ALL LEVELS

Built on the CCSS COMMON CORE

SRA

NUMBER WORLDS™

Accelerate Math Success

Implementation Guide

Mc Graw Hill Education

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Implementation Guide

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Preview Program

SRA Number Worlds is a comprehensive math prevention/intervention program, proven to bring struggling students up to grade-level performance and beyond. ***SRA Number Worlds***©2015 retains the strengths of earlier editions, while enhancing the program's ability to help all students master the Common Core State Standards. Built on the Common Core (including the Standards for Mathematical Practice), ***SRA Number Worlds*** accelerates mathematical understanding through five researched-based instructional principles: (1) build upon children's current knowledge; (2) follow natural developmental progressions when selecting new knowledge to teach; (3) teach computational fluency and conceptual understanding; (4) provide plenty of opportunity for hands-on exploration, problem-solving, and communication; (5) expose students to major ways we represent and talk about numbers. Based on these principles, the curriculum can effectively close achievement gaps for students in grades Pre–K to 8.

In ***SRA Number Worlds,*** learners are actively engaged in developing computational fluency and conceptual understanding by building skills and mastering strategies through a variety of interactive learning experiences designed to bring fun into the math classroom.

- Hands-on learning and digital games help students apply concepts in a variety of contexts.
- Purposeful activities bring real-world scenarios into every lesson, making math more tangible for students.
- Meaningful discussions promote a deeper understanding of math concepts.
- Project-Based Learning materials challenge students to extend their learning beyond simple practice and application, encouraging critical thinking and preparing students to take more responsibility for their own learning.
- Differentiated instruction and alternative grouping options provide the tools needed to tailor instruction to the needs of each classroom and student.

Embedded digital resources, including interactive whiteboard activities, expanded ***Building Blocks™*** activities, virtual manipulatives, **Digital Math Tools,** and a new teacher dashboard, make the research-based instruction even more powerful. Online assessments, progress monitoring, and reporting all make tracking student progress and accelerating math success even easier.

The easy-to-use placement and assessment resources help pinpoint where students should begin the curriculum, and allow teachers to monitor at-risk students. Through identification and development of key Common Core concepts, ***SRA Number Worlds*** accelerates learning and student achievement. Weekly and unit tests, along with informal assessments, ensure accurate progress monitoring, and help teachers move students into the regular math curriculum.

Prevention Levels (A–C) focus on foundations in number sense to support students who start school with a weak conceptual background in mathematics. Intervention Levels (D–J) help intervention students unlock the Common Core by focusing on key standards in grades 2– 8 as well as the foundational skills and concepts students need to access these standards.

Building Blocks for Math™ is an online component of the ***SRA Number Worlds*** program that uses a variety of activities to develop early mathematical knowledge in children. By embedding mathematics in daily activities, ***Building Blocks™*** helps children make connections between informal mathematical knowledge and formal mathematical concepts.

Prevention

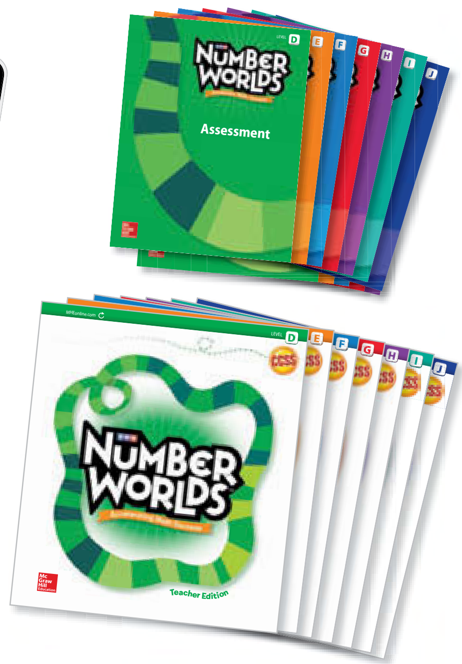
Prevention Levels A–C prepare students with foundational skills and concepts necessary to be successful with more complex mathematics in the future. Each Prevention Level consists of 32 weeks of daily instruction – now with additional instruction on time and money!

Level A Building Foundations for Grade Pre-K	**Level B** Grade K CCSS Key Standards	**Level C** Grade 1 CCSS Key Standards
Students acquire well-developed counting and quantity schemas.	Students develop a well-consolidated central conceptual structure for single-digit numbers.	Students link their central conceptual structure of number to the formal number system.

Intervention

Intervention Levels D–J help intervention students at each grade level learn the foundational skills and concepts needed to access key Common Core State Standards. Designed for flexibility, the five six-week units can be taught in any order or in isolation with placement tests to help identify student needs.

	Level D Grade 2	Level E Grade 3	Level F Grade 4	Level G Grade 5	Level H Grade 6	Level I Grade 7	Level J Grade 8
Unit 1	Number Sense within 100	Number Sense	Number Sense	Number Sense	Number Sense	Number Sense	Number Sense
Unit 2	Number Sense to 1,000	Addition	Addition & Subtraction	Multiplication & Division	Operations Sense	Operations Sense	Operations Sense
Unit 3	Addition	Subtraction	Multiplication	Operations with Decimals	Algebra	Algebra	Algebra
Unit 4	Subtraction	Multiplication & Division	Division	Operations with Fractions	Statistical Analysis	Statistical Analysis	Statistical Analysis
Unit 5	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement	Geometry & Measurement



Prepare for the Common Core State Standards

The Common Core State Standards identify the knowledge students should attain and the concepts students should understand at each grade level. Domains are used to group related Common Core State Standards. For Grades K–8, the standards are organized into 4 or 5 domains at each grade level.

CCSS Domains

Grades K–5	• Counting & Cardinality • Operations & Algebraic Thinking • Number & Operations in Base Ten • Number & Operations—Fractions • Measurement & Data • Geometry
Grades 6–8	• Ratios & Proportional Relationships • The Number System • Expressions & Equations • Functions • Geometry • Statistics & Probability

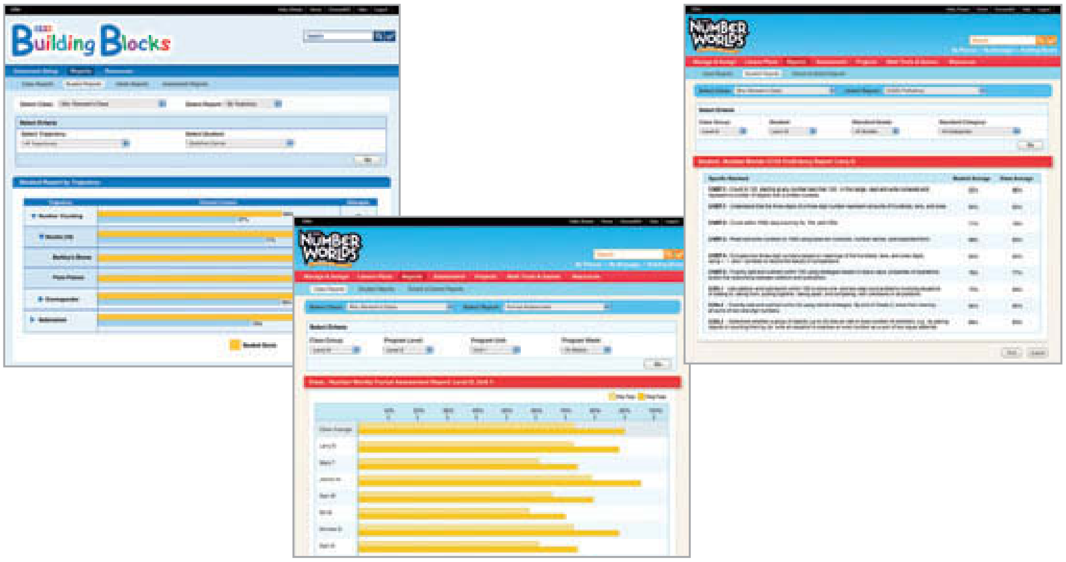
Engage Your Students

In addition to defining the mathematics that students should learn, the Common Core State Standards designate eight Standards for Mathematical Practice. These standards define the ways in which all students should engage in mathematics as they progress through a mathematics curriculum.

Throughout the ***SRA Number Worlds*** program, students are engaged in activities that employ the Common Core Mathematical Practices. The use of concrete models to build conceptual understanding is a cornerstone of the ***SRA Number Worlds*** curriculum. In addition, students are provided with daily opportunities to work collaboratively and then internalize lesson concepts through individual reflection.

Assess to Accelerate Student Success

The variety of formal and informal assessment options in the ***SRA Number Worlds*** program makes it easy to monitor student progress. Assessments are available in print and digital forms. Assessment types range from traditional paper-and-pencil tests to interactive whiteboard activities, computer-based games, and unit projects based on real-world problems.



Prepare Students to Be Mathematically Proficient

Students of all ages should be actively engaged on an individual level and with classmates as they deepen their understanding of mathematics. The Common Core State Standards for Mathematical Practice define eight sets of behaviors that should be developed in conjunction with mathematical content knowledge.

Common Core State Standards for Mathematical Practice

1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
5	Use appropriate tools strategically.
6	Attend to precision.
7	Look for and make use of structure.
8	Look for and express regularity in repeated reasoning.

Mathematically proficient students use manipulatives to develop conceptual understanding, use available tools appropriately, create multiple representations to model mathematical ideas, justify their solutions, and use math to model real-world problems. They use mathematical language and symbols precisely and express answers with appropriate degrees of precision. In addition, mathematically proficient students look for, use, and generalize patterns found in mathematical structures.

The tables on the following pages present the Common Core Standards for Mathematical Practices in their entirety and an overview of ***SRA Number Worlds*** features that support and promote these mathematical practices.

1	Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

With *SRA Number Worlds*, students employ Mathematical Practice 1 by:

- participating in activities that promote the use of various problem-solving strategies, and
- using multiple representations, including physical models, diagrams, tables, graphs, and graphic organizers, to organize information, make sense of problems, and analyze relationships.

2	Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

With *SRA Number Worlds*, students employ Mathematical Practice 2 by:

- participating in activities that use manipulatives, including virtual manipulatives, to explore, conceptualize, and generalize mathematical concepts,
- including referents, labels, or units for solutions to all real-world problems, and
- developing fluency in manipulating symbolic representations using a variety of print and digital practice opportunities.

3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

With *SRA Number Worlds*, students employ Mathematical Practice 3 by:

- using manipulatives, drawings, diagrams, and graphs to model, support, and justify solutions,
- providing written explanations and justifications to prompts and questions,
- listening to others, evaluating explanations/justifications, and asking clarifying questions, and
- reviewing solutions to real-world problems to determine if the solutions make sense with respect to their real-world contexts.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

With *SRA Number Worlds*, students employ Mathematical Practice 4 by:

- considering ways in which mathematics apply to everyday-life contexts,
- participating in activities and solving problems that use multiple representations, including concrete models, diagrams, tables, graphs, and flowcharts to analyze mathematical relationships, and
- applying and demonstrating mastery of concepts and skills to complete real-world tasks.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

With *SRA Number Worlds*, students employ Mathematical Practice 5 by:

- participating in hands-on activities in which various tools are used to explore mathematical concepts,
- using diagrams and graphs to analyze situations and make predictions,
- performing calculations using pencil and paper, estimations, and/or calculators, depending on the tasks at hand, and
- using technology, including digital tools, virtual manipulatives, online games, and spreadsheets, to develop fluency and improve/enhance the understanding of mathematical concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

With *SRA Number Worlds*, students employ Mathematical Practice 6 by:

- participating in activities and answering questions requiring precise mathematical language,
- using precise mathematical language when writing answers requiring explanations/justifications,
- participating in activities and solving problems that require the specification of units of measurement, such as activities that involve determining length, area, and volume, and
- participating in activities and solving problems in which students express numerical answers with specified place values, such as area and volume problems involving pi or computations involving money.

7	Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

With *SRA Number Worlds*, students employ Mathematical Practice 7 by:

- participating in activities that involve sorting and classifying objects,
- using manipulatives, digital tools, and online resources to develop conceptual understanding, such as using pattern blocks to sort and classify polygons according to various properties,
- using tables and graphic organizers to recognize patterns and make use of structures, and
- using manipulatives, diagrams, and drawings to transition from concrete representations to conceptual understanding.

8	Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

With *SRA Number Worlds*, students employ Mathematical Practice 8 by:

- participating in activities that involve looking for patterns that lead to generalizations about mathematical concepts, such as the understanding of multiplication as repeated addition,
- responding to questions that involve generalizing results to write rules (such as the rules for integer operations) and derive formulas (such as formulas to determine area, surface area, and volume), and
- making connections between what is known and new situations which require similar thinking.

Prepare Students for the Common Core Content Standards

The Common Core State Standards for Mathematical Content define the procedures students should be able to perform and the concepts students should understand at each grade level. Instruction in core procedures and concepts begins in the early grades. As students continue their study of core concepts and procedures, their mathematical understanding deepens and their problem-solving abilities become more sophisticated.

SRA Number Worlds helps students unlock the Common Core by focusing on Key Standards at each grade level. Prevention Levels A–C and Intervention Levels D–J focus on concepts aligned to the mathematical domains of the Common Core. At all levels, each Lesson Week focuses on one or two key Common Core State Standards. By developing a mastery of key foundational skills and concepts, students attain the prerequisite skills and background knowledge needed to access on-level Common Core State Standards.

Levels D–J contain five six-week, intensive units of instruction organized around the following mathematical topics.

Mathematical Topics

Level **D**	Number Sense within 100, Number Sense to 1,000, Addition, Subtraction, Geometry and Measurement
Level **E**	Number Sense, Addition, Subtraction, Multiplication and Division, Geometry and Measurement
Level **F**	Number Sense, Addition and Subtraction, Multiplication, Division, Geometry and Measurement
Level **G**	Number Sense, Multiplication and Division, Operations with Decimals, Operations with Fractions, Geometry and Measurement
Levels **H–J**	Number Sense, Operations Sense, Algebra, Statistical Analysis, Geometry and Measurement

Key Standards by Lesson Week

Level A			
Week	**Lesson**	**Domain**	**Key Standard**
1	**Counting Objects**	Counting and Cardinality	**K.CC.1**
2	**Counting and Sorting Objects**	Measurement and Data	**K.MD.3**
3	**More Counting and Sorting Objects**	Counting and Cardinality	**K.CC.6**
4	**Sequencing Sets**	Counting and Cardinality	**K.CC.3**
5	**Shapes**	Geometry	**K.G.2**
6	**More Shapes**	Geometry	**K.G.2**
7	**Learning about Set Size**	Counting and Cardinality	**K.CC.3**
8	**More Sets**	Operations and Algebraic Thinking	**K.OA.1**
9	**Number Sequence**	Counting and Cardinality	**K.CC.1**
10	**Identifying Pattern and Quantity**	Counting and Cardinality	**K.CC.6**
11	**Matching Patterns and Quantities**	Counting and Cardinality	**K.CC.3**
12	**Introduction to the Number Line**	Counting and Cardinality	**K.CC.2** **K.CC.4c**
13	**Position on the Number Line**	Counting and Cardinality	**K.CC.4c**
14	**Sequence on the Number Line**	Counting and Cardinality	**K.CC.4a**
15	**Using the Number Line**	Counting and Cardinality	**K.CC.4a**
16	**Moving Up and Down**	Counting and Cardinality	**K.CC.7**
17	**Counting Rotations**	Counting and Cardinality	**K.CC.2**
18	**Adding to a Set**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.1** **K.OA.1**
19	**Subtracting from a Set**	Operations and Algebraic Thinking	**K.OA.1**
20	**Comparing Sets**	Counting and Cardinality	**K.CC.6**
21	**Using Graphs**	Measurement and Data	**1.MD.4**
22	**More Subtraction**	Operations and Algebraic Thinking	**K.OA.1**
23	**Depicting Numbers**	Counting and Cardinality	**K.CC.6**
24	**Comparing Quantities**	Counting and Cardinality	**K.CC.6**
25	**Making Comparisons**	Counting and Cardinality	**K.CC.6**
26	**Adding and Subtracting**	Counting and Cardinality	**K.CC.6**
27	**More Graphs**	Counting and Cardinality Measurement and Data	**K.CC.6** **1.MD.4**
28	**Higher and Lower**	Counting and Cardinality	**K.CC.2**
29	**More Rotations**	Counting and Cardinality	**K.CC.6**
30	**Sequencing Numbers**	Counting and Cardinality	**K.CC.4c**
31	**Understanding the Analog Clock**	Counting and Cardinality	**K.CC.2**
32	**The Dollar Bill and the Penny**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.4a** **K.OA.1**

Key Standards by Lesson Week

Level B			
Week	**Lesson**	**Domain**	**Key Standard**
1	**Counting Basics**	Counting and Cardinality	**K.CC.1**
2	**Counting Objects**	Counting and Cardinality	**K.CC.1**
3	**Adding One**	Operations and Algebraic Thinking	**K.OA.1**
4	**Subtracting One**	Operations and Algebraic Thinking	**K.OA.1**
5	**Counting and Comparing**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.6** **K.OA.1**
6	**More Counting and Comparing**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.6** **K.OA.1**
7	**Comparing Sets**	Counting and Cardinality	**K.CC.6**
8	**Making Comparisons**	Measurement and Data	**K.MD.3**
9	**Matching Sets**	Counting and Cardinality	**K.CC.3**
10	**Matching Quantities and Numbers**	Counting and Cardinality	**K.CC.3**
11	**Comparing Quantities**	Counting and Cardinality	**K.CC.6** **K.CC.7**
12	**Numbers on a Vertical Scale**	Counting and Cardinality	**K.CC.4c**
13	**Numbers on a Line**	Counting and Cardinality	**K.CC.4c**
14	**Adding and Subtracting on a Line**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.6** **K.OA.1**
15	**Comparing Positions on a Line**	Counting and Cardinality	**K.CC.4c**
16	**Moving on a Vertical Scale**	Counting and Cardinality	**K.CC.4c**
17	**Predicting Quantities**	Operations and Algebraic Thinking	**K.OA.1**
18	**Numeral Magnitude**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.3** **K.CC.4c** **K.OA.4**
19	**Moving and Comparing on a Number Line**	Counting and Cardinality	**K.CC.4c**
20	**Comparing Representations**	Counting and Cardinality Measurement and Data	**K.CC.3** **K.MD.3**
21	**Numbers on a Dial**	Counting and Cardinality	**K.CC.4b** **K.CC.6**
22	**Adding and Subtracting on a Dial**	Operations and Algebraic Thinking	**K.OA.1**
23	**Understanding Numerals**	Counting and Cardinality	**K.CC.5**
24	**Adding and Subtracting on a Vertical Scale**	Operations and Algebraic Thinking	**K.OA.1**
25	**More Operations on a Vertical Scale**	Operations and Algebraic Thinking	**K.OA.1**
26	**Making Bar Graphs**	Measurement and Data	**1.MD.4**
27	**Recording Data on a Number Line**	Measurement and Data	**K.MD.3**
28	**Problem Solving**	Counting and Cardinality	**K.CC.6**
29	**More Vertical Bar Graphs**	Measurement and Data	**1.MD.4**
30	**Creating Sets**	Counting and Cardinality	**K.CC.3**
31	**Understanding the Analog Clock**	Counting and Cardinality	**K.CC.2**
32	**The Five-Dollar Bill and the Nickel**	Operations and Algebraic Thinking	**K.OA.2**

Key Standards by Lesson Week

Level C			
Week	**Lesson**	**Domain**	**Key Standard**
1	**Counting**	Counting and Cardinality	**K.CC.1**
2	**Counting and Comparing**	Counting and Cardinality	**K.CC.3**
3	**More Counting and Comparing**	Counting and Cardinality	**K.CC.3**
4	**Matching Dot Sets to Numerals**	Counting and Cardinality	**K.CC.1**
5	**Number Sequence and Number Lines**	Counting and Cardinality	**K.CC.1**
6	**More Number Sequence and Number Lines**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.1** **K.OA.1**
7	**Number Neighborhoods**	Counting and Cardinality	**K.CC.1**
8	**More Number Neighborhoods**	Counting and Cardinality	**K.CC.1**
9	**Adding Numbers**	Operations and Algebraic Thinking	**K.OA.1**
10	**More Adding**	Operations and Algebraic Thinking	**K.OA.1**
11	**Sequencing Numbers**	Operations and Algebraic Thinking	**1.OA.8**
12	**Writing Equations**	Operations and Algebraic Thinking	**K.OA.1**
13	**Counting and Adding**	Operations and Algebraic Thinking	**1.OA.6**
14	**Making Equations**	Operations and Algebraic Thinking	**K.OA.1**
15	**Graphing and Comparing Numbers**	Operations and Algebraic Thinking	**K.OA.1**
16	**More Counting and Adding**	Operations and Algebraic Thinking	**K.OA.1**
17	**Solving Equations**	Operations and Algebraic Thinking	**K.OA.1**
18	**Adding and Subtracting**	Operations and Algebraic Thinking	**K.OA.1**
19	**Subtracting**	Operations and Algebraic Thinking	**K.OA.1**
20	**Subtracting and Predicting**	Operations and Algebraic Thinking	**1.OA.8**
21	**Adding and Comparing**	Counting and Cardinality	**K.CC.4c**
22	**Subtracting to Zero**	Operations and Algebraic Thinking	**1.OA.6**
23	**More Adding and Subtracting**	Operations and Algebraic Thinking	**1.OA.6**
24	**Numbers to 100**	Counting and Cardinality Operations and Algebraic Thinking	**K.CC.1** **K.OA.1**
25	**More Numbers to 100**	Counting and Cardinality	**K.CC.1**
26	**Addition Stories**	Operations and Algebraic Thinking	**K.OA.2**
27	**Tens and Ones**	Number and Operations in Base Ten	**1.NBT.2a** **1.NBT.2c**
28	**Adding and Subtracting Length**	Operations and Algebraic Thinking	**K.OA.1**
29	**Addition and Subtraction Stories**	Operations and Algebraic Thinking	**K.OA.1** **1.OA.1**
30	**Making a Map**	Operations and Algebraic Thinking	**K.OA.1** **1.OA.1**
31	**Understanding the Analog Clock**	Operations and Algebraic Thinking	**1.OA.5**
32	**The Ten-Dollar Bill and the Dime**	Operations and Algebraic Thinking	**1.OA.1**

Key Standards by Lesson Week

Level D			
Week	**Lesson**	**Domain**	**Key Standard**
Unit 1 Number Sense within 100			
1	**Constructing Whole Numbers**	Number and Operations in Base Ten	**1.NBT.2a**
2	**Numbers on a Line**	Number and Operations in Base Ten	**2.NBT.1**
3	**Tens and Ones**	Number and Operations in Base Ten	**1.NBT.2**
4	**Visualizing and Constructing Whole Numbers**	Number and Operations in Base Ten	**2.NBT.1**
5	**Number Patterns**	Number and Operations in Base Ten	**2.NBT.2**
6	**Whole Number Relationships**	Number and Operations in Base Ten	**2.NBT.4**
Unit 2 Number Sense to 1,000			
1	**Understanding the Base-Ten Number System**	Number and Operations in Base Ten	**2.NBT.2**
2	**Constructing Whole Numbers to 999**	Number and Operations in Base Ten	**2.NBT.3**
3	**Representing Number Systems**	Number and Operations in Base Ten	**2.NBT.1**
4	**Place Value to 1,000**	Number and Operations in Base Ten	**2.NBT.3**
5	**Skip Counting within 1,000**	Number and Operations in Base Ten	**2.NBT.2**
6	**Comparing Whole Numbers**	Number and Operations in Base Ten	**2.NBT.4**
Unit 3 Addition			
1	**Addition Fundamentals**	Operations and Algebraic Thinking	**2.OA.2**
2	**Mastering the Basic Facts**	Operations and Algebraic Thinking	**2.OA.2**
3	**Solving Addition Problems**	Number and Operations in Base Ten	**2.NBT.5**
4	**Addition Tools and Strategies**	Number and Operations in Base Ten	**2.NBT.5**
5	**Addition Word Problems within 100**	Operations and Algebraic Thinking	**2.OA.1**
6	**Solving Addition Word Problems within 1,000**	Number and Operations in Base Ten	**2.NBT.7**
Unit 4 Subtraction			
1	**Subtraction Fundamentals**	Operations and Algebraic Thinking	**2.OA.2**
2	**Mastering Basic Subtraction Facts**	Operations and Algebraic Thinking	**2.OA.2**
3	**Solving Subtraction Problems**	Number and Operations in Base Ten	**2.NBT.5**
4	**Subtraction Tools and Strategies**	Number and Operations in Base Ten	**2.NBT.5**
5	**Subtraction Word Problems within 100**	Operations and Algebraic Thinking	**2.OA.1**
6	**Solving Subtraction Word Problems within 1,000**	Number and Operations in Base Ten	**2.NBT.7**
Unit 5 Geometry and Measurement			
1	**Linear Measurement**	Measurement and Data	**2.MD.3**
2	**Measurement Tools**	Measurement and Data	**2.MD.1**
3	**Time Measurement to the Half Hour**	Measurement and Data	**2.MD.7**
4	**Time Measurement to the Nearest Five Minutes**	Measurement and Data	**2.MD.7**
5	**Attributes of Shapes**	Geometry	**2.G.1**
6	**Graphs**	Measurement and Data	**2.MD.10**

Key Standards by Lesson Week

Level E			
Week	**Lesson**	**Domain**	**Key Standard**
Unit 1 Number Sense			
1	**Place Values to 999**	Number and Operations in Base Ten	**3.NBT.1**
2	**Round to the Nearest Ten or Hundred**	Number and Operations in Base Ten	**3.NBT.1**
3	**Skip Counting**	Number and Operations in Base Ten	**3.NBT.3**
4	**Introduction to Fractions**	Number and Operations- Fractions	**3.NF.1**
5	**Representations of Fractions**	Number and Operations- Fractions	**3.NF.1** **3.NF.3d**
6	**Comparing Fractions**	Number and Operations- Fractions	**3.NF.3a** **3.NF.3b** **3.NF.3d**
Unit 2 Addition			
1	**Mental Addition Strategies**	Number and Operations in Base Ten	**3.NBT.2**
2	**Grouping Strategies**	Number and Operations in Base Ten	**3.NBT.2**
3	**Regrouping Strategies**	Number and Operations in Base Ten	**3.NBT.2**
4	**Solving Addition Problems to 1,000**	Number and Operations in Base Ten	**3.NBT.2**
5	**Computational Estimation**	Operations and Algebraic Thinking	**3.OA.8**
6	**Variables and Equality**	Operations and Algebraic Thinking	**3.OA.8**
Unit 3 Subtraction			
1	**Subtraction Strategies**	Number and Operations in Base Ten	**3.NBT.2**
2	**More Subtraction Strategies**	Number and Operations in Base Ten	**3.NBT.2**
3	**Subtraction with Regrouping**	Number and Operations in Base Ten	**3.NBT.2**
4	**More Subtraction with Regrouping**	Number and Operations in Base Ten	**3.NBT.2**
5	**Solving Subtraction Problems within 100**	Operations and Algebraic Thinking	**3.OA.8**
6	**Solving Word Problems within 1,000**	Operations and Algebraic Thinking	**3.OA.8**
Unit 4 Multiplication and Division			
1	**Models for Multiplication**	Operations and Algebraic Thinking	**3.OA.1**
2	**Number Lines and Arrays**	Operations and Algebraic Thinking	**3.OA.5**
3	**Building Multiplication Facts**	Operations and Algebraic Thinking	**3.OA.7**
4	**Beyond the Basic Facts**	Operations and Algebraic Thinking	**3.OA.5**
5	**Constructing Division**	Operations and Algebraic Thinking	**3.OA.2**
6	**Solving Word Problems**	Operations and Algebraic Thinking	**3.OA.3**
Unit 5 Geometry and Measurement			
1	**Measuring Weight**	Measurement and Data	**3.MD.2**
2	**Finding Area**	Measurement and Data	**3.MD.6**
3	**Solving Problems Involving Area**	Measurement and Data	**3.MD.7**
4	**Finding Perimeter**	Measurement and Data	**3.MD.8**
5	**Understanding Shapes**	Geometry	**3.G.1**
6	**Creating and Interpreting Graphs**	Measurement and Data	**3.MD.3**

Key Standards by Lesson Week

Week	Lesson	Domain	Key Standard
Level F			
Unit 1 Number Sense			
1	**Place Value**	Number and Operations in Base Ten	**4.NBT.2**
2	**Rounding**	Number and Operations in Base Ten	**4.NBT.3**
3	**Prime Numbers and Factors**	Operations and Algebraic Thinking	**4.OA.4**
4	**Equivalent Fractions**	Number and Operations- Fractions	**4.NF.1**
5	**Working with Fractions**	Number and Operations- Fractions	**4.NF.2**
6	**Number Patterns**	Operations and Algebraic Thinking	**4.OA.5**
Unit 2 Addition and Subtraction			
1	**Adding and Subtracting One- and Two-Digit Numbers**	Number and Operations in Base Ten	**4.NBT.4**
2	**Addition and Subtraction with Regrouping**	Operations and Algebraic Thinking	**4.OA.3**
3	**Adding and Subtracting Fractions**	Number and Operations- Fractions	**4.NF.3d**
4	**Adding Mixed Numbers**	Number and Operations- Fractions	**4.NF.3c**
5	**Subtracting Mixed Numbers without Regrouping**	Number and Operations- Fractions	**4.NF.3d**
6	**Subtracting Mixed Numbers with Regrouping**	Number and Operations- Fractions	**4.NF.3c**
Unit 3 Multiplication			
1	**Products of Whole Numbers**	Number and Operations in Base Ten	**4.NBT.5**
2	**Multiplication Strategies**	Number and Operations in Base Ten	**4.NBT.5**
3	**Solving Multiplication Problems**	Number and Operations in Base Ten	**4.NBT.5**
4	**Multiplying Multi-Digit Numbers**	Number and Operations in Base Ten	**4.NBT.5**
5	**Multistep Word Problems with Multiplication**	Operations and Algebraic Thinking	**4.OA.3**
6	**Multiplying Fractions by Whole Numbers**	Number and Operations- Fractions	**4.NF.4** **4.NF.4a** **4.NF.4b**
Unit 4 Division			
1	**Relating Multiplication to Division**	Number and Operations in Base Ten	**4.NBT.6**
2	**Dividing 2-Digit Numbers by 1-Digit Numbers**	Number and Operations in Base Ten	**4.NBT.6**
3	**Dividing 3-Digit Numbers by 1-Digit Numbers**	Number and Operations in Base Ten	**4.NBT.6**
4	**Dividing 2- and 3-Digit Numbers by 1-Digit Numbers**	Number and Operations in Base Ten	**4.NBT.6**
5	**Dividing 4-Digit by 1-Digit Numbers (with Remainders)**	Number and Operations in Base Ten	**4.NBT.6**
6	**Multistep Word Problems with Division**	Operations and Algebraic Thinking	**4.OA.3**
Unit 5 Geometry and Measurement			
1	**Area and Perimeter of a Rectangle**	Measurement and Data	**4.MD.3**
2	**Measuring Angles Using a Protractor**	Measurement and Data	**4.MD.6**
3	**Points, Lines, Line Segments, Angles, and Rays**	Geometry	**4.G.1**
4	**Classifying Polygons**	Geometry	**4.G.2**
5	**Converting Units of Time and Length within the Same System**	Measurement and Data	**4.MD.1**
6	**Converting Units within the Same System**	Measurement and Data	**4.MD.1**

Key Standards by Lesson Week

Built on the CCSS Common Core

Level G			
Week	**Lesson**	**Domain**	**Key Standard**
Unit 1 Number Sense			
1	**Working with Decimals**	Number and Operations in Base Ten	**5.NBT.3**
2	**Read and Write Decimals Using Expanded Form**	Number and Operations in Base Ten	**5.NBT.3.a**
3	**Place Value**	Number and Operations in Base Ten	**5.NBT.3b**
4	**Multiplying and Dividing by Powers of Ten**	Number and Operations in Base Ten	**5.NBT.3b**
5	**Converting Words to Mathematical Symbols**	Operations and Algebraic Thinking	**5.OA.2**
6	**Order of Operations**	Operations and Algebraic Thinking	**5.OA.1**
Unit 2 Multiplication and Division			
1	**Prime Numbers and Factorization**	Number and Operations in Base Ten	**5.NBT.5**
2	**Multiplication Properties**	Number and Operations in Base Ten	**5.NBT.5**
3	**Multiplication Strategies**	Number and Operations in Base Ten	**5.NBT.5**
4	**Division Basics**	Number and Operations in Base Ten	**5.NBT.6**
5	**Division Using Whole Numbers**	Number and Operations in Base Ten	**5.NBT.6**
6	**Tables and Graphs**	Number and Operations in Base Ten	**5.NBT.5** **5.NBT.6**
Unit 3 Operations with Decimals			
1	**Adding Decimals**	Number and Operations in Base Ten	**5.NBT.7**
2	**Subtracting Decimals**	Number and Operations in Base Ten	**5.NBT.7**
3	**Multiplication with Models**	Number and Operations in Base Ten	**5.NBT.7**
4	**Multiplying Decimals**	Number and Operations in Base Ten	**5.NBT.7**
5	**Division with Models**	Number and Operations in Base Ten	**5.NBT.7**
6	**Solving Problems with Division**	Number and Operations in Base Ten	**5.NBT.7**
Unit 4 Operations with Fractions			
1	**Understanding Fractions**	Number and Operations- Fractions	**5.NF.1**
2	**Adding/Subtracting Fractions with Unlike Denominators**	Number and Operations- Fractions	**5.NF.1**
3	**Adding and Subtracting Mixed Numbers**	Number and Operations- Fractions	**5.NF.1**
4	**Multiplying Whole Numbers by Fractions**	Number and Operations- Fractions	**5.NF.4**
5	**Multiplying Fractions by Fractions**	Number and Operations- Fractions	**5.NF.4**
6	**Division with Whole Numbers and Fractions**	Number and Operations- Fractions	**5.NF.7**
Unit 5 Geometry and Measurement			
1	**Converting Measurements within the Same System**	Measurement and Data	**5.MD.1**
2	**Volume of Rectangular Prisms**	Measurement and Data	**5.MD.4**
3	**Coordinate Grids, Quadrant I**	Geometry	**5.G.1**
4	**Coordinate Grids**	Geometry	**5.G.1**
5	**Graphing Ordered Pairs**	Operations and Algebraic Thinking	**5.OA.3**
6	**Classifying Polygons**	Geometry	**5.G.4**

Key Standards by Lesson Week

Level H			
Week	**Lesson**	**Domain**	**Key Standard**
Unit 1 Number Sense			
1	**Comparing Fractions**	Ratios and Proportional Relationships	**6.RP.1**
2	**Comparing Decimals**	The Number System	**6.NS.3**
3	**Ratios and Rates**	Ratios and Proportional Relationships	**6.RP.2**
4	**Using Ratios and Rates**	Ratios and Proportional Relationships	**6.RP.3b**
5	**Integers**	The Number System	**6.NS.6**
6	**Using Integers**	The Number System	**6.NS.6**
Unit 2 Operations Sense			
1	**Operations with Whole Numbers**	The Number System	**6.NS.2**
2	**Decimals**	The Number System	**6.NS.3**
3	**Fractions**	The Number System	**6.NS.1**
4	**Order of Operations**	Expressions and Equations	**6.EE.3**
5	**Creating Function Tables**	Expressions and Equations	**6.EE.7**
6	**Functions and Graphs**	Expressions and Equations	**6.EE.7**
Unit 3 Algebra			
1	**Exponents**	Expressions and Equations	**6.EE.1**
2	**Expressions**	Expressions and Equations	**6.EE.2**
3	**Equivalent Expressions**	Expressions and Equations	**6.EE.3**
4	**Creating Equations**	Expressions and Equations	**6.EE.7**
5	**Solving Linear Equations**	Expressions and Equations	**6.EE.7**
6	**Inequalities**	Expressions and Equations	**6.EE.8**
Unit 4 Statistical Analysis			
1	**Showing Data in Line Plots**	Statistics and Probability	**6.SP.4**
2	**Data Displays 1**	Statistics and Probability	**6.SP.4**
3	**Measures of Center**	Statistics and Probability	**6.SP.4**
4	**Data Displays 2**	Statistics and Probability	**6.SP.4**
5	**Describing and Comparing Data**	Statistics and Probability	**6.SP.5** **6.SP.5a** **6.SP.5b** **6.SP.5c** **6.SP.5d**
6	**Statistical Measurements**	Statistics and Probability	**6.SP.5** **6.SP.5a** **6.SP.5b** **6.SP.5c** **6.SP.d**
Unit 5 Geometry and Measurement			
1	**Areas of Triangles**	Geometry	**6.G.1**
2	**Areas of Quadrilaterals and Polygons**	Geometry	**6.G.1**
3	**Volume of Right Rectanglular Prisms**	Geometry	**6.G.2**
4	**Volume of Prisms**	Geometry	**6.G.2**
5	**Nets**	Geometry	**6.G.4**
6	**Graphing Polygons**	Geometry	**6.G.3**

Key Standards by Lesson Week

Built on the CCSS COMMON CORE

Week	Lesson	Domain	Key Standard
Level I			
Unit 1 Number Sense			
1	**Rational Numbers**	The Number System	**7.NS.3**
2	**Fractions, Decimals, and Percents**	The Number System	**7.NS.2d**
3	**Ratios and Rates**	Ratios and Proportional Relationships	**7.RP.1**
4	**Unit Rates**	Ratios and Proportional Relationships	**7.RP.1**
5	**Percents, Discounts, and Commissions**	Ratios and Proportional Relationships	**7.RP.3**
6	**Markups, Markdowns, and Simple Interest**	Ratios and Proportional Relationships	**7.RP.3**
Unit 2 Operations Sense			
1	**Adding and Subtracting Integers**	The Number System	**7.NS.1**
2	**Properties of Addition and Subtraction**	The Number System	**7.NS.1d**
3	**Applications Using Addition and Subtraction**	The Number System	**7.NS.3**
4	**Multiplying and Dividing Integers**	The Number System	**7.NS.2**
5	**Properties of Operations for Rational Numbers**	The Number System	**7.NS.2c**
6	**Applications with Rational Numbers**	The Number System	**7.NS.3**
Unit 3 Algebra			
1	**Algebraic Expressions**	Expressions and Equations	**7.EE.4**
2	**Solving Simple Equations**	Expressions and Equations	**7.EE.4**
3	**Solving Two-Step Equations**	Expressions and Equations	**7.EE.4**
4	**Equations for Real-World Problems**	Expressions and Equations	**7.EE.4a**
5	**Solving Inequalities**	Expressions and Equations	**7.EE.4b**
6	**Inequalities for Real-World Problems**	Expressions and Equations	**7.EE.4b**
Unit 4 Statistical Analysis			
1	**Introduction to Probability**	Statistics and Probability	**7.SP.7**
2	**Data and Probability**	Statistics and Probability	**7.SP.7**
3	**Theoretical and Experimental Probability**	Statistics and Probability	**7.SP.8**
4	**Applications Using Probability**	Statistics and Probability	**7.SP.8**
5	**Box Plots**	Statistics and Probability	**7.SP.4**
6	**Interpreting Box Plots**	Statistics and Probability	**7.SP.4**
Unit 5 Geometry and Measurement			
1	**Angles and Shapes**	Geometry	**7.G.5**
2	**Parallel and Perpendicular Lines**	Geometry	**7.G.5**
3	**Pi, Circumference, and Diameter**	Geometry	**7.G.4**
4	**Area**	Geometry	**7.G.6**
5	**Surface Area**	Geometry	**7.G.6**
6	**Space Figures**	Geometry	**7.G.6**

Key Standards by Lesson Week

Week	Lesson	Domain	Key Standard
Level J			
Unit 1 Number Sense			
1	Rational Numbers	The Number System	8.NS.1
2	Exponents	Expressions and Equations	8.EE.1
3	Operations with Exponents	Expressions and Equations	8.EE.1
4	Scientific Notation, Greater Than 1	Expressions and Equations	8.EE.4
5	Scientific Notation, Between 1 and 0	Expressions and Equations	8.EE.4
6	Real Numbers	The Number System	8.NS.1
Unit 2 Operations Sense			
1	Squares	Expressions and Equations	8.EE.2
2	Cubes	Expressions and Equations	8.EE.2
3	Adding and Subtracting Integers	Expressions and Equations	8.EE.7
4	Multiplying and Dividing Integers	Expressions and Equations	8.EE.7
5	Solving Equations	Expressions and Equations	8.EE.7
6	Word Problems with Equations	Expressions and Equations	8.EE.7
Unit 3 Algebra			
1	Solving Multistep Equations	Expressions and Equations	8.EE.7b
2	Investigating Slope	Functions	8.F.3
3	Functions and Graphs	Functions	8.F.1
4	Linear Functions and Graphs	Functions	8.F.3
5	Comparing Lines	Functions	8.F.5
6	Nonlinear Functions and Graphs	Functions	8.F.3
Unit 4 Statistical Analysis			
1	Box Plots	Statistics and Probability	8.SP.1
2	Scatter Plots	Statistics and Probability	8.SP.1
3	Scatter Plot Relationships	Statistics and Probability	8.SP.1
4	Line of Best Fit	Statistics and Probability	8.SP.2
5	Using One-Way Tables	Statistics and Probability	8.SP.4
6	Using Two-Way Tables	Statistics and Probability	8.SP.4
Unit 5 Geometry and Measurement			
1	Pythagorean Theorem	Geometry	8.G.7
2	Symmetry	Geometry	8.G.3
3	Translations	Geometry	8.G.3
4	Rotations and Dilations	Geometry	8.G.3
5	Circles and Cylinders	Geometry	8.G.9
6	Cones, Cylinders, and Spheres	Geometry	8.G.9

Engage Your Students

Engage with Variety

A hallmark of ***SRA Number Worlds*** is that students who participate in the program become more and more engaged in mathematics. When students are engaged in learning, they will be more successful.

Engage is the heart of the daily routines that comprise ***SRA Number Worlds*** lesson instruction. Instead of a predictable pattern of activity in math class of studying a model-worked example and practicing that model, **Engage** offers a variety of instructional techniques that are appropriate for the concept being taught and appeal to a variety of learners.

Techniques include:

- Hands-On Activities,
- Games,
- Digital Activities,
- Interactive Whiteboard Activities,
- Guided Discussions,
- Free Response Activities,
- Writing Prompts, and
- Paper-and-Pencil Activities.

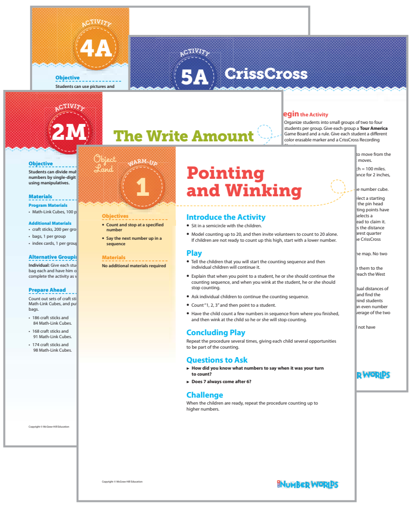
Engage through Collaboration and Higher-Order Thinking

A major focus of the Common Core Standards for Mathematical Practice is on the communication of mathematical ideas. Mathematically proficient students use language and symbols precisely to explain solutions, justify conclusions, and critique the reasoning of others. ***SRA Number Worlds*** provides students with daily opportunities to communicate mathematical ideas and engage in higher-order thinking.

The Teacher Edition and Instructional Activity Cards suggest alternative groupings that support oral communication, as well as discussion questions that promote higher-order thinking. The Reflect questions in the ***Student Workbook*** provide students with an opportunity to communicate about mathematical ideas in writing, and challenge students to gain deeper understandings of lesson concepts.

SRA Number Worlds weaves problem solving throughout each lesson week. Students are encouraged to find their own strategies to solve problems, and to work collaboratively to develop additional problem-solving strategies. Throughout ***SRA Number Worlds***, students develop, compare, contrast, and evaluate their own efforts to solve problems.

Through Project-Based Learning activities, students take this one step further, applying and translating mathematical skills and strategies in many different contexts. These activities encourage learners to take a deeper ownership of the concepts while, at the same time, requiring students to become thinkers and communicators of mathematics. This ongoing flow of ideas within the classroom is a powerful practice for ensuring student mastery of mathematics.



Engage with Digital Resources

The ***SRA Number Worlds*** program includes online resources for you and your students. Digital resources are designed to make classroom management effective and efficient, as well as make instruction fun and engaging.

The Teacher Dashboard makes it easier than ever to assess and track student progress. Online resources include progress monitoring and reporting tools, as well as digital assessments.

Digital resources for students include digitized versions of game boards, playing cards, manipulatives, Digital Math Tools, and ***Building Blocks***™ activities. The table below outlines the Digital Math Tools that are available for you and your students to explore concepts and solve problems.

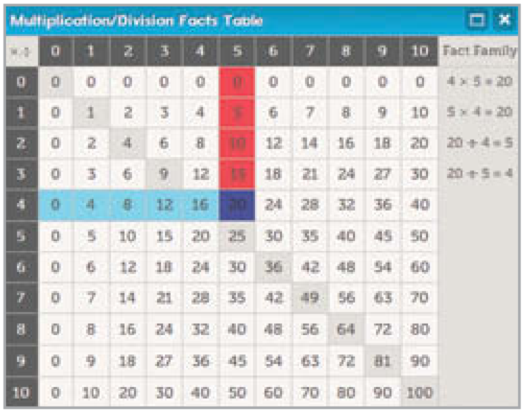
Digital Math Tools

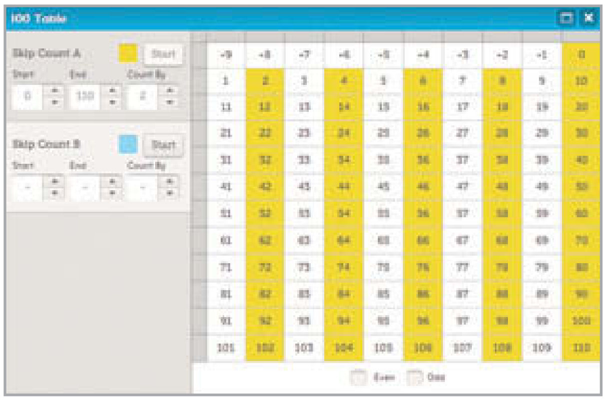
Calculation and Counting Tools	100 Table, Addition Table, Arrays, Base 10 Blocks, Calculator, Coins and Money, Fraction Tool, Function Machine, Multiplication Table, Number Line, Number Stairs, Set Former
Data Organization and Display Tools	Coordinate Grid, Graphing and Spreadsheet Tool, Pictograph, Venn diagram
Geometric Exploration Tools	Geoboard, Geometry Sketch Tool, Net Tool, Pythagorean Theorem, Shape Creator, Shape Tool, Tessellation Tool
Measurement and Conversion Tools	Calendar, Double Pan Balance, Estimating Proportions Tool, Metric and Customary Conversion Tool, Pitcher, Stopwatch, Thermometer
Probability Tools	Coin Flip, Probability

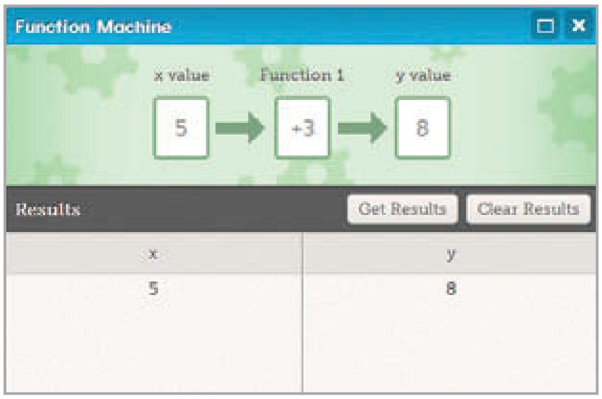
Building Blocks*™**, which is an integral part of the ***SRA Number Worlds program, is the result of National Science Foundation-funded research. Learning trajectories allow you to build the mathematical thinking of your students as it develops naturally. ***Building Blocks*** includes research-based computer tools with activities and a management system that guides students through research-based learning trajectories.

The ***Building Blocks*** program is designed to:

- Build upon young students' experiences with mathematics with activities that integrate ways to explore and represent mathematics,
- Involve students in "doing mathematics",
- Establish a strong foundation,
- Develop a strong conceptual framework,
- Emphasize students' mathematical thinking and reasoning abilities, and
- Encourage learning in line with Common Core State Standards.







Engage with Effective, Efficient Planning

The Lesson Plans in the Teacher Dashboard provide an overview of each selected lesson, suggested groupings for differentiation, as well as the resources and materials needed for instruction. Digital materials can be aligned to your teaching time by selecting plans that are tailored to 30-, 45-, or 60-minute instructional blocks. Lessons can be assigned to students and added to a customizable calendar for quicker access and pacing support.

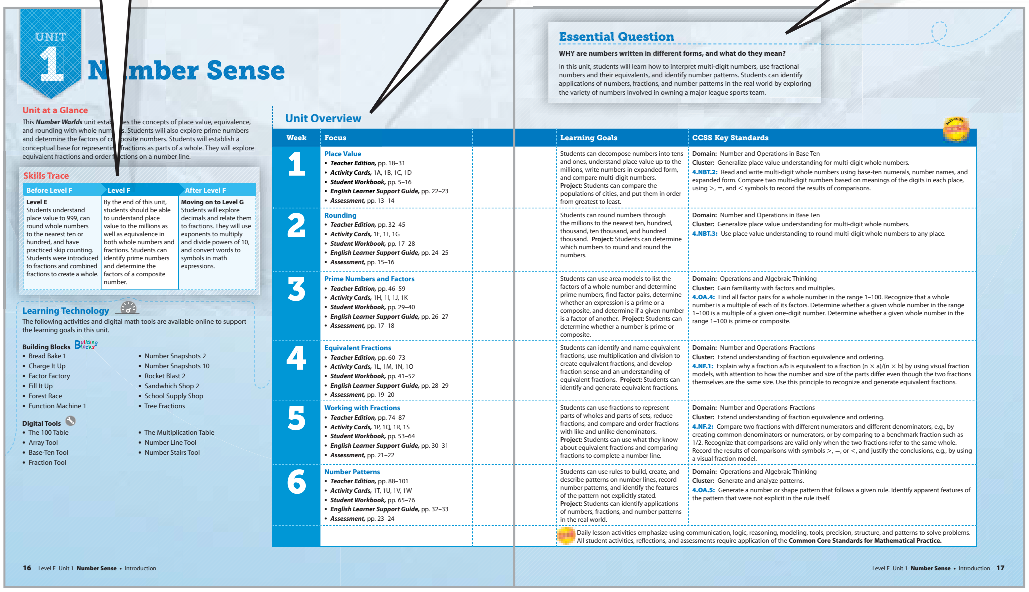
In the Teacher Edition, Unit Overviews and Weekly Planners help teachers plan lessons and prepare learning materials for students. Unit Overviews indicate the instructional focus of each week. Each Unit Overview includes a week-by-week summary of the learning goals and Common Core State Standards that are the focus of the unit. Unit Overviews also list the needed ***Activity Cards, Student Workbook*** pages, ***Practice*** pages, and ***Assessment*** pages.

Each Unit Overview also contains a summary of key mathematical concepts and a Skills Trace. Each Skills Trace describes skills learned in previous units and how the skills learned in the unit will be used in future lessons. This feature ensures coherence of instruction as students progress through the math curriculum.

Skills Trace gives an overview of the skills and concepts students will explore in the unit, and describes how unit topics connect to similar topics in earlier and later ***SRA Number Worlds*** levels.

Unit Overview provides information about each week's focus, learning goals, and targeted Common Core State Standard(s).

The Essential Question helps focus instruction on main ideas. Lessons and cumulative projects are developed around each unit's Essential Question.



UNIT 1 N[illegible]mber Sense

Unit at a Glance

This ***Number Worlds*** unit estab[illegible]es the concepts of place value, equivalence, and rounding with whole num[illegible]s. Students will also explore prime numbers and determine the factors of co[illegible]posite numbers. Students will establish a conceptual base for representin[illegible] fractions as parts of a whole. They will explore equivalent fractions and order f[illegible]ctions on a number line.

Skills Trace

Before Level F	Level F	After Level F
Level E Students understand place value to 999, can round whole numbers to the nearest ten or hundred, and have practiced skip counting. Students were introduced to fractions and combined fractions to create a whole.	By the end of this unit, students should be able to understand place value to the millions as well as equivalence in both whole numbers and fractions. Students can identify prime numbers and determine the factors of a composite number.	**Moving on to Level G** Students will explore decimals and relate them to fractions. They will use exponents to multiply and divide powers of 10, and convert words to symbols in math expressions.

Learning Technology

The following activities and digital math tools are available online to support the learning goals in this unit.

Building Blocks

- Bread Bake 1
- Charge It Up
- Factor Factory
- Fill It Up
- Forest Race
- Function Machine 1
- Number Snapshots 2
- Number Snapshots 10
- Rocket Blast 2
- Sandwhich Shop 2
- School Supply Shop
- Tree Fractions

Digital Tools

- The 100 Table
- Array Tool
- Base-Ten Tool
- Fraction Tool
- The Multiplication Table
- Number Line Tool
- Number Stairs Tool

16 Level F Unit 1 **Number Sense** • Introduction

Essential Question

WHY are numbers written in different forms, and what do they mean?

In this unit, students will learn how to interpret multi-digit numbers, use fractional numbers and their equivalents, and identify number patterns. Students can identify applications of numbers, fractions, and number patterns in the real world by exploring the variety of numbers involved in owning a major league sports team.

Unit Overview

Week	Focus	Learning Goals	CCSS Key Standards
1	**Place Value** • ***Teacher Edition,*** pp. 18–31 • ***Activity Cards,*** 1A, 1B, 1C, 1D • ***Student Workbook,*** pp. 5–16 • ***English Learner Support Guide,*** pp. 22–23 • ***Assessment,*** pp. 13–14	Students can decompose numbers into tens and ones, understand place value up to the millions, write numbers in expanded form, and compare multi-digit numbers. **Project:** Students can compare the populations of cities, and put them in order from greatest to least.	**Domain:** Number and Operations in Base Ten **Cluster:** Generalize place value understanding for multi-digit whole numbers. **4.NBT.2:** Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
2	**Rounding** • ***Teacher Edition,*** pp. 32–45 • ***Activity Cards,*** 1E, 1F, 1G • ***Student Workbook,*** pp. 17–28 • ***English Learner Support Guide,*** pp. 24–25 • ***Assessment,*** pp. 15–16	Students can round numbers through the millions to the nearest ten, hundred, thousand, ten thousand, and hundred thousand. **Project:** Students can determine which numbers to round and round the numbers.	**Domain:** Number and Operations in Base Ten **Cluster:** Generalize place value understanding for multi-digit whole numbers. **4.NBT.3:** Use place value understanding to round multi-digit whole numbers to any place.
3	**Prime Numbers and Factors** • ***Teacher Edition,*** pp. 46–59 • ***Activity Cards,*** 1H, 1I, 1J, 1K • ***Student Workbook,*** pp. 29–40 • ***English Learner Support Guide,*** pp. 26–27 • ***Assessment,*** pp. 17–18	Students can use area models to list the factors of a whole number and determine prime numbers, find factor pairs, determine whether an expression is a prime or a composite, and determine if a given number is a factor of another. **Project:** Students can determine whether a number is prime or composite.	**Domain:** Operations and Algebraic Thinking **Cluster:** Gain familiarity with factors and multiples. **4.OA.4:** Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
4	**Equivalent Fractions** • ***Teacher Edition,*** pp. 60–73 • ***Activity Cards,*** 1L, 1M, 1N, 1O • ***Student Workbook,*** pp. 41–52 • ***English Learner Support Guide,*** pp. 28–29 • ***Assessment,*** pp. 19–20	Students can identify and name equivalent fractions, use multiplication and division to create equivalent fractions, and develop fraction sense and an understanding of equivalent fractions. **Project:** Students can identify and generate equivalent fractions.	**Domain:** Number and Operations-Fractions **Cluster:** Extend understanding of fraction equivalence and ordering. **4.NF.1:** Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
5	**Working with Fractions** • ***Teacher Edition,*** pp. 74–87 • ***Activity Cards,*** 1P, 1Q, 1R, 1S • ***Student Workbook,*** pp. 53–64 • ***English Learner Support Guide,*** pp. 30–31 • ***Assessment,*** pp. 21–22	Students can use fractions to represent parts of wholes and parts of sets, reduce fractions, and compare and order fractions with like and unlike denominators. **Project:** Students can use what they know about equivalent fractions and comparing fractions to complete a number line.	**Domain:** Number and Operations-Fractions **Cluster:** Extend understanding of fraction equivalence and ordering. **4.NF.2:** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
6	**Number Patterns** • ***Teacher Edition,*** pp. 88–101 • ***Activity Cards,*** 1T, 1U, 1V, 1W • ***Student Workbook,*** pp. 65–76 • ***English Learner Support Guide,*** pp. 32–33 • ***Assessment,*** pp. 23–24	Students can use rules to build, create, and describe patterns on number lines, record number patterns, and identify the features of the pattern not explicitly stated. **Project:** Students can identify applications of numbers, fractions, and number patterns in the real world.	**Domain:** Operations and Algebraic Thinking **Cluster:** Generate and analyze patterns. **4.OA.5:** Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

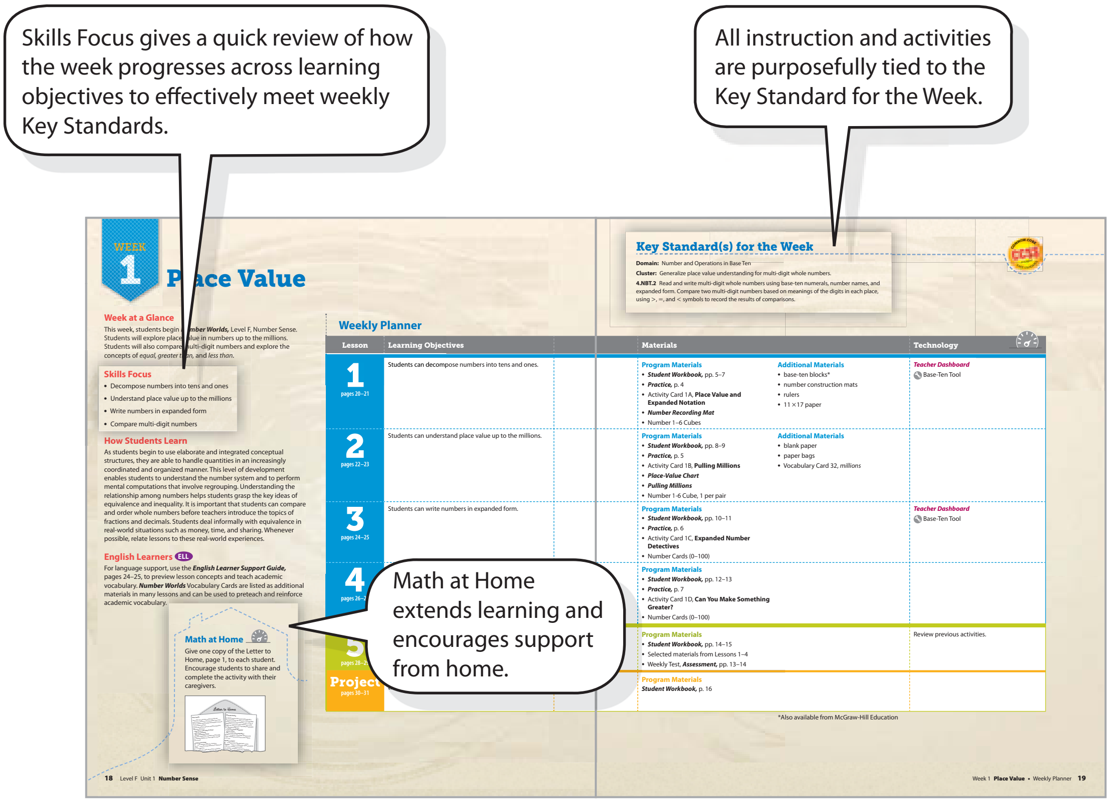
Daily lesson activities emphasize using communication, logic, reasoning, modeling, tools, precision, structure, and patterns to solve problems. All student activities, reflections, and assessments require application of the **Common Core Standards for Mathematical Practice.**

Level F Unit 1 **Number Sense** • Introduction 17

Weekly Planners provide background information for key mathematical concepts and offer lesson-by-lesson overviews of the learning goals and Common Core State Standards that are the focus of each lesson. In addition to listing the ***Activity Cards***, ***Student Workbook*** pages, ***Practice*** pages, and ***Assessment*** pages needed for instruction, the Weekly Planners indicate additional materials that will be used and technology that supports and enhances lessons. Each weekly planner also directs you to a Math at Home activity.

In the Teacher Editon, Lesson Plans indicate each lesson's objective, key standard, and vocabulary. Each Lesson Plan also provides a 4-part lesson structure outlining that day's instruction and offers suggestions for creating context for English Learners.

In Lessons 1–4, **Engage** provides two practice options, Independent Practice and Supported Practice. Data from the Unit Pretest can be used to determine which students would benefit from working in a supported group and which would benefit from an independent practice opportunity.



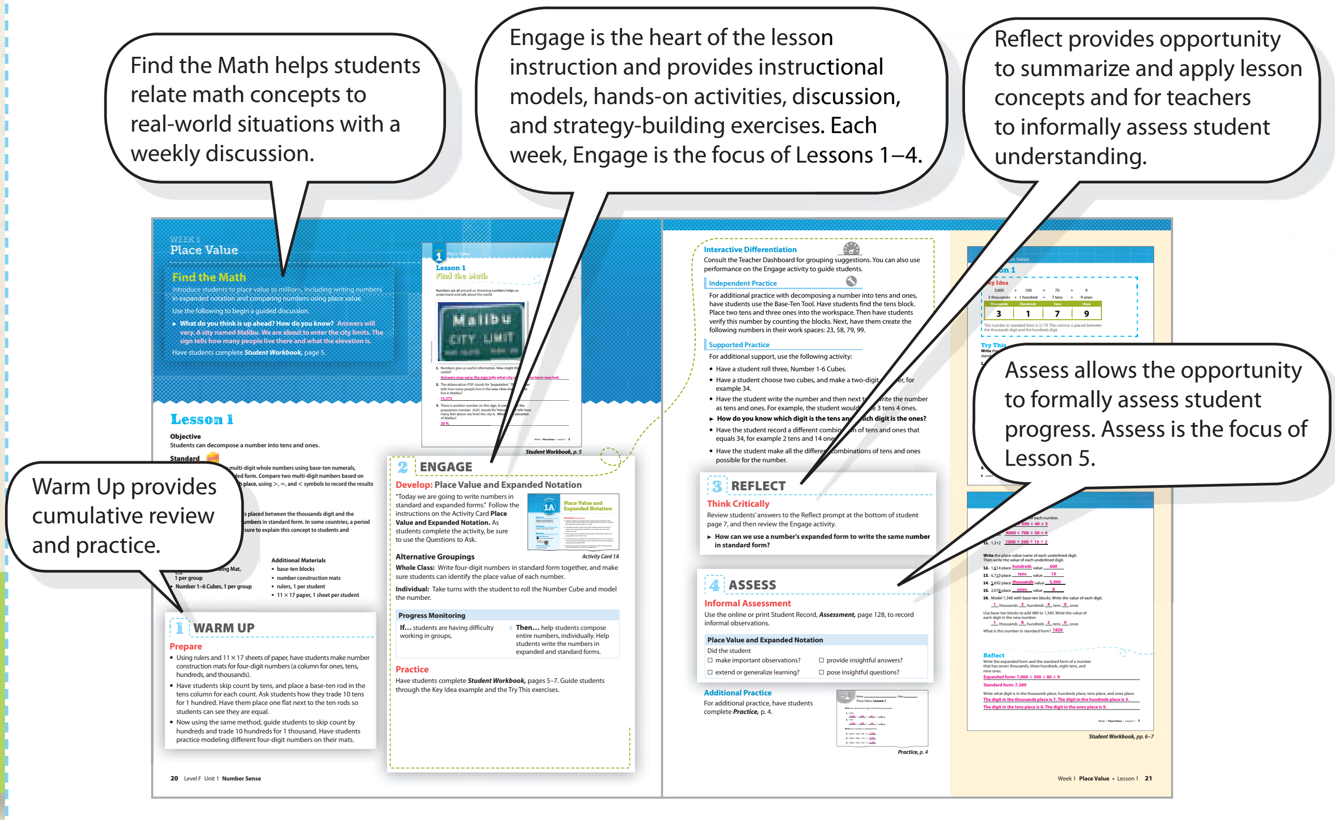
Engage by Using the 4-Part Lesson Structure

Every lesson in ***SRA Number Worlds*** is organized in the same way. Lessons begin with an overview of the day's objective and Common Core State Standard(s), along with the materials needed to facilitate the day's activities.

In Levels D–J, each lesson week begins with an opportunity for students to connect mathematics with real-world experiences. In Find the Math, students respond to questions based on a real-world context that relates to the week's mathematical focus.

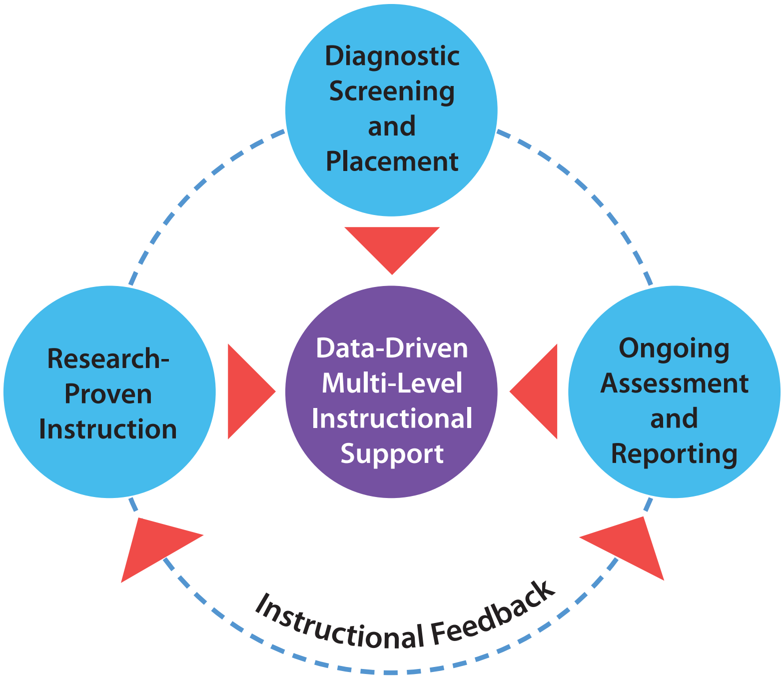
At all levels, lesson instruction is divided into four distinct sections for simplified time management in the classroom: Warm Up, Engage, Reflect, and Assess.

The Teacher Edition offers point-of-use suggestions to differentiate instruction, ask effective questions, and form collaborative groups. Strategies are provided to monitor student progress during instruction and assess end-of-the lesson understanding.



Engage through Intervention

An intervention is any instructional or practice activity designed to help students who are not making adequate progress. ***SRA Number Worlds*** is designed for both Tier 2 and Tier 3 students. Levels A–C are also appropriate for on-level Tier 1 students.



Layers of intervention are organized in tiers to enable teachers to respond to student needs. Tier 2 students may spend a brief time in the program learning a key concept and then be quickly reintegrated back into the core instructional program. Students in Tier 3 will most likely need to complete the entire ***SRA Number Worlds*** curriculum at their learning levels. Tier 3 requires intensive intervention for students with low skills and a sustained lack of adequate progress within Tiers 1 and 2.

SRA Number Worlds mathematics intervention offers flexible implementation options, instruction built on the Common Core State Standards, engaging resources, and assessment and reporting for placed students that differentiate instruction and measure progress toward learning goals and standards.

Engage with Flexible Implementation Options

The 4-part structure of each lesson allows for flexible implementation to meet your students' needs. The table below gives suggested time allocations for 30-minute, 45-minute, and 60-minute instructional periods. A 60-minute instructional period is recommended for Tier 3 students.

Engage is the heart of Lessons 1–4, and **Assess** is the focus of Lesson 5. In each Teacher Edition, the focus of a lesson is indicated with a dashed outline. **Engage** and **Assess** should receive the greatest allocation of time on Days 1–4 and Day 5, respectively.

If You Have 30 Minutes		Levels A-C		Levels D-J	
		TIME	ACTIVITY	TIME	ACTIVITY
Days 1-4	ENGAGE	30	Activity Card	15	Activity Card (Develop)
				15	Student Workbook (Practice)
Day 5	ENGAGE	15	Free-Choice Activity Card	15	Practice
	ASSESS	15	Formal Assessment	15	Formal Assessment

If You Have 45 Minutes		Levels A-C		Levels D-J	
		TIME	ACTIVITY	TIME	ACTIVITY
Days 1-4	ENGAGE	30	Activity Card	15	Activity Card (Develop)
				15	Student Workbook (Practice)
		15	Interactive Differentiation	15	Interactive Differentiation
Day 5	ENGAGE	15	Free-Choice Activity Card	15	Student Workbook (Practice)
	ASSESS	15	Formal Assessment	15	Formal Assessment
		15-20	Project-Based Learning	15-20	Project-Based Learning

If You Have 60 Minutes		Levels A-C		Levels D-J	
		TIME	ACTIVITY	TIME	ACTIVITY
Days 1-4	WARM UP	5	Warm Up Card	5	Prepare
	ENGAGE	30	Activity Card	15	Activity Card (Develop)
				15	Student Workbook (Practice)
		15	Interactive Differentiation	15	Interactive Differentiation
	REFLECT	5	Extended Response	5	Think Critically
	ASSESS	5	Informal Assessment	5	Informal Assessment
Day 5	WARM UP	5	Warm Up Card	5	Prepare
	ENGAGE	15	Free-Choice Activity Card	15	Student Workbook (Practice)
	REFLECT	5-10	Extended Response	5-10	Think Critically
	ASSESS	15	Formal Assessment	15	Formal Assessment
		15-20	Project-Based Learning	15-20	Project-Based Learning

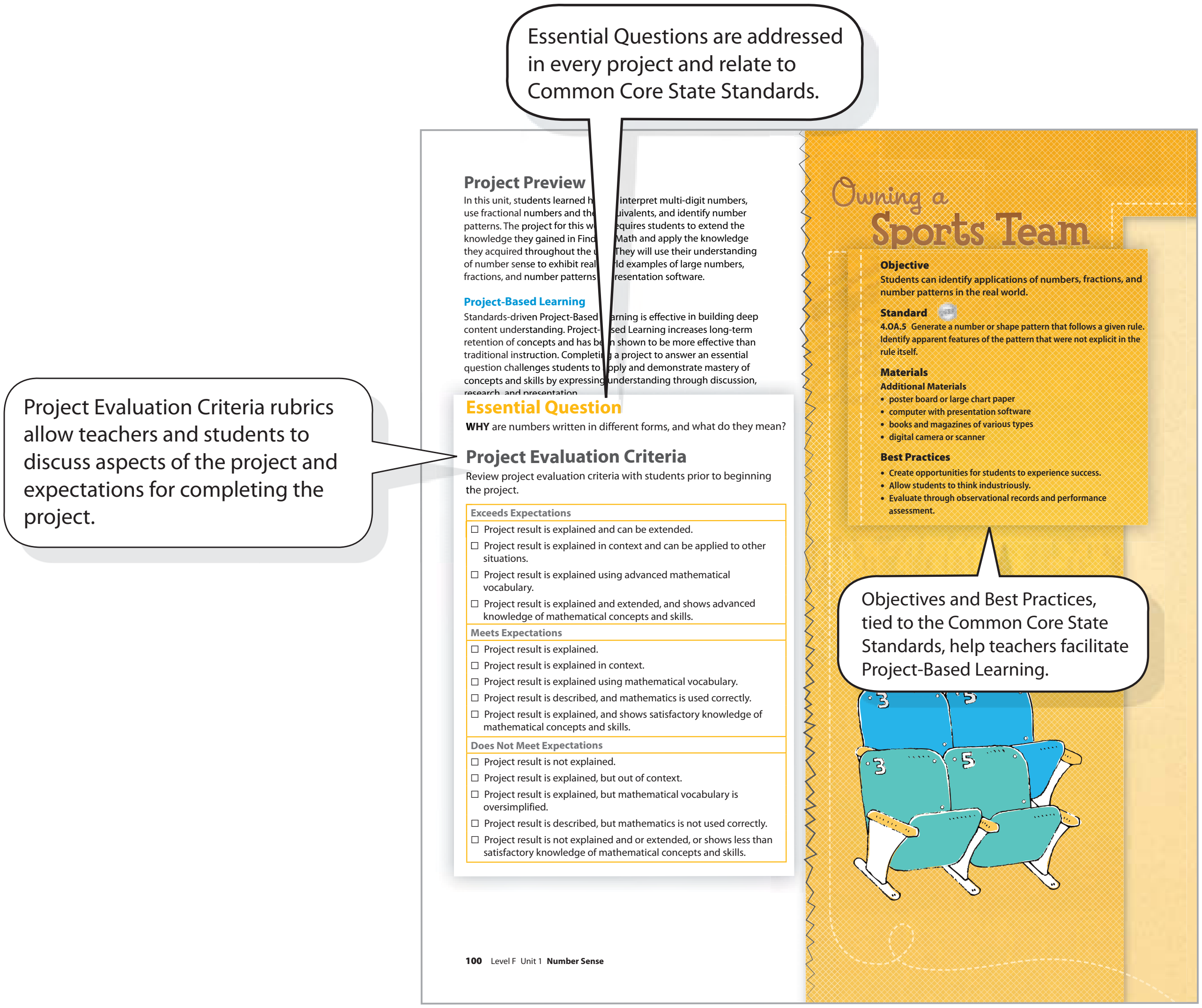
Because of ***SRA Number Worlds'*** hands-on, engaging activities, the program can be used effectively in a variety of instructional settings. The table below offers suggestions for program use in situations ranging from whole class instruction to tutoring and summer school settings.

Situation	Suggested Use
Whole Class	Levels A–C, in particular, are designed for whole-class settings, but all levels can be used for whole-class instruction.
Math Resource Room	***SRA Number Worlds*** is effective in small group pull-out sessions with a teacher or teacher's aide.
Special Education	Students with special needs benefit from ***SRA Number Worlds'*** intensive, concrete, multi-sensory developmental progression of abstract concepts.
After School	Because the activities are engaging, ***SRA Number Worlds'*** intensive math activities work well for both intervention and regular education students in after-school programs.
Summer School	***SRA Number Worlds*** is ideal for mathematics summer school.
Tutoring	***SRA Number Worlds*** is highly effective when used in an individualized tutoring session in which a teacher or an aide participates in activities.

Assess with Project-Based Learning

Project-Based Learning, or PBL, is an instructional approach in which students work collaboratively on a real-world based problem or challenge. As students explore real-world situations, group activities are facilitated by a teacher. At the conclusion of an investigation, students demonstrate mastery of concepts and skills in a variety of ways, such as creating displays, making presentations, and creating models.

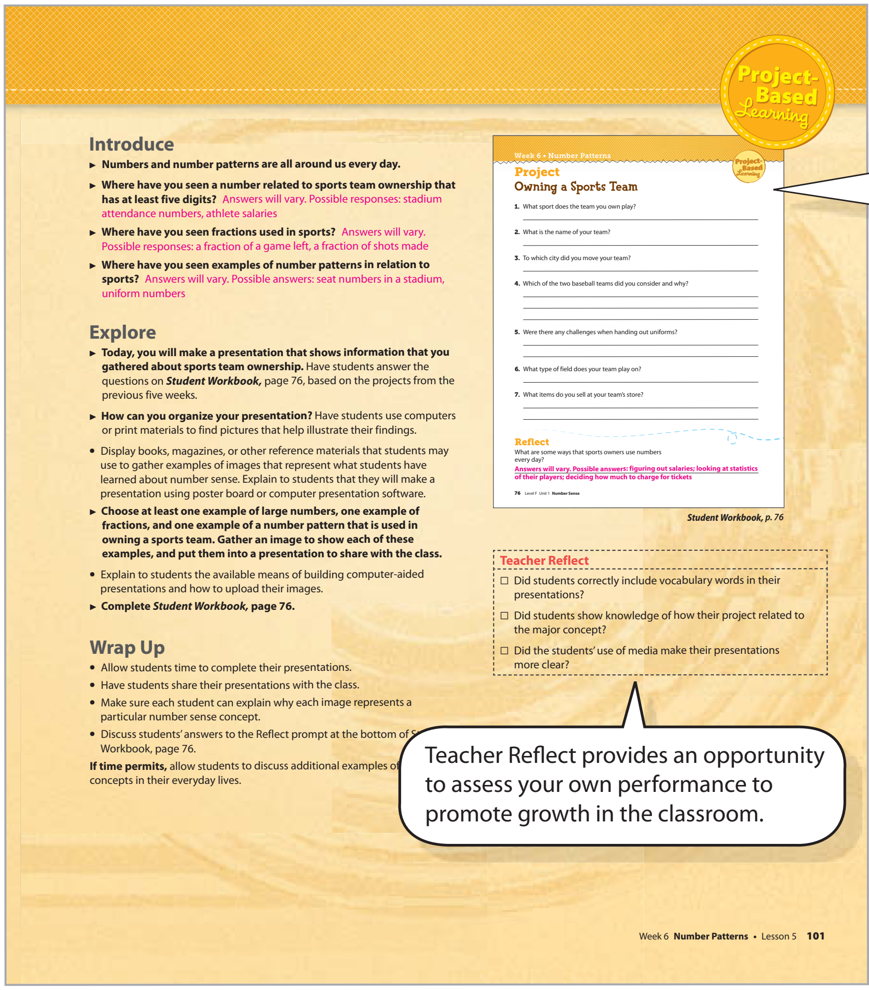
Project-Based Learning has been shown to increase student motivation and enhance student engagement. In addition, because students are engaging in authentic learning, Project-Based Learning increases long-term retention of concepts and has been shown to be more effective than traditional instruction.



SRA Number Worlds' lessons are developed around Essential Questions, which can be found on the Project pages for Levels A–C and on the Unit Opener pages for Levels D–J. In addition, each Lesson 1 in Levels D–J begins with a Find the Math real-world connection. Essential Questions and Find the Math features are first links to ***SRA Number Worlds*** projects.

In Levels A–C, students participate in activities for three weeks that build to a cumulative project at the end of every fourth week. In Levels D–J, students participate in weekly activities that culminate in Unit Projects.

For each project, the Teacher Edition provides guiding questions and directions you can use to facilitate student explorations. It also provides a rubric to evaluate student projects.



Project-Based Learning

Introduce

- **Numbers and number patterns are all around us every day.**
- **Where have you seen a number related to sports team ownership that has at least five digits?** Answers will vary. Possible responses: stadium attendance numbers, athlete salaries
- **Where have you seen fractions used in sports?** Answers will vary. Possible responses: a fraction of a game left, a fraction of shots made
- **Where have you seen examples of number patterns in relation to sports?** Answers will vary. Possible answers: seat numbers in a stadium, uniform numbers

Explore

- **Today, you will make a presentation that shows information that you gathered about sports team ownership.** Have students answer the questions on ***Student Workbook,*** page 76, based on the projects from the previous five weeks.
- **How can you organize your presentation?** Have students use computers or print materials to find pictures that help illustrate their findings.
- Display books, magazines, or other reference materials that students may use to gather examples of images that represent what students have learned about number sense. Explain to students that they will make a presentation using poster board or computer presentation software.
- **Choose at least one example of large numbers, one example of fractions, and one example of a number pattern that is used in owning a sports team. Gather an image to show each of these examples, and put them into a presentation to share with the class.**
- Explain to students the available means of building computer-aided presentations and how to upload their images.
- **Complete *Student Workbook,* page 76.**

Wrap Up

- Allow students time to complete their presentations.
- Have students share their presentations with the class.
- Make sure each student can explain why each image represents a particular number sense concept.
- Discuss students' answers to the Reflect prompt at the bottom of S Workbook, page 76.

If time permits, allow students to discuss additional examples of concepts in their everyday lives.

Week 6 • Number Patterns

Project

Owning a Sports Team

1. What sport does the team you own play?
2. What is the name of your team?
3. To which city did you move your team?
4. Which of the two baseball teams did you consider and why?
5. Were there any challenges when handing out uniforms?
6. What type of field does your team play on?
7. What items do you sell at your team's store?

Reflect

What are some ways that sports owners use numbers every day?

Answers will vary. Possible answers: figuring out salaries; looking at statistics of their players; deciding how much to charge for tickets

76 Level F Unit 1 **Number Sense**

Student Workbook, p. 76

Teacher Reflect

- ☐ Did students correctly include vocabulary words in their presentations?
- ☐ Did students show knowledge of how their project related to the major concept?
- ☐ Did the students' use of media make their presentations more clear?

Week 6 **Number Patterns** • Lesson 5 **101**

Student Workbooks provide a place for students to communicate thoughts and ideas throughout the project and develop writing skills.

Teacher Reflect provides an opportunity to assess your own performance to promote growth in the classroom.

Assess to Meet Students' Needs

There are two placement tests that can be used to place students into the appropriate ***SRA Number Worlds*** program level: the Number Knowledge Test and the Placement Test. The Number Knowledge Test can be used as an initial screener in Grades Pre–K to 1 to place students in one of the three prevention levels. The Number Knowledge Test is a validated, research-based test that assesses students' basic understanding of number sense and operations. The test is separated into four sections, with a minimum score for each section that students need to reach or exceed in order to move on to the next section. It is an oral test which is given individually and takes about 5 to 10 minutes to complete. Once a student completes the test or fails to reach the maximum score for one of the sections, a raw score conversation chart can be used to place the student in Levels A–G.

The ***SRA Number Worlds*** Placement Test can be used for student placement in any of the program levels. Much like the Number Knowledge Test, the Placement Test is separated into sections. Each section represents a program level, with each section containing a maximum of 10 questions. A student's raw score on a section will either place that student in that specific level or indicate that another section of the test should be completed to determine an appropriate placement. If a score is too low or too high, a student should take the section before or after the section just completed. Although this is a manually scored test, multiple students can be given the Placement Test at the same time, even if students begin the test at different sections.

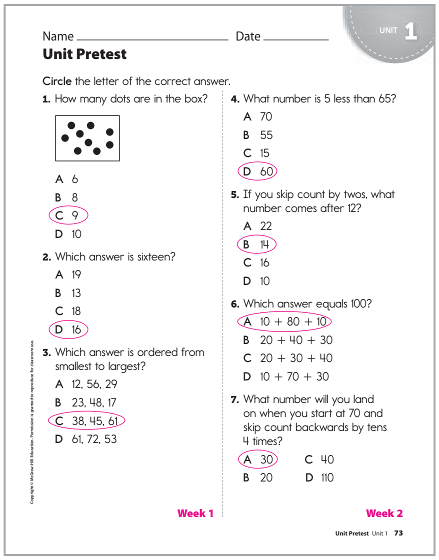
The Number Knowledge Test's Developmental Conversion Chart

Raw Test Score	Developmental Age Score (Chronological Age Equivalents)	Grade Level Equivalents	***SRA Number Worlds* Level**
1–6	3–4 years	Preschool	Level A
7–8	4–5 years	PreK–K	Level B
9–14	5–6 years	K–1	Level C
15–19	6–7 years	1–2	Level D
20–25	7–8 years	2–3	Level E
26–28	8–9 years	3–4	Level F
29–30	9–10 years	4–5	Level G

After using the Placement Test to identify students' levels within ***Number Worlds,*** teachers can assign a unit pretest to determine students' baseline measures for the selected unit. When students have completed the unit, teachers can evaluate students' mastery by assigning the posttest.

Although items in each unit's pretest and posttest differ, the tests are identical in structure and content. Available online, both assessments are multiple choice tests built on the Common Core State Standards. Both are scored automatically, and teachers may view students' results in the Assessment and Reporting area of the online platform.

Unlike the ***Number Worlds*** Placement Test, the pretests are not placement tests; instead, they focus on specific topics at particular levels. Finally, the posttests are valuable progress-monitoring tools, especially when teachers use them in conjunction with other data sources, such as *Student Workbook* and *Practice Book* pages, Weekly Tests, Projects, and in-class observations.


UNIT 1

Name ______ Date ______

Unit Pretest

Circle the letter of the correct answer.

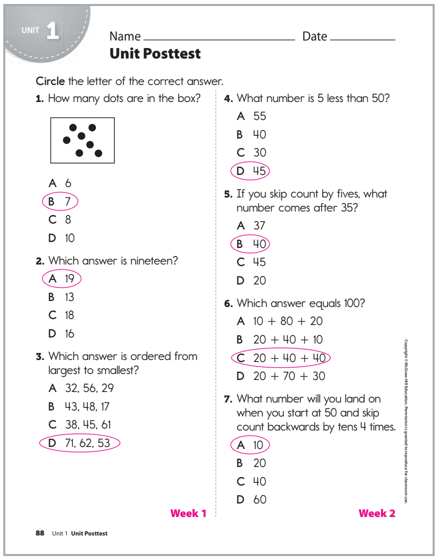
1. How many dots are in the box?
   A 6
   B 8
   C 9
   D 10
2. Which answer is sixteen?
   A 19
   B 13
   C 18
   D 16
3. Which answer is ordered from smallest to largest?
   A 12, 56, 29
   B 23, 48, 17
   C 38, 45, 61
   D 61, 72, 53

Week 1

4. What number is 5 less than 65?
   A 70
   B 55
   C 15
   D 60
5. If you skip count by twos, what number comes after 12?
   A 22
   B 14
   C 16
   D 10
6. Which answer equals 100?
   A 10 + 80 + 10
   B 20 + 40 + 30
   C 20 + 30 + 40
   D 10 + 70 + 30
7. What number will you land on when you start at 70 and skip count backwards by tens 4 times?
   A 30
   B 20
   C 40
   D 110

Week 2

Unit Pretest Unit 1 73


UNIT 1

Name ______ Date ______

Unit Posttest

Circle the letter of the correct answer.

1. How many dots are in the box?
   A 6
   B 7
   C 8
   D 10
2. Which answer is nineteen?
   A 19
   B 13
   C 18
   D 16
3. Which answer is ordered from largest to smallest?
   A 32, 56, 29
   B 43, 48, 17
   C 38, 45, 61
   D 71, 62, 53

Week 1

4. What number is 5 less than 50?
   A 55
   B 40
   C 30
   D 45
5. If you skip count by fives, what number comes after 35?
   A 37
   B 40
   C 45
   D 20
6. Which answer equals 100?
   A 10 + 80 + 20
   B 20 + 40 + 10
   C 20 + 40 + 40
   D 20 + 70 + 30
7. What number will you land on when you start at 50 and skip count backwards by tens 4 times?
   A 10
   B 20
   C 40
   D 60

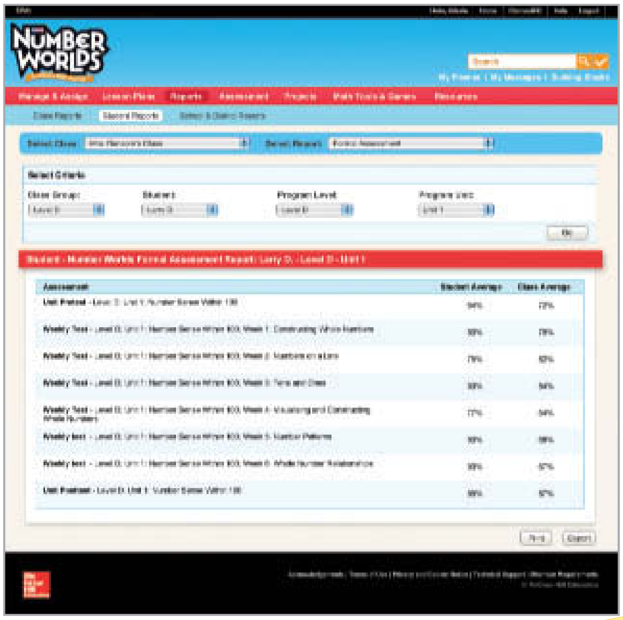
Week 2

88 Unit 1 Unit Posttest

Assess to Monitor Progress

Each Prevention level contains weekly and cumulative assessments. For each level, there are 32 weekly assessments and eight cumulative assessments, available in digital and print versions. The print versions contain a wide variety of question types that reflect concepts learned in each week. The digital versions closely reflect the question types in the printed versions, but with interactive questions and auto-scoring. If digital assessments are assigned, scores will be reported in the Assessment and Reporting areas of the teacher dashboard. If print assessments are used, scores can be recorded using the Student Assessment Record blackline masters or the online Assessment feature.

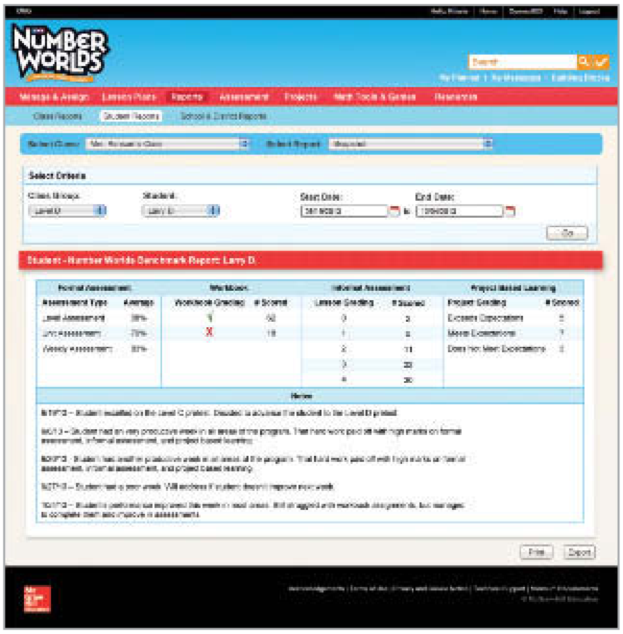
Each Intervention level contains weekly assessments, unit pretests, and unit posttests. For each level, there are 30 weekly assessments, five unit pretests, and five unit posttests. All of these assessments are offered both digitally and in print. These assessments have the same assignment and scoring features as the Prevention level assessments. The unit pretests can also be effectively used to place students in Tier 2, module-based learning. Using the program in this way allows for concentrated, topic-based instruction.



In addition to formal assessment scores, there are also several elements of the program where qualitative, informal assessment scores can be observed and recorded. For Levels A and B, a score of 0–4 can be awarded for student performance in both the **Warm Up** and **Engage** activities on Days 1–4. There are rubrics to assist with scoring. A Project-Based Learning rubric can be used to determine weekly project scores.

For Level C, **Warm Up** and **Engage** activities are scored in the same way as levels A and B. Weekly project scoring is also the same. Starting in Level C, a grade of *complete* or *incomplete* can be entered for ***Student Workbook*** activities for Lessons 1, 2, and 5.

For Intervention Levels D–J, ***Student Workbook*** activities can be scored for all 5 lessons of each week. For Lessons 1–4, a rubric is provided to informally assess students' responses. Weekly project scoring is the same as for the prevention levels. For all levels, these scores can either be entered using the Student Assessment Record blackline masters or by using the online Assessment feature.



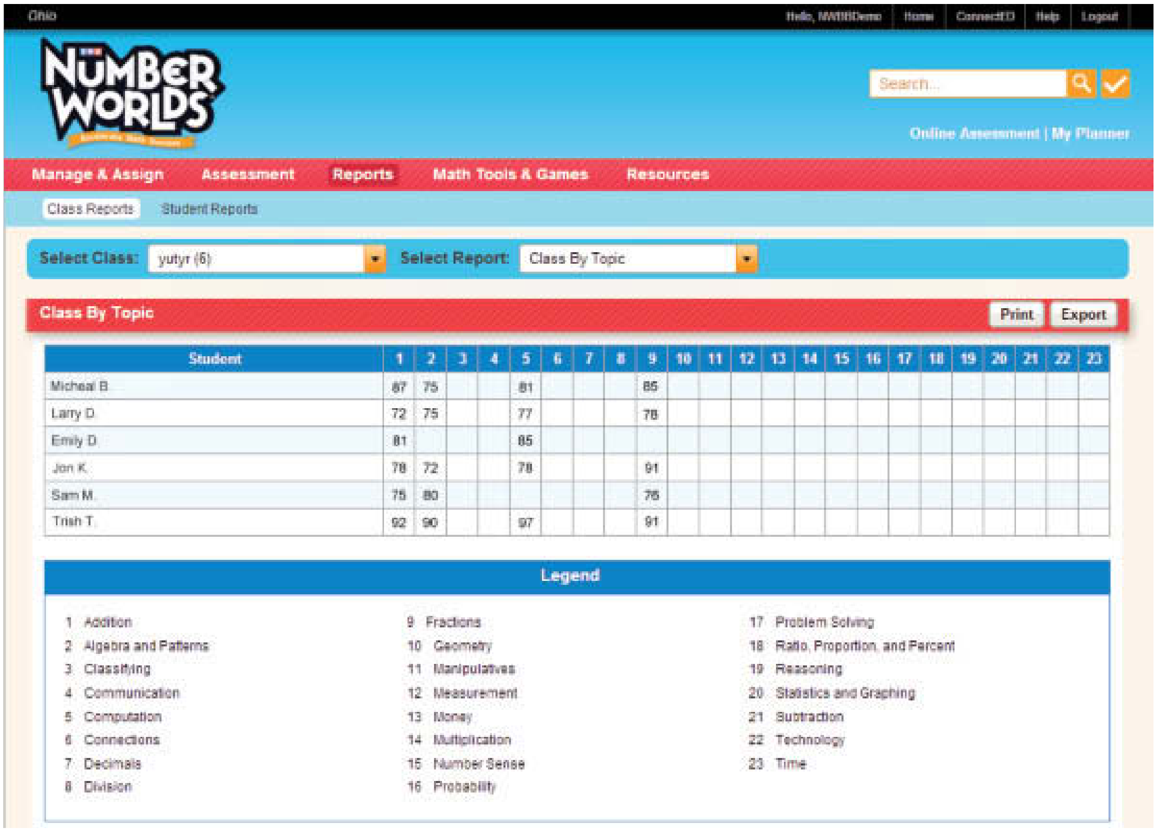
Assess with Digital Resources

Scores can be recorded in the Assessment section of the ***SRA Number Worlds*** Teacher Platform. Clicking on the Assessment tab allows for all formal and informal scores to be entered, edited, viewed, printed, and exported. An added benefit of entering scores online is the ability to enter a grade for an entire class and have that grade assigned to each student. Entering scores online also allows for a more complete picture of students' performance.

The ***SRA Number Worlds*** program also has a Reports section that offers a Common Core State Standard proficiency report, a Formal Assessment report, a Progress Over Time report, and ***Building Blocks*** reports at both the class level and the student level. The class reports show how students are performing compared to other students in the class as well as the class average. The student reports show how students are performing on a set of standards, on assessments, or over the course of the year. There are also administrator reports for users at the school and district levels.

With ***Building Blocks***, you can set up and customize the curriculum to meet the needs of every student. In the ***Building Blocks*** Learning Paths area of the ***Teacher Dashboard***, you can see an overview of each student's ***Building Blocks*** assignments, and adjust those assignments as needed.

You can also view reports to see how students are performing over time. The *Reports* section of the ***Teacher Dashboard*** offers various options, such as displaying student and class reports organized by activity, Common Core State Standards, topic, or trajectory. Since the reports provide real-time information, they allow you to monitor student progress as each student progresses through the adaptive curriculum at his or her own pace. These reports also help teachers evaluate students' needs and decide how to modify their assignments. For users at the school and district levels, Administrator Reports are also available.



Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Micheal B.	87	75			81				85														
Larry D.	72	75			77				78														
Emily D.	81				85																		
Jon K.	78	72			78				91														
Sam M.	75	80							76														
Trish T.	92	90			97				91														

Get Started with *Number Worlds*

Each week of instruction in ***SRA Number Worlds*** focuses on the skills and concepts germane to one or two key Common Core State Standards. For each lesson, the ***Teacher Edition*** presents the objective and relevant standards, as well as the academic vocabulary and context students need to know.

Below the standards, vocabulary words, and context information, the ***Teacher Edition*** also lists all of the resources needed for that day's lesson. These include materials, such as blackline masters, vocabulary cards, game boards, and manipulatives.

All ***SRA Number Worlds***' lessons have a 4-part lesson structure: **Warm Up, Engage, Reflect,** and **Assess**. This structure leads students to review previously learned skills and concepts, engage in activities that introduce new content, summarize and reflect on the day's concepts, and take an assessment on newly learned skills and concepts. Each of these sections supports teachers with engaging questions, instructional language, and meaningful activities to help students build skills, knowledge, and confidence. **Engage** is the primary instructional component for Days 1–4, while **Assess** is the core of the lesson on Day 5. Throughout the ***Teacher Edition,*** dashed outlines surrounding these sections remind teachers to allocate most of their instructional time to **Engage** and **Assess.** Furthermore, icons within these sections indicate the availability of digital resources, such as ***Building Blocks*** activities and ***Digital Math Tools.***

For each lesson, the ***Teacher Edition*** clearly identifies the resources and materials teachers need in order to facilitate student engagement and learning. In addition, **Engage** offers alternative grouping suggestions, differentiation options, ideas to facilitate student reflection, and tools to gauge student understanding.

SRA Number Worlds Prevention Levels A–C (for grades Pre-K to 1) give students with the foundational skills and concepts they need in order to be successful in mathematics. Intervention Levels D–J (for grades 2–8) help Tier 2 and Tier 3 students unlock the Key Common Core Standards by focusing on foundational skills and concepts.

Levels A–C are appropriate for whole-class instruction. Levels D–J can be used with Tier 2 students to target specific skills and concepts in short-term, pull-out settings. Alternatively, teachers may use Levels D-J as a supplemental mathematics program for Tier 3 students.

Prevention levels consist of 32 weeks of daily instruction. Intervention levels are comprised of five six-week units that can be taught in any order or in isolation. At all levels, weekly instruction is developed around one or two key Common Core State Standards for Mathematics Content. Because engagement is a hallmark of ***SRA Number Worlds,*** students at all levels have daily opportunities to employ the Common Core State Standards for Mathematical Practices.

As discussed previously, Project-Based Learning is an instructional approach in which students work collaboratively on real-world problems or challenges. All levels include projects that allow students to apply mathematical skills and concepts in real-world contexts. In Levels A–C, students participate in activities for three weeks that build to a cumulative project at the end of every fourth week. In Levels D–J, students participate in weekly activities that culminate in Unit Projects. The projects promote higher-level reasoning and provide opportunities for teachers to assess students as they practice 21st Century skills.

Get Started with the Common Core State Standards

The Common Core State Standards identify the knowledge students should attain and the concepts students should understand at each grade level. Every ***SRA Number Worlds*** Lesson Week is designed to focus on one or two key Common Core State Standards for Mathematical Content. By engaging in a sequence of activities designed to support the understanding of key foundational skills and concepts, students attain the prerequisite skills and background knowledge needed to access on-level Common Core standards.

Consider the following standard from the Grade 3 Operations and Algebraic Thinking domain.

> 3.OA.1 Interpret products of whole numbers, (e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each). *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

Lesson 1 lays a concrete foundation for the multiplication of whole numbers. Subsequent lessons build on this foundation one step at a time. By Day 5, the daily lessons have led to an on-level understanding of this key Common Core State Standard. For each lesson, strategies presented in the Teacher Edition can be used to enhance classroom instruction for all students.

Tier 2 students who have been identified as being deficient in this skill can receive targeted, short-term, pull-out instruction in 30–45 minute time increments to support the mastery and understanding of this key math concept. Tier 3 students can receive intensive instruction on this concept during a 60-minute supplemental intervention program.

Lesson	Objective
Lesson 1	Students use pictures of equal groups to create models for multiplication.
Lesson 2	Students use repeated addition to create models for multiplication.
Lesson 3	Students can use the × symbol to write multiplication problems.
Lesson 4	Students can describe groups that come in sets to build multiplication facts.

Get Started with Program Features

Unit Overviews and Weekly Planners (in the ***Teacher Edition***) and The Calendar (in the ***Teacher Dashboard***) provide general information about each unit's big ideas as well as the details needed for daily instruction. Both the ***Teacher Edition*** and ***the Teacher Dashboard*** also include specific information about the lessons' Common Core State Standards, vocabulary terms, resources needed, instructional activities, and strategies for differentiating instruction. In addition, the section called Creating Context offers strategies for supporting English Learners.

All lessons in Levels A–C, and Lessons 2–5 in Levels D–J, begin with **Warm Up**, activities that review previously learned concepts and skills. In Levels D-J, Lesson 1 of each week begins with Find the Math and then the **Warm Up**. Find the Math presents a real-world connection for the lesson's content in context. After students reply to the guided discussion question, they answer several related questions in their ***Student Workbook*** pages.

Engage is the heart of Lessons 1–4 of each week. During **Engage**, students participate in hands-on activities, discussions, and strategy-building exercises that give students firm foundations in critical skills. To help teachers introduce and facilitate meaningful activities and discussions, the program also includes Activity Cards; throughout each lesson, teachers can easily use an Activity Card for quick reference and instructional support.

SRA Number Worlds also ensures that teachers can easily monitor students' progress, differentiate instruction, and provide targeted, individualized intervention whenever it's needed. In the ***Teacher Edition***, for example, Progress Monitoring tips appear in every **Engage** section. Meanwhile, the ***Teacher Edition***, ***Teacher Dashboard*** and the Activity Cards help teachers create successful instructional groups. (See the Alternative Groupings in the ***Teacher Edition*** and in the Activity Cards; alternatively, group students based on performance data available in the ***Teacher Dashboard***.) Moreover, **Engage** includes Independent Practice and Supported Practice activity suggestions, so that teachers can provide each student with activities that meet his or her learning needs.

After the **Engage** activities, students can review and practice what they learned by completing the companion pages in the ***Student Workbook***. For each lesson, a Reflect prompt offers students opportunities to summarize and apply lesson concepts. Additional Reflect questions in the ***Teacher Edition*** involve critical thinking and real-world applications. These questions can be used as informal assessments of student understanding.

For Lessons 1–4 of each week, **Assess** provides several tools to evaluate student competency and progress, and to record informal observations. Students in need of additional practice can complete the lesson's ***Practice*** page. Each week, **Assess** is the core section of Lesson 5 and offers both digital and print formal assessment options. Weekly test results can be used to further differentiate instruction for students.

Get Started with the Standards for Mathematical Practice

The Common Core State Standards for Mathematical Practice define eight sets of behaviors that should be developed in conjunction with mathematical content knowledge. Because student engagement is a cornerstone of ***SRA Number Worlds***, the Common Core Standards for Mathematical Practices are embedded in ***SRA Number Worlds*** lessons, providing students with daily opportunities to use Common Core Mathematical Practices as they communicate with peers and engage in higher order thinking.

SRA Number Worlds uses research-based strategies built on field-test results to provide students with effective and engaging learning experiences. Strategies include: modeling mathematics through hands-on activities; game playing; participating in guided discussions; responding to verbal and written prompts; using math tools strategically, including Digital Math Tools and Interactive White Boards; solving problems using pencil-and-paper calculations; and Project-Based Learning.

The ***SRA Number Worlds*** program includes the digital and print resources needed to implement the ***SRA Number Worlds*** curriculum. Components include: the Teacher Edition, Activity Cards, Student Workbooks, Practice Blackline Masters, Assessment book, English Learner Support Guide, Vocabulary Cards, and the Number Knowledge Test.

Manipulatives are a key component of the ***SRA Number Worlds*** learning experience because they let students explore abstract mathematical concepts in a concrete way. They also allow students to demonstrate understanding of these concepts. Complete manipulative kits are available to support conceptual development at each level.

Get Started with Program Components and Resources Aligned with the Common Core

The ***SRA Number Worlds*** components provide a complete, comprehensive math intervention curriculum for students in grades PreK-8. The table describes program components and lists the the portions of the program for which each component is available.

Component	Levels	Description
Teacher Edition	A–J	research-based background and instructional strategies
Activity Cards	A–J	explicit instructions and questioning strategies to facilitate all activities
Student Workbooks	C–J	practice for all concepts
Practice Blackline Masters	D–J	extra practice or homework
Manipulative Kits	A–J	tools, resources, and manipulatives
Assessment	A–J	online and print assessment options
English Learner Support Guide	A–J	strategies for developing academic vocabulary
Vocabulary Cards	Packages 1 and 2	academic vocabulary in multiple languages with visuals and definitions
Placement Test Guide	Comprehensive	key questions designed to measure students' intuitive knowledge of number

The ***SRA Number Worlds*** program offers a variety of digital resources to enhance instruction and engage students.

Resources for Teachers	Resources for Students
calendar and assignment tools	***Building Blocks***™ activities
online assessments and reporting tools	online assessments and other materials as assigned by teachers
eTeacher Editions and eActivity Card	digital math tools, game boards, game pieces, and other virtual manipulatives
program implementation and professional development resources	discussion area to extend Project-Based Learning experiences

Get Started with *Building Blocks*™

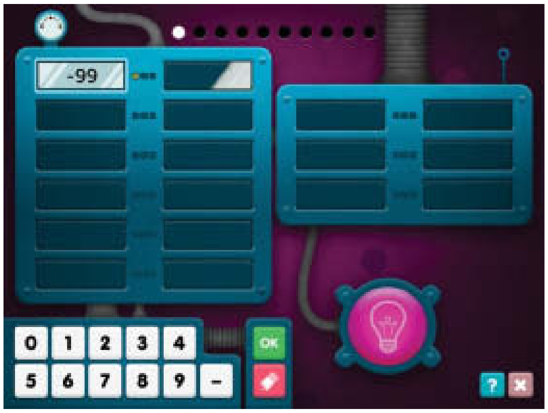
***Building Blocks*™** includes research-based computer tools with activities and a management system that guides students through research-based learning trajectories. Students use personalized screens to complete assignments, practice new concepts, and view their progress.

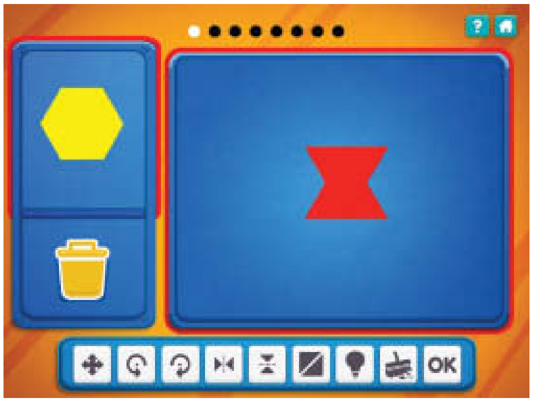
The teacher landing page provides a quick overview of assigned activities and the ability to monitor student progress. Its flexible management system allows you to view individual reports, and tailor activities to students' individual learning levels.

The program is designed to:

- build young students' experiences with mathematics through activities that integrate ways to explore and represent mathematics,
- involve students in "doing mathematics",
- develop a strong conceptual framework,
- emphasize students' mathematical thinking and reasoning abilities, and
- encourage learning in accordance with the Common Core State Standards.

Building Blocks increases students' attention and motivation with visual displays, animated graphics, virtual manipulatives, immediate feedback, and individualized tasks. Free Explores allow students to create their own scenarios, problems, and puzzles.





Learning trajectories allow you to build the mathematical thinking of your students as it develops naturally. Research has identified developmental paths, or sequences of activities, that effectively guide students through mathematical levels of thinking. ***Building Blocks'*** learning trajectories are built upon these developmental paths.

Learning trajectories have been identified for the skills and concepts listed below.

- Counting
- Comparing and Ordering Numbers
- Subitizing (instantly recognizing) Number
- Composing Number
- Adding and Subtracting
- Multiplying and Dividing
- Measuring Length and Area
- Recognizing Geometric Shapes
- Composing Geometric Shapes
- Spatial Sense and Motions
- Patterning and Algebraic Thinking
- Rational Numbers
- Classifying and Analyzing Data

Each ***Building Blocks*** level provides a natural developmental step. These levels help you assess, teach, and sequence activities.

Get Started with *Building Blocks*™

Following natural learning trajectories, ***Building Blocks*** activities address core math concepts at specific developmental levels. These activities provide engaging experiences that build students' mathematical understandings and skills. Through the ***Teacher Dashboard***, teachers can view and sort the activities in various ways, such as by trajectory, topic, and grade. Teachers can also assign individual activities or place students within specific learning sequences. From those points, students can move forward or backward, depending on their performance within each activity.

Building Blocks drills offer fluency practice and specific instruction in math skills. They target specific skills and concepts.

Practice activities are also included in ***Building Blocks***. These activities allow students to strengthen skills while participating in challenging games, such memory card games, races, and puzzles.

Free Explores are open-ended versions of the activities that give students a chance to develop their own shapes or explore various concepts. Free Explores are accessible after students have completed a series of activities. The table below lists the Free Explores and provides a description of each Free Explore.

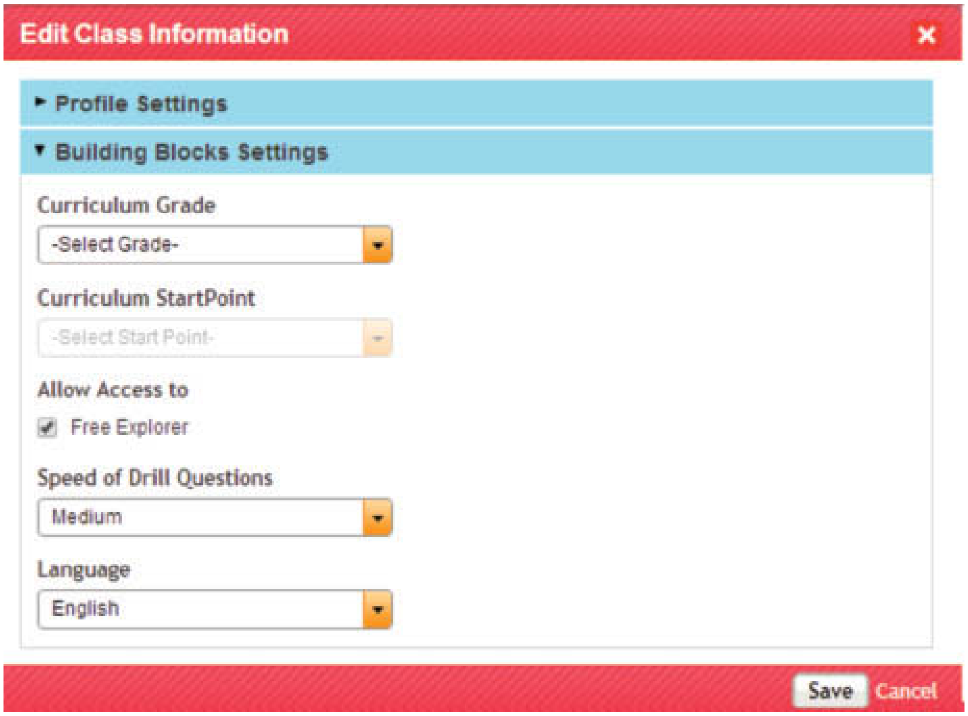
Free Explores

Build Stairs	Students explore counting, sequencing and ordering by building staircases.
Create a Scene	Students explore shapes by moving and manipulating them to make pictures.
Dino Shop	Students explore counting and related number topics by adding toy dinosaurs to boxes.
Mystery Pictures	Students freely construct pictures by assembling a variety of shapes.
Party Time	Students explore counting and related number topics by putting party items on a table.
Patterns	Students create rhythmic patterns of their own.
Piece Puzzler	Students create puzzles with shapes and have the option to save and recreate their picture.
Pizza Pizzazz	Students explore counting and related number topics by adding toppings to pizzas.

As a teacher, you can manage the ***Building Blocks*** resources in several ways:

(1) Along with other program resources in the *Lesson Plans* area, ***Building Blocks*** activities specified by ***Number Worlds*** lessons are automatically assigned to students. However, teachers can easily override this setting and adjust the assignments as needed.

(2) In the *My Classes* area, you can modify ***Building Blocks*** assignments for an entire class, including setting the curricular grade and starting point for all students, allowing access to Free Explores, setting the pace of drills, and selecting whether students work in English or Spanish.



(3) Within the *Manage and Assign* area, you can customize both the ***Building Blocks*** curriculum and trajectory starting points for each student. This screen shows the current Learning Path Assignments for all the students in your class. You can edit or un-assign an assignment by selecting the respective words in the *Action* column. To make a new assignment, select the *Assign an Activity* button. When creating an assignment for particular students, select those students by filling the check boxes to the left of their names, and choose a curriculum grade for the assignment. Once you select a grade, you have the option of assigning all trajectories or a specific trajectory within that grade. You will also have the option to assign all activities or a targeted activity within that trajectory.

For more information on assigning ***Number Worlds*** lessons and ***Building Blocks*** activities, refer to the *User's Guide*.

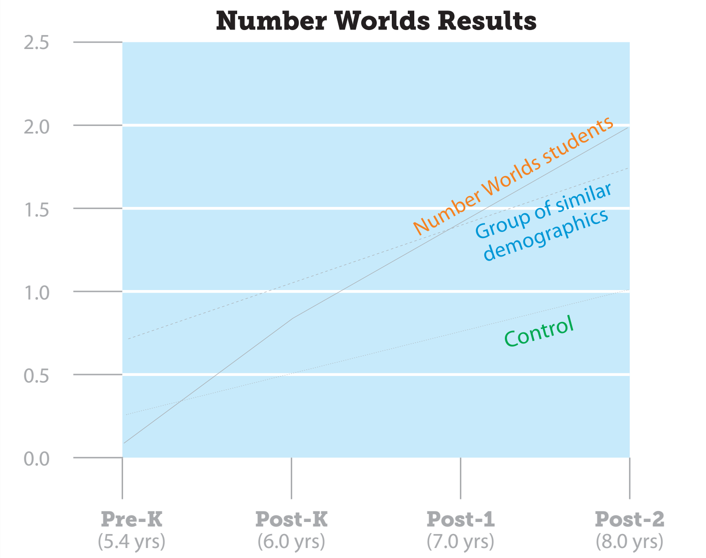
Building Blocks provides a wide range of data reports that show class and student progress through the various activities. Student reports can be viewed by topic, trajectory, learning activities, and Common Core State Standards. These clear, concise reports provide powerful analytical tools for you, your principal, and other school officials.

For topic and trajectory reports, student results are represented with bar charts. For each student selected, the bar chart shows the overall percentage of completed ***Building Blocks*** activities within a topic or trajectory. The column labeled *Attempts* indicates the number of activities (within a topic or standard) that the student has completed.

Another line on the graph indicates the average class score for that topic or trajectory. Under the Action Item column, a red flag indicates a recommendation for remedial action. The flexibility of the ***Building Blocks*** management system provides you with the data you need to individualize instruction, providing enrichment or remediation as needed.

The Activity Report for each student gives the student's average and whether the activity was mastered. Clicking on an activity name shows the student's responses for each question within that activity.

Meet the Authors



Longitudinal study showing mean developmenal scores in mathematical knowledge during Grades K-2

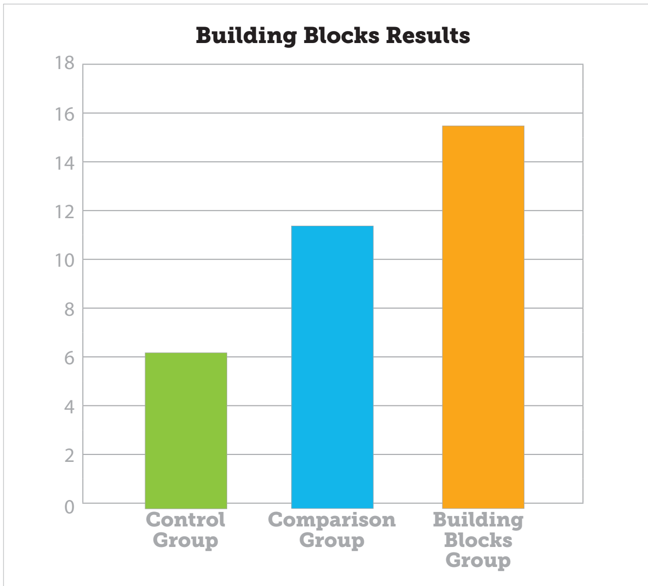
Sharon Griffin is Professor Emerita of Education and Psychology at Clark University in Worcester, Massachusetts. She received a B.A. in Psychology from McGill University, an M.A. in Education from the University of New Hampshire, and a Ph.D. in Cognitive Science from the University of Toronto. Before coming to Clark University in 1989, she worked as a Research Associate at the Ontario Institute for Studies in Education.

During the past 25 years, Dr. Griffin has received several research awards to study learning and achievement among at-risk children, to teach number sense, and to enable mathematics teachers to enhance their students' learning and achievement.

Dr. Griffin is the author of ***SRA Number Worlds***, a pre–K to grade 8 Prevention-Intervention mathematics curriculum, and the co-author of the book, *What Develops in Emotional Development? She is also* the author of numerous articles on the development of number sense, and in the fields of cognitive development, emotional development, and mathematics education.

Dr. Griffin has also served on several national and international advisory boards on projects designed to enhance the cognitive, mathematical, and language development of children, from birth through the elementary school years. As a member of the Mathematical Sciences Education Board at the National Academies of Science (NAS) as well as the Center of Education Research and Innovation at the Organization for Economic Collaboration and Development (OECD), she helped shape the direction of education research and policy for the United States, Canada, the United Kingdom, and several European countries.

Dr. Douglas Clements, Kennedy Endowed Chair in Early Childhood Learning and Professor at the University of Denver, is widely regarded as "the major scholar" in the field of early childhood mathematics education, with equal relevance to the academy, to the classroom, and to the educational policy arena. At the national level, his contributions have led to the development of new mathematics curricula, teaching approaches, teacher training initiatives, and models of "scaling up" interventions. His contributions have also had a tremendous impact on educational planning and policy, particularly in the area of mathematical literacy and access.



Julie Sarama, Kennedy Endowed Chair in Innovative Learning Technologies and Professor at the University of Denver, conducts research on young children's development of mathematical concepts and competencies, the implementation and scale-up of educational reform, professional development models and their influence on student learning, and the implementation and effects of software environments in mathematics classrooms. She has been Principal or Co-Principal Investigator on seven projects funded by the National Science Foundation (NSF), and she is Principal Investigator on her latest NSF award, entitled, *"Early Childhood Education in the Context of Mathematics, Science, and Literacy."* Dr. Sarama is currently co-directing three large-scale studies funded by the U.S. Education Department's Institute of Educational Studies (IES).

Build Number Sense with *Number Worlds*

A Mathematics Program for Young Children

What is number sense? We all know number sense when we see it but, if asked to define what it is and what it consists of, most of us, including the teachers among us, would have a much more difficult time. Yet this is precisely what we need to know to teach number sense effectively. Consider the answers three kindergarten children provide when asked the following question from the Number Knowledge Test (Griffin & Case, 1997): Which is bigger: seven or nine?"

Brie responds quickly, saying "Nine." When asked how she figured it out, she says, "Well, you go, 'seven' (pause) 'eight', 'nine' (putting up two fingers while saying the last two numbers). That means nine has two more than seven. So it's bigger."

Leah says, hesitantly, "Nine?" When asked how she figured it out, she says, "Because nine's a big number."

Caitlin looks genuinely perplexed, as if the question was not a sensible thing to ask, and says, "I don't know."

Kindergarten teachers will immediately recognize that Brie's answer provides evidence of a well-developed number sense for this age level and Leah's answer, a more fragile and less-developed number sense. The knowledge that lies behind this "sense" may be much less apparent. What knowledge does Brie have that enables her to come up with the answer in the first place and to demonstrate good number sense in the process?

1. Knowledge that underlies number sense

Research conducted with the Number Knowledge Test and several other cognitive developmental measures (see Griffin, 2002; Griffin & Case, 1997 for a summary of this research) suggests that the following understandings lie at the heart of the number sense that 5-year-olds like Brie are able to demonstrate on this problem. They know (a) that numbers indicate quantity and therefore, that numbers, themselves, have magnitude; (b) that the word "bigger" or "more" is sensible in this context; (c) that the numbers 7 and 9, like every other number from 1 to 10, occupy fixed positions in the counting sequence; (d) that 7 comes before 9 when you are counting up; (e) that numbers that come later in the sequence—that are higher up— indicate larger quantities and therefore, that 9 is bigger (or more) than 7.

Brie provided evidence of an additional component of number sense in the explanation she provided for her answer. By using the Count-On strategy to show that nine comes two numbers after seven and by suggesting that this means "it has two more than seven," Brie demonstrated that she also knows (f) that each counting number up in the sequence corresponds precisely to an increase of one unit in the size of a set. This understanding, possibly more than any of the others listed above, enables children to use the counting numbers alone, without the need for real objects, to solve quantitative problems involving the joining of two sets. In so doing, it transforms mathematics from something that can only be done out there (e.g., by manipulating real objects) to something that can be done in their own heads, and under their own control.

This set of understandings, the core of number sense, forms a knowledge network that Case and Griffin (1990), see also Griffin and Case (1997), have called a *central conceptual structure for number*. Research conducted by these investigators has shown that this structure is central in at least two ways (see Griffin, Case, & Siegler, 1994). First, it enables children to make sense of a broad range of quantitative problems across contexts and to answer questions, for example, about two times on a clock (Which is longer?), two positions on a path (Which is farther?), and two sets of coins (Which is worth more?). Second, it provides the foundation on which children's learning of more complex number concepts, such as those involving double-digit numbers, is built. For this reason, this network of knowledge is an important set of understandings that should be taught in the preschool years, to all children who do not spontaneously acquire them.

2. How can this knowledge be taught?

SRA Number Worlds, a mathematics program for young children (formerly called *Rightstart*), was specifically developed to teach this knowledge and to provide a test for the cognitive developmental theory (i.e., Central Conceptual Structure theory; see Case & Griffin, 1990) on which the program was based. Originally developed for kindergarten, the program (see Griffin & Case, 1995) was expanded to teach a broader range of understandings when research findings provided strong evidence that (a) children who were exposed to the program acquired the knowledge it was designed to teach (i.e., the central conceptual structure for number), and (b) the theoretical postulates on which the program was based were valid (see Griffin & Case, 1996; Griffin, Case, & Capodilupo, 1995; Griffin et al., 1994). Programs for grades one and two were developed to teach the more complex central conceptual structures that underlie base-ten understandings (see Griffin, 1997, 1998) and a program for preschool was developed (see Griffin, 2000) to teach the "precursor" understandings that lay the foundation for the development of the central conceptual structure for number.

Because the four levels of the program are based on a well-developed theory of cognitive development, they provide a finely graded sequence of activities (and associated knowledge objectives) that recapitulate the natural developmental progression for the age range of 3–9 years, and allow each child to enter the program at a point that is appropriate for his or her own development. To progress through the program to teach 20 or more children at any one time, every effort has been made in the construction of the ***SRA Number Worlds*** program to make it as easy as possible for teachers to accommodate the developmental needs of individual children (or groups of children) in their classroom. Five instructional principles that lie at the heart of the program are described below and are used to illustrate several features of the program that have already been mentioned and several that have not yet been introduced.

2.1. Principle 1: Build upon children's current knowledge

Each new idea that is presented to children must connect to their existing knowledge if it is going to make any sense at all. Children must also be allowed to use their existing knowledge to construct new knowledge that is within reach—that is one step beyond where they are now—and a set of bridging contexts and other instructional supports should be in place to enable them to do so.

In the examples of children's thinking presented earlier, three different levels of knowledge are apparent. Brie appears to have acquired the knowledge network that underlies number sense and to be ready, therefore, to move on to the next developmental level: to connect this set of understandings to the written numerals (i.e., the formal symbols) associated with each counting word. Leah appears to have some understanding of some of the components of this network (i.e., that numbers have magnitude) and to be ready to use this understanding as a base to acquire the remaining understandings (e.g., that a number's magnitude and its position in the counting sequence are directly related). Caitlin demonstrated little understanding of any element of this knowledge network, and she might benefit, therefore, from exposure to activities that will help her acquire the "precursor" knowledge needed to build this network, namely knowledge of counting (e.g., the one-to-one correspondence rule) and knowledge of quantity (e.g., an intuitive understanding of relative amounts). Although all three children are in kindergarten, each child appears to be at a different point in the developmental trajectory and to require a different set of learning opportunities—ones that will enable each child to use her existing knowledge to construct new knowledge at the next level up.

To meet these individual needs, teachers need (a) a way to assess children's current knowledge, (b) activities that are multi-leveled so children with different entering knowledge can all benefit from exposure to them, and (c) activities that are carefully sequenced and that span several developmental levels, so children with different entering knowledge can be exposed to activities that are appropriate for their level of understanding. These are all available in the ***SRA Number Worlds*** program and are illustrated in various sections of this paper.

2.2. Principle 2: Follow the natural developmental progression when selecting new knowledge to be taught

Researchers who have investigated the manner in which children between the ages of 3 and 9 years construct number knowledge have identified a common progression that most, if not all, children follow (Griffin, 2002; Griffin and Case, 1997). As suggested earlier, by the age of 4 years, most children have constructed two "precursor" knowledge networks—knowledge of counting and knowledge of quantity—that are separate in this stage and that provide the base for the next developmental stage. Sometime in kindergarten, children become able to integrate these knowledge networks—to connect the world of counting numbers to the world of quantity—and to construct the central conceptual understandings that were described earlier. Around the age of 6 or 7 years, children connect this integrated knowledge network to the world of formal symbols and, by the age of 8 or 9 years, most children become capable of expanding this knowledge network to deal with double-digit numbers and the base-ten system. A mathematics program that provides opportunities for children to use their current knowledge to construct new knowledge that is a natural next step, and that fits their spontaneous development, will have the best chance of helping children make maximum progress in their mathematics learning and development.

Because there are limits in development on the complexity of information children can handle at any particular age/stage (see Case, 1992), it makes no sense to attempt to speed up the developmental process by accelerating children through the curriculum. However, for children who are at an age when they should have acquired the developmental milestones but for some reason haven't, exposure to a curriculum that will give them ample opportunities to do so makes tremendous sense. It will enable them to catch up to their peers and thus, to benefit from the formal mathematics instruction that is provided in school. Children who are developing normally also benefit from opportunities to broaden and deepen the knowledge networks they are constructing, to strengthen these understandings, and to use them in a variety of contexts.

2.3. Principle 3: Teach computational fluency as well as conceptual understanding

Because computational fluency and conceptual understanding have been found to go hand in hand in children's mathematical development (see Griffin, 2003; Griffin et al., 1994), opportunities to acquire computational fluency, as well as conceptual understanding, are built into every ***SRA Number Worlds*** activity. This is nicely illustrated in the following activities, drawn from different levels of the program.

In *The Mouse and the Cookie Jar Game* (created for the preschool program and designed to give 3- to 4-year-olds an intuitive understanding of subtraction), children are given a certain number of counting chips (with each child receiving the same number but a different color) and told to pretend their chips are cookies. They are asked to count their cookies and, making sure they remember how many they have and what their color is, to deposit them in the cookie jar for safe keeping. While the children sleep, a little mouse comes along and takes one (or two) cookies from the jar. The problem that is then posed to the children is "How can we figure out whose cookie(s) the mouse took?"

Although children quickly learn that emptying the jar and counting the set of cookies that bears their own color is a useful strategy to use to solve this problem, it takes considerably longer for many children to realize that, if they now have four cookies (and originally had five), it means that they have one fewer and the mouse has probably taken one of their cookies. Children explore this problem by counting and recounting the remaining sets, comparing them to each other (e.g., by aligning them) to see who has the most or least, and ultimately coming up with a prediction. When a prediction is made, children search the mouse's hole to see whose cookie had been taken and to verify or revise their prediction. As well as providing opportunities to perfect their counting skills, this activity gives children concrete opportunities to experience simple quantity transformations and to discover how the counting numbers can be used to predict and explain differences in amount.

The *Dragon Quest Game* that was developed for the Grade 1 program teaches a much more sophisticated set of understandings. Children are introduced to Phase 1 activity by being told a story about a fire-breathing dragon that has been terrorizing the village where children live. The children playing the game are heroes who have been chosen to seek out the dragon and put out his fire. To extinguish this dragon's fire (as opposed to the other, more powerful dragons they will encounter in later phases) a hero will need at least 10 pails of water. If a hero enters into the dragon's area with less than 10 pails of water, he or she will become the dragon's prisoner and can only be rescued by one of the other players.

To play the game, children take turns rolling a die and moving their playing piece along the colored game board. If they land on a well pile (indicated by a star), they can pick a card from the face-down deck of cards, which illustrate, with images and symbols (e.g., +4) a certain number of pails of water. Children are encouraged to add up their pails of water as they receive them and they are allowed to use a variety of strategies to do so, ranging from mental math (which is encouraged) to the use of tokens to keep track of the quantity accumulated. The first child to reach the dragon's lair with at least 10 pails of water can put out the dragon's fire and free any teammates who have become prisoners.

As children play this game and talk about their progress, they have ample opportunity to connect numbers to several different quantity representations (e.g., dot patterns on the die; distance of their pawn along the path; sets of buckets illustrated on the cards; written numerals also provided on the cards) and to acquire an appreciation of numerical magnitude across these contexts. With repeated play, they also become capable of performing a series of successive addition operations in their heads and of expanding the well pile. When they are required to submit formal proof to the mayor of the village that they have amassed sufficient pails of water to put out the dragon's fire before they are allowed to do so, they become capable of writing a series of formal expressions to record the number of pails received and spilled over the course of the game. In contexts such as these children receive ample opportunity to use the formal symbol system in increasingly efficient ways to make sense of quantitative problems they encounter in the course of their own activity.

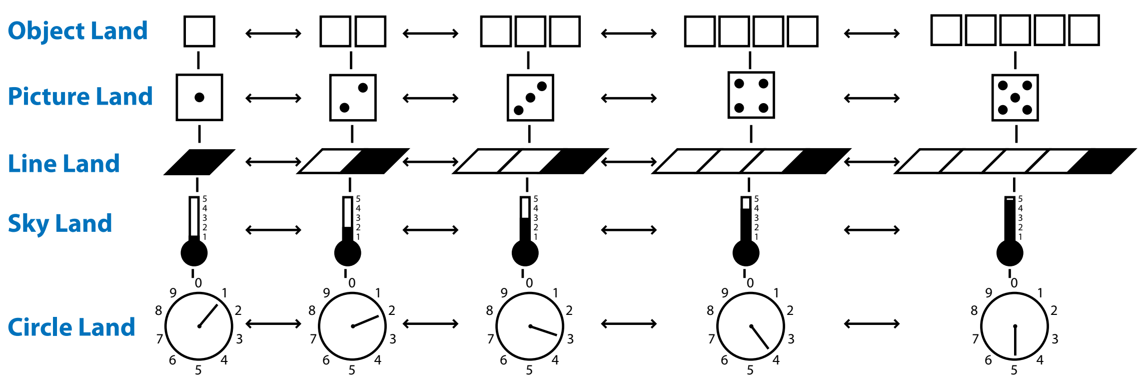
2.4. Principle 4: Provide plenty of opportunity for hands-on exploration, problem-solving, and communication

Like the *Dragon Quest Game* that was just described, many of the activities created for the ***SRA Number Worlds*** program are set in a game format that provides plenty of opportunity for hands-on exploration of number concepts, for problem-solving and for communication. Communication is explicitly encouraged in a set of question prompts that are included with each small group game (e.g., How far are you now? How many more buckets do you need to put out the dragon's fire? How do you know?) as well as in a more general set of dialogue prompts that are included in the teacher's guide. Opportunities for children to discuss what they learned during game play each day, to share their knowledge with their peers, and to make their reasoning explicit are also provided in a Wrap-Up session that is included at the end of each math lesson.

Finally, in the whole group games and activities that were developed for the Warm-Up portion of each math lesson, children are given ample opportunity to count (e.g., up from 1 and down from 10) and to solve mental math problems, in a variety of contexts. In addition to developing computational fluency, these activities expose children to the language of mathematics and give them practice using it. Although this is valuable for all children, it is especially useful for English learners, who may know how to count in their native language but not yet in English. Allowing children to take turns in these activities and to perform individually gives teachers opportunities to assess each child's current level of functioning, important for instructional planning, and gives children opportunities to learn from each other.

2.5. Principle 5: Expose children to the major way number is represented and talked about in developed societies

Representation for Numbers in Different Lands



Number is represented in our culture in five major ways: as a group of objects, a dot-set pattern, a position on a line, a position on a scale (e.g., a thermometer), and a point on a dial. In each of these contexts, number is also talked about in different ways, with a larger number (and quantity) described as "more" in the world of dot-sets, as "further along" in the world of paths and lines, as "higher up" in the world of scale measures, and as "further around" in the world of dials. Children who are familiar with these forms of representation and the language used to talk about number in these contexts have a much easier time making sense of the number problems they encounter inside and outside of school.

In the ***SRA Number Worlds*** program, children are systematically exposed to these forms of representations as they explore five different "lands." Learning activities developed for each land share a particular form of number representation while they simultaneously address specific knowledge goals for each grade level. Many of the games, like *Dragon Quest,* also expose children to multiple representations of number in one activity so children can gradually come to see the ways they are equivalent.

3. Discussion

Children who have been exposed to the ***SRA Number Worlds*** program do very well on number questions like the one presented in the introduction and on the Number Knowledge Test (Griffin & Case, 1997) from which this question was drawn. In several evaluation studies conducted with children from low-income communities, children who received the ***SRA Number Worlds*** program made significant gains in conceptual knowledge of number and in number sense, when compared to matched-control groups who received readiness training of a different sort. These gains enabled them to start their formal schooling in grade one on an equal footing with their more advantaged peers, to perform as well as groups of children from China and Japan on a computation test administered at the end of grade one, and to keep pace with their more advantaged peers (an even outperform them on some measures) as they progressed through the first few years of formal schooling (Griffin & Case, 1997).

Teachers also report positive gains from using the ***SRA Number Worlds*** program and from exposure to the instructional principles on which it is based. Although all teachers acknowledge that implementing the program and putting the principles into action is not an easy task, many claim that their teaching of all subjects has been transformed in the process. They now facilitate discussion rather than dominating it; they pay much more attention to what children say and do; and they now allow children to take more responsibility for their own learning, with positive and surprising results. Above all, they now look forward to teaching math and they and their students are eager to do more of it.

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Leverage Learning Trajectories

Children follow natural developmental progressions in learning, developing mathematical ideas in their own way. Curriculum research has revealed sequences of activities that are effective in guiding children through these levels of thinking. These developmental paths are the basis for ***Building Blocks*** Learning Trajectories. Learning trajectories have three parts: a mathematical goal, a developmental path through which children develop to reach that goal, and a set of activities matched to each of those levels that help children develop the next level. Thus, each learning trajectory has levels of understanding, each more sophisticated than the last, with tasks that promote growth from one level to the next. The ***Building Blocks*** Learning Trajectories give simple labels, descriptions, and examples of each level. Complete learning trajectories describe the goals of learning, the thinking and learning processes of children at various levels, and the learning activities in which they might engage. This document provides only the developmental levels.

Each column in the table shown on pages 66–78, such as "Counting," represents a main developmental progression that underlies the learning trajectory for that topic.

For some topics, there are "subtrajectories"—strands within the topic. In most cases, the names make this clear. For example, in Comparing and Ordering, some levels are about the "Comparer" levels, and others about building a "Mental Number Line." Similarly, the related subtrajectories of "Composition" and "Decomposition" are easy to distinguish. Sometimes, for clarification, subtrajectories are indicated with a note in italics after the title. For example, in "Shapes," Parts and Representing are subtrajectories within the Shapes trajectory.

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Frequently Asked Questions

1. **Why use learning trajectories?**
   Learning trajectories allow teachers to build the *mathematics of children— the thinking of children as it develops naturally.* So, we know that all the goals and activities are within the developmental capacities of children. We know that each level provides a natural *developmental building block* to the next level. Finally, we know that the activities provide the *mathematical building blocks* for school success, because the research on which they are based typically involves higher-income children.

2. **When are children "at" a level?**
   Children are at a certain level when most of their behaviors reflect the thinking—ideas and skills—of that level. Often, they show a few behaviors from the next (and previous) levels as they learn.

3. **Can children work at more than one level at the same time?**
   Yes, although most children work mainly at one level or in transition between two levels (naturally, if they are tired or distracted, they may operate at a much lower level). Levels are not "absolute stages." They are "benchmarks" of complex growth that represent distinct ways of thinking. So, another way to think of them is as a sequence of different patterns of thinking. Children are continually learning, within levels and moving between them.

4. **Can children jump ahead?**
   Yes, especially if there are separate "sub-topics." For example, we have combined many counting competencies into one "Counting" sequence with sub-topics, such as verbal counting skills. Some children learn to count to 100 at age 6 after learning to count objects to 10 or more; others may learn that verbal skill earlier. The sub-topic of verbal counting skills would still be followed.

5. **How do these developmental levels support teaching and learning?**
   The levels help teachers, as well as curriculum developers, assess, teach, and sequence activities. *Teachers who understand learning trajectories and the developmental levels that are at their foundation are more effective and efficient*. Through planned teaching and encouraging informal, incidental mathematics, teachers help children learn *at an appropriate and deep level*.

6. **Should I plan to help children develop just the levels that correspond to my children's ages?**
   No! The ages in the table are typical ages children develop these ideas. *But these are rough guides only*—children differ widely. Furthermore, the ages below are lower bounds of what children achieve without instruction. So, these are *"starting levels" not goals*. We have found that children who are provided high-quality mathematics experiences are capable of developing to levels one or more years beyond their peers.

Developmental Levels for Counting

The ability to count with confidence develops over the course of several years. Beginning in infancy, children show signs of understanding numbers. With instruction and number experience, most children can count fluently by age 8, with much progress in counting occurring in kindergarten and first grade. Most children follow a natural developmental progression in learning to count with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
1–2	Pre-Counter (Verbal)	1	At the earliest level a child shows no verbal counting. The child may name some number words with no sequence.
1–2	Chanter (Verbal)	2	At this level, a child may sing-song or chant indistinguishable number words.
2	Reciter (Verbal)	3	At this level, the child may verbally count with separate words, but not necessarily in the correct order.
3	Reciter (10)	4	A child at this level may verbally count to 10 with some correspondence with objects. He or she may point to objects to count a few items, but then lose track.
3	Corresponder	5	At this level, a child may keep one-to-one correspondence between counting words and objects—at least for small groups of objects laid in a line. A corresponder may answer "how many" by recounting the objects, starting over with one each time.
4	Counter (Small Numbers)	6	At around 4 years of age, the child may begin to count meaningfully. He or she may accurately count objects in a line to 5 and answer the "how many" question with the last number counted. When objects are visible, and especially with small numbers, the child begins to understand cardinality (that numbers tell how many).
4	Producer (Small Numbers)	7	The next level after counting small numbers is to count out objects to 5. When asked to show four of something, for example, this child may give four objects.
4	Counter (10)	8	This child may count structured arrangements of objects to 10. He or she may be able to write or draw to represent 1–10. A child at this level may be able to tell the number just after or just before another number, but only by counting up from 1.
5	Counter and Producer—Counter to (10+)	9	Around 5 years of age, a child may begin to count out objects accurately to 10 and then beyond to 30. He or she has explicit understanding of cardinality (that numbers tell how many). The child may keep track of objects that have and have not been counted, even in different arrangements. He or she may write or draw to represent 1 to 10 and then 20 and 30, and may give the next number to 20 or 30. The child also begins to recognize errors in others' counting and is able to eliminate most errors in his or her own counting.
5	Counter Backward from 10	10	Another milestone at about age 5 is being able to count backward from 10 to 1, verbally, or when removing objects from a group.

Age Range	Level Name	Level	Description
6	Counter from N (N+1, N–1)	11	Around 6 years of age, the child may begin to count on, counting verbally and with objects from numbers other than 1. Another noticeable accomplishment is that a child may determine the number immediately before or after another number without having to start back at 1.
6	Skip Counting by 10s to 100	12	A child at this level may count by 10s to 100 or beyond with understanding.
6	Counter to 100	13	A child at this level may count by 1s to 100. He or she can make decade transitions (for example, from 29 to 30) starting at any number.
6	Counter On Using Patterns	14	At this level, a child may keep track of a few counting acts by using numerical patterns (spatial, auditory, or rhythmic).
6	Skip Counter	15	At this level, the child can count by 5s and 2s with understanding.
6	Counter of Imagined Items	16	At this level, a child may count mental images of hidden objects to answer, for example, "how many" when 5 objects are visible and 3 are hidden.
6	Counter On Keeping Track	17	A child at this level may keep track of counting acts numerically, first with objects, then by counting counts. He or she counts up one to four more from a given number.
6	Counter of Quantitative Units	18	At this level, a child can count unusual units, such as "wholes" when shown combinations of wholes and parts. For example, when shown three whole plastic eggs and four halves, a child at this level will say there are five whole eggs.
6	Counter to 200	19	At this level, a child may count accurately to 200 and beyond, recognizing the patterns of ones, tens, and hundreds.
7	Number Conserver	20	A major milestone around age 7 is the ability to conserve number. A child who conserves number understands that a number is unchanged even if a group of objects is rearranged. For example, if there is a row of ten buttons, the child understands there are still ten without recounting, even if they are rearranged in a long row or a circle.
7	Counter Forward and Back	21	A child at this level may count in either direction and recognize that sequence of decades mirrors single-digit sequence.

Developmental Levels for Comparing and Ordering Numbers

Comparing and ordering sets is a critical skill for children as they determine whether one set is larger than another in order to make sure sets are equal and "fair." Prekindergartners can learn to use matching to compare collections or to create equivalent collections. Finding out how many more or fewer in one collection is more demanding than simply comparing two collections. The ability to compare and order sets with fluency develops over the course of several years. With instruction and number experience, most children develop foundational understanding of number relationships and place value at ages four and five. Most children follow a natural developmental progression in learning to compare and order numbers with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
2	Object Corresponder	1	At this early level, a child puts objects into one-to-one correspondence, but may not fully understand that this creates equal groups. For example, a child may know that each carton has a straw, but does not necessarily know there are the same numbers of straws and cartons.
2	Perceptual Comparer	2	At this level, a child can compare collections that are quite different in size (for example, one is at least twice the other) and know that one has more than the other. If the collections are similar, the child can compare very small collections.
2–3	First-Second Ordinal Counter	3	At this level the child can identify the "first" and often "second" object in a sequence.
3	Nonverbal Comparer of Similar Items	4	At this level, a child can identify that different organizations of the same number are equal and different from other sets (1–4 items). For example, a child can identify ••• and ∴ as equal and different from •• or •..
3	Nonverbal Comparer of Dissimilar Items	5	At this level, a child can match small, equal collections of dissimilar items, such as shells and dots, and show that they are the same number.
4	Matching Comparer	6	As children progress, they begin to compare groups of 1–6 by matching. For example, a child gives one toy bone to every dog and says there are the same number of dogs and bones.
4	Knows-to-Count Comparer	7	A significant step occurs when the child begins to count collections to compare. At the early levels, children are not always accurate when a larger collection's objects are smaller in size than the objects in the smaller collection. For example, a child at this level may accurately count two equal collections, but when asked, says the collection of larger blocks has more.
4	Counting Comparer (Same Size)	8	At this level, children make accurate comparisons via counting, but only when objects are about the same size and groups are small (about 1–5 items).
5	Counting Comparer (5)	9	As children develop their ability to compare sets, they compare accurately by counting, even when a larger collection's objects are smaller. A child at this level can figure out how many more or less.
5	Ordinal Counter	10	At this level, a child identifies and uses ordinal numbers from "first" to "tenth." For example, the child can identify who is "third in line."
6	Counting Comparer (10)	11	This level can be observed when the child compares sets by counting, even when a larger collection's objects are smaller, up to 10. A child at this level can accurately count two collections of 9 items each, and says they have the same number, even if one collection has larger blocks.
6	Mental Number Line to 10	12	As children move into this level, they begin to use mental images and knowledge of number relationships to determine relative size and position. For example, a child at this level can answer which number is closer to 6, 4 or 9 without counting physical objects.
6	Serial Orderer to 6+	13	At this level, the child orders lengths marked into units (1–6, then beyond). For example, given towers of cubes, this child can put them in order, 1 to 6.
7	Place Value Comparer	14	Further development is made when a child begins to compare numbers with place value understanding. For example, a child at this level can explain that "63 is more than 59 because six tens is more than five tens, even if there are more than three ones."
7	Mental Number Line to 100	15	Children demonstrate the next level when they can use mental images and knowledge of number relationships, including ones embedded in tens, to determine relative size and position. For example, when asked, "Which is closer to 45, 30 or 50?" a child at this level may say "45 is right next to 50, but 30 isn't."
8+	Mental Number Line to 1,000s	16	At about age 8, children may begin to use mental images of numbers up to 1,000 and knowledge of number relationships, including place value, to determine relative size and position. For example, when asked, "Which is closer to 3,500—2,000 or 7,000?" a child at this level may say "70 is double 35, but 20 is only fifteen from 35, so twenty hundreds, 2,000, is closer."

Developmental Levels for Recognizing Number and Subitizing (Instantly Recognizing)

The ability to recognize number values develops over the course of several years and is a foundational part of number sense. Beginning at about age two, children begin to name groups of objects. The ability to instantly know how many are in a group, called subitizing, begins at about age three. By age eight, with instruction and number experience, most children can identify groups of items and use place values and multiplication skills to count them. Most children follow a natural developmental progression in learning to count with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
2	Small Collection Namer	1	The first sign occurs when the child can name groups of 1 to 2, sometimes 3. For example, when shown a pair of shoes, this young child says, "two shoes."
3	Maker of Small Collections	2	At this level, a child can nonverbally make a small collection (no more than 4, usually 1 to 3) with the same number as another collection. For example, when shown a collection of 3, the child makes another collection of 3.
4	Perceptual Subitizer to 4	3	Progress is made when a child instantly recognizes collections up to 4 and verbally names the number of items. For example, when shown 4 objects briefly, the child says "4."
5	Perceptual Subitizer to 5	4	This level is the ability to instantly recognize collections up to 5 and verbally name the number of items. For example, when shown 5 objects briefly, the child says "5."
5	Conceptual Subitizer to 5+	5	At this level, the child can verbally label all arrangements to about 5, when shown only briefly. For example, a child at this level might say, "I saw 2 and 2, and so I saw 4."

Age Range	Level Name	Level	Description
5	Conceptual Subitizer to 10	6	This step is when the child can verbally label most arrangements to 6 shown briefly, then up to 10, using groups. For example, a child at this level might say, "In my mind, I made 2 groups of 3 and 1 more, so 7."
6	Conceptual Subitizer to 20	7	Next, a child can verbally label structured arrangements up to 20 shown briefly, using groups. For example, the child may say, "I saw 3 fives, so 5, 10, 15."
7	Conceptual Subitizer with Place Value and Skip Counting	8	At this level, a child is able to use groups, skip counting, and place value to verbally label structured arrangements shown briefly. For example, the child may say, "I saw groups of tens and twos, so 10, 20, 30, 40, 42, 44, 46 . . . 46!"
8+	Conceptual Subitizer with Place Value and Multiplication	9	As children develop their ability to subitize, they use groups, multiplication, and place value to verbally label structured arrangements shown briefly. At this level, a child may say, "I saw groups of tens and threes, so I thought, 5 tens is 50 and 4 threes is 12, so 62 in all."

Developmental Levels for Numerals

Age Range	Level Name	Level	Description
3	Quantity Representer	1	Represents and recalls sets with pictographic, iconic, representations of quantity. However, they may not incorporate written symbols into their own acting and thinking.
4	Numeral Representer	2	Can match small sets (1–5) with the corresponding numbers and represent and recall the size of sets using those numerals.
4–5	Functional Numeral User	3	Can use numerals to represent and communicate quantity. For example, can use numerals to remember results of counting or to compare quantities

Age Range	Level Name	Level	Description
6	Teen/Ten + Recognizer	4	Understands that a teen number is composed of a ten and one, two, three, ..., seven, eight or nine ones.
6–7	Decade Number Identifier	5	Understands decade words (e.g., sixty = 6 tens).
7	Digit Identifier	6	Understands that the two digits of a two-digit number represent amounts of tens and ones. In 29, for example, the 2 represents two tens and the 9 represents nine ones.

Developmental Levels for Composing (Knowing Combinations of Numbers)

Composing and decomposing are combining and separating operations that allow children to build concepts of "parts" and "wholes." Most prekindergartners can "see" that two items and one item make three items. Later, children learn to separate a group into parts in various ways and then to count to produce all of the number "partners" of a given number. Eventually children think of a number and know the different addition facts that make that number. Most children follow a natural developmental progression in learning to compose and decompose numbers with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
4	Pre-Part-Whole Recognizer	1	At the earliest levels of composing, a child only nonverbally recognizes parts and wholes. For example, when shown 4 red blocks and 2 blue blocks, a young child may intuitively appreciate that "all the blocks" includes the red and blue blocks, but when asked how many there are in all, the child may name a small number, such as 1.
5	Inexact Part-Whole Recognizer	2	A sign of development is that the child knows a whole is bigger than parts, but does not accurately quantify. For example, when shown 4 red blocks and 2 blue blocks and asked how many there are in all, the child may name a "large number," such as 5 or 10.
5	Composer to 4, then 5	3	At this level, a child knows number combinations. A child at this level quickly names parts of any whole, or the whole given the parts. For example, when shown 4, then 1 is secretly hidden, and then shown the 3 remaining, the child may quickly say "1" is hidden.

Age Range	Level Name	Level	Description
6	Composer to 7	4	The next sign of development is when a child knows number combinations to totals of 7. A child at this level quickly names parts of any whole, or the whole when given parts, and can double numbers to 10. For example, when shown 6, then 4 are secretly hidden, and then shown the 2 remaining, the child may quickly say "4" are hidden.
6	Composer to 10	5	This level is when a child knows number combinations to totals of 10. A child at this level may quickly name parts of any whole, or the whole when given parts, and can double numbers to 20. For example, this child would be able to say "9 and 9 is 18."
7	Composer with Tens and Ones	6	At this level, the child understands two-digit numbers as tens and ones, can count with dimes and pennies, and can perform two-digit addition with regrouping. For example, a child at this level may explain, "17 and 36 is like 17 and 3, which is 20, and 33, which is 53."
7–8	+/− Fact Fluency to 20	7	Quickly produces combinations (addends to 1–10).

Developmental Levels for Adding and Subtracting

Single-digit addition and subtraction are generally characterized as "math facts." It is assumed children must memorize these facts, yet research has shown that addition and subtraction have their roots in counting, counting on, number sense, the ability to compose and decompose numbers, and place value. Research has also shown that learning methods for addition and subtraction with understanding is much more effective than rote memorization of seemingly isolated facts. Most children follow an observable developmental progression in learning to add and subtract numbers with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
1	Pre +/−	1	At the earliest level, a child shows no sign of being able to add or subtract.
3	Nonverbal +/−	2	The first sign is when a child can add and subtract very small collections nonverbally. For example, when shown 2 objects, then 1 object being hidden under a napkin, the child identifies or makes a set of 3 objects to "match."
4	Small Number +/−	3	This level is when a child can find sums for joining problems up to 3 + 2 by counting with objects. For example, when asked, "You have 2 balls and get 1 more. How many in all?" the child may count out 2, then count out 1 more, then count all 3: "1, 2, 3, 3!"
5	Find Result +/−	4	**Addition** Evidence of this level in addition is when a child can find sums for joining (you had 3 apples and get 3 more; how many do you have in all?) and part-part-whole (there are 6 girls and 5 boys on the playground; how many children were there in all?) problems by direct modeling, counting all, with objects. For example, when asked, "You have 2 red balls and 3 blue balls. How many in all?" the child may count out 2 red, then count out 3 blue, then count all 5. **Subtraction** In subtraction, a child can also solve take-away problems by separating with objects. For example, when asked, "You have 5 balls and give 2 to Tom. How many do you have left?" the child may count out 5 balls, then take away 2, and then count the remaining 3.
5	Find Change +/−	5	**Addition** At this level, a child can find the missing addend (5 + _ = 7) by adding on objects. For example, when asked, "You have 5 balls and then get some more. Now you have 7 in all. How many did you get?" The child may count out 5, then count those 5 again starting at 1, then add more, counting "6, 7," then count the balls added to find the answer, 2. **Subtraction** A child can compare by matching in simple situations. For example, when asked, "Here are 6 dogs and 4 balls. If we give a ball to each dog, how many dogs will not get a ball?" a child at this level may count out 6 dogs, match 4 balls to 4 of them, then count the 2 dogs that have no ball.
5	Make It +/−	6	A significant advancement occurs when a child is able to count on. This child can add on objects to make one number into another without counting from 1. For example, when told, "This puppet has 4 balls, but she should have 6. Make it 6," the child may put up 4 fingers on one hand, immediately count up from 4 while putting up 2 fingers on the other hand, saying, "5, 6," and then count or recognize the 2 fingers.
6	Counting Strategies +/−	7	This level occurs when a child can find sums for joining (you had 8 apples and get 3 more . . .) and part-part-whole (6 girls and 5 boys . . .) problems with finger patterns or by adding on objects or counting on. For example, when asked "How much is 4 and 3 more?" the child may answer "4 . . . 5, 6, 7. 7!" Children at this level can also solve missing addend (3 + _ = 7) or compare problems by counting on. When asked, for example, "You have 6 balls. How many more would you need to have 8?" the child may say, "6, 7 [puts up first finger], 8 [puts up second finger]. 2!"
6	Part-Whole +/−	8	Further development has occurred when the child has part-whole understanding. This child can solve problems using flexible strategies and some derived facts (for example, "5 + 5 is 10, so 5 + 6 is 11"), can sometimes do start- unknown problems (_ + 6 = 11), but only by trial and error. When asked, "You had some balls. Then you get 6 more. Now you have 11 balls. How many did you start with?" this child may lay out 6, then 3, count, and get 9. The child may put 1 more, say 10, then put 1 more. The child may count up from 6 to 11, then recount the group added, and say, "5!"
6	Numbers-in-Numbers +/−	9	Evidence of this level is when a child recognizes that a number is part of a whole and can solve problems when the start is unknown (_ + 4 = 9) with counting strategies. For example, when asked, "You have some balls, then you get 4 more balls, now you have 9. How many did you have to start with?" this child may count, putting up fingers, "5, 6, 7, 8, 9." The child may then look at his or her fingers and say, "5!"
7	Deriver +/−	10	At this level, a child can use flexible strategies and derived combinations (for example, "7 + 7 is 14, so 7 + 8 is 15") to solve all types of problems. For example, when asked, "What's 7 plus 8?" this child thinks: 7 + 8 = 7 + [7 + 1] = [7 + 7] + 1 = 14 + 1 = 15. The child can also solve multidigit problems by incrementing or combining 10s and 1s. For example, when asked "What's 28 + 35?" this child may think: 20 + 30 = 50; + 8 = 58; 2 more is 60, and 3 more is 63. He or she can also combine 10s and 1s: 20 + 30 = 50. 8 + 5 is like 8 plus 2 and 3 more, so it is 13. 50 and 13 is 63.
8+	Problem Solver +/−	11	As children develop their addition and subtraction abilities, they can solve by using flexible strategies and many known combinations. For example, when asked, "If I have 13 and you have 9, how could we have the same number?" this child may say, "9 and 1 is 10, then 3 more makes 13. 1 and 3 is 4. I need 4 more!"

Age Range	Level Name	Level	Description
8+	Multidigit +/−	12	Further development is shown when children can use composition of 10s and all previous strategies to solve multidigit +/− problems. For example, when asked, "What's 37 −18?" this child may say, "Take 1 ten off the 3 tens; that's 2 tens. Take 7 off the 7. That's 2 tens and 0 . . . 20. I have one more to take off. That's 19." Or, when asked, "What's 28 + 35?" this child may think, 30 + 35 would be 65. But it's 28, so it's 2 less . . . 63.

Developmental Levels for Multiplying and Dividing

Multiplication and division builds on addition and subtraction understanding and is dependent upon counting and place-value concepts. As children begin to learn to multiply, they make equal groups and count them all. They then learn skip counting and derive related products from products they know. Finding and using patterns aids in learning multiplication and division facts with understanding. Children typically follow an observable developmental progression in learning to multiply and divide numbers with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
2	Non-quantitative Sharer "Dumper"	1	Multiplication and division concepts begin very early with the problem of sharing. Early evidence of these concepts can be observed when a child dumps out blocks and gives some (not an equal number) to each person.
3	Beginning Grouper and Distributive Sharer	2	Progression to this level can be observed when a child is able to make small groups (fewer than 5). This child can share by "dealing out," but often only between 2 people, although he or she may not appreciate the numerical result. For example, to share 4 blocks, this child may give each person a block, check that each person has one, and repeat this.
4	Grouper and Distributive Sharer	3	The next level occurs when a child makes small equal groups (fewer than 6). This child can deal out equally between 2 or more recipients, but may not understand that equal quantities are produced. For example, the child may share 6 blocks by dealing out blocks to herself and a friend one at a time.
5	Concrete Modeler ×/÷	4	As children develop, they are able to solve small-number multiplying problems by grouping—making each group and counting all. At this level, a child can solve division/sharing problems with informal strategies, using concrete objects—up to 20 objects and 2 to 5 people—although the child may not understand equivalence of groups. For example, the child may distribute 20 objects by dealing out 2 blocks to each of 5 people, then 1 to each, until the blocks are gone.
6	Parts and Wholes ×/÷	5	A new level is evidenced when the child understands the inverse relation between divisor and quotient. For example, this child may understand "If you share with more people, each person gets fewer."
7	Skip Counter ×/÷	6	As children develop understanding in multiplication and division, they begin to use skip counting for multiplication and for measurement division (finding out how many groups). For example, given 20 blocks, 4 to each person, and asked how many people, the children may skip count by 4, holding up 1 finger for each count of 4. A child at this level may also use trial and error for partitive division (finding out how many in each group). For example, given 20 blocks, 5 people, and asked how many each should get, this child may give 3 to each, and then 1 more.
8+	Deriver ×/÷	7	At this level, children use strategies and derived combinations to solve multidigit problems by operating on tens and ones separately. For example, a child at this level may explain "7 × 6, five 7s is 35, so 7 more is 42."
8+	Array Quantifier	8	Further development can be observed when a child begins to work with arrays. For example, given 7 × 4 with most of 5 × 4 covered, a child at this level may say, "There are 8 in these 2 rows, and 5 rows of 4 is 20, so 28 in all."
8+	Partitive Divisor	9	This level can be observed when a child is able to figure out how many are in each group. For example, given 20 blocks, 5 people, and asked how many each should get, a child at this level may say, "4, because 5 groups of 4 is 20."
8+	Multidigit ×/÷	10	As children progress, they begin to use multiple strategies for multiplication and division, from compensating to paper-and-pencil procedures. For example, a child becoming fluent in multiplication might explain that "19 times 5 is 95, because 20 fives is 100, and 1 less five is 95."

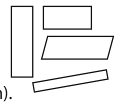
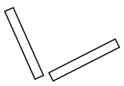
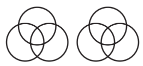
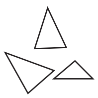
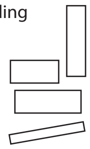
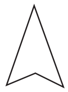
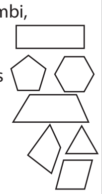
Developmental Levels for Measuring

Measurement is one of the main real-world applications of mathematics. Counting is a type of measurement which determines how many items are in a collection. Measurement also involves assigning a number to attributes of length, area, and weight. Prekindergarten children know that mass, weight, and length exist, but they do not know how to reason about these or to accurately measure them. As children develop their understanding of measurement, they begin to use tools to measure and understand the need for standard units of measure. Children typically follow an observable developmental progression in learning to measure with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

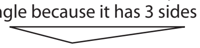
Age Range	Level Name	Level	Description
3	Length Quantity Recognizer	1	At the earliest level, children can identify length as an attribute. For example, they might say, "I'm tall, see?"
4	Length Direct Comparer	2	In this level, children can physically align 2 objects to determine which is longer or if they are the same length. For example, they can stand 2 sticks up next to each other on a table and say, "This one's bigger." 
5	Indirect Length Comparer	3	A sign of further development is when a child can compare the length of 2 objects by representing them with a third object. For example, a child might compare the length of 2 objects with a piece of string. Additional evidence of this level is that when asked to measure, the child may assign a length by guessing or moving along a length while counting (without equal-length units). For example, the child may move a finger along a line segment, saying 10, 20, 30, 31, 32.
6	Serial Orderer to 6+	4	At this level, a child can order lengths, marked in 1 to 6 units. For example, given towers of cubes, a child at this level may put them in order, 1 to 6.
6	End-to-End Length Measurer	5	At this level, the child can lay units end-to-end, although he or she may not see the need for equal-length units. For example, a child might lay 9-inch cubes in a line beside a book to measure how long it is.
7	Length Unit Relater and Repeater	6	At this level, a child can relate size and number of units. For example, the child may explain, "If you measure with centimeters instead of inches, you'll need more of them because each one is smaller."
8+	Length Measurer	7	As a child develops measurement ability, they begin to measure, knowing the need for identical units, the relationships between different units, partitions of unit, and the zero point on rulers. At this level, the child also begins to estimate. The children may explain, "I used a meterstick 3 times, then there was a little left over. So, I lined it up from 0 and found 14 centimeters. So, it's 3 meters, 14 centimeters in all."
8+	Conceptual Ruler Measurer	8	Further development in measurement is evidenced when a child possesses an "internal" measurement tool. At this level, the child mentally moves along an object, segmenting it, and counting the segments. This child also uses arithmetic to measure and estimates with accuracy. For example, a child at this level may explain, "I imagine one meterstick after another along the edge of the room. That's how I estimated the room's length to be 9 meters."

Developmental Levels for Recognizing Geometric Shapes

Geometric shapes can be used to represent and understand objects. Analyzing, comparing, and classifying shapes helps create new knowledge of shapes and their relationships. Shapes can be decomposed or composed into other shapes. Through their everyday activities, children build both intuitive and explicit knowledge of geometric figures. Most children can recognize and name basic two-dimensional shapes at four years of age. However, young children can learn richer concepts about shape if they have varied examples and nonexamples of shape, discussions about shapes and their characteristics, a wide variety of shape classes, and interesting tasks. Children typically follow an observable developmental progression in learning about shapes with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

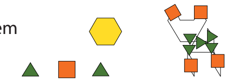
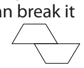
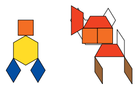
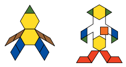
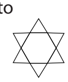
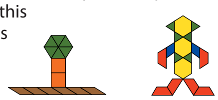
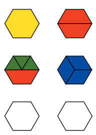
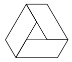
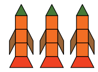
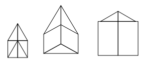
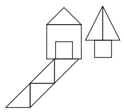
Age Range	Level Name	Level	Description
2	Shape Matcher—Identical	1	The earliest sign of understanding shape is when a child can match basic shapes (circle, square, typical triangle) with the same size and orientation.
	Shape Matcher—Sizes		A sign of development is when a child can match basic shapes with different sizes.
	Shape Matcher—Orientations		This level of development is when a child can match basic shapes with different orientations.
3	Shape Recognizer—Typical	2	A sign of development is when a child can recognize and name a prototypical circle, square, and, less often, a typical triangle. For example, the child names this a square.  Some children may name different sizes, shapes, and orientations of rectangles, but also accept some shapes that look rectangular but are not rectangles. Children name these shapes "rectangles" (including the nonrectangular parallelogram). 
3	Shape Matcher—More Shapes	3	As children develop understanding of shape, they can match a wider variety of shapes with the same size and orientation.
	Shape Matcher—Sizes and Orientations		The child matches a wider variety of shapes with different sizes and orientations. 
	Shape Matcher—Combinations		The child matches combinations of shapes to each other. 
4	Shape Recognizer—Circles, Squares, and Triangles	4	This sign of development is when a child can recognize some nonprototypical squares and triangles and may recognize some rectangles, but usually not rhombi (diamonds). Often, the child does not differentiate sides/corners. The child at this level may name these as triangles. 
4	Constructor of Shapes from Parts—Looks Like *Representing*	5	A significant sign of development is when a child represents a shape by making a shape "look like" a goal shape. For example, when asked to make a triangle with sticks, the child may create the following: .
5	Shape Recognizer—All Rectangles	6	As children develop understanding of shape, they recognize more rectangle sizes, shapes, and orientations of rectangles. For example, a child at this level may correctly name these shapes "rectangles." 
5	Side Recognizer *Parts*	7	A sign of development is when a child recognizes parts of shapes and identifies sides as distinct geometric objects. For example, when asked what this shape is, the child may say it is a quadrilateral (or has 4 sides) after counting and running a finger along the length of each side. 
5	Angle (Corner) Recognizer *Parts*	8	At this level, a child can recognize angles as separate geometric objects. For example, when asked, "Why is this a triangle," the child may say, "It has three angles" and count them, pointing clearly to each vertex (point at the corner).
5	Shape Recognizer—More Shapes	9	As children develop, they are able to recognize most basic shapes and prototypical examples of other shapes, such as hexagon, rhombus (diamond), and trapezoid. For example, a child can correctly identify and name all the following shapes: 
6	Shape Identifier	10	At this level, the child can name most common shapes, including rhombi, without making mistakes such as calling ovals circles. A child at this level implicitly recognizes right angles, so distinguishes between a rectangle and a parallelogram without right angles. A child may correctly name all the shapes shown. 
6	Angle Matcher *Parts*	11	A sign of development is when the child can match angles concretely. For example, given several triangles, the child may find two with the same angles by laying the angles on top of one another.

Leverage Learning Trajectories

Age Range	Level Name	Level	Description
7	Parts of Shapes Identifier	12	At this level, the child can identify shapes in terms of their components. For example, the child may say, "No matter how skinny it looks, that's a triangle because it has 3 sides and 3 angles." 
7	Constructor of Shapes from Parts—Exact *Representing*	13	A significant step is when the child can represent a shape with completely correct construction, based on knowledge of components and relationships. For example, when asked to make a triangle with sticks, the child may create the figure shown. 
8	Shape Class Identifier	14	As children develop, they begin to use class membership (for example, to sort) not explicitly based on properties. For example, a child at this level may say, "I put the triangles over here, and the quadrilaterals, including squares, rectangles, rhombi, and trapezoids, over there."
8	Shape Property Identifier	15	At this level, a child can use properties explicitly. For example, a child may say, "I put the shapes with opposite sides that are parallel over here, and those with 4 sides but not both pairs of sides parallel over there."
8	Angle Size Comparer	16	The next sign of development is when a child can separate and compare angle sizes. For example, the child may say, "I put all the shapes that have right angles here, and all the ones that have bigger or smaller angles over there."
8	Angle Measurer	17	A significant step in development is when a child can use a protractor to measure angles.
8	Property Class Identifier	18	The next sign of development is when a child can use class membership for shapes (for example, to sort or consider shapes "similar") explicitly based on properties, including angle measure. For example, the child may say, "I put the equilateral triangles over here, and the right triangles over here."
8	Angle Synthesizer	19	As children develop understanding of shape, they can combine various meanings of angle (turn, corner, slant). For example, a child at this level could explain, "This ramp is at a 45° angle to the ground."

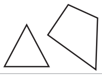
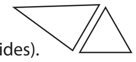
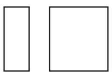
Developmental Levels for Composing Geometric Shapes

Children move through levels in the composition and decomposition of two-dimensional figures. Very young children cannot compose shapes but then gain ability to combine shapes into pictures, synthesize combinations of shapes into new shapes, and eventually substitute and build different kinds of shapes. Children typically follow an observable developmental progression in learning to compose shapes with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
2	Pre-Composer	1	The earliest sign of development is when a child can manipulate shapes as individuals, but is unable to combine them to compose a larger shape. 
3	Pre-Decomposer	2	At this level, a child can decompose shapes, but only by trial and error. For example, given only a hexagon, the child can break it apart to make a simple picture by trial and error. 
4	Piece Assembler	3	Around age 4, a child can begin to make pictures in which each shape represents a unique role (for example, one shape for each body part) and shapes touch. A child at this level can fill simple outline puzzles using trial and error. 
5	Picture Maker	4	As children develop, they are able to put several shapes together to make one part of a picture (for example, 2 shapes for 1 arm). A child at this level uses trial and error and does not anticipate creation of the new geometric shape. The children can choose shapes using "general shape" or side length, and fill "easy" outline puzzles that suggest the placement of each shape (but note that the child is trying to put a square in the puzzle where its right angles will not fit). 
5	Simple Decomposer	5	A significant step occurs when the child is able to decompose ("take apart" into smaller shapes) simple shapes that have obvious clues as to their decomposition. 
5	Shape Composer	6	A sign of development is when a child composes shapes with anticipation ("I know what will fit!"). A child at this level chooses shapes using angles as well as side lengths. Rotation and flipping are used intentionally to select and place shapes. For example, in this puzzle, all angles are correct, and patterning is evident. 
6	Substitution Composer	7	A sign of development is when a child is able to make new shapes out of smaller shapes and uses trial and error to substitute groups of shapes for other shapes in order to create new shapes in different ways. For example, the child can substitute shapes to fill outline puzzles in different ways. 
6	Shape Decomposer (with Help)	8	As children develop, they can decompose shapes by using imagery that is suggested and supported by the task or environment. For example, given hexagons, the child can break them apart to make this shape. 
7	Shape Composite Repeater	9	This level is demonstrated when the child can construct and duplicate units of units (shapes made from other shapes) intentionally, and understands each as being both multiple, small shapes and one larger shape. For example, the child may continue a pattern of shapes that leads to tiling. 
7	Shape Decomposer with Imagery	10	A significant sign of development is when a child is able to decompose shapes flexibly by using independently generated imagery. For example, the child can break hexagons apart into shapes such as these. 
8	Shape Composer—Units of Units	11	Children demonstrate further understanding when they are able to build and apply units of units (shapes made from other shapes). For example, in constructing spatial patterns, the child can extend patterning activity to create a tiling with a new unit shape—a unit of unit shapes that he or she recognizes and consciously constructs. For example, the child may build Ts out of 4 squares, use 4 Ts to build squares, and use squares to tile a rectangle.
8	Shape Decomposer — Units of Units	12	As children develop understanding of shape, they can decompose shapes flexibly by using independently generated imagery and planned decompositions of shapes that themselves are decompositions. 

Developmental Levels for Comparing Geometric Shapes

As early as four years of age, children can create and use strategies, such as moving shapes to compare their parts or to place one on top of the other for judging whether two figures are the same shape. From PreK to Grade 2, they can develop sophisticated and accurate mathematical procedures for comparing geometric shapes. Children typically follow an observable developmental progression in learning about how shapes are the same and different with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
3	"Same Thing" Comparer	1	The first sign of understanding is when the child can compare real-world objects. For example, the children may say two pictures of houses are the same or different.
4	"Similar" Comparer	2	This sign of development occurs when the child judges two shapes to be the same if they are more visually similar than different. For example, the child may say, "These are the same. They are pointy at the top." 
4	Part Comparer	3	At this level, a child can say that two shapes are the same after matching one side on each. For example, a child may say, "These are the same" (matching the two sides). 
4	Some Attributes Comparer	4	As children develop, they look for differences in attributes, but may examine only part of a shape. For example, a child at this level may say, "These are the same" (indicating the top halves of the shapes are similar by laying them on top of each other). 
5	Most Attributes Comparer	5	At this level, the child looks for differences in attributes, examining full shapes, but may ignore some spatial relationships. For example, a child may say, "These are the same." 
7	Congruence Determiner	6	A sign of development is when a child determines congruence by comparing all attributes and all spatial relationships. For example, a child at this level may say that two shapes are the same shape and the same size after comparing every one of their sides and angles.
7	Congruence Superposer	7	As children develop understanding, they can move and place objects on top of each other to determine congruence. For example, a child at this level may say that two shapes are the same shape and the same size after laying them on top of each other.
8+	Congruence Representer	8	Continued development is evidenced as children refer to geometric properties and explain with transformations. For example, a child at this level may say, "These must be congruent because they have equal sides, all square corners, and I can move them on top of each other exactly."

Developmental Levels for Spatial Sense and Motions

Infants and toddlers spend a great deal of time learning about the properties and relations of objects in space. Very young children know and use the shape of their environment in navigation activities. With guidance they can learn to "mathematize" this knowledge. They can learn about direction, perspective, distance, symbolization, location, and coordinates. Children typically follow an observable developmental progression in developing spatial sense with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
4	Simple Turner	1	An early sign of spatial sense is when a child mentally turns an object to perform easy tasks. For example, given a shape with the top marked with color, the child may correctly identify which of three shapes it would look like if it were turned "like this" (90 degree turn demonstrated), before physically moving the shape.
5	Beginning Slider, Flipper, Turner	2	This sign of development occurs when a child can use the correct motions, but is not always accurate in direction and amount. For example, a child at this level may know a shape has to be flipped to match another shape, but flips it in the wrong direction.
6	Slider, Flipper, Turner	3	As children develop spatial sense, they can perform slides and flips, often only horizontal and vertical, by using manipulatives. For example, a child at this level may perform turns of 45, 90, and 180 degrees. For example, a child knows a shape must be turned 90 degrees to the right to fit into a puzzle.
7	Diagonal Mover	4	A sign of development is when a child can perform diagonal slides and flips. For example, a children at this level may know a shape must be turned or flipped over an oblique line (45 degree orientation) to fit into a puzzle.
8	Mental Mover	5	Further signs of development occur when a child can predict results of moving shapes using mental images. A child at this level may say, "If you turned this 120 degrees, it would be just like this one."

Developmental Levels for Patterning and Early Algebra

Algebra begins with a search for patterns. Identifying patterns helps bring order, cohesion, and predictability to seemingly unorganized situations and allows one to make generalizations beyond the information directly available. The recognition and analysis of patterns are important components of the young children's intellectual development because they provide a foundation for the development of algebraic thinking. Although prekindergarten children engage in pattern-related activities and recognize patterns in their everyday environment, research has revealed that an abstract understanding of patterns develops gradually during the early childhood years. Children typically follow an observable developmental progression in learning about patterns with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
2	Pre-Explicit Patterner	1	A child at the earliest level does not recognize patterns. For example, a child may name a striped shirt with no repeating unit a "pattern."
3	Pattern Recognizer	2	At this level, the child can recognize a simple pattern. For example, a child at this level may say, "I'm wearing a pattern" about a shirt with black, white, black, white stripes.
4	Pattern Fixer	3	At this level the child fills in missing elements of a pattern, first with ABABAB patterns. When given items in a row with an item missing, such as ABAB_BAB, the child identifies and fills in the missing element (A).
4	Pattern Duplicator AB	4	A sign of development is when the child can duplicate an ABABAB pattern, although the children may have to work alongside the model pattern. For example, given objects in a row, ABABAB, the child may make his or her own ABABAB row in a different location.
4	Pattern Extender AB	5	At this level the child extends AB repeating patterns. For example, given items in a row—ABABAB—the child adds ABAB to the end of the row.
4	Pattern Duplicator	6	At this level, the child is able to duplicate simple patterns (not just alongside the model pattern). For example, given objects in a row, ABBABBABB, the child may make his or her own ABBABBABB row in a different location.
5	Pattern Extender	7	A sign of development is when the child can extend simple patterns. For example, given objects in a row, ABBABBABB, he or she may add ABBABB to the end of the row.
6	Pattern Unit Recognizer	8	At this level, a child can identify the smallest unit of a pattern. For example, given objects in a ABBABBABB pattern, the child identifies the core unit of the pattern as ABB.
7	Numeric Patterner	9	Describes a pattern numerically, can translate between geometric and numeric representation of a series. For example, given objects in a geometric pattern, child describes the numeric progression.
7	Beginning Arithmetic Patterner	10	Recognizes and uses relatively transparent arithmetic patterns with perceptual or pedagogical support, first that involve properties of zero. Accepts number sentences not in the form of $a + b = c$ (e.g., $c = a + b$, or $a + b = c + d$); this represent a move from Pre-Equivalence, "equals-as-an-answer" to Equal Numbers Relater. For example, child recognizes and uses patterns (e.g., that can be symbolized as $a + b - b = a$).
8	Relational Thinker +/−	11	Recognizes and uses patterns that involve addition and subtraction and is an Equals Relater who can compare two sides of a number sentence with reasoning without actually carrying out the computations. For example, child recognizes $3 + 6 - 3 = 6$ as a true statement without performing computations.
8	Relational Thinker—Symbolic +/−	12	Recognizes and uses patterns that involve addition and subtraction and is an Equals Relater who can compare two sides of a number sentence with reasoning without actually carrying out the computations. For example, child recognizes $a + b = b + a$ (presented that way, symbolically) as a true statement in all cases.
9	Relational Thinker with Multiplication	13	Recognizes and uses patterns that involve multiplication as repeated addition and the use of the distributive property to partition number facts. For example, child recognizes $3 \times 6 + 3 = 4 \times 6$ as a true statement without performing all computations.

Developmental Levels for Classifying and Analyzing Data

Data analysis contains one big idea: classifying, organizing, representing, and using information to ask and answer questions. The developmental continuum for data analysis includes growth in classifying and counting to sort objects and quantify their groups. Children eventually become capable of simultaneously classifying and counting, for example, counting the number of colors in a group of objects. Children typically follow an observable developmental progression in learning about patterns with recognizable stages or levels. This developmental path can be described as part of a learning trajectory.

Age Range	Level Name	Level	Description
2	Similarity Recognizer	1	The first sign that a child can classify is when he or she recognizes, intuitively, two or more objects as "similar" in some way. For example, "that's another doggie."
2	Informal Sorter	2	A sign of development is when a child places objects that are alike in some attribute together, but switches criteria and may use functional relationships as the basis for sorting. A child at this level might stack blocks of the same shape or put a cup with its saucer.
3	Attribute Identifier	3	The next level is when the child names attributes of objects and places objects together with a given attribute, but cannot then move to sorting by a new rule. For example, the child may say, "These are both red."
4	Attribute Sorter	4	At the next level the child sorts objects according to given attributes, forming categories, but may switch attributes during the sorting. A child at this stage can switch rules for sorting if guided. For example, the child might start putting red beads on a string, but switches to spheres of different colors.
5	Consistent Sorter	5	A sign of development is when the child can sort consistently by a given attribute. For example, the child might put several identical blocks together.
6	Exhaustive Sorter	6	At the next level, the child can sort consistently and exhaustively by an attribute, given or created. This child can use terms "some" and "all" meaningfully. For example, a child at this stage would be able to find all the attribute blocks of a certain size and color.
6	Multiple Attribute Sorter	7	A sign of development is when the child can sort consistently and exhaustively by more than one attribute, sequentially. For example, a child at this level can put all the attribute blocks together by color, then by shape.
7	Classifier and Counter	8	At the next level, the child is capable of simultaneously classifying and counting. For example, the child counts the number of colors in a group of objects.

Age Range	Level Name	Level	Description
7	List Grapher	9	In the early stage of graphing, the child graphs by simply listing all cases. For example, the child may list each child in the class and each child's response to a question.
8+	Multiple Attribute Classifier	10	A sign of development is when the child can intentionally sort according to multiple attributes, naming and relating the attributes. This child understands that objects could belong to more than one group. For example, the child can complete a two-dimensional classification matrix or form subgroups within groups.
8+	Classifying Grapher	11	At the next level the child can graph by classifying data (e.g., responses) and represent it according to categories. For example, the child can take a survey, classify the responses, and graph the result.
8+	Classifier	12	A sign of development is when the child creates complete, conscious classifications logically connected to a specific property. For example, a child at this level gives a definition of a class in terms of a more general class and one or more specific differences and begins to understand the inclusion relation.
8+	Hierarchical Classifier	13	At the next level, the child can perform hierarchical classifications. For example, the child recognizes that all squares are rectangles, but not all rectangles are squares.
8+	Data Representer	14	Signs of development are when the child organizes and displays data through both simple numerical summaries such as counts, tables, and tallies, and graphical displays, including picture graphs, line plots, and bar graphs. At this level the child creates graphs and tables, compares parts of the data, makes statements about the data as a whole, and determines whether the graphs answer the questions posed initially.

Trajectory Progress Chart

Student's Name ____________________

Number

Age Range	Counting	Comparing and Ordering Number	Recognizing Number and Subitizing (instantly recognizing)	Composing Number (knowing combinations of numbers)	Adding and Subtracting	Multiplying and Dividing (sharing)
1 year	____ Pre-Counter ____ Chanter				____ Pre +/−	
2	____ Reciter	____ Object Corresponder ____ Perceptual Comparer	____ Small Collection Namer			____ Nonquantitative Sharer
3	____ Reciter (10) ____ Corresponder	____ First-Second Ordinal Counter ____ Nonverbal Comparer of Similar Items (1–4 items)	____ Nonverbal Subitizer ____ Maker of Small Collections		____ Nonverbal +/−	____ Beginning Grouper and Distributive Sharer
4	____ Counter (small numbers) ____ Producer (small numbers) ____ Counter (10)	____ Nonverbal Comparer of Dissimilar Items ____ Matching Comparer ____ Knows-to-Count Comparer ____ Counting Comparer (same size)	____ Perceptual Subitizer to 4	____ Pre-Part-Whole Recognizer	____ Small Number +/−	____ Grouper and Distributive Sharer
5	____ Counter and Producer (10+) ____ Counter Backward from 10	____ Counting Comparer (5) ____ Ordinal Counter	____ Perceptual Subitizer to 5 ____ Conceptual Subitizer to 5+ ____ Conceptual Subitizer to 10	____ Inexact Part-Whole Recognizer ____ Composer to 4, then 5	____ Find Result +/− ____ Find Change +/− ____ Make It *N* +/−	____ Concrete Modeler ×/÷
6	____ Counter from *N* (*N* + 1, *N* − 1) ____ Skip Counter by tens to 100 ____ Counter to 100 ____ Counter On Using Patterns ____ Skip Counter ____ Counter of Imagined Items ____ Counter On Keeping Track ____ Counter of Quantitative Units ____ Counter to 200	____ Counting Comparer (10) ____ Mental Number Line to 10 ____ Serial Orderer to 6+	____ Conceptual Subitizer to 20	____ Composer to 7 ____ Composer to 10	____ Counting Strategies +/− ____ Part-Whole +/−	____ Parts and Wholes ×/÷
7	____ Number Conserver ____ Counter Forward and Back	____ Place Value Comparer ____ Mental Number Line to 100	____ Conceptual Subitizer with Place Value and Skip Counting	____ Composer with Tens and Ones	____ Numbers-in-Numbers +/− ____ Deriver +/−	____ Skip Counter ×/÷
8+		____ Mental Number Line to 1,000s	____ Conceptual Subitizer with Place Value and Multiplication		____ Problem Solver +/− ____ Multidigit +/−	____ Deriver ×/÷ ____ Array Quantifier ____ Partitive Divisor ____ Multidigit ×/÷

Trajectory Progress Chart

Student's Name ____________________

Geometry

Age Range	Shapes	Composing Shapes	Comparing Shapes	Motions and Spatial Sense	Measuring	Patterning	Classifying and Analyzing Data
2 years	____ Shape Matcher—Identical ____ —Sizes ____ —Orientations					____ Pre-Patterner	____ Similarity Recognizer ____ Informal Sorter
3	____ Shape Recognizer —Typical ____ Shape Matcher— More Shapes ____ —Sizes and Orientations ____ —Combinations	____ Pre-Composer ____ Pre-Decomposer	____ "Same Thing" Comparer		____ Length Quantity Recognizer	____ Pattern Recognizer	____ Attribute Identifier
4	____ Shape Recognizer—Circles, Squares, and Triangles ____ Constructor of Shapes from Parts—Looks Like Representing	____ Piece Assembler	____ "Similar" Comparer ____ Part Comparer ____ Some Attributes Comparer	____ Simple Turner	____ Length Direct Comparer	____ Pattern Fixer ____ Pattern Duplicator AB ____ Pattern Extender AB ____ Pattern Duplicator	____ Attribute Sorter
5	____ Shape Recognizer— All Rectangles ____ Side Recognizer ____ Angle Recognizer ____ Shape Recognizer— More Shapes	____ Picture Maker ____ Simple Decomposer ____ Shape Composer	____ Most Attributes Comparer	____ Beginning Slider, Flipper, Turner	____ Indirect Length Comparer	____ Pattern Extender	____ Consistent Sorter
6	____ Shape Identifier ____ Angle Matcher Parts	____ Substitution Composer ____ Shape Decomposer (with help)		____ Slider, Flipper, Turner	____ Serial Orderer to 6+ ____ End-to-End Length Measurer		____ Exhaustive Sorter ____ Multiple Attribute Sorter
7	____ Parts of Shapes Identifier ____ Constructor of Shapes from Parts—Exact Representing	____ Shape Composite Repeater ____ Shape Decomposer with Imagery	____ Congruence Determiner ____ Congruence Superposer	____ Diagonal Mover	____ Length Unit Iterater ____ Length Unit Relater	____ Pattern Unit Recognizer	____ Classifier and Counter ____ List Grapher
8+	____ Shape Class Identifier ____ Shape Property Identifier ____ Angle Size Comparer ____ Angle Measurer ____ Property Class Identifier ____ Angle Synthesizer	____ Shape Composer— Units of Units ____ Shape Decomposer with Units of Units	____ Congruence Representer	____ Mental Mover	____ Length Measurer ____ Conceptual Ruler Measurer		____ Multiple Attribute Classifier ____ Classifying Grapher ____ Classifier ____ Hierarchical Classifier ____ Data Representer

ALL LEVELS

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