Built on the CCSS COMMON CORE

NUMBER WORLDS™

Accelerate Math Success

Teacher Edition

Mc Graw Hill Education

WEEK 6

Nonlinear Functions and Graphs

Week at a Glance

This week, students continue ***Number Worlds,*** Level J, Algebra. Students will explore nonlinear functions. Students will interpret functions of the form $y = x^2$ and $y = x^3$. They will graph these both of these forms in the first quadrant and in all four quadrants of the coordinate plane.

Skills Focus

- Interpret functions with x having an exponent.
- Graph functions of the form $y = x^2$ in the first quadrant of the coordinate grid.
- Graph functions of the form $y = x^3$ in all four quadrants of the coordinate grid.

How Students Learn

As students explore nonlinear functions and graphs, encourage them to make observations and generalizations. The difference between the graphs of nonlinear and linear functions has to do with the rate at which one quantity changes in relationship to the other. So far students have only graphed linear functions that result in a straight line. Since nonlinear functions produce graphs that form a curve, make sure students realize the points they graph aren't incorrect even though they aren't in a straight line.

English Learners ELL

For language support, use the ***English Learner Support Guide,*** pages 68–69, to preview lesson concepts and teach academic vocabulary. ***Number Worlds*** Vocabulary Cards are listed as additional materials in many lessons and can be used to preteach and reinforce academic vocabulary.

Math at Home

Give one copy of the Letter to Home, page 18, to each student. Encourage students to share and complete the activity with their caregivers.


Letter to Home

Weekly Planner

Lesson	Learning Objectives	
1 pages 262–263	Students can graph and interpret functions of the form $y = x^2$.	
2 pages 264–265	Students can graph and interpret functions of the form $y = x^2$ in the four quadrants of the coordinate plane.	
3 pages 266–267	Students can graph and interpret functions of the form $y = x^3$.	
4 pages 268–269	Students can graph and interpret functions of the form $y = x^3$ in the four quadrants of the coordinate plane.	
5 pages 270–271	**Review and Assess** Students review skills learned this week and complete the weekly assessment and project.	
Project pages 272–273	Students can graph nonlinear functions to understand the measurements of a pool.	

Key Standard for the Week

Domain: Functions

Cluster: Define, evaluate, and compare functions.

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Materials		Technology
Program Materials • ***Student Workbook,*** pp. 65–67 • ***Practice,*** p. 72 • Activity Card 3U, **Maura's Murals** • Graph Activity Sheet 1	**Additional Materials** Vocabulary Card 21, *exponent*	***Teacher Dashboard*** Coordinate Grid Tool
Program Materials • ***Student Workbook,*** pp. 68–69 • ***Practice,*** p. 73 • Activity Card 3V, **The Other Half** • Coordinate Grid (I–IV)	**Additional Materials** Vocabulary Card 21, *exponent*	***Teacher Dashboard*** Coordinate Grid Tool
Program Materials • ***Student Workbook,*** pp. 70–71 • ***Practice,*** p. 74 • Activity Card 3W, **Cube Factory** • Graph Activity Sheet 2	**Additional Materials** math-link cubes*	***Teacher Dashboard*** Coordinate Grid Tool
Program Materials • ***Student Workbook,*** pp. 72–73 • ***Practice,*** p. 75 • Activity Card 3X, **Curve Ball** • Coordinate Grid (I–IV) • GalaX,Y Game Board	**Additional Materials** erasable marker	***Teacher Dashboard*** Coordinate Grid Tool
Program Materials • ***Student Workbook,*** pp. 74–75 • Weekly Test, ***Assessment,*** pp. 47–48		Review previous activities.
Program Materials ***Student Workbook,*** p. 76	**Additional Materials** grid paper	

*Available from McGraw-Hill Education

Nonlinear Functions and Graphs

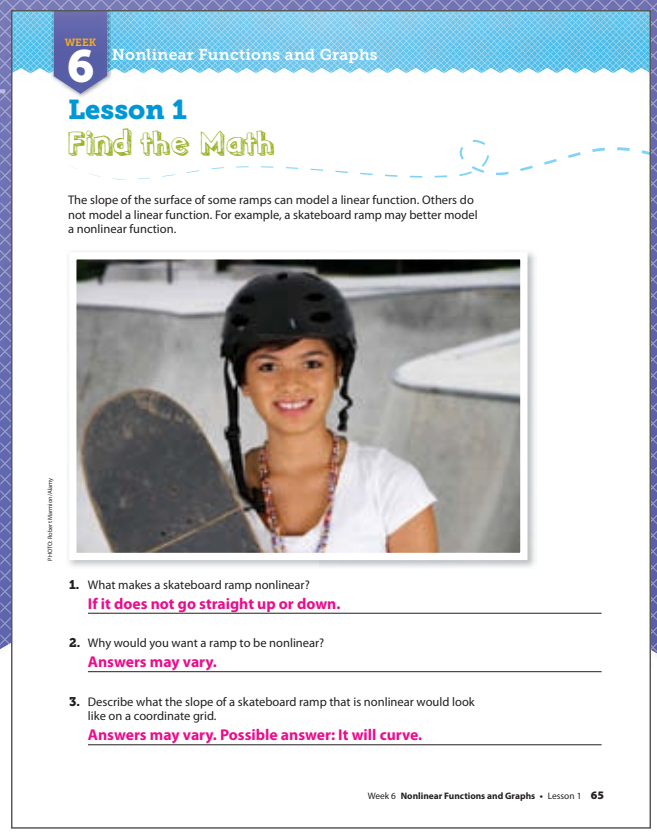
Find the Math

In this week, introduce students to writing, solving, and graphing nonlinear equations.

Use the following to begin a guided discussion:

▶ **What items do you think have a slope that is not a straight line?**
Answers may vary. Possible answers: skateboard ramp, water slide, rollercoaster hill.

Have students complete ***Student Workbook,*** page 65.



WEEK 6 Nonlinear Functions and Graphs

Lesson 1
Find the Math

The slope of the surface of some ramps can model a linear function. Others do not model a linear function. For example, a skateboard ramp may better model a nonlinear function.

PHOTO: Robert Marmon/Alamy

1. What makes a skateboard ramp nonlinear?
If it does not go straight up or down.

2. Why would you want a ramp to be nonlinear?
Answers may vary.

3. Describe what the slope of a skateboard ramp that is nonlinear would look like on a coordinate grid.
Answers may vary. Possible answer: It will curve.

Week 6 **Nonlinear Functions and Graphs** • Lesson 1 **65**

Student Workbook, p. 65

Lesson 1

Objective

Students can graph and interpret functions of the form $y = x^2$.

Standard

8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Vocabulary

- **coordinate grid**
- **exponent**
- **square of a number**

Creating Context

English Learners are sometimes reluctant to ask for explanations when it seems that everyone else understands. Discuss why it is important to ask for explanations when they do not understand, and establish a signal they can use to ask for help or information when they need it.

Materials

Program Materials	Additional Materials
Graph Activity Sheet 1, 1 per student	Vocabulary Card 21, *exponent*

1 WARM UP

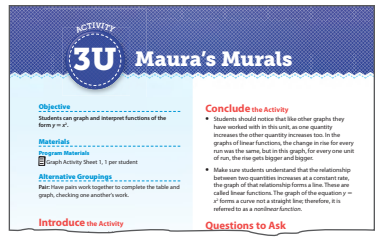
Prepare

- Remind student that a square of a number is the product of a number multiplied by itself.
- ▶ **What's 4 squared?** 16
- ▶ **What's 10 squared?** 100
- Review that an exponent is a numeral or symbol placed at the upper right side of another numeral or symbol to indicate the number of times it is to be used as a factor.
- Remind students what an exponent looks like by writing 4^2 and pointing out the exponent of 2 and the base of 4.

2 ENGAGE

Develop: Maura's Murals

"Today we are going to draw a nonlinear function." Follow the instructions on the Activity Card **Maura's Murals.** As students complete the activity, be sure to use the Questions to Ask.



ACTIVITY 3U Maura's Murals

Objective
Students can graph and interpret functions of the form $y = x^2$.

Materials
Program Materials
Graph Activity Sheet 1, 1 per student

Alternative Groupings
Pair: Have pairs work together to complete the table and graph, checking one another's work.

Introduce the Activity

Conclude the Activity
- Students should notice that like other graphs they have worked with in this unit, as one quantity increases the other quantity increases too. In the graphs of linear functions, the change in rise for every run was the same, but in this graph, for every one unit of run, the rise gets bigger and bigger.
- Make sure students understand that the relationship between two quantities increases at a constant rate, the graph of that relationship forms a line. These are called linear functions. The graph of the equation $y = x^2$ forms a curve not a straight line; therefore, it is referred to as a nonlinear function.

Questions to Ask

Activity Card 3U

Alternative Groupings

Pair: Have pairs work together to complete the table and graph, checking one another's work.

Progress Monitoring

If... students are having trouble computing x^2,	**Then...** write $x \times x$ on the board and review how this can be rewritten using exponents.

Practice

Have students complete ***Student Workbook,*** pages 66–67. Guide students through the Key Idea example and Try This exercises.

Interactive Differentiation

Consult the ***Teacher Dashboard*** for grouping suggestions. You can also use performance on the Engage activity to guide students.

Independent Practice

For additional practice with graphing nonlinear functions, have students graph the following functions and then compare and contrast the graphs: $y = 3x$ and $y = 3x^2$.

Supported Practice

For additional support with graphing nonlinear functions, have students use the Coordinate Grid Tool. Have them choose the 20-by-20 first quadrant grid and graph $y = 2x$ and then graph $y = x^2$. After the students graph the points, have them connect the points. Make sure students notice the curve $y = x^2$ makes.

3 REFLECT

Think Critically

Review students' answers to the Reflect prompt at the bottom of ***Student Workbook,*** page 67, and then review the Engage activity.

Discuss the Reflect activity to reinforce the concept of $y = x^2$ functions.

- **Why is $y = x^2$ a nonlinear equation?** Answers may vary. Possible answer: because the graph is not a line
- **As *x* increases, what happens to *y*?** it increases
- **Do both *x* and *y* increase at a constant rate?** no
- **How would you describe the graph for $y = x^2$?** Possible answer: it curves upwards

4 ASSESS

Informal Assessment

Use the online or print Student Record, ***Assessment,*** page 128, to record informal observations.

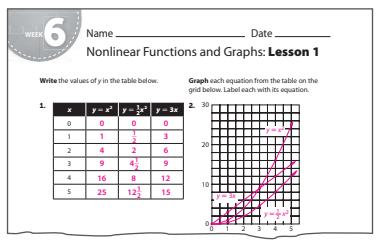
Maura's Murals

Did the student

- ☐ make important observations?
- ☐ provide insightful answers?
- ☐ extend or generalize learning?
- ☐ pose insightful questions?

Additional Practice

For additional practice, have students complete ***Practice,*** page 72.


WEEK 6

Name _____ Date _____

Nonlinear Functions and Graphs: **Lesson 1**

Write the values of *y* in the table below.

1.

x	$y = x^2$	$y = \frac{1}{2}x^2$	$y = 3x$
0	0	0	0
1	1	$\frac{1}{2}$	3
2	4	2	6
3	9	$4\frac{1}{2}$	9
4	16	8	12
5	25	$12\frac{1}{2}$	15

Graph each equation from the table on the grid below. Label each with its equation.

2.

Practice, p. 72

Lesson 1

Key Idea

When the relationship between two quantities increases at a constant rate, the graph of that relationship forms a line. These equations are called **linear functions.** In the equation $y = x^2$, the *x* is not increasing at a constant rate, therefore the graph of it is a curve instead of a line. This is referred to as a **nonlinear function.**

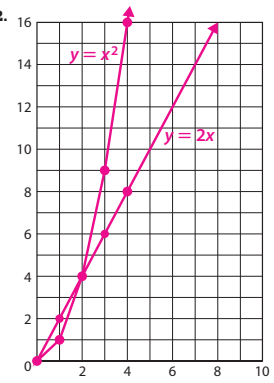
Try This

Fill in the values of *y* in the table below. Then, plot the points for each equation, connect each set of points, and label each with its equation.

1.

x	$y = 2x$	$y = x^2$
0	0	0
1	2	1
2	4	4
3	6	9
4	8	16

2.



Use the graph to the right to answer the following questions.

3. Does the line $y = 2x$ have a constant slope? If so, what is it? If not, what happens to the slope?
yes; The slope is constantly 2.

4. Why is $y = x^2$ a nonlinear function?
Answers may vary. Possible answer: It creates a curve when graphed.

Practice

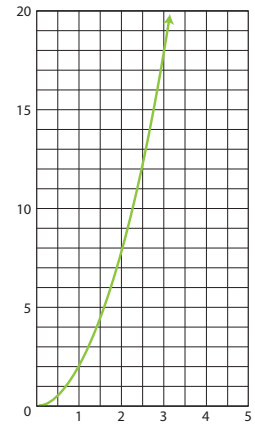
Use the graph on the opposite page to make predictions.

5. If you graphed $y = 3x$, which graph would it more closely resemble, $y = 2x$ or $y = x^2$?
$y = 2x$

6. If you graphed $y = 2x^2$, which graph would it more closely resemble, $y = 2x$ or $y = x^2$?
$y = x^2$

7. Which function is shown on the graph to the right? Circle the appropriate answer.

$y = 3x$ or $y = 2x^2$



Reflect

What are some similarities and differences between the graphs of $y = 2x$ and $y = x^2$.

Answers may vary. Possible similarities: two points are the same for both graphs; they both represent an equation. Possible differences: one is a curve while one is a line; one represents *x* with an exponent and one represents *x* with a coefficient.

Student Workbook, pp. 66–67

Lesson 2

Objective
Students can graph and interpret functions of the form $y = x^2$ in the four quadrants of the coordinate plane.

Standard
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Vocabulary
- coordinate grid
- exponent
- square of a number

Creating Context
Remind English Learners that they previously learned about and worked with linear functions and that this week's lesson is on nonlinear functions. Point out that the addition of the prefix *non-* (which means "not") to the word *linear* changes the meaning of the word to "not linear." Explain that this prefix reflects the difference between the two types of functions. When linear functions are graphed, they form a line. When nonlinear functions are graphed, they do not form a line.

Materials
Program Materials
Coordinate Grid (I–IV), 1 per student

Additional Materials
Vocabulary Card 21, *exponent*

Prepare Ahead
Prepare Coordinate Grid (I–IV) for display to the class.

1 WARM UP

Prepare
- ▸ **What does the graph of $y = x^2$ look like?** it is a curve
- Tell students that in this lesson they will continue to work with the graph of $y = x^2$, but they will expand their exploration to the other quadrants of the coordinate plane.
- Review the Key Idea section on student page 68.

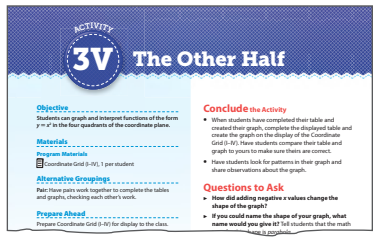
Just the Facts
Have students *raise their hands* for a true statement and *keep their hands down* for a false statement.

Use questions such as the following:

- ▸ **The point (−2,3) is in the second quadrant.** *raise hands*
- ▸ **The point (−2,−3) is in the third quadrant.** *raise hands*
- ▸ **The point (2,−3) is in the first quadrant.** *hands down*

2 ENGAGE

Develop: The Other Half
"Today we are going to graph a function that has x^2 using positive and negative values of x." Follow the instructions on the Activity Card, **The Other Half.** As students complete the activity, be sure to use the Questions to Ask.


ACTIVITY 3V The Other Half

Objective
Students can graph and interpret functions of the form $y = x^2$ in the four quadrants of the coordinate plane.

Materials
Program Materials
Coordinate Grid (I–IV), 1 per student

Alternative Groupings
Pair: Have pairs work together to complete the tables and graphs, checking each other's work.

Prepare Ahead
Prepare Coordinate Grid (I–IV) for display to the class.

Conclude the Activity
- When students have completed their table and created their graph, complete the displayed table and create the graph on the display of the Coordinate Grid (I–IV). Have students compare their table and graph to yours to make sure theirs are correct.
- Have students look for patterns in their graph and share observations about the graph.

Questions to Ask
- How did adding negative *x* values change the shape of the graph?
- If you could name the shape of your graph, what name would you give it? Tell students that the math name is parabola.

Activity Card 3V

Alternative Groupings
Pair: Have pairs work together to complete the tables and graphs, checking each other's work.

Progress Monitoring

If… students have negative numbers for *y*,
Then… review how multiplying two negative values yields a positive number.

Practice
Have students complete ***Student Workbook,*** pages 68–69. Guide students through the Key Idea example and Try This exercises.

Interactive Differentiation
Consult the ***Teacher Dashboard*** for grouping suggestions. You can also use performance on the Engage activity to guide students.

Independent Practice
For additional practice with graphing nonlinear functions, students should use the Coordinate Grid Tool. Students should pick the four quadrant grid from −20 to 20 and then graph $y = x^2$ from $x = -4$ to $x = 4$.

Supported Practice
For additional support with graphing nonlinear functions, students should practice filling in tables for equations such as $y = x^2 + 2$ and $y = x^2 - 2$. Once students master filling in the tables correctly, have them use copies of Coordinate Grid (I–IV) to graphs these equations.

3 REFLECT

Think Critically

Review students' answers to the Reflect prompt at the bottom of ***Student Workbook,*** page 69, and then review the Engage activity.

Discuss the Reflect activity to reinforce the concept of constants in nonlinear functions.

- **Were you surprised by the graph of $y = x^2$ when x is less than 0? What did you expect?** Answers may vary.
- **What is the relationship between the y values when $x = 3$ and $x = -3$.** they are the same

Real-World Application

Cell division, the change in human population over time, and the production of oil have all followed exponential growth patterns at one time.

- **Would the portion of the graph of $y = x^2$ where x is less than 0 make sense for human population? Explain.** No; Possible answer: Human population is measured over time, and you can't have negative time.

4 ASSESS

Informal Assessment

Use the online or print Student Record, ***Assessment,*** page 128, to record informal observations.

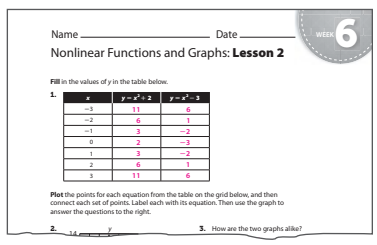
The Other Half

Did the student

- ☐ make important observations?
- ☐ provide insightful answers?
- ☐ extend or generalize learning?
- ☐ pose insightful questions?

Additional Practice

For additional practice, have students complete ***Practice,*** page 73.


Name _____ Date _____

WEEK 6

Nonlinear Functions and Graphs: **Lesson 2**

Fill in the values of y in the table below.

1.

x	$y = x^2 + 2$	$y = x^2 - 3$
−3	11	6
−2	6	1
−1	3	−2
0	2	−3
1	3	−2
2	6	1
3	11	6

Plot the points for each equation from the table on the grid below, and then connect each set of points. Label each with its equation. Then use the graph to answer the questions to the right.

2. y

3. How are the two graphs alike?

Practice, p. 73

Lesson 2

Key Idea

The nonlinear function $y = x^2$ can also have a constant. That function is written as $y = x^2 + b$.

A constant increases the value of y by a set value, unrelated to x^2.

Nonlinear functions can be graphed in all four quadrants. A graph of an equation that has x^2 is called a *parabola.*

Try This

Answer the following questions to make predictions about nonlinear functions.

1. What do you predict would happen to the graph of $y = x^2$ if a constant of 4 was added to x^2?
   Each point on the graph would be 4 units higher.

2. What do you predict would happen to the graph of $y = x^2$ if a constant of 5 was subtracted from x^2?
   Each point on the graph would be 5 units lower.

Fill in the values of y in the table below.

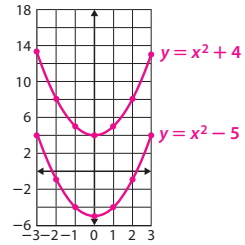
3.

x	$y = x^2 + 4$	$y = x^2 - 5$
−3	13	4
−2	8	−1
−1	5	−4
0	4	−5
1	5	−4
2	8	−1
3	13	4

Practice

Plot the points for each equation from the table on the opposite page on the grid below, and then connect each set of points. Label each with its equation.

4.



Use the graph above to answer the following questions.

5. How are the two graphs alike?
   They have the same shape.

6. How are the two graphs different?
   One graph is lower in position than the other.

7. Will $y = x^2 - 5$ ever cross over $y = x^2 + 3$? Why, or why not?
   No; they will always be separated by a distance of 9 because that is the difference between their constants.

8. What do you notice about the shape of the graphs on each side of the y-axis?
   They mirror one another.

Reflect

Describe how adding a constant to the equation affects the graph.

It moves the graph up or down the y-axis.

Student Workbook, pp. 68–69

Lesson 3

Objective
Students can graph and interpret functions of the form $y = x^3$.

Standard
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Vocabulary
- coordinate grid
- exponent

Creating Context
Review with English Learners the connection between the words *long* and *length*. Make a chart of the related adjectives and nouns *wide/width, long/length, tall/height*, and *deep/depth*.

Materials
Program Materials
Graph Activity Sheet 2, 1 per student

Additional Materials
math-link cubes

Prepare Ahead
Prepare two Graph Activity Sheet 2 for display to the class.

1 WARM UP

Prepare
- Review with students how to fill in a table and graph $y = x^2$.
- Show students a table with 0 through 3 for x. Fill in the table for y as a class.
- Display a Graph Activity Sheet 2.
- Have volunteers plot the points from the table onto the graph.
- Discuss with students what they have learned about these types of graphs.
- The completed graph will be used in the Activity.

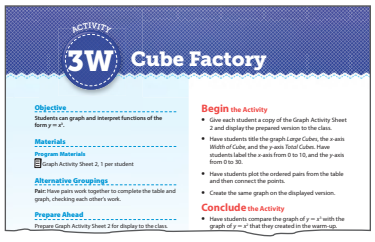
Just the Facts
Have students *stand up* for correct answers and *stay seated* for incorrect answers.

Use questions such as the following:

- **Two to the third power is 6.** *stay seated*
- **Three to the third power is 27.** *stand up*
- **The exponent tells how many times the base is used as a factor.** *stand up*

2 ENGAGE

Develop: Cube Factory
"Today we are going to graph a different type of nonlinear function." Follow the instructions on the Activity Card **Cube Factory.** As students complete the activity, be sure to use the Questions to Ask.



Activity Card 3W

Alternative Groupings
Pair: Have pairs work together to complete the table and graph, checking each other's work.

Progress Monitoring

If… students are drawing a series of straight lines,

Then… model for students how they can turn their lines into a curve.

Practice
Have students complete ***Student Workbook,*** pages 70–71. Guide students through the Key Idea and Try This exercises.

Interactive Differentiation
Consult the ***Teacher Dashboard*** for grouping suggestions. You can also use performance on the Engage activity to guide students.

Independent Practice
For additional practice with graphing nonlinear functions, have students compare and contrast the functions $y = 3x$ and $y = \frac{1}{3}x^3$ by graphing them using the Coordinate Grid Tool.

Supported Practice
For additional support with graphing nonlinear functions, have students explore finding the y values by using math-link cubes. Have them build a cube with 1, 2, and 3 as the width. Once students understand how to find the y values by cubing the width, have them make a table with 0 through 3 for x. If students are having difficulty graphing $y = x^3$ as well, have them practice graphing it from the table they just made.

3 REFLECT

Think Critically

Review students' answers to the Reflect prompt at the bottom of ***Student Workbook,*** page 71, and then review the Engage activity.

- **How is $y = x^2$ similar to $y = x^3$?** Answers may vary.
- **Does $y = x^3$ change at the same rate as $y = x^2$? Explain.** No; it changes at a greater rate because the exponent is greater.

Real-World Application

As you learned today, the volume of a cube is determined by $y = x^3$.

- **What is the volume of a box with a width of 2 inches?** 8 cubic inches
- **What is the volume of a box with a width of 4 inches?** 64 cubic inches
- **If you graphed the volume of these boxes, could you determine what the volume of a box with a width of 5 inches would be?** Answers may vary.

4 ASSESS

Informal Assessment

Use the online or print Student Record, ***Assessment,*** page 128, to record informal observations.

Cube Factory

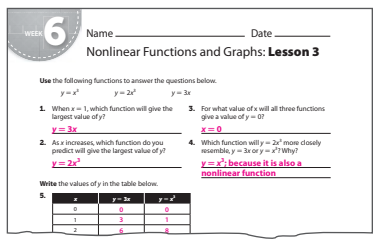
Did the student

☐ make important observations? ☐ provide insightful answers?

☐ extend or generalize learning? ☐ pose insightful questions?

Additional Practice

For additional practice, have students complete ***Practice,*** page 74.



Week 6 Name _____ Date _____

Nonlinear Functions and Graphs: **Lesson 3**

Use the following functions to answer the questions below.

$y = x^2$ $y = 2x^2$ $y = 3x$

1. When $x = 1$, which function will give the largest value of y? $y = 3x$
2. As x increases, which function do you predict will give the largest value of y? $y = 2x^2$
3. For what value of x will all three functions give a value of $y = 0$? $x = 0$
4. Which function will $y = 2x^2$ more closely resemble, $y = 3x$ or $y = x^2$? Why? $y = x^2$; because it is also a nonlinear function

Write the values of y in the table below.

5.

x	$y = 3x$	$y = x^2$
0	0	0
1	3	1
2	6	4

Practice, p. 74

Lesson 3

Key Idea

When the relationship between two quantities increases at a constant rate, the graph of that relationship forms a line. These are called **linear functions.**

The graph of the equation $y = x^3$ forms a curve, not a straight line; therefore, it is referred to as a **nonlinear function.**

Try This

Answer the following questions to make predictions about the following functions.

$y = 4x$ $y = x^3$ $y = \frac{1}{2}x^3$

1. As x increases, which function do you predict will give the largest values of y? $y = x^3$

2. When $x = 1$, which function will give the largest value of y? $y = 4x$

3. Will the function $y = \frac{1}{2}x^3$ more closely resemble $y = 4x$ or $y = x^3$? Why? $y = x^3$; Possible answer: because it is also a nonlinear function

Fill in the values of y in the table below.

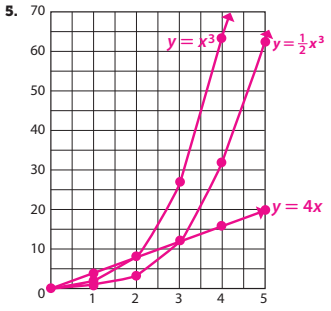
4.

x	$y = 4x$	$y = x^3$	$y = \frac{1}{2}x^3$
0	0	0	0
1	4	1	$\frac{1}{2}$
2	8	8	4
3	12	27	$13\frac{1}{2}$
4	16	64	32
5	20	125	$62\frac{1}{2}$

Practice

Plot the points from the table on the opposite page for each equation on the grid below, and then connect each set of points. Label each with its equation. Note: Some points might fall outside the range of the graph. You will not be able to plot any such points.

5.



Use the graph of $y = x^3$ to solve the following problem.

6. The volume of a cube (y) is equal to the length of each side cubed (x^3). If April is building a cube-shaped storage bin, and she wants the bin to have a volume of 27 cubic feet, how long will each side of the bin need to be? 3 feet long

Reflect

Which nonlinear function will increase the fastest as x increases: x^2 or x^3? Why?

x^3; because x^3 is multiplied one more time by x then x^2 is.

Student Workbook, pp. 70–71

Lesson 4

Objective
Students can graph and interpret functions of the form $y = x^3$ in the four quadrants of the coordinate plane.

Standard
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Vocabulary
- coordinate grid
- exponent

Creating Context
Show English Learners the coordinate grid and label each section as quadrant 1, quadrant 2, etc. Then write down and show pictures of the following words: *quarter, quadruplets,* and *quadriceps*. Challenge students to explain how all these different words are similar. Students should recognize that they all start with *quad-* and that the meaning of each word is related to the number four.

Materials
Program Materials
- Coordinate Grid (I–IV), 1 per student
- GalaX,Y Game Board

Additional Materials
erasable markers

Prepare Ahead
Prepare Coordinate Grid (I–IV) for display to the class.

1 WARM UP

Prepare
- Review with students what $y = x^2$ looks like when it is graphed in all four quadrants by showing a visual of the graph with 0 through 4 for the x values.
- Next display a graph of $y = x^3$ in the first quadrant only.
- ▸ **Can you predict what the shape of the graph of $y = x^3$ will look like if we include negative values for x, and plot the points on a four quadrant grid as we did for the equation $y = x^2$?**
  Answers may vary.
- Let students think about the question and then sketch and share their predictions.

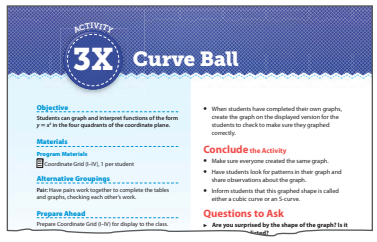
Just the Facts
Have students *clap* for correct statements and keep their *hands folded* for incorrect statements.

Use statements such as the following:

- ▸ **6 to the third power is 18** *hands folded*
- ▸ **2 to the third power is 8.** *clap*
- ▸ **1 to the third power is 1.** *clap*

2 ENGAGE

Develop: Curve Ball
"Today we are going to graph $y = x^3$ for negative values of x." Follow the instructions on the Activity Card **Curve Ball.** As students complete the activity, be sure to use the Questions to Ask.



Activity Card 3X

Alternative Groupings
Pair: Have pairs work together to complete the tables and graphs, checking each other's work.

Progress Monitoring
If… students are computing values for y that are positive when x is less than 0,

Then… remind students that the product of an even amount of negative numbers is positive, but an odd amount of negative numbers results in a negative answer.

Practice
Have students complete ***Student Workbook,*** pages 72–73. Guide students through the Key Idea example and Try This exercises.

Interactive Differentiation
Consult the ***Teacher Dashboard*** for grouping suggestions. You can also use performance on the Engage activity to guide students.

Independent Practice
For additional practice with graphing nonlinear functions, students should use the Coordinate Grid Tool. Students should pick the four quadrant grid from −30 to 30 and then graph $y = x^3$ from $x = -3$ to $x = 3$.

Supported Practice
For additional support with graphing nonlinear functions, students should use the GalaX,Y Game Board to graph $y = x^3$ from $x = -1$ to $x = 1$. First, have them fill out the table, and then have them draw the x-axis and the y-axis on the game board, using the pink star as the origin.

- ▸ **What type of shape is formed when the ordered pairs are graphed?** curve

3 REFLECT

Think Critically

Review students' answers to the Reflect prompt at the bottom of ***Student Workbook,*** page 73, and then review the Engage activity.

- **How is the graph of x^3 different from the graph of x^2 for values of x less than 0?** y is positive for x^2 but negative for x^3.
- **If you had a value of −9 for y, how could you use the graph to determine the value of x?** On the graph, find where the plot-line hits −9 for y, and then find what the value of x is at that position.

Real-World Application

Ronnie's job is to measure the amount of volume from the huge cubic aquarium. Ronnie considers the tank to have a water level of zero when it is filled correctly. She measures how many inches the water has moved up or down and finds the volume for the amount of water that has been added or removed.

- **If the water moves up 2 inches, what would the volume of the extra water be compared to the original volume?** 8 cubic inches
- **If the water moves down 2 inches, what would the volume of the missing water be compared to the original volume?** −8 cubic inches

4 ASSESS

Informal Assessment

Use the online or print Student Record, ***Assessment,*** page 128, to record informal observations.

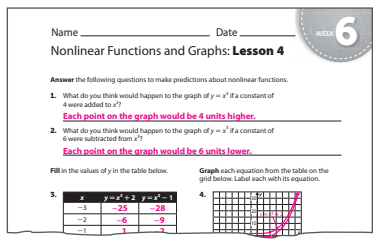
Curve Ball

Did the student

- ☐ make important observations?
- ☐ provide insightful answers?
- ☐ extend or generalize learning?
- ☐ pose insightful questions?

Additional Practice

For additional practice, have students complete ***Practice,*** page 75.



Name _____ Date _____

WEEK 6

Nonlinear Functions and Graphs: **Lesson 4**

Answer the following questions to make predictions about nonlinear functions.

1. What do you think would happen to the graph of $y = x^3$ if a constant of 4 were added to x^3?
   Each point on the graph would be 4 units higher.
2. What do you think would happen to the graph of $y = x^3$ if a constant of 6 were subtracted from x^3?
   Each point on the graph would be 6 units lower.

Fill in the values of y in the table below.

3.

x	$y = x^3 + 2$	$y = x^3 - 1$
−3	−25	−28
−2	−6	−9
−1	1	−2

Graph each equation from the table on the grid below. Label each with its equation.

4.

***Practice,* p. 75**

Week 6 • Nonlinear Functions and Graphs

Lesson 4

Key Idea

The nonlinear function $y = x^3$ can also have a constant. That function is written as $y = x^3 + b$.

A constant increases the value of y by a set value, unrelated to x^3.

Nonlinear functions can be graphed in all four quadrants. A graph of an equation that has x^3 is called a cubic curve or an S-curve.

Try This

Answer the following questions to make predictions about nonlinear functions.

1. What do you predict would happen to the graph of $y = x^3$ if a constant of 8 were added to x^3?
   Each point on the graph would be 8 units higher.

2. What do you predict would happen to the graph of $y = x^3$ if a constant of 10 were subtracted from x^3?
   Each point on the graph would be 10 units lower.

Fill in the values of y in the table below.

3.

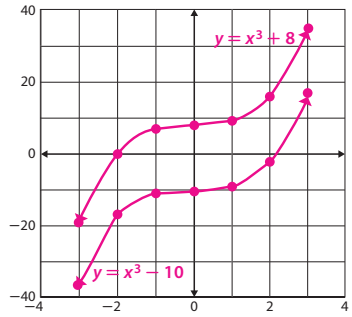
x	$y = x^3 + 8$	$y = x^3 - 10$
−3	−19	−37
−2	0	−18
−1	7	−11
0	8	−10
1	9	−9
2	16	−2
3	35	17

72 Level J Unit 3 **Algebra**

Practice

Plot the points from the table on the opposite page for each equation on the grid below, and then connect each set of points. Label each with its equation. Answer the questions that follow.

4.



5. How are the two graphs alike?
   Possible answer: They have the same shape.

6. How are the two graphs different?
   Possible answer: One graph is lower in position than the other.

Reflect

Describe the relationship between the graph of $y = x^3$ and the graph of $y = x^2$ for negative values of x.

Answer may vary. Possible answer: That side of the graph is rotated around the y-axis.

Week 6 **Nonlinear Functions and Graphs** • Lesson 4 **73**

***Student Workbook,* pp. 72–73**

Lesson 5 Review

Objective
Students review skills learned this week and complete the weekly assessment and project.

Standard
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Vocabulary
Review vocabulary introduced during the week.

Creating Context
To reinforce vocabulary acquisition with English Learners, you may wish to create a crossword puzzle using terms taught in this unit. Students can work independently or in pairs to fill out the puzzle.

1 WARM UP

Prepare
- **What is a nonlinear function?** Possible answers: a relationship that does not increase or decrease at a constant rate; a function that when graphed doesn't form a line
- **Why is $y = x^2$ a nonlinear function?** Possible answers: the rate of change is not constant; the graph does not produce a line
- **How is the graph of $y = x^3$ similar to the graph of $y = x^2$?** Answers may vary. Possible answer: The curve of each line is similar in the first quadrant.
- **How is the graph of $y = x^3$ different from the graph of $y = x^2$?** Answer may vary. Possible answer: When x is less than 0, the y is negative for $y = x^3$, which makes the left side of the y-axis look different.

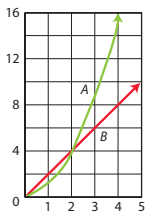
2 ENGAGE

Practice
Have students complete ***Student Workbook,*** pages 74–75.

Lesson 5 Review

This week you worked with nonlinear functions. You graphed and interpreted functions of the forms $y = x^2$ and $y = x^3$ in both the first quadrant and all four quadrants of a coordinate grid.

Lesson 1 **Answer** the questions that follow about nonlinear functions.



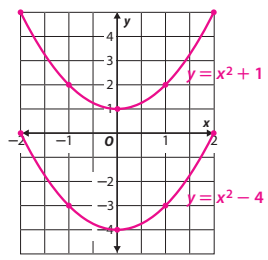
1. Write the letter of the graph for each equation.
   $y = 2x$ **B**
   $y = x^2$ **A**
2. If you graphed $y = 4x$, which graph would it more closely resemble, $y = 2x$ or $y = x^2$?
   $y = 2x$
3. If you graphed $y = 4x^2$, which graph would it more closely resemble, $y = 2x$ or $y = x^2$?
   $y = x^2$

Lesson 2 **Fill in** the values of y in the table below. Plot the points for each equation on the grid below, and then connect each set of points. Label each with its equation.

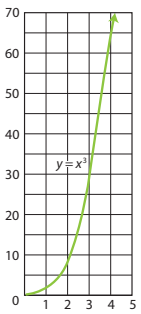
4.

x	$y = x^2 + 1$	$y = x^2 - 4$
−2	5	0
−1	2	−3
0	1	−4
1	2	−3
2	5	0

5.



Lesson 3 **Use** the graph of $y = x^3$ to solve the following problems.



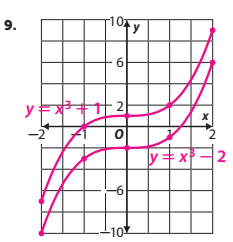
6. The volume of a cube-shaped aquarium (y) is equal to the length of each side cubed (x^3). If Jasmine's betta fish needs 64 cubic inches of space, how wide will the aquarium need to be?
   4 inches
7. If Cody is building a cube-shaped storage bin, and each side of the bin is 2 feet long, what will the volume be?
   8 cubic feet

Lesson 4 **Fill in** the values of y in the table below. Plot the points for each equation on the grid below, and then connect each set of points. Label each with its equation.

8.

x	$y = x^3 + 1$	$y = x^3 - 2$
−2	−7	−10
−1	0	−3
0	1	−2
1	2	−1
2	9	6

9.



Reflect
Do you think it is easier to graph a linear or nonlinear function? Why?
Answers may vary.

Student Workbook, pp. 74–75

3 REFLECT

Think Critically

Review students' answers to the Reflect prompt at the bottom of ***Student Workbook,*** page 75.

Discuss the answer with the group to reinforce Week 6 concepts.

4 ASSESS

Formal Assessment

Students may take the weekly assessment online.

As an alternative, students may complete the weekly test on ***Assessment,*** pages 47–48. Record progress using the Student Assessment Record, ***Assessment,*** page 128.

Going Forward

Use the ***Teacher Dashboard*** to view results of the online assessments, to input the results of print student assessments, and to review progress before making decisions about next steps. Use the weekly test results and observations to determine the next steps for each student.

Retention	
Student displays good grasp of this week's concepts and skills.	Have students fold a piece of paper in half and title one half *parabola* and the other half *cubic curve*. Have students give an example of each by writing down an equation, filling in a table, and graphing from the table. Have students use equations with a coefficient and ask them how having this in the equation affects the graph.
Remediation	
Student is still struggling with the week's concepts and skills.	If students are struggling with filling in the tables, have them practice squaring and cubing numbers. They should come up with strategies to remember what to do for each and what to do with negative numbers. If students are struggling with graphing the nonlinear functions, have them use the Coordinate Grid Tool to see how the equations are supposed to be graphed and the connections of the points is supposed to curve.

Suggestions for Re-Evaluation: If a student has struggled without success for several weeks, use observations and test results to place the student at a level in which he or she can find success and build confidence to move forward.

WEEK 6

Name _____ Date _____

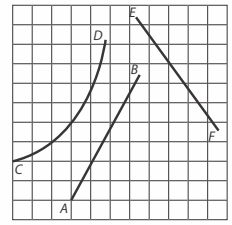
Nonlinear Functions and Graphs

1. Which of these is not a linear function? Circle your answer.

$y = 2x$ $y = 2x - 4$ $y = x^2$

2. Which line on the graph is not a linear function? Circle your answer.

AB CD EF

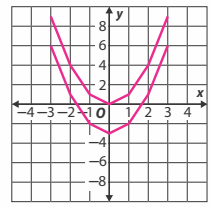


3. Complete the table below.

x	$y = x^2$	$y = x^2 - 3$
−3	9	6
−2	4	1
−1	1	−2
0	0	−3
1	1	−2
2	4	1
3	9	6

Use the table for question 3 to answer questions 4–6.

4. Plot the points for the two equations from the table above for question 4 on this grid. Connect each set of points.



Level J Unit 3 Week 6 **47**

WEEK 6

Name _____ Date _____

Nonlinear Functions and Graphs

5. Do the two graphed lines have similar shapes, *yes* or *no*? yes

6. How does the constant affect the two graphed lines?

Answers will vary.The constant is the difference between similar points on both lines.

7. Which of these will have the greatest value of y when x = 4? Circle your answer.

$y = x^2$ $y = x^2 + 3$ $y = x^3$

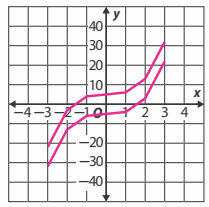
8. Which of these is a nonlinear function? Circle your answer.

$y = x^2$ $y = x + 2$ $y = 2 - x$

9. Complete the table below.

x	$y = x^3 + 5$	$y = x^3 - 5$
−3	−22	−32
−2	−3	−13
−1	4	−6
0	5	−5
1	6	−4
2	13	3
3	32	22

10. Plot the points from the table above on this grid. Connect each set of points.



48 Level J Unit 3 Week 6

Assessment, pp. 47–48

Project Preview

This week, students learned what a nonlinear function is and how to graph one. The project for this unit requires students to use what they know to graph functions that are nonlinear to show different area and volume.

Project-Based Learning

Standards-driven Project-Based Learning is effective in building deep content understanding. Project-Based Learning increases long-term retention of concepts and has been shown to be more effective than traditional instruction. Completing a project to answer an essential question challenges students to apply and demonstrate mastery of concepts and skills by expressing understanding through discussion, research, and presentation.

Essential Question

WHY do I need to be able to interpret the graph of a function?

Project Evaluation Criteria

Review project evaluation criteria with students prior to beginning the project.

Exceeds Expectations

- ☐ Project result is explained and can be extended.
- ☐ Project result is explained in context and can be applied to other situations.
- ☐ Project result is explained using advanced mathematical vocabulary.
- ☐ Project result is explained and extended, and shows advanced knowledge of mathematical concepts and skills.

Meets Expectations

- ☐ Project result is explained.
- ☐ Project result is explained in context.
- ☐ Project result is explained using mathematical vocabulary.
- ☐ Project result is described, and mathematics are used correctly.
- ☐ Project result is explained, and shows satisfactory knowledge of mathematical concepts and skills.

Does Not Meet Expectations

- ☐ Project result is not explained.
- ☐ Project result is explained, but out of context.
- ☐ Project result is explained, but mathematical vocabulary is oversimplified.
- ☐ Project result is described, but mathematics are not used correctly.
- ☐ Project result is not explained and/or extended, or shows less than satisfactory knowledge of mathematical concepts and skills.

The Pool

Objective

Students can graph nonlinear functions to understand the measurements of a pool.

Standard

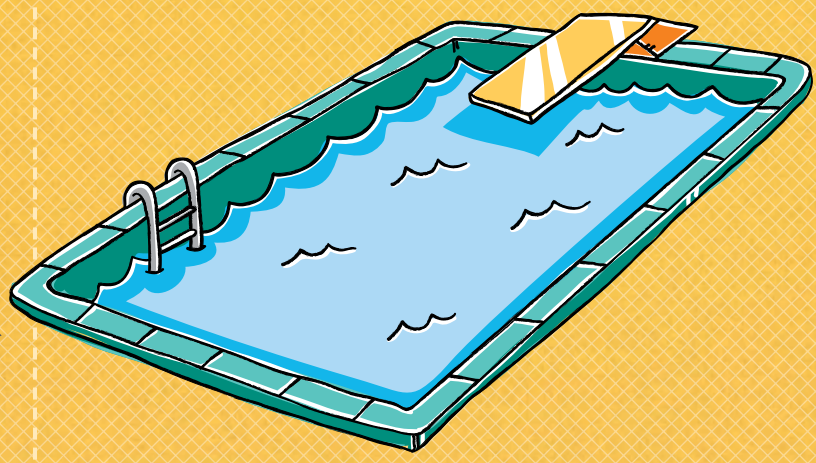
8.F.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Materials

grid paper

Best Practices

- Check for student understanding frequently.
- Create an energetic environment.
- Vary seating arrangements for functionality.



Project-Based Learning

Introduce

Eventually you want to build a pool for the gym, but you need some information before making a choice on the size of the pool.

- ► **What is the formula for the area of a square?**
- ► **What is the formula for the volume of a cube?**

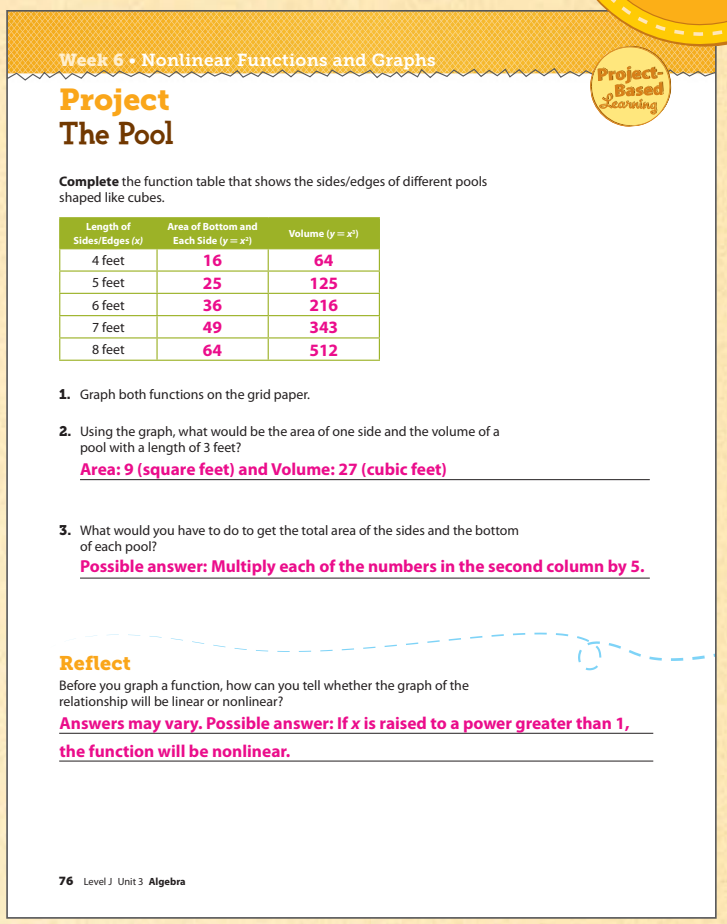
Explore

- ► **Today you will find the areas of walls and floors of different pools and the volumes of different pools that are cube shaped. In order to determine which pool to purchase, you need to know these measurements to project the cost of paint for the walls and floors and water to fill it.**
- Instruct the students to draw an *x*-axis and a *y*-axis on their grid paper for the first quadrant only. Label the *x*-axis from −10 through 10 by ones and the *y*-axis −5,000 through 5,000 by two hundreds. Students will graph both equations on one grid.
- ► **Complete *Student Workbook,* page 76.**

Wrap Up

- Have students work in pairs to compare their tables and graphs. Have them solve any discrepancies that arise between their work.
- Discuss with students how knowing the area of the walls and floors and knowing the volume could help in determining how big of a pool to get.
- Discuss students' answers to the Reflect prompt at the bottom of ***Student Workbook,*** page 76.

If time permits, allow each pair to determine the total area of the sides and bottom for each pool size.



Week 6 • Nonlinear Functions and Graphs

Project-Based Learning

Project
The Pool

Complete the function table that shows the sides/edges of different pools shaped like cubes.

Length of Sides/Edges (x)	Area of Bottom and Each Side ($y = x^2$)	Volume ($y = x^3$)
4 feet	16	64
5 feet	25	125
6 feet	36	216
7 feet	49	343
8 feet	64	512

1. Graph both functions on the grid paper.

2. Using the graph, what would be the area of one side and the volume of a pool with a length of 3 feet?
   Area: 9 (square feet) and Volume: 27 (cubic feet)

3. What would you have to do to get the total area of the sides and the bottom of each pool?
   Possible answer: Multiply each of the numbers in the second column by 5.

Reflect

Before you graph a function, how can you tell whether the graph of the relationship will be linear or nonlinear?
Answers may vary. Possible answer: If *x* is raised to a power greater than 1, the function will be nonlinear.

76 Level J Unit 3 **Algebra**

Student Workbook, p. 76

Teacher Reflect

- ☐ Did I define vocabulary words clearly and correctly?
- ☐ Did students use the appropriate available materials?
- ☐ Did students tell or show the steps when they explained how to do something?

LEVEL J

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