a single click.

➔ Pre-Requisite
🔄 Reinforce
🎯 On-Lesson
⊕ Extension
➔ Post-Requisite

Explore More Reports Unlock additional insights to support student growth and instruction with:

✓ **Actionable Insights to Track Progress:**
Review real-time data for *Arizona Reveal Math* practice and assessments, including item analysis by student and class.

✓ **Standards Performance Report:**
See class and student performance by standard, with cumulative scores to support instructional decisions.

Personalize Student Learning Like Never Before

Arizona Reveal Math[®] is more than just a curriculum—it's a complete learning ecosystem personalized to meet each student's needs.



Arizona Reveal Math Generates Rich Data in Real Time

As students answer questions, data from **Interim Assessments, Daily Core Practice & Assessment, Advanced Adaptive Learning, and Dynamic Personalized Practice** flows into the program's data system, generating powerful insights.



Real-Time Data Powers Personalized Learning Paths

Rich data automatically generates individualized learning paths for each student, grouping them by what they need and saving teachers valuable planning time.

As students' needs evolve, the program updates its personalized recommendations accordingly—new data generates new insights each and every day.

Dynamic Personalized Practice A B

Using real-time insights, *Arizona Reveal Math* automatically groups students and generates up to five levels of personalized recommendations to meet each student's needs. Teachers can easily assign mini-lesson, tools, and scaffolds with one click, ensuring all learners access grade-level content.

Advanced Adaptive Learning with ALEKS[®] C

ALEKS uses AI to assess and adjust in real time, providing practice tailored to each student's skill level. This targeted intervention or acceleration helps students focus on what they're ready to learn instead of standard grade-level content.

Collaborative and Independent Learning D

Students apply their understanding through both digital and hands-on small-group activities and independent practice, including ready-made games and tasks with STEM connections.



Arizona Reveal **MATH**[®]



Learn more at
mheducation.com/arizona