

Number Sense

UNIT

1

Level I

NUMBER WORLDS™

Lesson 1

Find the Math

Every purchase involves choices. For example, suppose you want to take dance lessons. Also suppose that one studio offers 5 lessons for \$40 while another studio offers 12 lessons for \$72. You have a decision to make.



PHOTO: Ariel Skelley/Getty Images

1. To decide which studio offers the better deal, we cannot simply compare the dollar amounts \$40 and \$72. Why not?

2. How can you decide which studio offers the better deal?

3. At which studio would you choose to take lessons? Explain?

Lesson 1

Key Idea

A **ratio** is a comparison of two quantities using division, such as 16 girls to 15 boys. A ratio can be written in three ways. For example, 16 girls to 15 boys can also be written as $\frac{16}{15}$ and 16:15.

A **rate** is a ratio comparing two quantities with unlike units. Some examples include miles per gallon, heartbeats per minute, and dollars per hour.

A **unit rate** is a rate with a denominator of 1. To figure out a unit rate, simplify a rate so that it has a denominator of 1. Here are three examples:

Rate	Unit Rate
$\frac{100 \text{ miles}}{2 \text{ hours}}$	$\frac{50 \text{ miles}}{1 \text{ hour}}$
$\frac{\$400}{2 \text{ weeks}}$	$\frac{\$200}{1 \text{ week}}$
$\frac{650 \text{ heart beats}}{10 \text{ minutes}}$	$\frac{65 \text{ heart beats}}{1 \text{ minute}}$

Try This

Write the unit rate of miles per gallon (mpg) for the trips shown in the table below.

	Miles Traveled	Gallons of Fuel	Unit Rate
1.	240	10	
2.	360	18	
3.	850	25	

Write the unit rate of miles per hour (mph) for the trips shown in the table below.

	Miles Traveled	Hours	Unit Rate
4.	250	5	
5.	360	9	
6.	770	14	

Practice

Write the unit rate in dollars per hour for the types of jobs listed in the table below.

	Type of Job	Dollars Earned	Hours Worked	Unit Rate
7.	Full-Time	\$70	7	
8.	Part-Time	\$70	10	

Answer Questions 9–11 using the nutritional information shown in the table below. Round decimals to the nearest tenth.

FOOD per Serving	FAT 1 gram = 9 calories	CARBOHYDRATES 1 gram = 4 calories	PROTEIN 1 gram = 4 calories	TOTAL CALORIES
1% Low Fat Milk	2.5 g	13 g	8 g	106.5
Reduced Fat Peanut Butter	11 g	15 g	8 g	191
Rice Cereal	0 g	29 g	2 g	124

9. Compare the calories to grams of carbohydrates in reduced fat peanut butter.

_____ cal : _____ g \approx _____ cal : 1 g

10. Compare the calories to grams of protein in rice cereal.

_____ cal : _____ g = _____ cal : 1 g

11. Compare the calories to grams of protein in 1% low fat milk.

_____ cal : _____ g \approx _____ cal : 1 g

Reflect

In the real world, gas mileage rarely comes out to whole numbers. If you want to know the cost of gas for a trip, would you prefer to use a rate of 22.7 mpg or 23 mpg for your calculations? Explain the advantages both choices.

Lesson 2

Key Idea

You can use rates and unit rates to convert between units of length.
The **customary system of measurement** is the measurement system used most often in the United States.

1 foot (ft) = 12 inches (in.)
1 yard (yd) = 3 feet or 36 inches
1 mile (mi) = 5,280 feet

Example: How many inches are in 4 feet?

Step 1 Write the relationship between units as a unit rate.

$$\frac{12 \text{ inches}}{1 \text{ foot}}$$

Step 2 Set up a proportion with the unit rate and the units you know. Use x for the unknown units.

$$\frac{12 \text{ inches}}{1 \text{ foot}} = \frac{x \text{ inches}}{4 \text{ feet}}$$

Step 3 Cross multiply to solve the proportion.

$$\begin{array}{c} \frac{12 \text{ inches}}{1 \text{ foot}} \quad \times \quad \frac{x \text{ inches}}{4 \text{ feet}} \\ 12 \times 4 = 1 \times x \\ 48 = x \end{array}$$

Answer: There are 48 inches in 4 feet.

Try This

Use a proportion to convert each measurement. Show your work.

1. 30 inches to feet

2. 5 yards to inches

3. 2 miles to feet

Practice

Convert each measurement to complete the table.

	Inches	Feet	Yards
4.	36		
5.	72		
6.			3
7.	144		
8.		1.5	

Use conversions to solve each real-world problem.

9. Miguel needs to cut 2 feet of wood. How many inches of wood should he cut?

10. A bird built a nest 4 yards above the ground. How many feet above the ground is the nest?

11. Isabella ran one-half of a mile. How many feet did she run?

12. A spool of thread is 120 feet long. How many yards long is it?

Reflect

How would you use a unit rate to convert a unit of measure?
Give an example.

Lesson 3

Key Idea

Most countries have their own systems of money, or currencies. The basic unit of the United States' money system is the U.S. dollar. You can use **rates** to convert between currencies.

Example: If 1 U.S. dollar equals 54 gabloons, how many gabloons equal 100 U.S. dollars?

Step 1 Write the relationship between the currencies as a unit rate.

$$\frac{54 \text{ gabloons}}{1 \text{ dollar}}$$

Step 2 Set up a **proportion** with the unit rate and the units you want to convert.

$$\frac{54 \text{ gabloons}}{1 \text{ dollar}} = \frac{x \text{ gabloons}}{100 \text{ dollars}}$$

Step 3 Cross multiply to solve the proportion.

$$\frac{54 \text{ gabloons}}{1 \text{ dollar}} = \frac{x \text{ gabloons}}{100 \text{ dollars}}$$

$$54 \times 100 = 1 \times x$$

$$5,400 = x$$

Answer: 100 U.S. dollars would equal 5,400 gabloons.

Try This

Set up a proportion to convert U.S. dollars to gabloons for each problem. Show your work.

1. 1 U.S. dollar = 3 gabloons

4 U.S. dollars = _____ gabloons

2. 1 U.S. dollar = 98 gabloons

10 U.S. dollars = _____ gabloons

3. 1 U.S. dollar = 12 gabloons

15 U.S. dollars = _____ gabloons

Practice

Complete the table below. Use this exchange rate:
1 unit of Currency A = 8 units of Currency B.

	Currency A	Currency B
4.	15	
5.	9	
6.	50	
7.		24
8.		200

Use the table below to answer Questions 9–11. The table shows the exchange rate between four different currencies and the U.S. dollar.

Currency	Number of Units Exchanged for 1 U.S. Dollar
W	2
X	5
Y	9
Z	1.5

9. Bryan is traveling to a country that uses Currency Y. How many units of currency will he get in exchange for 5 U.S. dollars?
- _____
10. Shaquori is sending 50 U.S. dollars to his friend in a country using Currency X. How many units of that currency will his friend get?
- _____
11. Tamika is ordering a gift from a country that uses Currency Z. The price, in Currency Z, is 30 units. What is the price in U.S. dollars?
- _____

Reflect

If you were to go from the U.S. to Gablonia, which exchange rate would be better for you: 10 gabloons for 1 dollar or 10 dollars for 1 gabloon? Explain.

Lesson 4

Key Idea

You can use **ratios** and **rates** to solve problems involving **fractions**.

Example: You walk $\frac{1}{2}$ mile in $\frac{1}{4}$ hour. What is your unit rate in miles per hour?

Step 1 Write the rate as a fraction.

$$\frac{\frac{1}{2} \text{ mile}}{\frac{1}{4} \text{ hour}}$$

Step 2 Divide the numerator by the denominator. Simplify.

$$\begin{aligned} \frac{\frac{1}{2}}{\frac{1}{4}} &= \frac{1}{2} \div \frac{1}{4} \\ &= \frac{1}{2} \times \frac{4}{1} \\ &= \frac{4}{2} \text{ or } \frac{2}{1} \end{aligned}$$

The unit rate is 2 miles per hour.

Try This

Use the figure below to answer questions about a vegetable garden. Write each answer in simplest form. Show your work.



Width = $\frac{1}{6}$ ft

Length = $\frac{1}{3}$ ft

1. What is the ratio of the width to the length?

2. What is the ratio of the length to the width?

Practice

Calculate each of the following unit rates.

3. A hose spreads $\frac{4}{5}$ gallon of water on a garden each $\frac{2}{5}$ second.
At what rate does water flow out of the hose?

4. Georgia ran $\frac{1}{6}$ mile in $\frac{1}{5}$ hour. At what rate did she run?

5. A factory machine produces $\frac{2}{5}$ pound of clay in $\frac{1}{10}$ hour. At what rate does the machine produce clay?

Use a unit rate to solve each problem.

6. A painter uses $\frac{1}{3}$ gallon of paint to cover $\frac{1}{9}$ of a wall. At that rate, how many gallons does the painter need to cover the entire wall?

7. In $\frac{1}{2}$ minute, a dog eats $\frac{1}{4}$ of his food. At this rate, how long will the dog need to finish all of the food?

8. Elijah raked $\frac{1}{2}$ of the lawn in $\frac{1}{5}$ hour. At that rate, how long will it take him to rake the entire lawn?

Reflect

How is calculating unit rates expressed as fractions the same as calculating unit rates expressed as whole numbers? How is it different?

Lesson 5 Review

This week you used unit rates to solve real-world problems. You used them to convert between units of length in the customary system and to convert between currencies. You also solved ratio and rate problems involving fractions.

Lesson 1 Write the unit rate in dollars per hour.

	Dollars Earned	Hours Worked	Unit Rate
1.	21	3	
2.	99	9	
3.	350	10	

Use unit rates to answer each question. Round decimals to the nearest tenth.

4. Car A can travel for 7 hours on 21 gallons of fuel. At this rate, how many gallons of fuel will Car A use on a one hour trip?

5. Car B can travel for 9 hours on 31.5 gallons of fuel. At this rate, how many gallons of fuel will Car B use on a one hour trip?

Lesson 2 Use a proportion to convert each measurement. Show your work.

6. 5 yards = _____ feet

7. 48 inches = _____ feet

8. 24 feet = _____ yards

9. 3.5 miles = _____ feet

Lesson 3 Convert each amount of currency using this rate of exchange:
1 U.S. dollar is equal to 8 gabloons.

10. 10 dollars _____

11. 88 gabloons _____

12. 25 dollars _____

13. 144 gabloons _____

Lesson 4 Answer each question by figuring out the unit rate.

14. Enrique drives $\frac{2}{10}$ of his trip in $\frac{1}{2}$ hour. At this rate, how much of the trip will Enrique drive in 1 hour?

15. Val uses $\frac{3}{8}$ pound of flour to make $\frac{1}{4}$ of a recipe of muffins. At this rate, how many pounds of flour does she need to make one full recipe?

16. Tina can perform $\frac{1}{5}$ of a dance routine in $\frac{3}{10}$ of a minute. At this rate, how much of the routine can Tina to perform in one minute?

Reflect

Describe a time when you used a unit rate to solve a real-world problem.
If you cannot remember one, make up a realistic one.

Project Lemonade Empire

Complete each of the following tasks.

1. Roll a number cube to determine a cost for each supply shown in the table below. Record your values in the table. The cost of the water will be \$0.00.
2. Add up the costs to determine the total cost of one batch of lemonade.

Supply	Cost
Lemon juice	
Water	\$0.00
Sugar	
Cups	
Total	

3. Choose a number card at random to decide the number of cups of lemonade that each batch will make. _____
4. Calculate the cost per cup. Show your work, and record your answer.

Lemonade Cost per Unit

\$_____ per cup

Reflect

What are some possible ways to reduce the unit cost of a product, such as lemonade?



Student Workbook

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